A New Model for Interactive Group Decision Making with Intuitionistic Fuzzy Preference Relations

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Abstract. A new method is proposed to solve the interactive group decision making problem in which the preference information takes the form of intuitionistic fuzzy preference relations. Firstly, we aggregate all individual intuitionistic fuzzy preference relations into a collective one. Then, a method to determine the experts' weights by utilizing the compatibility measures of the individual intuitionistic fuzzy preference relations and the collective one is proposed. Furthermore, a practical interactive procedure is developed, in which the intuitionistic fuzzy association coefficient is used to rank the given alternatives. Finally, this study presents a numerical example to illustrate the availability of the developed approach and compare it to another method.

Key words: intuitionistic fuzzy preference relations, group decision making problem, compatibility measures, consensus, interaction.

1. Introduction

Due to the high complexity of socioeconomic environments, it is difficult and impracticable for a single decision maker to consider all important aspects in practical decision making problems. Therefore, group decision making (GDM), especially with preference information, has caught attention widely in the decision making field and many desirable results have been derived over the last few decades (Cebi and Kahraman, 2010; Fan *et al.*, 2006; Herrera-Viedma *et al.*, 2007; Xia and Xu, 2011; Wan *et al.*, 2015; Wu and Chiclana, 2014; Zeng *et al.*, 2016). Each assessment value in a preference relation is given by a group member (decision maker, DM) for expressing his/her preference degree of one object over another. Owing to inherent complexity and uncertainty in real-world decision problems, it is often impractical to require a DM to provide his/her judgement in precise numerical values. To characterize this vagueness and uncertainty,

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different uncertain preference relations have been proposed (Xu, 2007a), such as interval multiplicative preference relations (Saaty and Vargas, 1987, 2007), interval fuzzy preference relations (Xu, 2004), triangular fuzzy preference relations (Van Laarhoven and Pedrycz, 1983) and linguistic fuzzy preference relations (Herrera *et al.*, 1996; Xu, 2005). Often, decision making under uncertainty necessitates the decision makers (DMs) who come from different backgrounds, levels, skills, experience, and personality. Sometimes, they also face some hesitancy in expressing their evaluations. The inherent uncertainty in the nature of decision-making process and vague knowledge of experts makes it necessary to resort to qualitative rather than quantitative evaluation of alternatives. As the evaluation values in a preference relation described previously cannot be used to completely express all the information in the problems considered, motivated by the idea of Atanassov's intuitionistic fuzzy set (IFS) (Atanassov, 1986), Szmidt and Kacprzyk (2002) generalized the fuzzy preference relation to the intuitionistic fuzzy preference relation, and investigated how to reach consensus with intuitionistic fuzzy preference relations in group decision making. Then, Xu (2007a) investigated the properties of intuitionistic fuzzy preference relations by constructing the score matrix and accuracy matrix, and he also gave the research of the group decision method with the intuitionistic fuzzy preference relations. Xu (2009) developed a method for estimating criteria weights from intuitionistic preference relations. Gong (2008) proposed the least squares and the goal programming model to derive the priority vector of the intuitionistic preference relations. Gong *et al.* (2010) investigated additive consistent properties of the intuitionistic fuzzy preference relation. Wang (2013) proposed linear goal programming models for deriving intuitionistic fuzzy weights from intuitionistic fuzzy preference relations. Xu and Yager (2009) introduced a similarity measure between IFSs and applied this measure to consensus analysis in group decisions with intuitionistic fuzzy preference relations. Xu (2013) investigated the compatibility of intuitionistic preference relations, proposed some novel compatibility measures of intuitionistic fuzzy information, and used them to put forward a consensus reaching procedure in group decision making with intuitionistic preference relations.

Usually, the DMs come from various research domains or have different knowledge backgrounds, and they necessitate different weights in deciding group preferences. Thus, how to estimate DMs' weights from intuitionistic preference relations is an interesting and important issue, which no investigation has been devoted to. In this paper, we shall develop a method based on compatibility measures for estimating DMs' weights from intuitionistic preference relations. We first utilize the intuitionistic fuzzy weighted averaging operator to aggregate all individual intuitionistic fuzzy preference relations into a collective intuitionistic fuzzy preference relation. Then, we develop an approach to derive the experts' weights directly from the degree of compatibility of the individual intuitionistic fuzzy preference relations and the collective one. Furthermore, we present a practical interactive procedure for group decision making with intuitionistic fuzzy preference relations, and develop a method based on compatibility measures to rank and select the alternatives. The interactive procedure can not only produce the DMs' weights automatically from the given intuitionistic fuzzy preference relations, but also improve the consensus agree between the individual intuitionistic fuzzy preference relations and the collective intuition-

istic fuzzy preference relation. Therefore, the evaluation results are more objective and unbiased than those individually assessed.

In order to do so, the remainder of this paper is structured as follows. Section 2 reviews some basic concepts and compatibility measures related to intuitionistic fuzzy sets. Section 3 develops an approach to interactive group decision making with intuitionistic fuzzy preference relations. Section 4 provides a practical example to illustrate the developed approaches, and Section 5 concludes the paper.

2. Preliminaries

Let a set X be fixed, an intuitionistic fuzzy set A in X is given by Atanassov (1986) as an object having the following form:

$$A = \left\{ \langle x, \mu_A(x), v_A(x) \rangle \big| x \in X \right\} \tag{1}$$

where the functions $\mu_A(x) : X \to [0, 1]$ and $v_A(x) : X \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in X$, such that $0 \leqslant \mu_A(x) + v_A(x) \leqslant 1$ for all $x \in X$. In addition $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy of x to A, or called the degree of hesitancy of x to A. It is obvious that $0 \leqslant \pi_A(x) \leqslant 1$ for every $x \in A$.

Clearly, a prominent characteristic of IFS is that it assigns to each element a membership degree, a nonmembership degree and a hesitation degree, and thus, IFS constitutes an extension of Zadeh's fuzzy set (Zadeh, 1965) which only assigns to each element a membership degree. Thanks to its effectiveness in dealing with vagueness and uncertainty, the IFS theory has been widely investigated and applied to a variety of fields, such as cluster analysis (Chaira, 2011; Xu *et al.*, 2008; Zhao *et al.*, 2012), pattern recognition (Boran and Akay, 2014; Chen and Randyanto, 2013; Mitchell, 2003; Papacostas *et al.*, 2013; Ye, 2011), medical diagnosis (Boran and Akay, 2014; Chen and Randyanto, 2013; De *et al.*, 2001) and multi-attribute decision making (Baležentis and Zeng, 2013; Chen, 2014; Liu and Wang, 2014; Hajiagha *et al.*, 2013; Wan and Dong, 2014; Wan *et al.*, 2016; Yu, 2015; Zeng *et al.*, 2013; Zeng and Xiao, 2016; Zhao and Wei, 2013; Zhou and He, 2014).

For an intuitionistic fuzzy set A and a given x, the triplet $(\mu_A(x), v_A(x), \pi(x))$ is called an intuitionistic fuzzy value (IFV) (Xu, 2007b; Xu and Yager, 2006; Zeng, 2013), and for convenience, we denote an IFV by $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$, where

$$\mu_\alpha \in [0, 1], \quad v_\alpha \in [0, 1], \quad \mu_\alpha + v_\alpha \leqslant 1, \quad \pi_\alpha = 1 - \mu_\alpha - v_\alpha. \tag{2}$$

For any two IFVs $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i})$ $(i = 1, 2)$, the following operational laws are valid (Xu and Yager, 2006):

(1) $\alpha_1 \oplus \alpha_2 = \left(\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1} \cdot \mu_{\alpha_2}, v_{\alpha_1} \cdot v_{\alpha_2}, (1 - \mu_{\alpha_1})(1 - \mu_{\alpha_2}) - v_{\alpha_1} \cdot v_{\alpha_2} \right)$;

(2) $\lambda\alpha = \left(1 - (1 - \mu_{\alpha_1})^\lambda, v_{\alpha_1}^\lambda, (1 - \mu_{\alpha_1})^\lambda - v_{\alpha_1}^\lambda \right)$.

Based on the operations (1) and (2), Xu (2007b) introduced an intuitionistic fuzzy weighted averaging (IFWA) operator as follows:

DEFINITION 1. Let $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i})$ $(i = 1, 2, \ldots, n)$ be a collection of IFVs, an intuitionistic fuzzy weighted averaging (IFWA) operator of dimension n is a mapping IFWA: $\Omega^n \to \Omega$ that has an associated weighting vector $w = (w_1, w_2, \ldots, w_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$IFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = w_1\alpha_1 \oplus w_2\alpha_2 \oplus \cdots \oplus w_n\alpha_n. \tag{3}$$

Especially, if $w = (1/n, 1/n, \ldots, 1/n)$, then the IFWA operator is reduced to an intuitionistic fuzzy averaging (IFA) operator of dimension n, which is defined as follows:

$$IFA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{n}(\alpha_1 \oplus \alpha_2 \oplus \cdots \oplus \alpha_n). \tag{4}$$

Compatibility measure is an efficient and important tool which can be used to measure the consensus of opinions within a group of decision makers. The lack of compatibility can lead to unsatisfied or even incorrect results because there are unavoidable differences and even contradictions among the preference relations provided by decision makers in GDM. Chen *et al.* (2011) proposed the compatibility degree of uncertain additive linguistic preference relations. Saaty and Vargas (2007) put forward the compatibility to judge the difference between two multiplicative preference relations. Jiang *et al.* (2013) developed some compatibility measures for intuitionistic multiplicative values and intuitionistic multiplicative preference relations in GDM. Recently, Xu (2013) defined the compatibility degree of IFVs as follows:

DEFINITION 2. Let $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2}, \pi_{\alpha_2})$ be two IFVs, then we call

$$c(\alpha_1, \alpha_2) = \frac{\mu_{\alpha_1}\mu_{\alpha_2} + v_{\alpha_1}v_{\alpha_2} + \pi_{\alpha_1}\pi_{\alpha_2}}{\max\{(\mu_{\alpha_1})^2 + (v_{\alpha_1})^2 + (\pi_{\alpha_1})^2, (\mu_{\alpha_2})^2 + (v_{\alpha_2})^2 + (\pi_{\alpha_2})^2\}} \tag{5}$$

a compatibility degree between α_1 and α_2.

It is clear that the larger the value of $c(\alpha_1, \alpha_2)$, the greater the compatibility degree of α_1 and α_2.

Theorem 1. *(See* Xu, 2013*.) The compatibility degree $c(\alpha_1, \alpha_2)$ derived from Eq.* (5) *satisfies the properties:*

   (1) $0 \leqslant c(\alpha_1, \alpha_2) \leqslant 1$;
   (2) $c(\alpha_1, \alpha_2) = 1$ *if and only if* $\alpha_1 = \alpha_2$;
   (3) $c(\alpha_1, \alpha_2) = c(\alpha_2, \alpha_1)$.

3. An Interactive Procedure for Group Decision Making with Intuitionistic Fuzzy Preference Relations

In this section, we present an approach to group decision making with intuitionistic fuzzy preference relations based on the intuitionistic fuzzy compatibility measures.

For a group decision making problem, let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of alternatives, $E = \{e_1, e_2, \ldots, e_m\}$ be the set of decision makers, whose weight vector is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, where $\lambda_k \geqslant 0$ and $\sum_{k=1}^{m} \lambda_k = 1$. The weights λ_k of e_k $(k = 1, 2, \ldots, m)$ are not predefined. In the process of decision making, an expert usually needs to provide his/her preference information over alternatives. Especially, in some real-life situations, such as negotiation processes, the high technology project investment of venture capital firms, supply chain management, etc., the expert may provide his/her preferences over alternatives to a certain degree, but it is possible that he/she is not so sure about it (Deschrijver and Kerre, 2003). Thus, it is very suitable to express the expert's preference values with intuitionistic fuzzy values. Xu (2007a) introduced the concepts of intuitionistic fuzzy preference relation as below:

DEFINITION 3. An intuitionistic fuzzy preference relation R on the set X is represented by a matrix $R = (r_{ij})_{n \times n} \subset X \times Y$ with $r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$ for all $i, j = 1, 2, \ldots, n$, r_{ij} is an IFV, composed by the certainty degree μ_{ij} to which x_i is preferred to x_j, the certainty degree v_{ij} to which x_i is non-preferred to x_j, and the hesitancy degree π_{ij} of x_i is preferred to x_j. Furthermore, μ_{ij}, v_{ij} and π_{ij} satisfy the following characteristics:

$$0 \leqslant \mu_{ij} + v_{ij} \leqslant 1, \quad \mu_{ji} = v_{ij}, \quad v_{ji} = \mu_{ij},$$

$$\pi_{ij} = \pi_{ji} = 1 - \mu_{ij} - v_{ij} = 1 - \mu_{ji} - v_{ji},$$

$$\mu_{ii} = v_{ii} = 0.5, \quad \pi_{ii} = 0, \quad i, j = 1, 2, \ldots, n. \tag{6}$$

Suppose that the experts e_k $(k = 1, 2, \ldots, m)$ provide intuitionistic fuzzy preferences for each pair of alternatives, and construct the intuitionistic fuzzy preference relation $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, \ldots, m)$. Then, we have:

Theorem 2. *(See Xu and Yager, 2009; Xu, 2013.) Let $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, \ldots, m)$ be m intuitionistic fuzzy preference relations given by the experts e_k $(k = 1, 2, \ldots, m)$ respectively, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ be the weight vector of experts, where $\lambda_k \geqslant 0$ and $\sum_{k=1}^{m} \lambda_k = 1$, then the aggregation $R = (r_{ij})_{n \times n}$ of $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, \ldots, m)$ is also an intuitionistic fuzzy preference relation, where*

$$r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij}), \qquad \mu_{ij} = \sum_{k=1}^{m} \lambda_k \mu_{ij}^{(k)}, \qquad v_{ij} = \sum_{k=1}^{m} \lambda_k v_{ij}^{(k)},$$

$$\pi_{ij} = \sum_{k=1}^{m} \lambda_k \pi_{ij}^{(k)}, \qquad \mu_{ii} = v_{ii} = 0.5, \qquad \pi_{ii} = 0, \ i, j = 1, 2, \ldots, n.$$

Based on the Definition 2, we can define the compatibility measure for the intuitionistic preference relations:

DEFINITION 4. Let $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, \ldots, m)$ be m intuitionistic fuzzy preference relations, and $R = (r_{ij})_{n \times n}$ of $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ be their aggregated (or collective) intuitionistic fuzzy preference relation. Then we call

$$c(R^{(k)}, R)$$
$$= \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left(\mu_{ij}^{(k)} \mu_{ij} + v_{ij}^{(k)} v_{ij} + \pi_{ij}^{(k)} \pi_{ij} \right)}{\max \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \left((\mu_{ij}^{(k)})^2 + (v_{ij}^{(k)})^2 + (\pi_{ij}^{(k)})^2 \right), \sum_{i=1}^{n} \sum_{j=1}^{n} \left((\mu_{ij})^2 + (v_{ij})^2 + (\pi_{ij})^2 \right) \right\}} \quad (7)$$

the compatibility degree of $R^{(k)}$ and R. Furthermore, if $c(R^{(k)}, R) > \alpha_0$, then $R^{(k)}$ and R are matrices of acceptable compatibility, where α_0 is the threshold value of acceptable compatibility. In general, we take $\alpha_0 \in [0.5, 1]$ in practical applications.

Clearly, the larger the value of $c(R^{(k)}, R) > \alpha_0$, the greater the compatibility degree of $R^{(k)}$ and R. By the Definition 4, we have the following properties:

(1) $0 \leqslant c(R^{(k)}, R) \leqslant 1$;
(2) $c(R^{(k)}, R) = c(R, R^{(k)})$;
(3) $c(R^{(k)}, R) = 1$ iff $R^{(k)} = R$.

In the process of group decision making, if $c(R^{(k)}, R) \leqslant \alpha$ then we shall return $R^{(k)}$ together with R to the expert e_k, and at the same time, inform him/her of some elements of $R^{(k)}$ with small degrees of compatibility, which need to be revaluated. We repeat this procedure until $R^{(k)}$ and R are of acceptable similarity. In general, the compatibility measure $c(R^{(k)}, R)$ reflects the degree of consensus between the individual intuitionistic fuzzy preference relation $R^{(k)}$ and the collective R. In many actual situations, some experts may provide unduly high or unduly low preference arguments for their preferred or repugnant objects, which may result in the low degrees of consensus among the collective intuitionistic fuzzy preference relation and the individual intuitionistic fuzzy preference relations. In such cases, we shall assign low weights to these experts in the process of decision making. That is, the more the compatibility degree $c(R^{(k)}, R)$, the more the weight of the expert e_k. As a result, we propose a formula for determining the experts' weights as following:

$$\lambda_k = \frac{c(R^{(k)}, R)}{\sum_{k=1}^{m} c(R^{(k)}, R)}, \quad k = 1, 2, \ldots, m. \quad (8)$$

The weight determined by this method has the following desirable characteristic: the larger the value of $c(R^{(k)}, R)$, the greater the compatibility degree of $R^{(k)}$ and R, and the larger the weight of e_k. This can avoid the unduly high or low evaluation values induced by decision makers' limited knowledge or expertise.

Based on the above analysis, we can develop a practical interactive procedure for group decision making with intuitionistic fuzzy preference relations as follows.

Step 1: Consider a group decision making problem with intuitionistic fuzzy preference relations. The expert $e_k \in E$ compares each pair of alternatives in X by using IFVs, and constructs an intuitionistic fuzzy preference relation $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$. Suppose that the weights of the experts e_k $(k = 1, 2, \ldots, m)$ are completely unknown. In such case, all experts are assigned originally the same weight, i.e. $\lambda = (1/m, 1/m, \ldots, 1/m)$. Assume that the experts predefine the dead line of acceptable compatibility as α_0.

Step 2: Utilize intuitionistic fuzzy weighted averaging (IFWA) operator to aggregate all individual intuitionistic fuzzy preference relation $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, \ldots, m)$ into the collective intuitionistic fuzzy preference relation $R = (r_{ij})_{n \times n}$. For ease of calculation, let $r_i = (r_{i1}, r_{i2}, \ldots, r_{in})$ be the collective preference vector corresponding to the alternative x_i.

Step 3: Utilize Eq. (7) to calculate the compatibility degree $c(R^{(k)}, R)$ $(k = 1, 2, \ldots, m)$ for the individual intuitionistic fuzzy preference relation $R^{(k)}$ and the collective intuitionistic fuzzy preference relation R.

Step 4: If $c(R^{(k)}, R) > \alpha_0$ $(k = 1, 2, \ldots, m)$, then go to Step 5, otherwise we recalculate the weights of the experts e_k $(k = 1, 2, \ldots, m)$ by using Eq. (8), and return $R^{(k)}$ together with R to the expert e_k, and at the same time, inform him/her of some elements of $R^{(k)}$ with small degrees of compatibility, which need to be revaluated. For convenience, we also denote $R^{(k)}$ as the revaluated intuitionistic fuzzy preference relation. Then go to Step 2.

Step 5: Define the positive ideal intuitionistic fuzzy solution (PIIFS): $r^+ = (r_1^+, r_2^+, \ldots, r_n^+)$ where $r_k^+ = (1, 0, 0)$ and the negative ideal intuitionistic fuzzy solution (NIIFS): $r^- = (r_1^-, r_2^-, \ldots, r_n^-)$ where $r_k^- = (0, 1, 0)$.

Step 6: Calculate the compatibility measures between the collective preference vector r_i and the PIIFS r^+ and NIIFS r^- by using the Eq. (7) as follows:

$$c(r_i, r^+) = \frac{\sum_{j=1}^n (\mu_{ij} \cdot 1 + v_{ij} \cdot 0 + \pi_{ij} \cdot 0)}{\max\left\{\sum_{j=1}^n (\mu_{ij}^2 + v_{ij}^2 + \pi_{ij}^2), \sum_{j=1}^n (1 + 0 + 0)\right\}} = \frac{\sum_{j=1}^n \mu_{ij}}{n}, \quad (9)$$

$$c(r_i, r^-) = \frac{\sum_{j=1}^n (\mu_{ij} \cdot 0 + v_{ij} \cdot 1 + \pi_{ij} \cdot 0)}{\max\left\{\sum_{j=1}^n (\mu_{ij}^2 + v_{ij}^2 + \pi_{ij}^2), \sum_{j=1}^n (0 + 1 + 0)\right\}} = \frac{\sum_{j=1}^n v_{ij}}{n}. \quad (10)$$

Step 7: Calculate the association degree $C(r_i)$ for each alternative as follows:

$$C(r_i) = \frac{c(r_i, r^-)}{c(r_i, r^+) + c(r_i, r^-)} = \frac{\sum_{j=1}^n v_{ij}}{\sum_{j=1}^n (\mu_{ij} + v_{ij})}, \quad i = 1, 2, \ldots, n. \quad (11)$$

Step 8: Rank all the alternatives x_i $(i = 1, 2, \ldots, n)$ based on the calculated association degree $C(r_i)$ $(i = 1, 2, \ldots, n)$, where the greater value is the better alternative.

Table 1
Intuitionistic fuzzy preference relation $R^{(1)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.2, 0.6, 0.2)	(0.4, 0.4, 0.2)	(0.5, 0.4, 0.1)
x_2	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.3, 0.4, 0.3)	(0.6, 0.2, 0.2)	(0.7, 0.2, 0.1)
x_3	(0.6, 0.2, 0.2)	(0.4, 0.3, 0.3)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.6, 0.3, 0.1)
x_4	(0.4, 0.4, 0.2)	(0.2, 0.6, 0.2)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.2, 0.5, 0.3)
x_5	(0.4, 0.5, 0.1)	(0.2, 0.7, 0.1)	(0.3, 0.6, 0.1)	(0.5, 0.2, 0.3)	(0.5, 0.5, 0)

Table 2
Intuitionistic fuzzy preference relation $R^{(2)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.3, 0.4, 0.3)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)	(0.5, 0.3, 0.2)
x_2	(0.4, 0.3, 0.3)	(0.5, 0.5, 0)	(0.4, 0.4, 0.2)	(0.5, 0.3, 0.2)	(0.6, 0.4, 0)
x_3	(0.4, 0.4, 0.2)	(0.4, 0.4, 0.2)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.6, 0.3, 0.1)
x_4	(0.3, 0.6, 0.1)	(0.3, 0.5, 0.2)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)
x_5	(0.3, 0.5, 0.2)	(0.4, 0.6, 0)	(0.3, 0.6, 0.1)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)

4. Numerical Example

In this section, a group decision making problem with intuitionistic fuzzy preference relations involves the selection of production strategies in a company. Assume an enterprise that produces cars is looking for its general strategy the next year and they consider that it should be useful for them to create a new production plant in order to be bigger and more competitive in the market. After careful evaluation of the information, the group of experts of the company considers the following countries where it could be interesting to create a new production plant.

(1) x_1: produce in Russia;
(2) x_2: produce in China;
(3) x_3: produce in India;
(4) x_4: produce in Brazil;
(5) x_5: produce in Nigeria.

One main criterion used is benefit. There are four decision makers e_k $(k = 1, 2, 3, 4)$. The decision makers compare these five strategies with respect to the criterion benefit by using IFVs, and construct, respectively, the intuitionistic fuzzy preference relation $R^{(k)} = (r_{ij}^{(k)})_{n \times n}$ $(k = 1, 2, 3, 4)$, as listed in Tables 1–4. In what follows, we apply the developed procedure to the selection of best production strategies from the potential countries x_i $(i = 1, 2, \ldots, 5)$.

Step 1: Set the original weight vector of the experts e_k $(k = 1, 2, 3, 4)$, $\lambda = (1/4, 1/4, 1/4, 1/4)$, and assume that the experts predefine the dead line of acceptable compatibility as $\alpha_0 = 0.9$.

Table 3
Intuitionistic fuzzy preference relation $R^{(3)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.4, 0.5, 0.1)	(0.2, 0.6, 0.2)
x_2	(0.4, 0.5, 0.1)	(0.5, 0.5, 0)	(0.9, 0.1, 0)	(0.3, 0.6, 0.1)	(0.2, 0.7, 0.1)
x_3	(0.2, 0.7, 0.1)	(0.1, 0.9, 0)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)
x_4	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.2, 0.8, 0)
x_5	(0.6, 0.2, 0.2)	(0.7, 0.2, 0.1)	(0.4, 0.4, 0.2)	(0.8, 0.2, 0)	(0.5, 0.5, 0)

Table 4
Intuitionistic fuzzy preference relation $R^{(4)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.4, 0.3, 0.3)	(0.2, 0.5, 0.3)	(0.6, 0.2, 0.2)	(0.4, 0.5, 0.1)
x_2	(0.3, 0.4, 0.3)	(0.5, 0.5, 0)	(0.4, 0.4, 0.2)	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.2)
x_3	(0.5, 0.2, 0.3)	(0.4, 0.4, 0.2)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.6, 0.1, 0.3)
x_4	(0.2, 0.6, 0.2)	(0.2, 0.6, 0.2)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.4, 0.6, 0)
x_5	(0.5, 0.4, 0.1)	(0.3, 0.5, 0.2)	(0.1, 0.6, 0.3)	(0.6, 0.4, 0)	(0.5, 0.5, 0)

Table 5
Collective intuitionistic fuzzy preference relation R.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.43, 0.35, 0.22)	(0.42, 0.39, 0.19)	(0.51, 0.33, 0.16)	(0.41, 0.44, 0.15)
x_2	(0.35, 0.43, 0.22)	(0.5, 0.5, 0)	(0.60, 0.28, 0.12)	(0.51, 0.29, 0.20)	(0.53, 0.36, 0.11)
x_3	(0.39, 0.42, 0.19)	(0.28, 0.60, 0.12)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.56, 0.24, 0.20)
x_4	(0.33, 0.51, 0.16)	(0.29, 0.51, 0.20)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.34, 0.52, 0.14)
x_5	(0.44, 0.41, 0.15)	(0.36, 0.53, 0.11)	(0.24, 0.56, 0.20)	(0.52, 0.34, 0.14)	(0.5, 0.5, 0)

Step 2: Utilize the IFWA operator to aggregate all individual intuitionistic fuzzy preference relations $R^{(k)} = (r_{ij}^{(k)})_{5\times5}$ ($k = 1, 2, \ldots, 4$) into the collective intuitionistic fuzzy preference relation $R = (r_{ij})_{5\times5}$ (see Table 5).

Step 3: Utilize Eq. (7) to calculate the compatibility degrees $c(R^{(k)}, R)$ of the individual intuitionistic fuzzy preference relation $R^{(k)}$ and the collective intuitionistic fuzzy preference relation R:

$$c(R^{(1)}, R) = 0.946, \qquad c(R^{(2)}, R) = 0.971,$$
$$c(R^{(3)}, R) = 0.838, \qquad c(R^{(4)}, R) = 0.960.$$

Step 4: $c(R^{(k)}, R) > \alpha_0$ ($k = 1, 2, 4$), but $c(R^{(3)}, R) < \alpha_0$. Then by Eq. (5), we calculate all the compatibility degrees $c(r_{ij}^{(3)}, r_{ij})$ ($i, j = 1, 2, 3, 4, 5$):

$$c(r_{11}^{(3)}, r_{11}) = 1, \qquad c(r_{12}^{(3)}, r_{12}) = 0.898, \qquad c(r_{13}^{(3)}, r_{13}) = 0.724,$$
$$c(r_{14}^{(3)}, r_{14}) = 0.917, \qquad c(r_{15}^{(3)}, r_{15}) = 0.855, \qquad c(r_{21}^{(3)}, r_{21}) = 0.898,$$

Table 6
Revaluated intuitionistic fuzzy preference relations $R^{(3)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	$(0.5, 0.5, 0)$	$(0.6, 0.3, 0.1)$	$(0.5, 0.4, 0.1)$	$(0.4, 0.5, 0.1)$	$(0.5, 0.5, 0)$
x_2	$(0.3, 0.6, 0.1)$	$(0.5, 0.5, 0)$	$(0.4, 0.4, 0.2)$	$(0.6, 0.3, 0.1)$	$(0.5, 0.4, 0.1)$
x_3	$(0.4, 0.5, 0.1)$	$(0.4, 0.5, 0.2)$	$(0.5, 0.5, 0)$	$(0.5, 0.3, 0.2)$	$(0.6, 0.2, 0.2)$
x_4	$(0.5, 0.4, 0.1)$	$(0.3, 0.6, 0.1)$	$(0.3, 0.5, 0.2)$	$(0.5, 0.5, 0)$	$(0.3, 0.6, 0.1)$
x_5	$(0.5, 0.5, 0)$	$(0.4, 0.5, 0.1)$	$(0.2, 0.6, 0.2)$	$(0.6, 0.3, 0.1)$	$(0.5, 0.5, 0)$

Table 7
Collective intuitionistic fuzzy preference relations R.

	x_1	x_2	x_3	x_4	x_5
x_1	$(0.5, 0.5, 0)$	$(0.43, 0.33, 0.24)$	$(0.30, 0.48, 0.22)$	$(0.49, 0.36, 0.15)$	$(0.47, 0.43, 0.10)$
x_2	$(0.33, 0.43, 0.24)$	$(0.5, 0.5, 0)$	$(0.37, 0.4, 0.23)$	$(0.57, 0.25, 0.18)$	$(0.57, 0.33, 0.10)$
x_3	$(0.48, 0.30, 0.22)$	$(0.4, 0.37, 0.23)$	$(0.5, 0.5, 0)$	$(0.52, 0.3, 0.18)$	$(0.6, 0.23, 0.17)$
x_4	$(0.36, 0.49, 0.15)$	$(0.25, 0.57, 0.18)$	$(0.3, 0.52, 0.18)$	$(0.5, 0.5, 0)$	$(0.33, 0.51, 0.16)$
x_5	$(0.43, 0.47, 0.10)$	$(0.33, 0.57, 0.10)$	$(0.23, 0.6, 0.17)$	$(0.51, 0.33, 0.16)$	$(0.5, 0.5, 0)$

$$c\big(r_{22}^{(3)}, r_{22}\big) = 1, \qquad c\big(r_{23}^{(3)}, r_{23}\big) = 0.693, \qquad c\big(r_{24}^{(3)}, r_{24}\big) = 0.754,$$

$$c\big(r_{25}^{(3)}, r_{25}\big) = 0.683, \qquad c\big(r_{31}^{(3)}, r_{31}\big) = 0.724, \qquad c\big(r_{32}^{(3)}, r_{32}\big) = 0.693,$$

$$c\big(r_{33}^{(3)}, r_{33}\big) = 1, \qquad c\big(r_{34}^{(3)}, r_{34}\big) = 1, \qquad c\big(r_{35}^{(3)}, r_{35}\big) = 0.876,$$

$$c\big(r_{41}^{(3)}, r_{41}\big) = 0.917, \qquad c\big(r_{42}^{(3)}, r_{42}\big) = 0.754, \qquad c\big(r_{43}^{(3)}, r_{43}\big) = 1,$$

$$c\big(r_{44}^{(3)}, r_{44}\big) = 1, \qquad c\big(r_{45}^{(3)}, r_{45}\big) = 0.712, \qquad c\big(r_{51}^{(3)}, r_{51}\big) = 0.855,$$

$$c\big(r_{52}^{(3)}, r_{52}\big) = 0.683, \qquad c\big(r_{53}^{(3)}, r_{53}\big) = 0.876, \qquad c\big(r_{54}^{(3)}, r_{54}\big) = 0.712,$$

$$c\big(r_{55}^{(3)}, r_{55}\big) = 1.$$

Thus, we need to utilize Eq. (8) to recalculate the weights of the expert e_k ($k = 1, 2, 3, 4$): $\lambda = (0.255, 0.261, 0.226, 0.258)$, and return $R^{(3)}$ together with R to the experts e_3, respectively, and suggest him/her to revaluate the elements in $R^{(3)}$, especially those with the compatibility degrees less than α_0, such as $r_{12}, r_{15}, r_{23}, r_{24}, r_{25}$. Assume that the expert e_3 provides the revaluated intuitionistic fuzzy preference relation as Table 6 (for convenience, we also denote $R^{(3)}$ as the intuitionistic fuzzy relation).

Then, we utilize Eq. (3) to fuse all individual intuitionistic preference relations $R^{(1)}$, $R^{(2)}$, $R^{(4)}$ and the revaluated $R^{(3)}$ into the collective intuitionistic fuzzy preference relation $R = (r_{ij})_{5 \times 5}$ (see Table 7).

Using Eq. (7), we calculate the compatibility degrees:

$$c\big(R^{(1)}, R\big) = 0.944, \qquad c\big(R^{(2)}, R\big) = 0.973,$$

$$c\big(R^{(3)}, R\big) = 0.951, \qquad c\big(R^{(4)}, R\big) = 0.962.$$

Then all $c(R^{(k)}, R) > \alpha_0$ $(k = 1, 2, 3, 4)$, and thus, the group reaches an acceptable consensus.

Step 5: Define the PIIFS: r^+ and the NIIFS: r^-:

$$r^+ = \big((1, 0, 0), (1, 0, 0), (1, 0, 0), (1, 0, 0), (1, 0, 0)\big),$$
$$r^- = \big((0, 1, 0), (0, 1, 0), (0, 1, 0), (0, 1, 0), (0, 1, 0)\big).$$

Step 6: Utilize Eq. (11) to calculate the association degree of each alternative x_i $(i = 1, 2, 3, 4, 5)$:

$$C(r_1) = 0.490, \qquad C(r_2) = 0.430, \qquad C(r_3) = 0.324,$$
$$C(r_4) = 0.598, \qquad C(r_5) = 0.557.$$

Step 7: Since

$$C(r_4) > C(r_5) > C(r_1) > C(r_2) > C(r_3).$$

Then the rank of the alternatives is: $x_4 \succ x_5 \succ x_1 \succ x_2 \succ x_3$, and thus x_4 is the best choice.

From the above example, we can see that the weights of the experts can be deduced automatically from the intuitionistic fuzzy preference relations. The higher the compatibility degree $c(R^{(k)}, R)$, the larger the weight of the expert e_k, which can improve the consensus agreements between the individual intuitionistic fuzzy preference relation and the collective intuitionistic fuzzy preference relation. In addition, the proposed method to compare the alternatives only employs the intuitionistic fuzzy association coefficient between an alternative and the positive ideal alternative and negative ideal alternative. Therefore, it is very simple and convenient to use in practical applications.

If we use Xu and Yager's (2009) method to solve the problem, then in a similar way, we utilize IFWA operator to aggregate all individual intuitionistic fuzzy preference relations $R^{(k)} = (r_{ij}^{(k)})_{5 \times 5}$ $(k = 1, 2, \ldots, 4)$ into the collective intuitionistic fuzzy preference relation $R = (r_{ij})_{5 \times 5}$, and use the following formula proposed by Xu and Yager (2009)

$$s\big(r_{ij}^{(k)}, r_{ij}\big) = \begin{cases} 0.5, & \text{if } r_{ij}^{(k)} = r_{ij} = r_{ij}^{(c)}, \\ \dfrac{d(r_{ij}^{(k)}, r_{ij}^{(c)})}{d(r_{ij}^{(k)}, r_{ij}^{(c)}) + d(r_{ij}^{(k)}, r_{ij})}, & \text{otherwise} \end{cases} \tag{12}$$

to calculate the similarity degree between $r_{ij}^{(k)}$ and r_{ij}:

$$s\big(r_{11}^{(1)}, r_{11}\big) = s\big(r_{22}^{(1)}, r_{22}\big) = s\big(r_{33}^{(1)}, r_{33}\big) = s\big(r_{44}^{(1)}, r_{44}\big) = s\big(r_{55}^{(1)}, r_{55}\big) = 0.5,$$
$$s\big(r_{12}^{(1)}, r_{12}\big) = s\big(r_{21}^{(1)}, r_{21}\big) = 0.68, \qquad s\big(r_{13}^{(1)}, r_{13}\big) = s\big(r_{31}^{(1)}, r_{31}\big) = 0.46,$$
$$s\big(r_{14}^{(1)}, r_{14}\big) = s\big(r_{41}^{(1)}, r_{41}\big) = 0.5, \qquad s\big(r_{15}^{(1)}, r_{15}\big) = s\big(r_{51}^{(1)}, r_{51}\big) = 0.40,$$

$$s\left(r_{23}^{(1)}, r_{23}\right) = s\left(r_{32}^{(1)}, r_{32}\right) = 0.40, \qquad s\left(r_{24}^{(1)}, r_{24}\right) = s\left(r_{42}^{(1)}, r_{42}\right) = 0.78,$$

$$s\left(r_{25}^{(1)}, r_{25}\right) = s\left(r_{52}^{(1)}, r_{52}\right) = 0.67, \qquad s\left(r_{34}^{(1)}, r_{34}\right) = s\left(r_{43}^{(1)}, r_{43}\right) = 0.5,$$

$$s\left(r_{35}^{(1)}, r_{35}\right) = s\left(r_{53}^{(1)}, r_{53}\right) = 0.78, \qquad s\left(r_{45}^{(1)}, r_{45}\right) = s\left(r_{54}^{(1)}, r_{54}\right) = 0.67,$$

$$s\left(r_{12}^{(2)}, r_{12}\right) = s\left(r_{21}^{(2)}, r_{21}\right) = 0.38, \qquad s\left(r_{13}^{(2)}, r_{13}\right) = s\left(r_{31}^{(2)}, r_{31}\right) = 0.5,$$

$$s\left(r_{14}^{(2)}, r_{14}\right) = s\left(r_{41}^{(2)}, r_{41}\right) = 0.75, \qquad s\left(r_{15}^{(2)}, r_{15}\right) = s\left(r_{51}^{(2)}, r_{51}\right) = 0.44,$$

$$s\left(r_{23}^{(2)}, r_{23}\right) = s\left(r_{32}^{(2)}, r_{32}\right) = 0.5, \qquad s\left(r_{24}^{(2)}, r_{24}\right) = s\left(r_{42}^{(2)}, r_{42}\right) = 0.95,$$

$$s\left(r_{25}^{(2)}, r_{25}\right) = s\left(r_{52}^{(2)}, r_{52}\right) = 0.69, \qquad s\left(r_{34}^{(2)}, r_{34}\right) = s\left(r_{43}^{(2)}, r_{43}\right) = 0.5,$$

$$s\left(r_{35}^{(2)}, r_{35}\right) = s\left(r_{53}^{(2)}, r_{53}\right) = 0.78, \qquad s\left(r_{45}^{(2)}, r_{45}\right) = s\left(r_{54}^{(2)}, r_{54}\right) = 0.21,$$

$$s\left(r_{12}^{(3)}, r_{12}\right) = s\left(r_{21}^{(3)}, r_{21}\right) = 0.56, \qquad s\left(r_{13}^{(3)}, r_{13}\right) = s\left(r_{31}^{(3)}, r_{31}\right) = 0.53,$$

$$s\left(r_{14}^{(3)}, r_{14}\right) = s\left(r_{41}^{(3)}, r_{41}\right) = 0.29, \qquad s\left(r_{15}^{(3)}, r_{15}\right) = s\left(r_{51}^{(3)}, r_{51}\right) = 0.53,$$

$$s\left(r_{23}^{(3)}, r_{23}\right) = s\left(r_{32}^{(3)}, r_{32}\right) = 0.67, \qquad s\left(r_{24}^{(3)}, r_{24}\right) = s\left(r_{42}^{(3)}, r_{42}\right) = 0.24,$$

$$s\left(r_{25}^{(3)}, r_{25}\right) = s\left(r_{52}^{(3)}, r_{52}\right) = 0.33, \qquad s\left(r_{34}^{(3)}, r_{34}\right) = s\left(r_{43}^{(3)}, r_{43}\right) = 0.5,$$

$$s\left(r_{35}^{(3)}, r_{35}\right) = s\left(r_{53}^{(3)}, r_{53}\right) = 0.5, \qquad s\left(r_{45}^{(3)}, r_{45}\right) = s\left(r_{54}^{(3)}, r_{54}\right) = 0.62,$$

$$s\left(r_{12}^{(4)}, r_{12}\right) = s\left(r_{21}^{(4)}, r_{21}\right) = 0.62, \qquad s\left(r_{13}^{(4)}, r_{13}\right) = s\left(r_{31}^{(4)}, r_{31}\right) = 0.77,$$

$$s\left(r_{14}^{(4)}, r_{14}\right) = s\left(r_{41}^{(4)}, r_{41}\right) = 0.70, \qquad s\left(r_{15}^{(4)}, r_{15}\right) = s\left(r_{51}^{(4)}, r_{51}\right) = 0.60,$$

$$s\left(r_{23}^{(4)}, r_{23}\right) = s\left(r_{32}^{(4)}, r_{32}\right) = 0.5, \qquad s\left(r_{24}^{(4)}, r_{24}\right) = s\left(r_{42}^{(4)}, r_{42}\right) = 0.78,$$

$$s\left(r_{25}^{(4)}, r_{25}\right) = s\left(r_{52}^{(4)}, r_{52}\right) = 0.72, \qquad s\left(r_{34}^{(4)}, r_{34}\right) = s\left(r_{43}^{(4)}, r_{43}\right) = 0.5,$$

$$s\left(r_{35}^{(4)}, r_{35}\right) = s\left(r_{53}^{(4)}, r_{53}\right) = 0.77, \qquad s\left(r_{45}^{(4)}, r_{45}\right) = s\left(r_{54}^{(4)}, r_{54}\right) = 0.65.$$

Then using the following formula,

$$s(R^{(k)}, R) = \frac{1}{5^2} \sum_{i=1}^{5} \sum_{j=1}^{5} s(r_{ij}^{(k)}, r_{ij}) \tag{13}$$

we calculate the similarity degree $s(R^{(k)}, R)$ between the individual intuitionistic fuzzy preference relation $R^{(k)}$ and the collective intuitionistic fuzzy preference relation R:

$$s\left(R^{(1)}, R\right) = 0.57, \qquad s\left(R^{(2)}, R\right) = 0.56,$$
$$s\left(R^{(3)}, R\right) = 0.48, \qquad s\left(R^{(4)}, R\right) = 0.63.$$

Suppose that the threshold value $\alpha_0 = 0.5$, since $s(R^{(3)}, R) < \alpha_0$, then we utilize Eq. (14) to recalculate the weights of the expert e_k $(k = 1, 2, 3, 4)$: $\lambda = (0.25, 0.25, 0.22, 0.28)$,

Table 8
Revaluated intuitionistic fuzzy preference relations $R^{(3)}$.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.3, 0)	(0.2, 0.6, 0.2)
x_2	(0.4, 0.5, 0.1)	(0.5, 0.5, 0)	(0.9, 0.1, 0)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)
x_3	(0.3, 0.6, 0.1)	(0.1, 0.9, 0)	(0.5, 0.5, 0)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)
x_4	(0.3, 0.7, 0)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.5, 0.5, 0)	(0.2, 0.8, 0)
x_5	(0.6, 0.2, 0.2)	(0.3, 0.6, 0.1)	(0.4, 0.4, 0.2)	(0.8, 0.2, 0)	(0.5, 0.5, 0)

Table 9
Collective intuitionistic fuzzy preference relations R.

	x_1	x_2	x_3	x_4	x_5
x_1	(0.5, 0.5, 0)	(0.43, 0.34, 0.23)	(0.36, 0.44, 0.2)	(0.58, 0.29, 0.13)	(0.42, 0.43, 0.15)
x_2	(0.34, 0.43, 0.23)	(0.5, 0.5, 0)	(0.58, 0.29, 0.13)	(0.55, 0.26, 0.19)	(0.6, 0.29, 0.11)
x_3	(0.44, 0.36, 0.2)	(0.29, 0.58, 0.13)	(0.5, 0.5, 0)	(0.59, 0.27, 0.14)	(0.57, 0.23, 0.2)
x_4	(0.29, 0.58, 0.13)	(0.26, 0.55, 0.19)	(0.27, 0.59, 0.14)	(0.5, 0.5, 0)	(0.35, 0.51, 0.14)
x_5	(0.43, 0.42, 0.15)	(0.29, 0.6, 0.11)	(0.23, 0.57, 0.2)	(0.51, 0.35, 0.14)	(0.5, 0.5, 0)

and return $R^{(3)}$ together with R to the experts e_3, respectively, and suggest him/her to revaluate the elements in $R^{(3)}$, especially those with the similarity degrees less than α_0, including $s(r_{14}^{(3)}, r_{14})$, $s(r_{41}^{(3)}, r_{41})$, $s(r_{24}^{(3)}, r_{24})$, $s(r_{42}^{(3)}, r_{42})$, $(r_{25}^{(3)}, r_{25})$, $s(r_{52}^{(3)}, r_{52})$.

$$\lambda_k = \frac{s(R^{(k)}, R)}{\sum_{k=1}^{4} s(R^{(k)}, R)}, \quad k = 1, 2, \dots, 4. \tag{14}$$

Assume that the expert e_3 provides the revaluated intuitionistic fuzzy preference relation as Table 8 (for convenience, we also denote $R^{(3)}$ as the intuitionistic fuzzy relation). Then we aggregate $R^{(1)}$, $R^{(2)}$, $R^{(4)}$ and the revaluated $R^{(3)}$ into the collective intuitionistic fuzzy preference relation $R = (r_{ij})_{5 \times 5}$ (see Table 9).

By Eqs. (12) and (13), it follows that $s(R^{(1)}, R) = 0.62$, $s(R^{(2)}, R) = 0.58$, $s(R^{(3)}, R) = 0.60$, $s(R^{(4)}, R) = 0.65$. Since $s(R^{(k)}, R) > \alpha_0$ $(k = 1, 2, 3, 4, 5)$, then all $R^{(k)}$ $(k = 1, 2, 3, 4, 5)$ and R are of acceptable similarity.

The numerical examples above show that the individuals with the highest discrepancies from the collective preference relation are different across the two measures employed, namely similarity degree and compatibility degree. This is because Xu and Yager's (2009) method mainly examines whether the values under comparison are similar to each other on the basis of the relative distances defined in terms of intuitionistic fuzzy values and their complements, whereas the method proposed in this paper considers the compatibility degrees for each pair of intuitionistic fuzzy values.

5. Conclusions

In this paper, we have utilized the intuitionistic preference compatibility measures to develop an interactive approach for group decision making situations where the preference

information given by the decision makers is expressed as intuitionistic preference relations. We first utilize the intuitionistic fuzzy weighted averaging operator to aggregate all individual intuitionistic fuzzy preference relations into a collective one, and then we developed an approach to determine the experts' weights by utilizing the consensus degree among the individual intuitionistic fuzzy preference relation and the collective intuitionistic fuzzy preference relation. Furthermore, we have developed a practical interactive procedure for group decision making with intuitionistic fuzzy preference relations, and illustrated the proposed procedure with the group decision making problem about the selection of production strategies in a company. The interactive procedure can not only produce the experts' weights automatically from the given intuitionistic fuzzy preference information, but also improve the consensus agreements between the individual intuitionistic fuzzy preference relation and the collective intuitionistic fuzzy preference relation. In addition, we develop an effective evaluation formula to compare the alternatives, by which the most desirable alternatives can be selected or ranked according to the intuitionistic fuzzy association coefficient between an alternative and the positive ideal alternative and negative ideal alternative. The numerical analysis has showed the feasibility and practicality of the approach.

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Naujas modelis interaktyviems grupiniams sprendimams pagal intuityviuosius neraiškiuosius pirmumo santykius priimti

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Siūlomas naujas metodas interaktyviems grupinio sprendimo priėmimo uždaviniams spręsti, kuriuose informacija apie pirmumą pateikiama intuityviųjų neraiškiųjų pirmumo santykių forma. Pirmiausia mes sujungiame visus atskirus intuityviuosius pirmenybės santykius į vieną kolektyvinį. Po to siūlomas metodas eksperto svoriams nustatyti taikant atskirų intuityviųjų neraiškiųjų pirmumo santykių, o po to ir kolektyvinių suderinamumo priemones. Vėliau sukuriama praktinė interaktyvioji procedūra įvertinti, kurioje intuityviosios neraiškiosios sąsajos koeficientas yra taikomas pateiktoms alternatyvoms įvertinti. Galiausiai šis tyrimas pateikia skaitmeninį pavyzdį sukurto būdo galimybėms pavaizduoti ir palyginti su kitu metodu.