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A NEW METHODOLOGY FOR MAGNETIC FORCE CALCULATIONS BETWEEN PLANAR SPIRAL COILS

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Abstract—Due to the special structure of current carrying planar spiral coils, precise calculation of the forces between them is complicated and time-consuming. To overcome these problems, in this paper a new and fast method is proposed for calculation of the magnetic forces between planar spiral coils. The advantage of the proposed method is that just by having the external dimension of coils and their number of turns, the force between them at different distances and with different currents can be calculated. The results obtained by direct and proposed calculation methods show the efficiency of the latter in simplicity and calculation time. The precision of the proposed method has been confirmed by experimental tests done on constructed coils.

1. INTRODUCTION

Regarding the extensive application of planar spiral coils in communication and robotics, determination of magnetic fields around them and forces between these coils are interesting for engineers. In these systems, to have a high inductance and flat configuration, spiral windings are used [1–3]. Besides, these coils have an extensive application in power electronics and dc/dc converters due to their flatness and special configuration; so they are better replacement for the ordinary inductances to reduce the volume of the converter [4–7].

In recent decades, spiral coils have been employed in casting industries to form the thin metal sheets. In[8], the finite difference method is employed to calculate the force between them; furthermore in this reference to calculate the force, spiral coils are replaced by

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concentric rings, but there is no study and discussion on the precision of the method. In [3], these forces have just been obtained by test. In [9, 10] the force between circular coaxial coils has been investigated. So, there is not enough investigation about calculation of the force between spiral coils in literature. In this paper, a new approach is presented to calculate the magnetic force between spiral coils. The aim of this work is to reduce the force calculation complexity and computational time in such coils. Using the results obtained by the numerical solution of direct force calculations, the precision of the proposed method is investigated and finally compared with experimental results.

2. CALCULATION OF THE MAGNETIC FORCE BETWEEN TWO PLANAR SPIRAL COILS

Suppose two spiral coils as shown in Figure 1. To calculate the magnetic force between them, first we calculate the vector magnetic potential resulting from one of the coils in any given point like P (Figure 2).

Vector magnetic potential of spiral coil 1 in any given point P is obtained by the following equation [11]:

$$A = \frac{\mu_0 I_1}{4\pi} \int \frac{dl'}{R_1} \tag{1}$$

where I_1, is the current of coil 1, dl' is the longitudinal differential component and R_1 is the distance between this differential component and point P, which in Cartesian coordinates is equal to:

$$R_1 = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \tag{2}$$

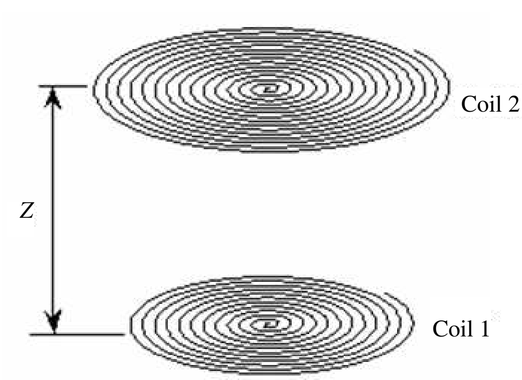

Figure 1. Two spiral coils in z distance of each other.

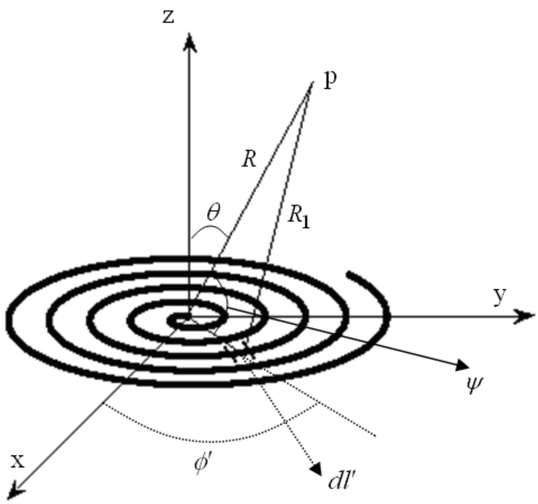

Figure 2. Calculation of vector magnetic potential of spiral coils in any given point P.

The coordinates marked by prime are related to the source. With suitable substitution for dl', the following equation for vector magnetic potential is obtained [12, 13]:

$$A=\frac{\mu_0 I_1}{4\pi}\int\frac{[-a_x\sin\phi'+a_y\cos\phi']r'd\phi'+[a_x\cos\phi'+a_y\sin\phi']dr'}{R_1} \tag{3}$$

To calculate the integral in Equation (3), one of the integral variables must be replaced by another according to the relations between them. The variables ϕ' and r' have a linear relation, so we can write [12, 14]:

$$\phi' = K_1 r' \tag{4}$$

where K_1, is a constant coefficient that is called "compression factor" of coil 1 in this paper. This factor is dependent on the diameter of the wire used and the structure of the coil and determines its compression. Having the vector magnetic potential, magnetic field is calculated using the following equation [11]:

$$B = \nabla \times A \tag{5}$$

Substituting A from Equation (3) in Equation (5) and simplifying the equation, we have:

$$B = a_x B_x + a_y B_y + a_z B_z \tag{6}$$

where:

$$B_x = -\frac{\partial A_y}{\partial z} \tag{7}$$

$$B_y = \frac{\partial A_x}{\partial z} \tag{8}$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \tag{9}$$

The force acting on the coil 2 is [11]:

$$F_{21} = I_2 \oint_{C_2} dl_2 \times B \tag{10}$$

In the above equation, dl_2 is longitudinal differential component on coil 2.

Substituting proper expression for dl_2 and employing Equation (6) in Equation (10) and doing some mathematical calculations, we get:

$$F_{21} = a_x f_x + a_y f_y + a_z f_z \tag{11}$$

where f_x, f_y and f_z are the components of the force in directions x, y and z, respectively and are equal to:

$$f_x = \frac{\mu_0 I_1 I_2}{4\pi} \int_{r_1}^{r_2} \int_{r'_1}^{r'_2} \frac{\left(\begin{array}{l} \left[r\sin(K_2 r - K_1 r') - K_1 r r' \cos(K_2 r - K_1 r') \right. \\ \left. +K_1 r'^2 \right] \left[\sin(K_2 r) + K_2 r \cos(K_2 r) \right] \end{array} \right)}{\left(\begin{array}{l} [(r\cos(K_2 r) - r'\cos(K_1 r'))^2 \\ +(r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2} \end{array} \right)} dr' dr \tag{12}$$

$$f_y = \frac{\mu_0 I_1 I_2}{4\pi} \int_{r_1}^{r_2} \int_{r'_1}^{r'_2} \frac{\left(\begin{array}{l} \left[r\sin(K_2 r - K_1 r') - K_1 r r' \cos(K_2 r - K_1 r') \right. \\ \left. +K_1 r'^2 \right] \left[\cos(K_2 r) + K_2 r \sin(K_2 r) \right] \end{array} \right)}{\left(\begin{array}{l} [(r\cos(K_2 r) - r'\cos(K_1 r'))^2 \\ +(r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2} \end{array} \right)} dr' dr \tag{13}$$

$$f_z = \frac{\mu_0 I_1 I_2}{4\pi} z \int_{r_1}^{r_2} \int_{r'_1}^{r'_2} \frac{\left(\begin{array}{l} (1 + K_1 K_2 r r') \cos(K_2 r - K_1 r') \\ -(K_2 r - K_1 r') \sin(K_2 r - K_1 r') \end{array} \right)}{\left(\begin{array}{l} [(r\cos(K_2 r) - r'\cos(K_1 r'))^2 \\ +(r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2} \end{array} \right)} dr' dr \tag{14}$$

In the above equations, the parameters r'_1 and r_1 are the inner radii of coils 1 and 2, respectively, and r'_2 and r_2 are the outer radii of coils 1 and 2, respectively. Also, the following equation has been used:

$$\phi = K_2 r \tag{15}$$

where, K_2 is compression factor of coil 2 which is determined with regard to the compression of the coil and the diameter of the wire used in it.

3. CALCULATION OF THE MAGNETIC FORCES BETWEEN TWO CONCENTRIC CIRCULAR FILAMENTS

Suppose a system of two current carrying rings as shown in Figure 3. To calculate the force between them, we use the concept of vector magnetic potential. The vector magnetic potential of ring 1 in any point P on ring 2, like Equation (1), is equal to [11]:

$$A = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{dl'}{R_1} \tag{16}$$

where μ_0 is the vacuum permeability, I_1 and C_1 are the current and the length of ring 1, respectively, and R_1, as shown in Figure 4, is the distance between the differential component of the source dl' at point P' and the field point P.

With appropriate substitutions for dl' and R_1 in Equation (16), the vector magnetic potential will be as follow [12]:

$$A = a_\phi f(R,\, \theta) \tag{17}$$

where the function $f(R,\, \theta)$ is equal to:

$$f(R,\, \theta) = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \frac{a \sin\phi'}{\sqrt{R^2 + a^2 - 2aR \sin\theta \sin\phi'}} d\phi' \tag{18}$$

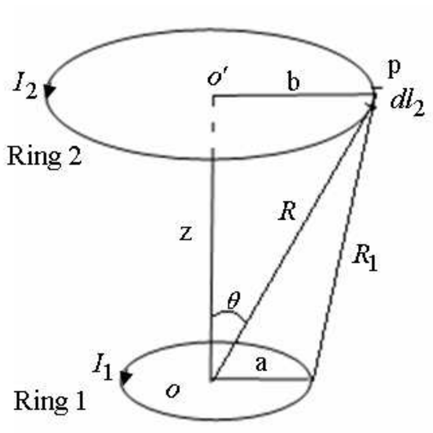

Figure 3. Two current carrying concentric rings.

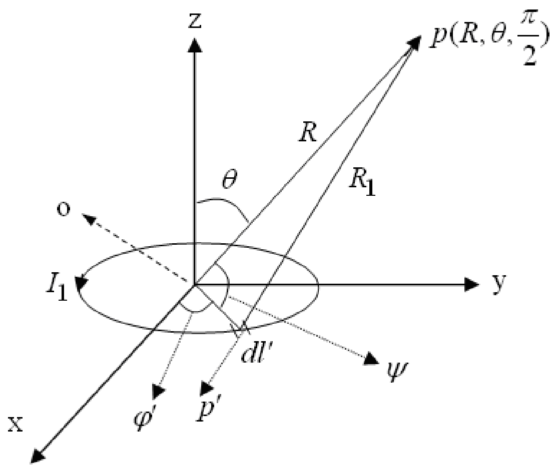

Figure 4. Determination of vector potential of a current carrying ring in any given point P.

In the above equation, a is the radius of ring 1. By obtaining the vector magnetic potential, the magnetic field is calculated using Equation (5).

Substituting Equations (17) and (18) in Equation (5) and doing some manipulations, we can get [12, 13]:

$$B = a_R g_1(R,\, \theta) + a_\theta g_2(R,\, \theta) \tag{19}$$

where $g_1(R,\, \theta)$ and $g_2(R,\, \theta)$ are:

$$g_1(R,\, \theta) = \frac{1}{R\sin\theta}\left[\cos\theta f(R,\, \theta) + \sin\theta\frac{\partial}{\partial\theta}f(R,\, \theta)\right] \tag{20}$$

$$g_2(R,\, \theta) = -\frac{1}{R}\left[f(R,\, \theta) + R\frac{\partial}{\partial R}f(R,\, \theta)\right] \tag{21}$$

To calculate the force of ring 1 exerted on ring 2, Equation (10) is employed. Using Equations (18), (19), (20) and (21) in Equation (10) and also substituting an appropriate expression for dl_2 and doing some simple mathematical calculations, the following equation for the force is obtained:

$$F_{21} = -a_z\frac{\mu_0 ab I_1 I_2 z}{2}\int_0^{2\pi}\frac{\sin\phi'}{[z^2 + a^2 + b^2 - 2ab\sin\phi']^{3/2}}d\phi' \tag{22}$$

In the above equation b is the radius of ring 2 and z is the axial distance between the two rings. The obtained force in Equation (22) has no analytical solution, so we can use numerical integration methods to solve it. By replacing the variable ϕ' by $\frac{3\pi}{2}+2\theta$ in Equation (22), the following equation for the force is obtained [12]:

$$F_{21} = a_z \left(\frac{\mu_0 I_1 I_2 z k}{2\sqrt{ab}(1-k^2)} \right) \left[(1-k^2)K(k) - (1-(1/2)k^2)E(k) \right] \quad (23)$$

In the above equation, k $(0 < k < 1)$ is a constant coefficient and is equal to:

$$k = \sqrt{\frac{4ab}{(a+b)^2 + z^2}} \quad (24)$$

And $K(k)$ and $E(k)$ are first and second order Elliptic Integrals respectively, which are defined as:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k^2\sin^2\theta)^{1/2}} \quad (25)$$

$$E(k) = \int_0^{\frac{\pi}{2}} (1-k^2\sin^2\theta)^{1/2} d\theta \quad (26)$$

In some references, special numerical and recursive methods are presented to solve the above integrals [15].

4. THE PROPOSED METHOD TO CALCULATE THE MAGNETIC FORCE BETWEEN TWO COILS

To calculate the force between two coils, we can use direct method presented in Section 2. But, using this method is complicated and time-consuming and also, requires knowledge of the compression factors in each coil [12]. Therefore, proposing a method to calculate the force between these types of coils is interesting for engineers. So, in this section, we propose a new simple and effective approach to calculate the force between two coils just by having the external specifications of the latter. The calculation time in the proposed method is very satisfactory.

Suppose the coils 1 and 2 with the turn numbers of N_1 and N_2, respectively. A cross-section of the coils is shown in Figure 5, where $r_{1,0}$ and $r_{2,0}$ are the inner radii; b_1 and b_2 are the radial thicknesses; a_1 and a_2 are the height of the coils 1 and 2, respectively.

As shown in Figure 5, the cross-section of each coil is divided into several segments. In this figure, coil 1 is divided into $n_{r1} \times n_{a1}$ cells, and coil 2 is divided into $n_{r2} \times n_{a2}$ cells. To calculate the force between these coils, the force between different filaments (in Figure 5, each filament is specified with two cells in both sides) of both coils is calculated and added together. Therefore, the force between the coils is calculated by the following equation:

$$F_{21} = \sum_{k=0}^{n_{r2}-1} \sum_{j=0}^{n_{r1}-1} \sum_{l=0}^{n_{a2}-1} \sum_{i=0}^{n_{a1}-1} f(k,\, j,\, l,\, i) \tag{27}$$

where, regarding the equation derived for the force between two filaments (Equation (23)), $f(k,\, j,\, l,\, i)$ is equal to:

$$f(k,j,l,i) = a_z \left(\frac{\mu_0 i_1 i_2 z_{il} k'}{2\sqrt{r_k r_j}(1-k'^2)} \right) \left[(1-k'^2)K(k') - (1-(1/2)k'^2)E(k') \right] \tag{28}$$

The above equation is the force between two filaments of the two coils, in which the current of each filament is supposed to concentrate at its center and the current density of the whole coil is supposed to be uniform and i_1 and i_2 are the currents of each filament in coils 1 and 2 respectively, which can be calculated using the following equations:

$$i_1 = \frac{N_1 I_1}{n_{r1} \times n_{a1}} \tag{29}$$

$$i_2 = \frac{N_2 I_2}{n_{r2} \times n_{a2}} \tag{30}$$

In the above equations, I_1 and I_2 are the currents of coils 1 and 2

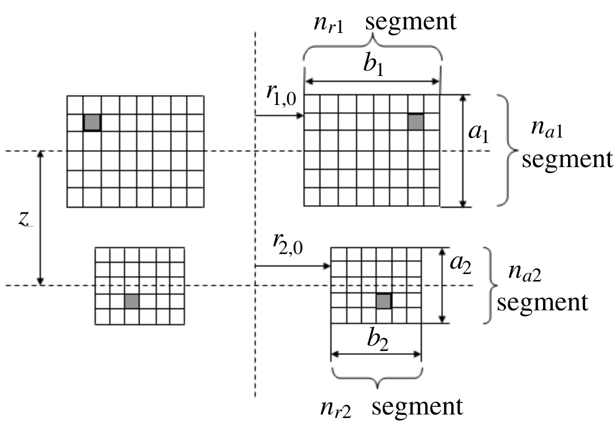

Figure 5. Division of the coils into different meshes to calculate the force between them.

respectively. Also, other parameters of Equation (28) are defined as:

$$r_k = r_{2,0} + \left(\frac{1}{2} + k\right)\left(\frac{b_2}{n_{r2}}\right) \tag{31}$$

$$r_j = r_{1,0} + \left(\frac{1}{2} + j\right)\left(\frac{b_1}{n_{r1}}\right) \tag{32}$$

$$z_{il} = z - \left[\left(\frac{a_2}{2}\right) + \left(\frac{a_1}{2}\right)\right] + \left(\frac{1}{2} + i\right)\left(\frac{a_1}{n_{a1}}\right) + \left(\frac{1}{2} + l\right)\left(\frac{a_2}{n_{a2}}\right) \tag{33}$$

$$k' = \sqrt{\frac{4 r_k r_l}{(r_k + r_l)^2 + {z_{il}}^2}} \tag{34}$$

z in Equation (33), is the distance between the two centers of the two coils.

5. CALCULATION RESULTS

In Section 2, the force between two spiral coils was analytically obtained (Equation (11)). Supposing that the compression factor of the coils is high. In other words, the radial growth of rings in each coil in any turn is not much more than the diameter of the wires used; then the force values in x and y directions are almost zero; just the component of the force in z direction is non-zero [12, 13], which is shown in Equation (14). The force in this equation is the force exerted on the upper coil (coil 2) from the lower coil (coil 1) as shown in Figure 1. It can be observed that, when the currents of the two coils have the same direction, the force between them is attractive, and by changing the direction of one of the currents, the force changes to a repelling type.

Although we use precise analytical relations to obtain the force in Equation (14), its integral has no analytical solution, and numerical integration techniques must be used to solve it. The integrand of the Equation (14) has some "semi-poles" which are dependent on the value of the coefficients K_1 and K_2. The curve of the integrand versus variables r and r' is shown in Figure 6 for variation of r and r' from 0 to 0.01. As seen in the figure, by increasing the value of r and r' from zero, the value of the integrand produces some sharp peaks (the semi-pole points). It is clear that integration of these surfaces is much more difficult, since for higher precisions, we need to increase the number of iterations of numerical integration intensively which, in turn, requires much longer computation time to solve such a problem.

Now we compare the results of the direct method of the calculation of the force using Equation (14) with that of proposed method (Equation (27)). To calculate the integral in Equation (14) we used recursive adaptive Simpson quadrature method. To calculate the force between two coils in Equation (27), the cross-section of the coils has to be divided into several segments. In order to investigate the effect of the number of divisions on precision of the calculations, the force between two spiral coils have been calculated by dividing their cross-section into different segments in radial directions. The results are summarized in Tables 1 and 2. In these tables, the force between two coils is calculated at different distances. In each distance, the force is calculated by direct method and compared with proposed method for a 100 turn numbers of the coils. The current in both coils is 10 Amperes; the diameter of wires used is 1 mm; the compression factor for both coils is assumed as $\frac{2\pi}{d}$, where d is the diameter of the wires in both coils, meaning that for each turn of coils or for change of 2π Radians in the value of variable ϕ in cylindrical coordinates, the change in the value of variable r (the radial growth of coils) is equal to diameter of the wires used in the coils. Table 1 presents the results for the coils with inner radii and compression factor of 0 and $\frac{2\pi}{0.001}$, respectively. In the 3rd column of Table 1, both of the coils are divided into 100 segments. In this case, the results are the same up to 4 decimal integers, and so the force error is zero. In the next column, both of the coils are divided radially into 50 segments. The force error is -0.015% in this case, but the calculation time is a quarter of the calculation time in the previous case. As the number of divisions decreases, the calculation time also decreases, while the error increases. Consequently, one can

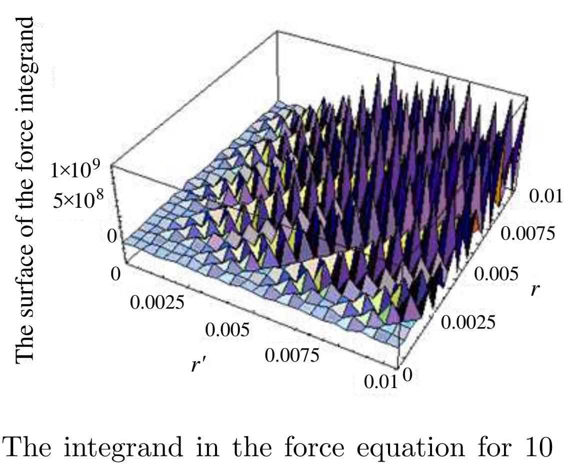

Figure 6. The integrand in the force equation for 10 turns in each coil.

decrease the number of divisions in order to save the computational time for a given force error range. In the next rows of Table 1, the force calculation is carried out for other distances between two coils. The results show that the more is the distance between two coils, the less is the calculated force error between them. In other words, dividing the cross-section of the coils generates less error at far distances between two coils. As a result, for a specified force error, one can use less division at far distances. The error has been calculated as follow:

$$\%error = \frac{f_d - f_p}{f_d} \times 100 \tag{35}$$

where f_p, is the calculated force by using the proposed method and f_d, is the force calculated by direct method.

Table 1. Comparison of the force between two coils with different divisions (inner radii and compression factor of the coils are 0 and $\frac{2\pi}{0.001}$, respectively).

Z=1 cm (direct method:1.3653 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	1.3653	1.3655	1.3702	1.3828	1.5834	3.0422
	Force error(%)	0	-0.015	-0.36	-1.28	-15.97	-122.82
Z=5 cm (direct method:0.4481 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	0.4481	0.4481	0.4484	0.4486	0.4504	0.4769
	Force error(%)	0	0	-0.067	-0.112	-0.51	-6.42
Z=10 cm (direct method:0.1403 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	0.1403	0.1403	0.1403	0.1402	0.1400	0.1390
	Force error(%)	0	0	0	+0.07	+0.21	+0.93
	(calculation time)/(calculation time of the direct method)	3×10^{-3}	7.5×10^{-4}	1.5×10^{-4}	3×10^{-5}	7.5×10^{-6}	1.2×10^{-6}

Table 2. Comparison of the force between two coils with different divisions (inner radii and compression factor of the coils are 1 cm and $\frac{2\pi}{0.001}$, respectively).

Z=1 cm (direct method:1.6909 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	1.6909	1.6912	1.6975	1.7134	1.9575	3.7192
	Force error(%)	0	-0.018	-0.39	-1.33	-15.77	-119.95
Z=5 cm (direct method:0.6070 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	0.6070	0.6071	0.6076	0.6082	0.6120	0.6559
	Force error(%)	0	-0.016	-0.10	-0.20	-0.82	-8.06
Z=10 cm (direct method:0.2061 N)	Number of divisions for coil 1	100	50	10	10	5	2
	Number of divisions for coil 2	100	50	50	10	5	2
	Force (N)	0.2061	0.2061	0.2061	0.2062	0.2064	0.2092
	Force error(%)	0	0	0	-0.05	-0.15	-1.50
	(calculation time)/ (calculation time of the direct method)	3×10^{-3}	7.5×10^{-4}	1.5×10^{-4}	3×10^{-5}	7.5×10^{-6}	1.2×10^{-6}

In Table 2, the same calculations have been done as for Table 1 but for 1 cm inner radii of coils 1 and 2. As can be seen in this table, the errors are quite negligible for divisions more than 10 for both coils. According to the results obtained, one can reduce the number of divisions in order to save calculation time, without having significant error in calculations.

It should be mentioned that the sign change in the calculated force error in Tables 1 and 2 is because of the changes of the radii of any parts of the coils involved in the calculations. The change of the radii of the coils changes the maximum amount of the force and its position in the distance axis.

In Tables 3 and 4, the results of calculation of the force with two methods for different turn numbers and distance of the coils are compared. In these tables, the current of the coils and diameter of the wires used are the same as previous case. For better comparison between the results of the direct method and the proposed one in Tables 3 and 4, the number of divisions is supposed to be the same as the number of turns of the coils. In Table 3, it is assumed that the coils start to grow from point $(0, 0)$. Comparing the results of the two methods in this table, one can see that for the fewer number of turns the error is high, but by increasing the number of turns, the error gradually decreases and when the turn number approaches to 100, the error becomes zero. In Tables 3 and 4, the precision of the calculations is adjusted according to the numerical value of the results. For instance, for the first column of Table 3 the calculated forces are in the range of 10^{-16} (their minimum); to have better comparison between the results of two methods, the precision of the calculations is chosen to be 10^{-16}. To compare the calculation time in two methods, it suffices to mention that the required calculation time using the adaptive Simpson method for 100 turns in Table 3 for precision of 10^{-4}, is 28000 times more than that of using proposed method. As it is seen from the table, the results are equal up to four decimal digits. Another interesting point about Tables 3 and 4 is that by increasing the distance between the two coils, the calculation error increases, showing that in large distances, the proposed method does not present a proper approximation of the force for lower turn numbers.

Table 3. Comparison of the force between two coils with different divisions (inner radii and compression factor of the coils are 0 and $\frac{2\pi}{0.001}$, respectively).

	Number of Turns or Rings Per Coil	1	2	5	10	100
Z=1 cm	Direct Method (N)	1.0453×10^{-7}	7.4636×10^{-7}	5.9312×10^{-5}	0.1409×10^{-2}	1.3653
	Proposed Method (N)	3.6553×10^{-9}	3.3540×10^{-7}	5.6863×10^{-5}	0.1402×10^{-2}	1.3653
	Error (%)	96.5	55.1	1.4	0.5	0.0
Z=5 cm	Direct Method (N)	4.0074×10^{-9}	1.6615×10^{-8}	2.5756×10^{-7}	9.7620×10^{-6}	0.4481
	Proposed Method (N)	5.9188×10^{-12}	5.8976×10^{-10}	1.5662×10^{-7}	9.3511×10^{-6}	0.4481
	Error (%)	99.8	96.5	39.2	4.2	0.0
Z=10 cm	Direct Method (N)	1.0005×10^{-9}	4.0386×10^{-9}	3.5066×10^{-8}	7.3663×10^{-7}	0.1403
	Proposed Method (N)	3.7006×10^{-12}	3.6973×10^{-11}	1.0003×10^{-8}	6.3569×10^{-7}	0.1403
	Error (%)	100	99.1	71.5	13.7	0

* Precision of the calculations in numerical integration for rings of 1 to 100 turns are 0.5×10^{-16}, 0.5×10^{-15}, 0.5×10^{-12}, 0.5×10^{-11} and 0.5×10^{-4}, respectively.

Table 4. Comparison of the force between two coils with different divisions (inner radii and compression factor of the coils are 1 cm and $\frac{2\pi}{0.001}$, respectively).

	Number of Turns or Rings Per Coil	1	2	5	10	100
Z=1 cm	Direct Method (N)	7.8358×10^{-5}	3.3733×10^{-4}	2.4909×10^{-2}	1.1712×10^{-2}	1.6909
	Proposed Method (N)	7.8384×10^{-5}	3.3745×10^{-4}	2.4916×10^{-2}	1.1714×10^{-2}	1.6909
	Error (%)	-3.3×10^{-2}	-3.6×10^{-2}	-2.8×10^{-2}	-1.7×10^{-2}	0.0
Z=5 cm	Direct Method (N)	9.4323×10^{-7}	4.4683×10^{-6}	4.4231×10^{-5}	3.3155×10^{-4}	0.6070
	Proposed Method (N)	9.3905×10^{-7}	4.4517×10^{-6}	4.4129×10^{-5}	3.3117×10^{-4}	0.6070
	Error (%)	0.44	0. 37	0.23	0.11	0.0
Z=10 cm	Direct Method (N)	6.9227×10^{-8}	3.3218×10^{-7}	3.4467×10^{-6}	2.8509×10^{-5}	0.2061
	Proposed Method (N)	6.8194×10^{-8}	3.2804×10^{-7}	3.4207×10^{-6}	2.8404×10^{-5}	0.2061
	Error (%)	1.5	1.3	0.76	0.37	0.0

* Precision of the calculations in numerical integration for rings of 1 to 100 turns are 0.5×10^{-12}, 0.5×10^{-12}, 0.5×10^{-10}, 0.5×10^{-10} and 0.5×10^{-4}, respectively.

In Table 4, the comparison between two methods is made for the case in which the inner radius of two coils are equal to 1 cm; in other words, the coils start to wind from $r = 1\,\text{cm}$. As it can be seen from the results of the table, the errors in this case are less than the corresponding errors in Table 3. For example, the force error for 1 turn coil in distance of 10 cm is reduced from 100% in Table 3 to 1.5% in Table 4. This fewer error for lower turn numbers, decreases to zero by increasing the turn numbers.

According to the results of Tables 3 and 4, generally for turn numbers higher than 10 turns in each coil, using the proposed method presents good approximations; while having much simpler and faster calculations compared to the direct method and using Equation (14).

Now suppose the case in which we have smaller compression factor for the coils compared with previous cases; i.e., for each turn of coils or for change of 2π Radians in the value of variable ϕ, the change in the value of variable r is more than the diameter of the wires used in the coils. For example, suppose that the growth of r is equal to 5 mm, in this case the compression factor for both coils will be: $K_1 = K_2 = \frac{2\pi}{0.005}$.

The results of the calculations of the force with the above mentioned conditions and by two methods of direct and proposed one are presented in Table 5. In this table, like the previous cases, the current of the coils is 10 Amperes.

It is interesting to compare the results of the Tables 3 and 5. In

Table 5. Comparison of the force between two coils with different divisions (inner radii and compression factor of the coils are 0 and $\frac{2\pi}{0.005}$, respectively).

	Number of Turns or Rings Per Coil	1	2	5	10	100
Z=1 cm	Direct Method (N)	4.2719×10^{-6}	6.2280×10^{-5}	1.4744×10^{-2}	1.0132×10^{-2}	1.8028
	Proposed Method (N)	1.7467×10^{-6}	5.5591×10^{-5}	1.4611×10^{-2}	1.0114×10^{-2}	1.8028
	Error (%)	59.1	10.7	0.9	0.2	0.0
Z=5 cm	Direct Method (N)	1.0453×10^{-7}	7.4636×10^{-7}	5.9312×10^{-5}	1.4089×10^{-2}	1.3653
	Proposed Method (N)	3.6553×10^{-9}	3.3540×10^{-7}	5.6863×10^{-5}	1.4023×10^{-2}	1.3653
	Error (%)	96.5	55.1	1.4	0.47	0.0
Z=10 cm	Direct Method (N)	2.5288×10^{-8}	1.2347×10^{-7}	5.9584×10^{-6}	2.3166×10^{-4}	1.0081
	Proposed Method (N)	2.3060×10^{-10}	2.2553×10^{-8}	5.3127×10^{-6}	2.2922×10^{-4}	1.0077
	Error (%)	99.1	81.7	10.8	1.1	0.04

* Precision of the calculations in numerical integration for rings of 1 to 100 turns are 0.5×10^{-16}, 0.5×10^{-15}, 0.5×10^{-12}, 0.5×10^{-11} and 0.5×10^{-4}, respectively.

Table 5, the trend of increasing and decreasing of the error with the increase of the distance between two coils and the number of turns is the same as Table 3; but in this case, the calculated percentage of relative error of the force is lower than the corresponding values in Table 3 (although the absolute error increases). At first, it seemed that by decreasing the compression factor calculation error increases, but this assumption is not true because by decreasing the compression factor, the relative error of calculations with proposed method decreases. This is also true for smaller compression factors [12].

6. THE EXPERIMENTAL RESULTS

To evaluate the precision of the proposed method in calculating the force between coils, two coils with different radii were precisely constructed in the laboratory with the characteristics presented in Table 6.

In order to precisely measure the repelling and attracting forces between the coils, a test, as illustrated in Figure 7, is arranged. In this figure, one of the coils is placed on a fiber board isolator whose permeability is the same as air, and the other coil is connected to a digital force meter via four pieces of string and a fiber board isolator. So, by applying current to the circuit of the two coils, the force exerted

Table 6. Characteristics of the constructed spiral coils.

	Number of turns	Inner radius (cm)	Diameter of wire used (mm)
Coil 1	54	2.15	1.6
Coil 2	55	2.0	1.6

on the higher coil, which is equal to the force on the lower coil, is measured precisely.

In Table 7, the results of the calculations and experimental results for different distances are presented. For the calculations in this table, due to large number of turns and inner radiuses for the coils, the proposed method with number of divisions equal to 25, is employed. Regarding the results and the explanations of the previous section, using this method in this case causes no significant error. As Table 7 shows, the results of the force measurement are in good agreement with the results of the calculations, validating the precision of the proposed method.

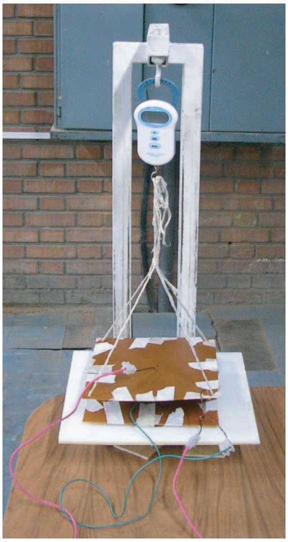

Figure 7. Measurement of the magnetic force between the two coils.

Table 7. The experimental results and their comparison with calculation results of the proposed method.

	Current of the Coils (A)	4	6	8	10	12
Z=1 cm	Measured Force (N)	0.0981	0.2256	0.3826	0.6180	0.8829
	Calculated Force (N)	0.0981	0.2208	0.3925	0.6133	0.8832
Z=5 cm	Measured Force (N)	0.0392	0.0883	0.1373	0.2158	0.3237
	Calculated Force (N)	0.0349	0.0786	0.1397	0.2182	0.3142
Z=10 cm	Measured Force (N)	0.0098	0.0294	0.0491	0.0785	0.1079
	Calculated Force (N)	0.0117	0.0264	0.0469	0.0733	0.1056

7. CONCLUSION

In this paper, a new method for calculation of the force between spiral coils is proposed. Generally, the direct method is employed in order to calculate the force between spiral coils. The direct method involves integrals with no analytical solutions. The numerical solution of these integrals, due to the fact that the integrands are not smooth, is difficult and time-consuming. To overcome this problem, in this paper a new method is developed with simpler calculations and shorter calculation time.

In this method just knowing the external dimensions and turn numbers of each coil is sufficient to calculate the force between them, and it is not necessary to know the arrangement of the turns in different layers and number of layers in the coils. The effect of the number of divisions in the cross-section of the coils on the precision of the calculations is investigated. The results show that one can reduce the number of divisions in order to save calculation time without having significant error in the force.

According to the obtained results, the calculation error of the proposed method for the number of turns more than 10 is negligible, and the method is effective. These errors are reduced by increasing the inner radius of spiral coils and have acceptable values even in the lower turn numbers. The experimental results confirm and validate the results obtained by the proposed method. The presented method can be useful and applicable for the calculation of the magnetic forces between any spiral type coils with different sizes.

REFERENCES

1. Ding, X., K. Kuribayashi, and T. Hashida, "Development of a new type tactile sensor using micro electromagnetic coil for human

robot," *IEEE International Symposium on Micromechatronics and Human Science*, 181–187, 2000.

2. Atluri, S. and M. Ghovanloo, "A wideband power-efficient inductive wireless link for implantable microelectronic devices using multiple carriers," *IEEE International Symp. on Circuits and Systems (ISCAS)*, 1131–1134, 2006.
3. Kawabe, K., H. Koyama, and K. Shirae, "Planar inductor," *IEEE Trans. on Mag.*, Vol. 20, No. 5, 1804–1811, 1984.
4. Sato, T., M. Hasegawa, T. Mizogochi, and M. Sahashi, "Study of high power planar inductor," *IEEE Trans. on Mag.*, Vol. 27, No. 6, 5277–5279, 1991.
5. Fukuda, Y., T. Inoue, T. Mizoguchi, S. Yatabe, and Y. Tachi, "Planar inductor with ferrite layers for DC-DC converter," *IEEE Trans. on Mag.*, Vol. 39, No. 4, 2057–2061, 2003.
6. Sasada, I., T. Yamaguchi, K. Harada, and Y. Notohara, "Planar inductors using NiZn ferrite thin plates and the application to high-frequency DC-DC converters," *IEEE Trans. on Mag.*, Vol. 29, No. 6, 3231–3233, 1993.
7. Kawase, I., T. Sato, K. Yamasawa, and Y. Miura, "A planar inductor using Mn-Zn ferrite/polyimide composite thick film for low-Voltage and large-current DC-DC converter," *IEEE Trans. on Mag.*, Vol. 41, No. 10, 3991–3993, 2005.
8. Meriched, A., M. Feliachi, and H. Mohellebi, "Electromagnetic forming of thin metal sheets," *IEEE Trans. on Mag.*, Vol. 36, No. 4, 1808–1811, 2000.
9. Akyel, C., S. I. Babic, S. Kincic, and J. P. Lagacé, "Magnetic force calculation of some circular coaxial coils in air," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 9, 1273–1283, 2007.
10. Babic, S. I. and C. Akyel, "Magnetic force calculation between thin coaxial circular coils in air," *IEEE Trans. on Magnetics*, Vol. 44, No. 4, 445–452, 2008.
11. Cheng, D. K., *Field and Wave Electromagnetics*, 2nd edition, Addison Wesley, 1989.
12. Shiri, A., "Calculation of magnetic forces between current carrying coils in two plates," Master of Science Thesis, Iran University of Science & Technology, September 2006 (in Persian).
13. Shiri, A. and A. Shoulaie, "Calculation of magnetic forces between planar spiral coils," *15th Iranian Conference on Electrical Engineering*, Tehran, Iran, 2007 (in Persian).
14. Spiegel, M. R., *Schaum's Outline Series: Mathematical Handbook*

of Formulas and Tables, McGraw-Hill, 1968.

15. Urankar, L. and P. Henninger, "Compact extended algorithms for elliptic integrals in electromagnetic field and potential computations part 1: Elliptic integrals of first and second kind with extended integration range," *IEEE Trans. on Mag.*, Vol. 27, No. 5, 4338–4342, 1991.