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A New Mixed Integer Linear Programming Formulation for Protection Relay Coordination Using Disjunctive Inequalities

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ABSTRACT Numerical optimization-based solution to directional overcurrent relay (DOCR) coordination problem has been a widely addressed research problem in the recent past. Many linear (LP), nonlinear (NLP), mixed integer nonlinear (MINLP), mixed integer linear (MILP), and quadratically constrained quadratic programming (QCQP)-based formulations have been presented in the past literature. This paper proposes a new MILP-based formulation using disjunctive inequalities. The nonlinear DOCR protection coordination model is formulated as MILP by linearizing the bilinear terms existing in the original formulation. One of the variables in each bilinear term is discretized over its interval into a fixed number of steps. After assigning binary variables to each discrete interval, the resulting bilinear terms with binary variables are written in terms of disjunctive inequalities. The results have shown that the proposed MILP formulation fetch better optimal solutions compared with past MILP and MINLP formulations. The MILP problem is programmed in GAMS package with CPLEX solver and tested on standard 3 bus, 9 bus, 15 bus, and 30 bus systems and results are found to be satisfactory.

INDEX TERMS Directional overcurrent relay, bilinear relaxation, mixed integer programming, current pickup settings.

NOMENCLATURE

SETS

F	Fault locations.
F^s	Feasible solutions.
G	Number of disjunctions.
N_r	Relays.

PARAMETERS

I	Fault Current.
q, r, s	interval step size.
TDS^{min}, TDS^{max}	Upper and lower bounds of TDS.
I_p^{min}, I_p^{max}	Upper and lower bounds of I_p.
t^{min}, t^{max}	Upper and lower bounds of t.
P^{min}, P^{max}	Upper and lower bounds of P.

VARIABLES

t	relay operating time.
T	Time delay setting.
I_p	Plug setting.
P	Intermediate continuous variable.
W, Y, Q	Intermediate continuous bilinear variables.
α, β, γ	Intermediate binary variables.
X	vector of variables.

I. INTRODUCTION

Protection of power delivery systems is a vital task to alleviate the effects of fault events by selectively isolating the portion of the affected network from the whole network to ensure reliable and good quality electrical power wheeled to the customers. In the context of network expansion and reconfiguration, the design of efficient protection schemes reduces the risk of failure. So, a protection engineer must optimally set the protection relay parameters so that, the relay tripping characteristics and the fault characteristics are matched. It is statistically evident that majority of relay trippings are due to unsuitable and poor relay settings than because of actual fault occurrence [1]. Inadequate relay settings also lead to mis-coordination of primary and backup relays. It is imperative to decide the sequence of relays in the pipeline and fix the settings to detect and trip for a particular fault. Hence, optimal relay coordination study is an important part of protection design.

A. MOTIVATION

As it is known that the prompt effect of any short-circuit fault is the rise of current, the protection relays are set to sense the fault current beyond a particular limit and issue trip command to the circuit breaker nearest to the fault. The relay closest to the fault location, called primary relay, is supposed to operate first. In case primary relay fails to pick up, the next relay in sequence, called the backup relay, picks up. While transmission systems rely on distance protection, directional overcurrent protection is economical and effective for sub-transmission and distribution system protection [2]. The tripping times of directional overcurrent relays (DOCRs) are based on two settings viz., time dial settings (*TDS*) and current pickup setting (I_p). To properly coordinate the DOCRs, their respective *TDS* and I_p values have to be set optimally so that the fault is isolated in minimum time resulting in no miscoordinations. Such DOCR protection coordination (DOCR-PC) problem can be written as a mathematical optimization problem with the objective of minimizing the operating times of DOCRs constrained with coordination and limit constraints.

B. LITERATURE REVIEW

Many DOCR coordination methods have been proposed in the literature which can be broadly categorized into three types. They are based on graph theory and functional dependencies, analytical methods and numerical optimization based coordination methods.

1) GRAPH THEORY BASE METHODS

In graphical theory-based approach, the network is taken as an oriented graph, and simple loop structures are found with the help of a set of minimum number of breakpoints to initiate the coordination process [3], [4]. The obtained relay settings are optimal based on available settings but not best in any sense. In [5], the coordination constraints relating to the operating times of primary and backup DOCRs are written as a set of functional dependencies similar to database systems.

2) ANALYTICAL TECHNIQUES

In analytical techniques [6]–[10], iterative numerical algorithms are developed in which coordination constraints and parameter limit constraints are considered as inequalities and solved iteratively. The algorithm is set to terminate when there is no significant change between two iterations.

3) NUMERICAL OPTIMIZATION BASED DOCR COORDINATION METHODS

After the advent, numerical optimization based DOCR coordination techniques has become quite popular providing better solutions to DOCR-PC problem. Various linear and nonlinear optimization methods [11]–[28] have been proposed in the literature which solves the coordination problem mathematically. Linear programming (LP) based solution methods are proposed in [11] and [12] in which I_p values are fixed and *TDS* values are optimally found. Though LP based solutions are simple and quickly converge, only *TDS* values can be optimized by fixing the I_p which does not give the global solution to the problem. By taking *TDS* and I_p as decision variables, the DOCR-PC problem can be formulated as a nonlinear programming (NLP) problem. Mixed integer nonlinear programming (MINLP) is proposed in [13] where I_p values are assumed as integer variables and *TDS* parameters are considered to be continuous and solved using Seeker's algorithm. [14] also formulated a MINLP based DOCR-PC model considering discrete steps of I_p settings. It is concluded in [14], that solving DOCR-PC in nonlinear programming (NLP) and then rounding the I_p values could end up in feeble solutions. In [15], mixed integer linear programming (MILP) based formulation with discrete I_p values is proposed based on relaxing the bilinear variables in terms of linear inequalities. In [16], sequential quadratic programming (SQP) based solution is proposed for DOCR coordination problem with both *TDS* and I_p as continuous variables. Reference [17] proposed a solution using linear interval programming (ILP) which considers various network topologies to find the best DOCR settings that fit for all scenarios.

Quadratically constrained quadratic programming (QCQP) based solution approach is proposed in [18] which obtained the optimal global solution for the DOCR coordination problem irrespective of the initial solution. Overall primary and backup DOCR operating times can be minimized by designing new DOCR characteristics which is an available feature in microprocessor-based relays. Dual-setting DOCR characteristics are proposed in [19], [20] which account for both forward and reverse relay operational characteristics and can reduce the backup DOCR operating times to minimum thus reducing the overall relay operating times. In [21], a new DOCR characteristic called current - voltage-time characteristics are proposed which is effective in sensing the faults not only based on current magnitude, but also bus voltage as an additional parameter. Naturally inspired optimization algorithms like GA [22], PSO [23], symbiotic organisms search [24], and Modified water cycle (MWCA) [25] algorithms and hybrid heuristic methods like GA-NLP [26], GSA-SQP [27], GA-ILP [28] have also been used to solve the DOCR optimal coordination problem.

C. CONTRIBUTION

In the MINLP [13], [14] and MILP [15] formulations proposed in the past literature, the DOCR-PC model is developed by considering I_p as discrete variable and *TDS* as a continuous variable. In most of the literature, I_p is deemed to have a fixed number of steps which, though reasonably good, but does not provide an overall solution and the relay operating times are comparatively high. This paper proposes a new MILP formulation for DOCR-PC model based on disjunctive inequalities which work better than past integer programming formulations and fetches better optimum. The NLP based DOCR-PC model is written in the bilinear form (both objective function and constraints as bilinear terms), and then one of the variables in each bilinear term is discretized about its interval. By assigning binary variables to each discrete bilinear term, disjunctive linear inequalities are posed and, thus, the whole NLP is transformed into a single stage MILP. It is to be noted that in the proposed MILP formulation the I_p remains continuous. Results show that the MILP formulation can get better optimal solutions compared to past propositions in the literature.

D. ORGANIZATION OF THE PAPER

The paper is divided into five sections. The proposed formulation of MILP based on disjunctive inequalities is discussed in section II. The DOCR coordination problem formulation which was commonly used in previous literature is presented in section III along with application using the proposed formulation. The details of case studies and simulated test results are presented in section IV, and finally, conclusions are drawn in section V.

II. MILP FORMULATION BASED ON DISJUNCTIVE INEQUALITIES

In this section, the procedure of formulating the MILP for DOCR PC model using disjunctive inequalities is discussed. The definitions and theorems supporting the reformulation are also given in this section.

A. THEORY OF DISJUNCTION

A generic LP optimization problem defined by,

$$min\ cx, \quad x \in F^S \tag{1}$$

where, $c, x \in \mathbb{R}^n$, and (1) is said to have an optimal solution x^* when $x^* \in F^S$. Let the search space be $G \supseteq F^S$. Then, one approach to find F^S (which contains x^*) in G is to create an inequality $dx \geq e$ which is satisfied by all feasible solutions of F^S such that x^* lies in the region $dx \leq e$ [29]. This addition of inequality $dx \geq e$ create two disjuctions, one is infeasible solution space which does not contain x^* and the other which contain x^*. Such a disjuction leads to a tighter relaxation $G' = \{G \cap x \in \mathbb{R}^n : dx \geq e\}$. This process is repeated iteratively to get tighter relaxations of the search space till x^* is found.

However, by creating finite number of disjunctions (G) of the search space and by assigning binary variables to each disjunction, the optimization problem can be converted into a single stage MILP.

B. DEFINITIONS AND THEOREMS

Definition 1: Any continuous variable v defined in the interval $V = [\underline{V}, \bar{V}]$ can be discretized into fixed number of m steps such that any step $v_j \in \{v_1, v_2, v_3 \ldots . v_m\}$ is defined as,

$$v_j = \underline{v} + S \sum_{\forall j \in m} \alpha_j \tag{2}$$

where, S is the fixed step size given by, $S = (\bar{V} - \underline{V})/m$, and α_j is a binary variable defined by $\alpha_j \in \{0, 1\}$.

Theorem 1: A bilinear term given by $t = \alpha T$, where α is a binary variable defined by $\alpha \in \{0, 1\}$ and T is a continuous variable defined in the interval $[\underline{T}, \bar{T}]$, then t can be linearized using a pair of disjunctive linear inequalities given by,

$$-B\alpha \leq t \leq B\alpha \tag{3a}$$

$$T - B(1 - \alpha) \leq t \leq T + B(1 - \alpha) \tag{3b}$$

where, B is a positive large constant.

C. MILP FORMULATION

Any NLP problem, handling a function $O(x, y) = \sum xy$, can be written as,

$$minimize\ Z = O(x, y) \tag{4a}$$

$$subjected\ to:\ g_{ineq}(x, y) \leq 0 \tag{4b}$$

$$h_{eq}(x, y) = 0 \tag{4c}$$

Assuming that g_{ineq} and h_{eq} contains bilinear terms of the form xy and O is also the sum of bilinear terms xy. So, each bilinear term in above NLP problem can be linearized using *definition*1 and *theorem*1 in discrete steps, thus transforming the entire NLP problem defined by (4) into an MILP problem given by,

$$minimize\ Z = CX \tag{5a}$$

$$subjected\ to:\ A_{con}X \leq B_{con} \tag{5b}$$

here X is the set of new variables of MILP representing the bilinear terms in O. A_{con} is the constraint coefficient matrix having continuous and binary constants as entries. B_{con} is a column vector which also has continuous and binary constants. X contains variables belonging to continuous ($\mathbb{R}$) and integer domains ($\mathbb{Z}$). The DOCR coordination problem is formulated and solved as MILP as explained above. The exact MILP formulation of DOCR-PC using disjunctive inequalities is discussed in the next section.

III. MILP FORMULATION OF DOCR COORDINATION PROBLEM

A. DOCR PROTECTION COORDINATION PROBLEM

The DOCR protection coordination (DOCR-PC) problem for a given network topology is defined with an objective of finding the time dial settings (TDS) and current pickup settings (I_p) of all the participating DOCRs in fault detection by subjecting to coordination and boundary constraints. The operating time of a DOCR, based on the IEC [30] characteristics is given as follows:

$$t_{i,f} = \frac{a(TDS_i)}{\left(\frac{I_{i,f}}{I_{p,i}}\right)^b - 1} \quad \forall i \in N_r,\ f \in F \tag{6}$$

where, i is the relay identifier and f is the fault location identifier. $t_{i,f}$ refers to operating time for i^{th} primary relay. F indicates total number of fault locations, N_r defines number of DOCRs. $I_{i,f}$ is fault current flowing through i^{th} relay for

f^{th} fault location. TDS_i and $I_{p,i}$ are i^{th} DOCR settings which are also the design variables of the NLP problem. a, b are constants which indicate the level of steepness of the inverse curve required by the user. In this work we take $a = 0.14$, and $b = 0.02$. The objective function denoting the PC model of DOCRs is given by:

$$min\ OF = \sum_{f=1}^{F}\sum_{i=1}^{N_r} t_{i,f} \tag{7}$$

The constraints of DOCR PC model are given as follows,

$$t_{i,f}^{k} - t_{i,f} \geq CTI \quad \forall (i,k) \in N_r \tag{8a}$$
$$TDS_i^{min} \leq TDS_i \leq TDS_i^{max} \tag{8b}$$
$$I_{p,i}^{min} \leq I_{p,i} \leq I_{p,i}^{max} \tag{8c}$$
$$t_i^{min} \leq t_{i,f} \leq t_i^{max} \tag{8d}$$

here, $k \in K$ and K is the number of backup relays for i^{th} primary relay. $t_{i,f}^{k}$ is the operating time of k^{th} backup relay of i^{th} primary relay for f^{th} fault location. $[TDS_i^{min}, TDS_i^{max}]$ and $[I_{p,i}^{min}, I_{p,i}^{max}]$ are the bounds on TDS and I_p values of i^{th} DOCR respectively. In (8a) CTI refers to coordination time interval. Similar to (8d), bounds for $t_{i,f}^{k}$ can be written as well.

B. PROPOSED MILP FORMULATION

Now, (7) and (8) defines the PC model of DOCRs which is a nonlinear, non-convex problem (NLP) with highly nonlinear constraints.

Defining $t_i = P_i T_i$ and $t_i^k = P_i^k T_i^k$, where,

$$T_i = TDS_i \tag{9}$$
$$P_i = \frac{a}{\left(\frac{I_{i,f}}{I_{p,i}}\right)^b - 1} \tag{10a}$$
$$P_i^k = \frac{a}{\left(\frac{I_{i,f}^k}{I_{p,i}^k}\right)^b - 1} \tag{10b}$$
$$P_i^{min} \leq P_i \leq P_i^{max} \tag{11a}$$
$$P_i^{k,min} \leq P_i^k \leq P_i^{k,max} \tag{11b}$$

Since TDS of DOCR remains same whether it operates in primary or backup mode, from (9) we can write,

$$T_i = T_i^k \quad \forall\ (i,k) \in N_r \tag{12}$$

By (9) and (12), the bounds of TDS given in (8b) holds good for T_i as well. P_i and P_i^k are expressed as shown in (10a) and (10b) respectively. The expressions for bounds of P_i and P_i^k are given as follow,

$$P_i^{min} = \frac{a}{\left(\frac{I_{i,f}}{I_{p,i}^{min}}\right)^b - 1} \tag{13a}$$
$$P_i^{max} = \frac{a}{\left(\frac{I_{i,f}}{I_{p,i}^{max}}\right)^b - 1} \tag{13b}$$
$$P_i^{k,min} = \frac{a}{\left(\frac{I_{i,f}^k}{I_{p,i}^{kmin}}\right)^b - 1} \tag{13c}$$
$$P_i^{k,max} = \frac{a}{\left(\frac{I_{i,f}^k}{I_{p,i}^{kmax}}\right)^b - 1} \tag{13d}$$

In (10b), (13c) and (13d), $I_{i,f}^k$ denotes the fault current seen by k^{th} backup relay of i^{th} primary relay. Now, the bilinear terms t_i and t_i^k can be linearized in a discrete manner (by (2)) as follows,

$$t_i = P_i^{min} T_i + r_i \sum_{\forall j \in m} W_{ij} \tag{14}$$
$$t_i^k = P_i^{k,min} T_i + s_i \sum_{\forall j \in m} Y_{ij} \tag{15}$$

where, $W_{ij} = \alpha_{ij} T_i$ and $Y_{ij} = \beta_{ij} T_i$. α_{ij} and β_{ij} are binary variables, $r_i = (P_i^{max} - P_i^{min})/m$, and $s_i = (P_i^{k,max} - P_i^{k,min})/m$. Now W_{ij} and Y_{ij} are bilinear terms which can be linearlized using (3) as,

$$-B\alpha_{ij} \leq W_{ij} \leq B\alpha_{ij} \tag{16a}$$
$$T_i - B(1-\alpha_{ij}) \leq W_{ij} \leq T_i + B(1-\alpha_{ij}) \tag{16b}$$
$$-B\beta_{ij} \leq Y_{ij} \leq B\beta_{ij} \tag{17a}$$
$$T_i - B(1-\beta_{ij}) \leq Y_{ij} \leq T_i + B(1-\beta_{ij}) \tag{17b}$$

Further, as the I_p values remain unchanged irrespective of whether the DOCR is operating as a primary or backup relay, hence we can write

$$I_{p,i} = I_{p,i}^k \quad \forall\ (i,k) \tag{18}$$

So, by combining (10a) and (10b), we get

$$aMP_i - aP_i^k - (1-M)Q_i = 0 \tag{19}$$

here, $M = exp(b\ ln(I_{i,f}/I_{i,f}^k))$ and $Q_i = P_i P_i^k$. Now, Q_i is another bilinear term which is linearized in the same manner as t_i and similar to (14) and (16), Q_i can be also be expressed in terms of disjunctive inequalities as given below,

$$Q_i = P_i^{min} P_i^k + q_i \sum_{\forall j \in m} \zeta_{ij} \tag{20}$$
$$-B\gamma_{ij} \leq \zeta_{ij} \leq B\gamma_{ij} \tag{21a}$$
$$P_i^k - B(1-\gamma_{ij}) \leq \zeta_{ij} \leq P_i^k + B(1-\gamma_{ij}) \tag{21b}$$

In (20), $\zeta_{ij} = \gamma_{ij} P_i^k$, $q_i = (P_i^{max} - P_i^{min})/m$, and γ_{ij} is a binary variable. Now, the decision variable vector is $X = [t_i, t_i^k, P_i, P_i^k, T_i, W_{ij}, Y_{ij}, Q_i, \zeta_{ij}, \alpha_{ij}, \beta_{ij}, \gamma_{ij}]$. In X, α_{ij}, β_{ij}, and γ_{ij} are binary decision variables ($\alpha_{ij}, \beta_{ij}, \gamma_{ij} \in \{0,1\}$) and remaining variables belong to continuous domain ($\mathbb{R}$). With X as decision variable vector, (7) becomes the objective function for the new MILP with (8a), (8b), (11), (16), (17) and (21) as inequality constraints, and (14), (15), (19), and (20) as equality constraints. So, for each test case, the proposed MILP is solved with N_r limit constraints corresponding to (8b), (11), $4N_r$ equality constraints, and $(2N_r \times m)$ inequality constraints, where m is the step size or number of disjunctions. The lower and upper bounds of Q_i can be calculated as follows,

$$Q_i^{min} \leq Q_i \leq Q_i^{max} \quad (22a)$$
$$Q_i^{min} = min(P_i^{min} P_i^{max}, P_i^{k,min} P_i^{k,max}, P_i^{k,min} P_i^{max}, P_i^{k,max} P_i^{min}) \quad (22b)$$
$$Q_i^{max} = max(P_i^{min} P_i^{max}, P_i^{k,min} P_i^{k,max}, P_i^{k,min} P_i^{max}, P_i^{k,max} P_i^{min}) \quad (22c)$$

C. METHOD OF CHOOSING m

The accuracy of the obtained solution from the proposed MILP method depends on the value of m. Smaller amounts of m sometimes do not fetch feasible solutions as it is evident that the number of created disjunctions (i.e., m) decide the possibility of obtaining the optimum. For larger values of m, though an optimal solution is obtained, the solver consumes a humongous amount of execution time. To trade off execution time against acquiring better optimum, the value of m is chosen from the knowledge of the test systems iteratively. The value m is set to 1 in the first trial run. The cost of m is incremented by 1 in each successive term until there is no further improvement in the objective function value. This procedure is shown in algorithm 1.

Algorithm 1 Iterative Algorithm to Choose m

Input: Fault current data, bounds of TDS, CPS, t_i, t_i^k, and primary backup relay sequence.
Define tolerance, ϵ, iteration count, k.
Set m=1
Calculate $[P_i^{min}, P_i^{max}]$, $[P_i^{k,min}, P_i^{k,max}]$, and $[Q_i^{min}, Q_i^{max}]$.
Formulate and solve MILP.
Store the OF.
Calculate $\sigma = |OF^k - OF^{k+1}|$
while $\sigma > \epsilon$
  Set $m^{k+1} = m^k + 1$.
  Calculate $[P_i^{min}, P_i^{max}]$, $[P_i^{k,min}, P_i^{k,max}]$, and $[Q_i^{min}, Q_i^{max}]$.
  Formulate and solve MILP.
  Store the OF.
  Calculate $\sigma = |OF^k - OF^{k+1}|$.
end while
Display the value of m

The absolute difference in objective function value in two successive iterations is shown as σ in algorithm 1. The threshold value of ϵ is set to a minimum amount of 1×10^{-3}. For each unit increment of m, the MILP is run, and σ is calculated. When σ falls below ϵ, it means that the best m for the particular test system is found an algorithm can be stopped. It should be noted that the value of m which is found using algorithm 1 is just enough to fetch the global solution. Values of m higher than that would be overestimated which will, eventually, increase the execution time of the program and consume more memory space in the computing system and there would not be much substantial decrement in the objective function value.

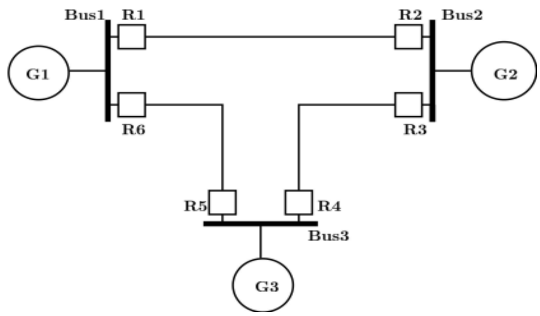

FIGURE 1. **Test case 1: 3 bus system.**

TABLE 1. **Optimal DOCR settings for 3 bus system.**

Relay	MINLP		MILP		Proposed MILP	
	TDS	I_p	TDS	I_p	TDS	I_p
1	0.1070	2.500	0.1066	2.500	0.1000	2.9367
2	0.1080	2.000	0.1082	2.000	0.1000	1.5000
3	0.1000	3.000	0.1000	3.000	0.1000	2.7782
4	0.1000	2.500	0.1000	2.500	0.1000	2.3420
5	0.1000	2.500	0.1000	2.500	0.1000	1.5000
6	0.1120	1.500	0.1119	1.500	0.1000	1.7817

IV. SIMULATION TEST RESULTS AND DISCUSSIONS

The proposed MILP formulation for DOCR-PC is programmed in GAMS coding platform using CPLEX solver on a 4GB, 3GHz, Intel Core2duo processor-based computing system. For the sake of fair comparison, the MILP [15] and MINLP [14] formulations are adapted and programmed on the same computing system using GAMS-CPLEX solver, and the results are compared. The proposed method is employed on various test systems among which the results of a three bus system, an eight bus, a 15 bus, and the IEEE 30 bus systems are explained in this section.

A. TEST CASE 1: 3 BUS SYSTEM

The proposed algorithm is tested on three bus system. The network structure is shown in Fig. 1 and system data are taken from [13]. The bounds of t_i and t_i^k are taken as [0.1, 1.1], and TDS bounds are considered as [0.1, 1.1]. As in the proposed formulation I_p is not discrete, the limits of I_p are fixed to be [1.5, 5]. For the sake of comparison of the proposed method, MILP and MINLP algorithms are also developed with same parameter limits, except, I_p is considered to be discrete variable with limits [1.5, 5] with a step size of 0.5. CTI is taken to be 0.2 sec for all test methods. The step size of m is found to be eight from algorithm 1. For this system, the proposed MILP comprises of 144 discrete variables, 180 continuous variables with 582 inequality constraints and 24 equality constraints.

The optimal DOCR settings obtained for the 3 bus system using the proposed algorithm, MILP and MINLP are presented in Table 1. The DOCR operating times for close-in line faults case on the test system are shown in Table 2. It can be observed that the difference between primary and backup DOCR operating times is clearly over minimum CTI of 0.2sec. To highlight the performance of the proposed algorithm the sum of the primary and backup relay operating

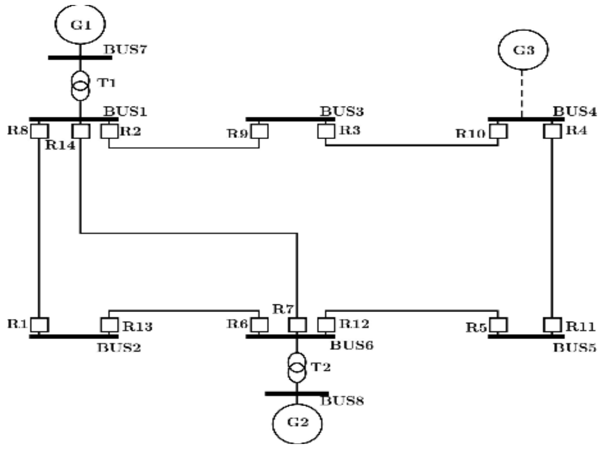

FIGURE 2. Test case 2: 8 bus system.

TABLE 2. Primary and backup DOCR tripping times of 3 bus system.

Relay Pairs		Proposed MILP		MILP	
$t_i(s)$	$t_i^k(s)$	$t_i(s)$	$t_i^k(s)$	$t_i(s)$	$t_i^k(s)$
1	5	0.2825	1.2439	0.2819	1.2439
2	4	0.2094	0.5356	0.2494	0.5356
3	1	0.2506	0.5514	0.2581	0.5201
4	6	0.2666	0.5694	0.2738	0.5694
5	3	0.2106	0.5948	0.2516	0.5948
6	2	0.2712	1.261	0.2835	1.261
$\sum(t_i + t_i^k)$		6.247		6.323	

times obtained from both proposed algorithm and MILP are given in the last row of Table 2. It is clear from the table that the obtained solution is better than MILP.

B. TEST CASE 2: 8 BUS SYSTEM

The second test system employed is the 8 bus test system constituting 14 DOCRs. The single line diagram of the 8 bus system is shown in Fig. 2, and the network short circuit data is taken from [13]. For the proposed algorithm, the limits of TDS and I_p are taken to be [0.01, 1.1] and [0.5, 2.5] respectively. For MILP and MINLP, discrete I_p values with limits [0.5, 2.5] is considered with steps of 0.5, 0.6, 0.8, 1, 1.5, 2 and 2.5 respectively. The step size m for the proposed MILP is obtained to be 8. CTI is taken as 0.3 sec for all test methods. The obtained optimal DOCR settings are shown in Table 3. Similar to test case 1, the proposed MILP fetched better optimal settings for this test system as compared to MINLP. A value of OF (s) is obtained as 8.0061 sec which is better compared to MINLP and Seeker [13]. Whereas, it is observed that for this test system the results of MILP and proposed algorithm are the same for all step sizes greater than $m = 8$. For values of $m < 8$ sub-optimal solutions are obtained with few constraint violations. This is obvious, because, with lesser number of discrete intervals the size of solution space (G') increases and include infeasible sub-optimal points and the solver ends up at its best sub-optimal point where all constraints may not satisfy. Hence, optimal selection of m is crucial in getting best optimal solutions.

TABLE 3. Optimal DOCR settings for 8 bus system.

Relay	MINLP		MILP		Proposed MILP	
	TDS	I_p	TDS	I_p	TDS	I_p
1	0.113	2.000	0.0772	2.500	0.0772	2.500
2	0.260	2.500	0.2548	2.500	0.2548	2.500
3	0.225	2.500	0.2185	2.500	0.2185	2.500
4	0.160	2.500	0.1531	2.500	0.1531	2.500
5	0.100	2.500	0.0929	2.500	0.0929	2.500
6	0.173	2.500	0.1506	2.500	0.1506	2.500
7	0.243	2.500	0.2391	2.500	0.2391	2.500
8	0.170	2.500	0.1484	2.500	0.1484	2.500
9	0.147	2.500	0.1473	2.500	0.1473	2.500
10	0.176	2.500	0.1759	2.500	0.1759	2.500
11	0.187	2.500	0.1869	2.500	0.1869	2.500
12	0.266	2.500	0.2664	2.500	0.2664	2.500
13	0.114	2.000	0.0784	2.000	0.0784	2.000
14	0.246	2.500	0.2459	2.500	0.2459	2.500

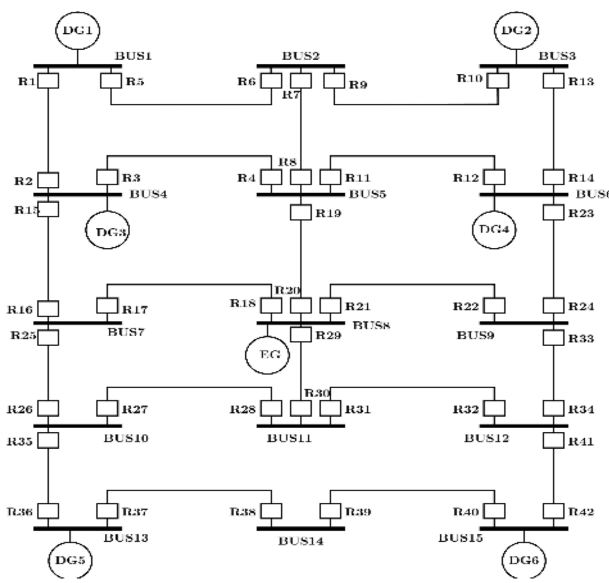

FIGURE 3. Test case 3: 15 bus system with DG.

C. TEST CASE 3: 15 BUS SYSTEM

The performance of proposed MILP is tested on large scale networks including distributed generation (DG). One such system is the 15 bus test system connected with DGs and having 42 DOCRs. The single line diagram of 15 bus system is shown in Fig. 3 and the network short circuit data are taken from [13]. The proposed algorithm is tested on this system with limits of TDS and I_p to be [0.1, 1] and [0.5, 2.5] respectively. For MILP and MINLP, discrete I_p values with limits [0.5, 2.5] are considered with a step size of 0.5. CTI is taken to be 0.3 sec for all test methods. The value of m for the proposed MILP is 5. The obtained optimal DOCR settings are shown in Table 4. From the table, it can be seen that the proposed MILP attained improved optimal settings for this test system compared to MILP and MINLP.

D. TEST CASE 4: IEEE 30 BUS SYSTEM WITH DG

Fig. 4 shows the distribution portion of the IEEE 30 bus system connected with DG and comprises of 38 DOCRs. The network parameters and short circuit data of this test system

TABLE 4. Optimal DOCR settings for 15 bus system.

Relay	MINLP		MILP		Proposed MILP	
	TDS	I_p	TDS	IP	TDS	I_p
1	0.1031	1.767	0.1000	2.000	0.10056	1.25652
2	0.1000	1.8509	0.1000	1.500	0.10269	0.95333
3	0.1179	2.4223	0.1163	2.500	0.10405	1.97319
4	0.1018	1.8765	0.1000	2.000	0.1045	1.20063
5	0.1356	2.2392	0.1223	2.500	0.1026	2.13336
6	0.1290	2.1934	0.1191	2.500	0.10294	2.02521
7	0.1264	2.2369	0.1162	2.500	0.10405	1.98154
8	0.1022	2.2847	0.1000	2.500	0.10328	1.52629
9	0.1080	2.4358	0.1070	2.500	0.10518	1.77072
10	0.1139	2.1855	0.1000	2.500	0.10348	1.66003
11	0.1014	1.8815	0.1000	2.000	0.10559	1.2011
12	0.1055	1.9004	0.1020	2.000	0.10405	1.35149
13	0.1467	2.2206	0.1071	2.000	0.10432	1.45836
14	0.1312	1.8499	0.1000	2.000	0.10436	1.06507
15	0.1000	1.838	0.1000	2.000	0.1000	1.15724
16	0.1000	2.3279	0.1000	2.500	0.10243	1.42128
17	0.1058	2.1499	0.1000	2.500	0.10384	1.51338
18	0.1000	1.6969	0.1000	2.000	0.1000	1.14174
19	0.1073	2.4192	0.1055	2.500	0.10348	1.74254
20	0.1066	1.7402	0.1000	2.000	0.10405	1.25264
21	0.1042	1.6991	0.1000	2.000	0.10006	1.14249
22	0.1008	2.4413	0.1000	2.500	0.10384	1.62202
23	0.1000	1.8169	0.1000	2.000	0.1000	1.14969
24	0.1104	1.8515	0.1028	2.000	0.10302	1.41642
25	0.1280	2.3049	0.1166	2.500	0.10175	1.99645
26	0.1082	2.301	0.1025	2.500	0.10512	1.62483
27	0.1179	2.4139	0.1121	2.500	0.10375	1.94082
28	0.1622	2.2284	0.1464	2.500	0.10543	2.5000
29	0.1001	2.3819	0.1000	2.500	0.1000	1.66298
30	0.1129	2.2054	0.1519	2.500	0.10593	1.62059
31	0.1112	2.4595	0.1069	2.500	0.10333	1.81184
32	0.1017	2.3891	0.1000	2.500	0.10084	1.59488
33	0.1582	2.2993	0.1451	2.500	0.1001	2.5000
34	0.1540	2.2145	0.1401	2.500	0.10768	2.5000
35	0.1302	2.3784	0.1225	2.500	0.1000	2.15394
36	0.1168	2.2476	0.1037	2.500	0.10358	1.7961
37	0.1546	2.2434	0.1429	2.500	0.10282	2.5000
38	0.1473	2.3745	0.1372	2.500	0.10553	2.5000
39	0.1392	2.356	0.1359	2.500	0.10235	2.5000
40	0.1570	2.2822	0.1422	2.500	0.10382	2.5000
41	0.1555	2.4317	0.1472	2.500	0.10384	2.5000
42	0.1021	2.2448	0.1000	2.500	0.10333	1.52349

can be found in [10]. The relay sequence is considered in the same manner as in [24]. The proposed algorithm is tested on this system with the bounds on TDS and I_p to be [0.1, 1] and [1.5, 6] respectively. For MILP and MINLP, discrete I_p values with limits [1.5, 6] are considered with a step size of 0.5. The value of m as obtained using algorithm 1 is 10. The obtained optimal DOCR settings are shown in Table 5. Compared to MILP and MINLP, and the proposed MILP delivered effective settings with reduced objective function value. The results of the proposed MILP has also surpassed the hybrid heuristic method GSA-SQP [24] under the same test conditions.

E. DISCUSSION ON OBTAINED RESULTS

To highlight the robustness of the proposed MILP technique, the objective function values OF (sec) are compared with the MILP [15], MINLP [14], which are also programmed in the same computing system, and also with Seeker [13], and GSA-SQP [24] techniques in the literature. The values of OF (sec) for all the test systems are shown in Table 6. The numerical superiority of the proposed algorithm over other test methods is highlighted in the table. For the 3-bus

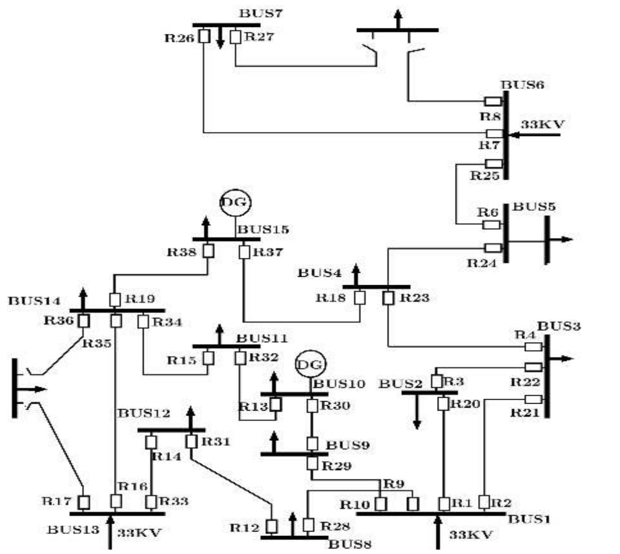

FIGURE 4. Test case 4: Distribution portion of 30 bus system with DG.

TABLE 5. Optimal DOCR settings for 30 bus system.

Relay	MINLP		MILP		Proposed MILP	
	TDS	I_p	TDS	I_p	TDS	I_p
1	0.4917	1.5000	0.1657	6.0000	0.1621	6.0000
2	0.3099	1.5000	0.1000	5.0000	0.1003	4.7223
3	0.3766	1.5000	0.1125	6.0000	0.1089	6.0000
4	0.3321	1.5000	0.1120	5.5000	0.1029	5.7737
5	0.2828	1.5000	0.1000	5.5000	0.1000	4.9175
6	0.1808	1.5000	0.1306	2.0000	0.1000	2.7981
7	0.1692	1.5000	0.1000	3.0000	0.1000	2.8742
8	0.1533	1.5000	0.1000	2.5000	0.1000	2.3168
9	0.4989	1.5000	0.1624	6.0000	0.1628	6.0000
10	0.4988	1.5000	0.1604	6.00000	0.1542	6.0000
11	0.3914	1.5000	0.1146	6.00000	0.1084	6.0000
12	0.3746	1.5000	0.1000	6.00000	0.1029	5.7961
13	0.3268	1.5000	0.1000	6.00000	0.1023	5.2792
14	0.2750	1.5000	0.1000	3.5000	0.1021	3.3185
15	0.1852	2.2980	0.1277	2.5000	0.1023	2.9478
16	0.3626	1.5000	0.1177	6.00000	0.1148	6.0000
17	0.1512	1.5000	0.1227	1.50000	0.1022	1.7278
18	0.1521	3.5355	0.1000	5.5000	0.1029	4.8272
19	0.3259	1.5000	0.1161	5.5000	0.1014	5.8447
20	0.1000	3.2917	0.1042	2.5000	0.1011	2.5623
21	0.1728	1.5000	0.1156	1.5000	0.1007	1.6792
22	0.1665	3.4857	0.1028	6.0000	0.1019	6.0000
23	0.1150	4.6565	0.1102	4.5000	0.1004	4.8478
24	0.1000	3.9050	0.1000	4.0000	0.1017	3.6416
25	0.1000	6.000	0.1533	4.0000	0.1000	6.0000
26	0.1000	1.5000	0.1000	1.5000	0.1000	1.5000
27	0.1000	1.5000	0.1000	1.5000	0.1000	1.5000
28	0.1000	4.6041	0.10361	3.5000	0.1008	3.5337
29	0.1000	4.1377	0.11059	3.0000	0.1018	3.2079
30	0.1218	5.0352	0.1000	6.0000	0.1018	5.2891
31	0.2402	2.4821	0.1000	5.0000	0.1017	4.8781
32	0.3078	1.5000	0.1018	5.5000	0.1018	5.2083
33	0.4033	1.5657	0.1277	6.0000	0.1274	6.0000
34	0.2078	4.5281	0.1313	6.0000	0.1263	6.0000
35	0.2500	2.3347	0.1000	5.0000	0.1018	4.132
36	0.1000	1.5432	0.1000	1.5000	0.1000	1.5000
37	0.2086	2.2869	0.1000	5.0000	0.1017	4.1561
38	0.1000	4.8972	0.1000	4.5000	0.1038	4.0117

TABLE 6. Objective function (OF(s) values for the test systems.

Test System	Proposed MILP	MILP[15]	MINLP[14]	Seeker [13]	GSA-SQP [27]
3 BUS	1.4909	1.5667	1.5997	1.5997	2.735
8 BUS	8.0061	8.0061	8.3610	8.4270	-
15 BUS	11.908	-	15.404	12.227	-
30 BUS	18.0183	-	26.06	-	26.8258

system, the proposed algorithm has better optimum compared to MILP[15], Seeker [13] as well as GSA-SQP [24]. For 8 bus system, however, the OF (sec) values of MILP [15], and proposed MILP are same which indicate that the obtained result for this system is indeed the global solution. Whereas, proposed MILP had outperformed MINLP [14] formulation and Seeker [13]. As shown in Table 6, the performance of the proposed algorithm for large scale test systems like 15 bus and 30 bus systems is also notably superior.

In terms of computational performance, the execution time of proposed MILP is dependent on the step size m which when increase, so do execution time as discussed in section III-C. For the 3 bus system which constitutes 144 discrete variables, 180 continuous variables with 582 inequality constraints and 24 equality constraints in the proposed MILP, The CPLEX solver consumed approximately four megabytes of memory space with 5.33 msec as execution time per iteration. The solution is obtained in 3 repetitions with a total execution time of 16 msec. Whereas, for the same test system, the traditional MILP formulation has 90 continuous variables, 30 discrete variables with 126 inequality constraints and 12 equality constraints and consumed approximately three megabytes of memory space with 1.153 msec execution time per iteration. The solution converged in 26 iterations making a total execution time of 0.03 sec. In MINLP formulation, there are 30 continuous and discrete variables each with six inequality constraints and 24 equality constraints. In terms of execution time per iteration MINLP is fastest with 0.645 msec but slowest in terms of number of iterations with 96 iterations. The above analysis tells that as the proposed MILP consists of more number of decision variables, it occupies more memory and high execution time per iteration. But, since it converges in the least amount of iterations compared with conventional MILP and MINLP formulations for all test systems the overall execution time is found to be less.

V. CONCLUSION

This work proposed a new MILP formulation for the coordination of directional overcurrent relays using disjunctive inequalities. The traditional nonlinear programming problem of directional overcurrent relay coordination is reformulated to MILP by discretizing the variables of bilinear terms and then linearizing using disjunctive inequalities. The proposed algorithm is formulated in GAMS package with CPLEX solver, and results are presented. The superiority of the proposed algorithm is tested on various small and large scale test systems, and results have shown that the proposed algorithm obtained significantly better solutions with less execution time compared to past integer programming formulations without considering current pickup settings as a discrete variable.

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