

Properties of the new $\mathcal{N} = 1$ AdS₄ vacuum of maximal supergravity

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ABSTRACT: The recent comprehensive numerical study of critical points of the scalar potential of four-dimensional $\mathcal{N} = 8$, SO(8) gauged supergravity using Machine Learning software in [1] has led to a discovery of a new $\mathcal{N} = 1$ vacuum with a triality-invariant SO(3) symmetry. Guided by the numerical data for that point, we obtain a consistent SO(3) \times \mathbb{Z}_2 -invariant truncation of the $\mathcal{N} = 8$ theory to an $\mathcal{N} = 1$ supergravity with three chiral multiplets. Critical points of the truncated scalar potential include both the $\mathcal{N} = 1$ point as well as two new non-supersymmetric and perturbatively unstable points not found by previous searches. Studying the structure of the submanifold of SO(3) \times \mathbb{Z}_2 -invariant supergravity scalars, we find that it has a simple interpretation as a submanifold of the 14-dimensional \mathbb{Z}_2^3 -invariant scalar manifold (SU(1,1)/U(1))⁷, for which we find a rather remarkable superpotential whose structure matches the single bit error correcting (7,4) Hamming code. This 14-dimensional scalar manifold contains approximately one quarter of the known critical points. We also show that there exists a smooth supersymmetric domain wall which interpolates between the new $\mathcal{N} = 1$ AdS₄ solution and the maximally supersymmetric AdS₄ vacuum. Using holography, this result indicates the existence of an $\mathcal{N} = 1$ RG flow from the ABJM SCFT to a new strongly interacting conformal fixed point in the IR.

KEYWORDS: AdS-CFT Correspondence, Supergravity Models

ARXIV EPRINT: [1909.10969](https://arxiv.org/abs/1909.10969)

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1 Introduction

The four-dimensional gauged supergravity of de Wit and Nicolai [2] has proven to be a remarkably rich theory with a plethora of applications. It has an $\text{SO}(8)$ gauge group and the maximal $\mathcal{N} = 8$ supersymmetry. Much of the interesting physics in this theory arises from a highly nontrivial potential for the 70 scalar fields. It is perhaps fair to say that unlocking that physics is tantamount to understanding the structure of the potential. In

particular, it has been a long standing problem to determine all of its critical points, which lead to AdS₄ vacuum solutions of the theory.

A systematic study of the nontrivial critical points, that is other than the maximally supersymmetric SO(8)-invariant one with vanishing scalar fields, was initiated in [3, 4], where several AdS₄ vacua were found by imposing certain symmetry constraints and thus effectively reducing the 70-dimensional scalar manifold to a smaller one, which could be fully analyzed.

As shown in [5–7], the four-dimensional SO(8) gauged supergravity¹ is a consistent truncation of the eleven-dimensional supergravity on S^7 . Therefore, classifying the critical points of the four-dimensional theory amounts to finding a large class of AdS₄ equilibria of eleven-dimensional supergravity with internal space that is topologically S^7 .

Those AdS₄ vacua of the SO(8) gauged supergravity and their uplifts to eleven dimensions are also interesting from the point of view of holography. Indeed, such backgrounds are dual to conformal field theories arising on the worldvolume of coincident M2-branes. The most studied and well-understood example is the ABJM theory [8], and its BLG version [9, 10], which has maximal supersymmetry. Another example is the so called mABJM SCFT which has $\mathcal{N} = 2$ supersymmetry and arises as a particular mass deformation of ABJM [11–13].

The vacuum structure of the SO(8) gauged supergravity suggests that there are also conformal phases of M2-branes with $\mathcal{N} = 1$ and $\mathcal{N} = 0$ supersymmetry. In particular, the G₂-invariant and the U(1) × U(1)-invariant critical points found in [4] and [14], respectively, preserve $\mathcal{N} = 1$ supersymmetry. The SO(3) × SO(3)-invariant critical point found in [3] has no supersymmetry but it is perturbatively stable [14]. There is a conjecture that all non-supersymmetric AdS₄ vacua are unstable [15]. However, it has not been explicitly shown how the non-perturbative instability of the SO(3) × SO(3)-invariant critical point might arise.

Due to the low amount of supersymmetry not much is known about these $\mathcal{N} = 0$ and $\mathcal{N} = 1$ strongly interacting CFTs. Nevertheless, it is interesting to understand whether there are any other supersymmetric or non-supersymmetric perturbatively stable critical points of the SO(8) gauged supergravity since this will amount to non-trivial predictions for the IR phases of the ABJM theory.

The consistent truncation of the maximal supergravity using a suitable symmetry as introduced by Warner [3, 4] has remained the cornerstone for the analytic studies of the potential since 1983. Between 2008 and 2010, there has been also a considerable progress in developing numerical techniques to search for the critical points in the full 70-parameter space. Those methods were used by one of us to explore the vacuum structure of maximal gauged supergravity theories in three dimensions [16, 17] and then ported to four dimensions in [18–20]. In particular, a new $\mathcal{N} = 1$ supersymmetric critical point S1200000² was discovered in [18] and, using the numerical data as a guide, subsequently confirmed analytically in [14].

¹Throughout this paper, the SO(8) gauged supergravity means the original de Wit-Nicolai theory [2].

²Following [20], we label the critical points by the first 7 digits of the critical value of the potential.

A new method for determining critical points in gauged supergravities based on the embedding tensor formalism (see, e.g., [21], and the references therein) was proposed in [22] and [23, 24] in 2011. While this method has not yet led to any new stable vacua of the potential of the de Wit-Nicolai theory,³ it has been used (see, e.g., [23, 25–27]) to obtain new analytic results for critical points in the deformed SO(8) gauged supergravities constructed in [28]. It also provided for a simple proof in [29] that these theories, including the original SO(8) gauged supergravity, have no supersymmetric vacua with $\mathcal{N} > 2$ except for the maximally supersymmetric one.

Recently, a new numerical approach based on Machine Learning (ML) software libraries, such as Google’s TensorFlow [30], was employed in [1] to simplify the analysis of the potential resulting in the total of 192 critical points together with a precise information about those points that includes the mass spectra of small fluctuations and unbroken (super)symmetries. It is expected that this list of critical points should be nearly complete.

Perhaps the most interesting result of the search in [1] is a discovery of yet another $\mathcal{N} = 1$ supersymmetric critical point, **S1384096**, which is invariant under a triality symmetric SO(3) subgroup of the SO(8) gauge group. Moreover, this point gives rise to the only new AdS₄ solution that is perturbatively stable. Therefore, it is most interesting to understand how to construct it using a more analytic approach. This is our goal in this paper.

The numerical data for the $\mathcal{N} = 1$ critical point, **S1384096**, in [1] point towards additional symmetry, which we identify as a discrete \mathbb{Z}_2 subgroup of the SO(8) gauge group. The resulting SO(3) \times \mathbb{Z}_2 -invariant truncation of the $\mathcal{N} = 8$ supergravity can be constructed analytically. Its bosonic sector consists of the metric and three complex scalar fields. Despite the small scalar sector, the potential in this truncation has 15 inequivalent critical points with **S1384096** amongst them. Surprisingly, two of those 15 critical points, **S2096313** and **S2443607**, were not found by the numerical search in [1] and thus they represent new AdS₄ equilibria. However, they are not supersymmetric and are perturbatively unstable.

We also study some of the properties of the supersymmetric point **S1384096** in more detail. In particular, we compute the mass spectrum of excitations for all bosonic and fermionic fields of the $\mathcal{N} = 8$ supergravity around this point. Using holography, we map it to the spectrum of operators in the dual $\mathcal{N} = 1$ three-dimensional SCFT, which are then organized into multiplets of $\mathcal{N} = 1$ superconformal symmetry. Our explicit analytic construction of the truncation with three complex scalar fields also allows us to initiate the study of the web of holographic RG flows connecting the four supersymmetric critical points in this model. We find explicit domain wall solutions which interpolate between **S1384096** and the maximally supersymmetric critical point of the $\mathcal{N} = 8$ supergravity.

In the next section we discuss two SO(3)- and SO(3) \times \mathbb{Z}_2 -invariant truncations of the maximal supergravity. In section 3 we show that the new $\mathcal{N} = 1$ AdS₄ solution found in [1] corresponds to a critical point in the SO(3) \times \mathbb{Z}_2 -invariant truncation. In section 4, we apply the same numerical technique as in [1] to the potential in the SO(3) \times \mathbb{Z}_2 -invariant truncation and find the total of 15 critical points that also include two non-supersymmetric and perturbatively unstable ones that were missed by previous searches. We show how other

³See, however, the construction of a new SO(4)-invariant point in [25].

well-known critical points arise in our truncation. In section 6 we perform a preliminary study of the holographic RG flows to the new $\mathcal{N} = 1$ point. In section 7 we present an $\mathcal{N} = 1$ supergravity truncation of the maximal supergravity with 7 complex scalar fields which contains all perturbatively stable critical points. We conclude with some comments in section 8. Some group theory details, the full spectrum of four-dimensional supergravity fields around the new $\mathcal{N} = 1$ AdS₄ vacuum, as well as more details on the two new non-supersymmetric critical points are given in the appendices.

2 A consistent truncation

The starting point of our analytic search for the new $\mathcal{N} = 1$ critical point, S1384096, is the precise information about its symmetry that is given, together with the numerical data for the position of the point and the spectrum of supergravity fluctuations, in [1]. Specifically, we know that S1384096 lies within an SO(3)-invariant sector of the 70-dimensional scalar manifold, $E_{7(7)}/(SU(8)/\mathbb{Z}_2)$, of the $\mathcal{N} = 8$ supergravity. The particular SO(3) symmetry is identified as the triality invariant subgroup of the SO(8) gauge group specified by the following branchings of the three fundamental representations:

$$\mathbf{8}_{v,s,c} \longrightarrow \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{1} \oplus \mathbf{1}. \quad (2.1)$$

Using the standard group theory summarized in appendix A, this completely determines the embedding of that SO(3) into both SO(8) and $E_{7(7)}$ at the level of Lie algebras:

$$\mathfrak{so}(8) \supset \mathfrak{so}(3) \times \mathfrak{u}(1) \times \mathfrak{u}(1) \quad \text{and} \quad \mathfrak{e}_{7(7)} \supset \mathfrak{so}(3) \times \mathfrak{g}_{2(2)} \times \mathfrak{su}(1,1). \quad (2.2)$$

Indeed, those embeddings are confirmed by the $U(1) \times U(1)$ unbroken gauge symmetry and the presence of $8 + 2 = 10$ scalar fluctuations⁴ that are SO(3) singlets in the $\mathcal{N} = 8$ supergravity spectrum around that point.

It follows from (2.2) that the scalar manifold spanned by the SO(3)-invariant scalars is the coset⁵

$$\frac{G_{2(2)}}{SO(4)} \times \frac{SU(1,1)}{U(1)}. \quad (2.3)$$

In fact, keeping track of all invariant fields in the $\mathcal{N} = 8$ supergravity, one finds that the (consistent) SO(3)-invariant truncation is to a four-dimensional $\mathcal{N} = 2$ gauged supergravity coupled to an Abelian vector multiplet and two hypermultiplets. The scalars in the hyper and the vector multiplets parametrize the first and second factor in (2.3), respectively. While considerably simpler than the full $\mathcal{N} = 8$ theory, this truncation is still too complicated to effectively work with.

The crucial hint that allows us to further simplify the analytic search for S1384096 comes from the numerical values of the scalar fields at that point. It has been observed in [1] that one can specify the position of this point in terms of 6 independent scalars when

⁴See, table 3 in appendix B.

⁵Incidentally, the first factor in the coset has appeared in the SU(3)-invariant truncation of the type IIB supergravity [31, 32] and in the SO(3)-invariant consistent truncation of the maximal five-dimensional SO(6) supergravity [33].

using a certain parametrization of the $E_{7(7)}/SU(8)$ coset. This suggests the presence of an additional discrete symmetry that might allow further truncation of the scalar manifold to a smaller subspace.

To identify that discrete symmetry we will use the standard parametrization of the scalar manifold of the $\mathcal{N} = 8$ theory in which the scalar 56-bein is given by [2, 34]

$$\mathcal{V} \equiv \begin{pmatrix} u_{ij}^{IJ} & v_{ijIJ} \\ v^{klIJ} & u^{kl}_{KL} \end{pmatrix} = \exp \begin{pmatrix} 0 & -\frac{1}{4}\sqrt{2}\phi_{ijkl} \\ -\frac{1}{4}\sqrt{2}\bar{\phi}^{ijkl} & 0 \end{pmatrix} \in E_{7(7)}, \quad (2.4)$$

where the scalar fields, ϕ_{ijkl} , are components of a complex selfdual 4-form in \mathbb{R}^8 ,

$$\Phi = \frac{1}{24}\sqrt{2}\phi_{ijkl} dx^i \wedge dx^j \wedge dx^k \wedge dx^l, \quad (2.5)$$

and the indices i, j, \dots transform in the $\mathbf{8}_v$ representation of $SO(8)$. To simplify the notation, the wedge product above will be denoted by dx^{ijkl} .

We take the $SO(3)$ symmetry of the truncation to act diagonally on the indices (123) and (456) with the singlets (7) and (8). The invariant forms under this action are spanned by:

$$\begin{aligned} \Phi^{(1)} &= \omega_1 (dx^{1267} - dx^{1357} + dx^{2347}) + \bar{\omega}_1 (dx^{1568} - dx^{2468} + dx^{3458}), \\ \Phi^{(2)} &= \omega_2 dx^{4567} + \bar{\omega}_2 dx^{1238}, \\ \Phi^{(3)} &= \omega_3 (dx^{1245} + dx^{1346} + dx^{2356}) + \bar{\omega}_3 (dx^{1478} + dx^{2578} + dx^{3678}), \end{aligned} \quad (2.6)$$

and

$$\begin{aligned} \Phi^{(4)} &= \vartheta_1 (dx^{1268} - dx^{1358} + dx^{2348}) - \bar{\vartheta}_1 (dx^{1567} - dx^{2467} + dx^{3457}), \\ \Phi^{(5)} &= \vartheta_2 dx^{4568} - \bar{\vartheta}_2 dx^{1237}, \end{aligned} \quad (2.7)$$

where

$$\omega_a = s_{2a-1} + i s_{2a}, \quad a = 1, 2, 3, \quad \vartheta_j = t_{2j-1} + i t_{2j}, \quad j = 1, 2. \quad (2.8)$$

Each $\Phi^{(\alpha)}$ contains a scalar and a pseudoscalar that parametrize an $SU(1,1)/U(1)$ coset. The $SU(1,1)$ subalgebras of $E_{7(7)}$ generated by $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{(3)}$ mutually commute as do the two subalgebras corresponding to $\Phi^{(4)}$ and $\Phi^{(5)}$. The $\Phi^{(1)}$, $\Phi^{(2)}$, $\Phi^{(4)}$ and $\Phi^{(5)}$ correspond to the noncompact generators of $G_{2(2)}$ and $\Phi^{(3)}$ to the $SU(1,1)$ that commutes with that $G_{2(2)}$.

This truncation contains the smaller $SU(3)$ -invariant truncation used by Warner [4] and more recently discussed in [35]. It is obtained by setting $\omega_1 = -\omega_2$ and $\vartheta_1 = -\vartheta_2$, which results in the noncompact group $SU(2,1) \times SU(1,1)$.

Using the numerical data for S1384096 in [1], we find that, modulo a suitable $SO(8)$ gauge rotation, the critical point of the potential lies within the 6-dimensional subspace spanned by $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{(3)}$. We also find that the fluctuation of the gravitino field, ψ_μ^8 , remains massless as required by the unbroken $\mathcal{N} = 1$ supersymmetry.

Consider the following discrete symmetry

$$g_S : (x^1, x^2, x^3, x^4, x^5, x^6, x^7, x^8) \longrightarrow (x^1, x^2, x^3, -x^4, -x^5, -x^6, -x^7, x^8). \quad (2.9)$$

Clearly, g_S is an $SO(8)$ rotation that does not belong to the $SO(3)$ symmetry subgroup. Under the action of g_S , the forms in (2.6) are even while the ones in (2.7) are odd, and the correct supersymmetry is preserved. This provides us with an additional discrete symmetry for the truncation to 6 scalar fields, where the truncation simply amounts to setting

$$t_1 = t_2 = t_3 = t_4 = 0. \tag{2.10}$$

In the next section we will confirm directly that the critical point **S1384096** indeed lies in the $SO(3) \times \mathbb{Z}_2$ invariant sector, where the \mathbb{Z}_2 is generated by the $SO(8)$ rotation g_S .

To summarize, we have been led by the group theory and numerical data to consider a $SO(3) \times \mathbb{Z}_2$ -invariant truncation of the $\mathcal{N} = 8$ $d = 4$ supergravity with the scalar coset

$$\frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)}. \tag{2.11}$$

The first two factors above are embedded in the first factor in (2.3) and the last factor above corresponds to the second factor in (2.3). The resulting theory is an $\mathcal{N} = 1$ $d = 4$ supergravity coupled to 3 scalar multiplets. The $\mathcal{N} = 8$ fields that remain in this truncation are indicated by the star in tables 3–6.

To recast the bosonic sector of this $\mathcal{N} = 1$ $d = 4$ supergravity in a canonical form [36], we use the usual complex coordinates, z_a , on each $SU(1,1)/U(1)$. First set

$$s_{2a-1} + i s_{2a} = \lambda_a e^{i\varphi_a}, \quad a = 1, 2, 3, \tag{2.12}$$

with the s_a given in (2.8), and then define

$$z_a = \tanh\left(\frac{1}{2}\lambda_a\right) e^{i\varphi_a}, \quad a = 1, 2, 3. \tag{2.13}$$

In this parametrization of the scalar fields, the bosonic Lagrangian of the truncated four-dimensional supergravity is given by

$$\mathcal{L} = \frac{1}{2}R - \mathcal{K}^{a\bar{b}} \partial_\mu z_a \partial^\mu \bar{z}_{\bar{b}} - g^2 \mathcal{P}. \tag{2.14}$$

The scalar kinetic term is determined by the Kähler metric of the coset (2.11) with Kähler potential

$$\mathcal{K} = -3 \log(1 - z_1 \bar{z}_1) - \log(1 - z_2 \bar{z}_2) - 3 \log(1 - z_3 \bar{z}_3), \tag{2.15}$$

where the integer coefficients of the logarithms are the embedding indices of the numerator $SU(1,1)$'s in (2.11) in $E_{7(7)}$. The Kähler metric and its inverse are defined by

$$\mathcal{K}_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K}, \quad \mathcal{K}^{a\bar{b}} = (\mathcal{K}_{a\bar{b}})^{-1}. \tag{2.16}$$

The potential can be succinctly written as

$$\mathcal{P} = 2e^{\mathcal{K}} (\mathcal{K}^{a\bar{b}} \nabla_a \mathcal{W} \nabla_{\bar{b}} \bar{\mathcal{W}} - 3\mathcal{W} \bar{\mathcal{W}}), \tag{2.17}$$

where the holomorphic superpotential is⁶

$$\mathcal{W} = (z_3 - 1)(z_1^3 z_2 z_3^2 + z_1^3 z_2 z_3 + z_1^3 z_2 + 3z_3 z_1^2 - 3z_1 z_2 z_3 - z_3^2 - z_3 - 1), \quad (2.18)$$

and the Kähler covariant derivative is given by

$$\nabla_a(\cdot) = \partial_a(\cdot) + (\cdot)\partial_a \mathcal{K}. \quad (2.19)$$

3 The new $\mathcal{N} = 1$ critical point

In this section we will look first for those critical points of the potential (2.17) that preserve the $\mathcal{N} = 1$ (or more) supersymmetry of our truncation. The expectation is that those points should include the new supersymmetric point, S1384096.

Supersymmetric critical points correspond to the ‘‘covariant extrema’’ of the holographic superpotential (2.18) satisfying

$$\nabla_a \mathcal{W} = 0, \quad \nabla_{\bar{b}} \bar{\mathcal{W}} = 0, \quad a, \bar{b} = 1, 2, 3. \quad (3.1)$$

It is easy to check that any solution to (3.1) is also a critical point of the potential (2.17).

In the domain $|z_a| < 1$, (3.1) unpack to the following system of septic polynomial equations

$$\begin{aligned} z_1^2 z_2 + 2z_1 z_3 - z_2 z_3 + z_1^2 z_2 z_3 + z_1^2 z_2 z_3^2 - \bar{z}_1(1 + z_3 - z_1^2 z_3 + 2z_1 z_2 z_3 + z_3^2) &= 0, \\ z_1^3 - 3z_1 z_3 + 3z_1 z_3^2 - z_1^3 z_3^3 - \bar{z}_2(1 - 3z_1^2 z_3 + 3z_1^2 z_3^2 - z_3^3) &= 0, \\ z_1^2 - z_1 z_2 - 2z_1^2 z_3 + 2z_1 z_2 z_3 + z_3^2 - z_1^3 z_2 z_3^2 & \\ - \bar{z}_3(1 - z_1^3 z_2 - 2z_1^2 z_3 + 2z_1 z_2 z_3 + z_1^2 z_3^2 - z_1 z_2 z_3^2) &= 0, \end{aligned} \quad (3.2)$$

plus the three complex conjugate equations.

Finding all solutions to the system (3.2) analytically, if feasible at all, will require techniques that go beyond what is employed in the present article. However, one can fully analyze (3.2) using standard numerical routines such as `NSolve[]` in Mathematica [37]. In this way we recover three known supersymmetric critical points with the $\text{SO}(8)$, G_2 and $\text{SU}(3) \times \text{U}(1)$ invariance, respectively. As we discuss in section 4.2, each can be found analytically by performing a further truncation that reduces (3.2) to a simpler system.

We also find four numerical solutions at approximately

$$\begin{aligned} z_1 &= 0.1696360 \pm 0.1415740 i, & z_1 &= -0.1696360 \mp 0.1415740 i, \\ z_2 &= 0.4833214 \pm 0.3864058 i, & z_2 &= -0.4833214 \mp 0.3864058 i, \\ z_3 &= -0.3162021 \pm 0.5162839 i, & z_3 &= -0.3162021 \pm 0.5162839 i. \end{aligned} \quad (3.3)$$

The value of the potential at these points is

$$\mathcal{P} \approx -13.840964, \quad (3.4)$$

which is the same as at S1384096 in [1].

⁶The holomorphic superpotential can be read-off from the component of the A_{ij}^1 -tensor of the $\mathcal{N} = 8$ supergravity along the unbroken supersymmetry.

As we discuss in more detail in appendix C, there is a residual action of the SO(8) gauge symmetry on the coset (2.11). Indeed, solutions in the two columns in (3.3) are related by the rotation, g_H , defined in (C.3). In turn, solutions within each column are related by complex conjugation. This degeneracy is expected given the corresponding invariance of the system of equations in (3.1) or, equivalently, (3.2). However, those complex conjugate solutions are *not* related by an SO(8) rotation and thus represent two distinct critical points of the potential in the $\mathcal{N} = 8$ supergravity.⁷ To complete the identification of the solutions (3.3) with the new supersymmetric point, S1384096, and its complex conjugate, $\bar{S}1384096$, in [1], one can also perform an explicit change of variables (D.1) accompanied by an SO(8) rotation.

The critical values (3.3) of the coordinates, z_a , $a = 1, 2, 3$, can be efficiently determined to an arbitrary precision from the roots of the following system of integer coefficient polynomials:⁸

$$\begin{aligned}
 P_{\text{Re } z_1}(x) &= 256x^{24} + 5632x^{22} + 78592x^{20} - 2135808x^{18} - 543360x^{16} - 4684032x^{14} \\
 &\quad - 12045600x^{12} - 15419808x^{10} - 6033744x^8 - 1553904x^6 - 222264x^4 \\
 &\quad - 17496x^2 + 729, \\
 P_{\text{Im } z_1}(y) &= 16y^{12} - 96y^{11} + 144y^{10} + 848y^9 - 944y^8 + 2000y^7 - 3504y^6 + 3456y^5 \\
 &\quad - 2028y^4 + 740y^3 - 164y^2 + 20y - 1.
 \end{aligned} \tag{3.5}$$

$$\begin{aligned}
 P_{\text{Re } z_2}(x) &= 60715264x^{24} - 256862720x^{22} + 937708288x^{20} - 2138845440x^{18} \\
 &\quad + 3067400064x^{16} - 2992061952x^{14} + 1409137632x^{12} - 388837152x^{10} \\
 &\quad + 229269744x^8 - 95418000x^6 + 4147848x^4 + 1627128x^2 + 59049, \\
 P_{\text{Im } z_2}(y) &= 7792y^{12} + 4320y^{11} + 26256y^{10} + 16832y^9 + 62032y^8 + 107504y^7 + 70872y^6 \\
 &\quad + 37872y^5 + 14172y^4 + 19880y^3 + 4900y^2 - 1372y - 2401,
 \end{aligned} \tag{3.6}$$

$$\begin{aligned}
 P_{\text{Re } z_3}(x) &= 16x^{17} + 96x^{16} + 496x^{15} + 672x^{14} + 456x^{13} - 1584x^{12} - 1384x^{11} - 816x^{10} \\
 &\quad + 1388x^9 - 1512x^8 + 462x^7 + 2028x^6 + 537x^5 + 810x^4 - 819x^3 - 639x^2 \\
 &\quad - 180x - 27, \\
 P_{\text{Im } z_3}(y) &= 768y^{24} - 48128y^{22} + 1018112y^{20} - 2517248y^{18} + 4496192y^{16} - 8476736y^{14} \\
 &\quad + 7496864y^{12} - 8223008y^{10} + 4957568y^8 - 1487120y^6 + 233460y^4 \\
 &\quad - 18900y^2 + 675.
 \end{aligned} \tag{3.7}$$

Polynomials $P_{\text{Re } z_a}(x)$ and $P_{\text{Im } z_a}(y)$, $a = 1, 2$, have precisely two real roots, $\pm x_a^*$, and one real root, y_a^* , respectively, such that $z_a = x_a^* + i y_a^*$, $x_a^*, y_a^* > 0$, lie in the unit disk. Similarly, $P_{\text{Re } z_3}(x)$ and $P_{\text{Im } z_3}(y)$ have one real root, x_3^* , and two real roots, $\pm y_3^*$, $y_3^* > 0$, with $|z_3^*| < 1$. Then

$$(z_1^*, z_2^*, z_3^*), \quad (-z_1^*, -z_2^*, z_3^*), \quad (\bar{z}_1^*, \bar{z}_2^*, \bar{z}_3^*), \quad (-\bar{z}_1^*, -\bar{z}_2^*, \bar{z}_3^*), \tag{3.8}$$

⁷Since such ‘‘conjugate’’ critical points have the same values of the potential and the same mass spectra of fluctuations around them, they were identified as a single point in the numerical searches [1, 20].

⁸See, section 5. In appendix E.1, we also give the minimal polynomials for the complex coordinates (3.5)–(3.7).

are the four solutions to (3.1) that we found above in (3.3). By acting on these solutions with the $SO(8)$ rotation g_C defined in (C.3), we obtain 4 additional solutions that exhaust the critical points of the potential (2.17) with this critical value. Those 8 critical points of (2.17) represent two different critical points of the $\mathcal{N} = 8$ supergravity.

The exact value of the potential at these critical points is given by the negative real root of the following polynomial [1],

$$5^{15} v^{12} - (2^8 \cdot 3^4 \cdot 7 \cdot 53 \cdot 107 \cdot 887 \cdot 1567) v^8 + (2^{15} \cdot 3^{17} \cdot 210719) v^4 - 2^{20} \cdot 3^{30} = 0. \quad (3.9)$$

The mass spectra of the fermion and scalar fluctuations of $\mathcal{N} = 8$ supergravity are the same around each of the points and have been obtained as part of the numerical search in [1]. Those results are summarized in tables 4, 6 and 3 in appendix B. In particular, the presence of a single unit mass gravitino mode in the spectrum shows that there is $\mathcal{N} = 1$ unbroken supersymmetry. In appendix B we also find the masses of the fluctuations of the vector fields, which allows us to verify explicitly that the entire spectrum of operators in the dual three-dimensional superconformal field theory can be arranged into multiplets of the $\mathcal{N} = 1$ superconformal algebra, $\mathfrak{osp}(1|4)$. This provides a nontrivial consistency test for the calculation of the spectra and of the unbroken supersymmetry.

4 All critical points of the truncated potential

Although the truncated potential (2.17) can be written in a closed analytic form, to determine all of its critical points we have to resort to numerical methods outlined in section 5. Here, we summarize the results of that search, which yielded critical points with 15 different values of the cosmological constant, including the two new ones that were not captured by the search in [1].

Given a critical point at (z_1, z_2, z_3) , the reality of the potential implies that there is another (conjugate) critical point at $(\bar{z}_1, \bar{z}_2, \bar{z}_3)$. In appendix C, we argue that unless z_3 is real, the two points are not related by an $SO(8)$ rotation and thus represent two distinct critical points of the potential in the $\mathcal{N} = 8$ supergravity. In addition, we show that each point in the coset (2.11) lies on an orbit of the discrete subgroup of $SO(8)$ that preserves the coset. This discrete subgroup is generated by the rotations g_H and g_C defined in (C.3). Generically, the corresponding orbit through a point (z_1, z_2, z_3) consists of 4 points:

$$(z_1, z_2, z_3), \quad (-z_1, -z_2, z_3), \quad (\bar{z}_1, \bar{z}_2, z_3), \quad (-\bar{z}_1, -\bar{z}_2, z_3), \quad (4.1)$$

obtained by applying the rotations 1, g_H , g_C and $g_H g_C$, respectively. Clearly, when z_1 and z_2 ($z_1 z_2 \neq 0$) are both either real or imaginary, that orbit degenerates to just two points. For some points we also find additional discrete $SO(8)$ rotations (C.1) that preserve the coset (2.11) at that particular point giving rise to additional critical points of the potential (2.17).

The end result is that for each critical value of the potential (2.17) at (z_1, z_2, z_3) , there are either two orbits or a single orbit of critical points, namely (4.1) and its complex

conjugate, or just (4.1) when z_3 is real. In $\mathcal{N} = 8$ supergravity those $\text{SO}(8)$ orbits correspond two “conjugate” critical points, $\mathbf{S}n_1 \dots n_7$ and $\bar{\mathbf{S}}n_1 \dots n_7$, or a single point, $\mathbf{S}n_1 \dots n_7$, respectively.

All points in this section have at least an $\text{SO}(3)$ symmetry, which is the continuous symmetry of the truncation. We should note that there are other critical points in [1] that are $\text{SO}(3)$ -invariant: **S0847213**, **S1075828**, **S1195898**, **S1271622**, **S2503105**. However, their symmetry is incompatible with the symmetry of our truncation since it corresponds to a different embedding of $\text{SO}(3)$ in $\text{SO}(8)$.

In the following we list all the critical points of the potential (2.17). For each point we give its location, the value of the potential, the continuous symmetry, and the $\text{SO}(8)$ rotations for the orbit(s). All but two of those points were discovered in previous searches as indicated by the references where they first appeared. For all but two points the location and the critical value of the potential are known in either a closed analytic form or via a minimal polynomial. In some cases where the explicit analytic form is too involved, we do not list it in the text.

The mass spectra of scalar fluctuations around the known points can be found in [1].⁹ For the two new points, the mass spectra are given in appendix D. Perhaps unsurprisingly, the only perturbatively stable points, that is with the scalar masses satisfying the Breitenlohner-Freedman bound [38], are the supersymmetric ones and the non-supersymmetric $\text{SO}(3) \times \text{SO}(3)$ -invariant point, **S1400000**. In particular, both new points are perturbatively unstable. Overall, for the solutions **S0668740**, **S0698771**, **S0800000**, and **S1424025**, all instabilities in this truncation are due to modes that are not $\text{SO}(3)$ singlets and thus can be seen only within the full $\mathcal{N} = 8$ supergravity. For the $\text{SU}(4)$ -invariant solution **S0800000** this observed previously in [35].

4.1 The critical points

S0600000 [2]

$$z_1 = z_2 = z_3 = 0, \tag{4.2}$$

$$\mathcal{P} = -6. \tag{4.3}$$

Symmetry: $\text{SO}(8)$, $\mathcal{N} = 8$. Orbit: $\langle 1 \rangle$.

S0668740 [39]

$$z_1 = -z_2 = -z_3 = \frac{1}{2} \left(3 + \sqrt{5} - \sqrt{10 + 6\sqrt{5}} \right) \approx 0.1985088, \tag{4.4}$$

$$\mathcal{P} = -2 \cdot 5^{3/4} \approx -6.687403. \tag{4.5}$$

Symmetry: $\text{SO}(7)^+$, $\mathcal{N} = 0$. Orbit: $\langle 1, g_H \rangle$.

$$\mathcal{P}_{\text{Re } z_{1,2,3}}(x) = x^4 + 6x^3 + 6x^2 + 6x + 1. \tag{4.6}$$

⁹Also, see earlier work referred to in [1].

S0698771 & $\bar{\text{S0698771}}$ [40, 41]

$$z_1 = -z_2 = -z_3 = -i(2 - \sqrt{5}) \approx 0.2360680i, \quad (4.7)$$

$$\mathcal{P} = -\frac{5^{5/2}}{8} \approx -6.987712. \quad (4.8)$$

Symmetry: $\text{SO}(7)^-$, $\mathcal{N} = 0$. Orbits: $\langle 1, g_H \rangle$.

$$P_{\text{Im } z_{1,2,3}}(x) = x^2 + 4x - 1. \quad (4.9)$$

S0719157 & $\bar{\text{S0719157}}$ [5]

$$z_1 = -z_2 = -z_3 = \frac{1}{4} \left(3 + \sqrt{3} - 3^{1/4} \sqrt{10} \right) \left(1 - i 3^{-1/4} \sqrt{2 + \sqrt{3}} \right) \quad (4.10)$$

$$\approx 0.1425648 + 0.2092695i,$$

$$\mathcal{P} = -\frac{2^{7/2} \cdot 3^{13/4}}{5^{5/2}} \approx -7.191576. \quad (4.11)$$

Symmetry: G_2 , $\mathcal{N} = 1$. Orbits: $\langle 1, g_H, g_C, g_H g_C \rangle$.

$$P_{\text{Re } z_{1,2,3}}(x) = 8x^4 + 24x^3 + 24x^2 + 24x + 3, \quad (4.12)$$

$$P_{\text{Im } z_{1,2,3}}(x) = 64x^8 - 704x^6 + 240x^4 - 32x^2 + 1.$$

S0779422 [4]

$$z_1 = -z_2 = i \sqrt{5 - 2\sqrt{6}} \approx 0.3178372i, \quad z_3 = \sqrt{3} - 2 \approx -0.2679492, \quad (4.13)$$

$$\mathcal{P} = -\frac{9\sqrt{3}}{2} \approx -7.794229. \quad (4.14)$$

Symmetry: $\text{SU}(3) \times \text{U}(1)$, $\mathcal{N} = 2$. Orbit: $\langle 1, g_H \rangle$.

$$P_{\text{Im } z_{1,2}}(x) = x^4 - 10x^2 + 1, \quad P_{\text{Re } z_3}(x) = x^2 + 4x + 1. \quad (4.15)$$

S0800000 [4]

$$z_1 = -z_2 = i(\sqrt{2} - 1) \approx 0.4142136i, \quad z_3 = \bar{z}_3 = 0, \quad (4.16)$$

$$\mathcal{P} = -8. \quad (4.17)$$

Symmetry: $\text{SU}(4)$, $\mathcal{N} = 0$. Orbit: $\langle 1, g_H \rangle$.

$$P_{\text{Im } z_{1,2}}(x) = x^2 + 2x - 1. \quad (4.18)$$

S0869597 [20]

$$z_1 = i \sqrt{9 + 2\sqrt{21} - 2\sqrt{41 + 9\sqrt{21}}} \approx 0.1659702i,$$

$$z_2 = \frac{i}{67} \sqrt{7521 + 738\sqrt{21} - 2\sqrt{11962961 + 2775249\sqrt{21}}} \approx 0.4641278i, \quad (4.19)$$

$$z_3 = \frac{1}{4} \left(-1 - \sqrt{21} + \sqrt{2(3 + \sqrt{21})} \right) \approx -0.4220824,$$

$$\mathcal{P} = -\frac{4}{5} \sqrt{54 + 14\sqrt{21}} \approx -8.695969. \quad (4.20)$$

Symmetry: $SO(3) \times U(1)$, $\mathcal{N} = 0$. Orbit $\langle 1, g_H \rangle$.

$$\begin{aligned} P_{\text{Im } z_1}(x) &= x^8 - 36x^6 - 10x^4 - 36x^2 + 1, \\ P_{\text{Im } z_2}(x) &= 4489x^8 - 30084x^6 + 49190x^4 - 30084x^2 + 4489, \\ P_{\text{Re } z_3}(x) &= x^4 + x^3 - 3x^2 + x + 1. \end{aligned} \tag{4.21}$$

S0880733 [14]

$$\begin{aligned} z_1 = z_2 &= i \sqrt{3 + 2\sqrt{3} - 2\sqrt{5 + 3\sqrt{3}}} \approx 0.2789600i, \\ z_3 &= -\frac{1}{2} + \frac{3^{1/4}}{\sqrt{2}} - \frac{\sqrt{3}}{2} \approx -0.435421, \end{aligned} \tag{4.22}$$

$$\mathcal{P} = -2\sqrt{9 + 6\sqrt{3}} \approx -8.807339. \tag{4.23}$$

Symmetry: $SO(4)$, $\mathcal{N} = 0$. Orbit: $\langle 1, g_H, g_R, g_H g_R \rangle$.

$$\begin{aligned} P_{\text{Im } z_{1,2}}(x) &= x^8 - 12x^6 - 10x^4 - 12x^2 + 1, \\ P_{\text{Re } z_3}(x) &= x^4 + 2x^3 + 2x + 1. \end{aligned} \tag{4.24}$$

S0983994 & $\bar{\text{S0983994}}$ [20]

$$z_1 \approx 0.184246, \quad z_2 \approx 0.5073269, \quad z_3 \approx 0.1331835 + 0.4676097i, \tag{4.25}$$

$$\mathcal{P} = -5 \cdot 15^{1/4} \approx -9.839948. \tag{4.26}$$

Symmetry: $SO(3) \times U(1)$, $\mathcal{N} = 0$. Orbits: $\langle 1, g_H \rangle$.

$$\begin{aligned} P_{\text{Re } z_1}(x) &= x^8 - 28x^6 - 42x^4 - 28x^2 + 1, \\ P_{\text{Re } z_2}(x) &= 289x^8 - 1372x^6 + 1302x^4 - 1372x^2 + 289, \\ P_{\text{Re } z_3}(x) &= 1397x^4 - 2380x^3 + 4350x^2 - 2380x + 245, \\ P_{\text{Im } z_3}(x) &= 1951609x^8 - 12150144x^6 + 22045824x^4 - 17915904x^2 + 2985984. \end{aligned} \tag{4.27}$$

S1039230 & $\bar{\text{S1039230}}$ [25]

$$z_1 = z_2 = \sqrt{5 - 2\sqrt{6}} \approx 0.3178372, \quad z_3 = -i \sqrt{2 - \sqrt{3}} \approx -0.517638i, \tag{4.28}$$

$$\mathcal{P} = -6\sqrt{3} \approx -10.39230. \tag{4.29}$$

Symmetry: $SO(4)$, $\mathcal{N} = 0$. Orbits: $\langle 1, g_H, g'_R, g_H g'_R \rangle$.

$$P_{\text{Re } z_{1,2}}(x) = x^4 - 10x^2 + 1, \quad P_{\text{Im } z_3}(x) = x^4 - 4x^2 + 1. \tag{4.30}$$

S1384096 & $\bar{\text{S1384096}}$ [1]

$$\begin{aligned} z_1 &\approx 0.1696360 + 0.1415740i, \\ z_2 &\approx 0.4833214 + 0.3864058i, \end{aligned} \tag{4.31}$$

$$\begin{aligned} z_3 &\approx -0.3162021 - 0.5162839i, \\ \mathcal{P} &\approx -13.840964. \end{aligned} \tag{4.32}$$

Symmetry: $\text{SO}(3)$, $\mathcal{N} = 1$. Orbit: $\langle 1, g_H, g_C, g_H g_C \rangle$.

Comment: see section 3.

S1400000 [3]

$$z_1 = z_2 = \frac{1}{2}(1+i)\sqrt{3-\sqrt{5}} \approx 0.4370160(1+i), \quad z_3 = \bar{z}_3 = 0, \quad (4.33)$$

$$\mathcal{P} = -14. \quad (4.34)$$

Symmetry: $\text{SO}(3) \times \text{SO}(3)$, $\mathcal{N} = 0$. Orbit: $\langle 1, g_H, g_C, g_H g_C \rangle$.

$$P_{\text{Re } z_{1,2}}(x) = P_{\text{Im } z_{1,2}}(x) = 4x^4 - 6x^2 + 1. \quad (4.35)$$

Comment: this point is non-supersymmetric, but perturbatively stable [14].

S1424025 [20]

$$z_1 \approx 0.4490422 + 0.4843455 i, \quad (4.36)$$

$$z_2 \approx 0.3750597 + 0.2850151 i, \quad (4.36)$$

$$z_3 \approx -0.04539020,$$

$$\mathcal{P} \approx -14.24026. \quad (4.37)$$

Symmetry: $\text{SO}(3)$, $\mathcal{N} = 0$. Orbit: $\langle 1, g_H, g_C, g_H g_C \rangle$.

Comment: coordinates are known algebraically, cf. appendix E.2.

S2096313 & $\bar{\text{S}}2096313$

$$z_1 = -i(2 - \sqrt{5}) \approx 0.2360680 i, \quad (4.38)$$

$$z_2 = -\frac{i}{2}(1 - \sqrt{5}) \approx 0.6180340 i, \quad (4.38)$$

$$z_3 = \frac{2}{41}(15 - 2\sqrt{5}) - \frac{i}{41}\sqrt{21(49 - 12\sqrt{5})} \approx 0.5135543 - 0.5262366 i,$$

$$\mathcal{P} = -\frac{75}{8}\sqrt{5} \approx -20.96314. \quad (4.39)$$

Symmetry: $\text{SO}(3) \times \text{U}(1)$, $\mathcal{N} = 0$. Orbits: $\langle 1, g_H \rangle$.

$$P_{\text{Im } z_1}(x) = x^2 - 4x - 1, \quad P_{\text{Im } z_2}(x) = x^2 - x - 1, \quad (4.40)$$

$$P_{\text{Re } z_3}(x) = 41x^2 - 60x + 20, \quad P_{\text{Im } z_3}(x) = 1681x^4 - 2058x^2 + 441.$$

S2443607 & $\bar{\text{S}}2443607$

$$z_1 \approx 0.2187103 + 0.1800635 i, \quad (4.41)$$

$$z_2 \approx -0.2046730 + 0.4973759 i, \quad (4.41)$$

$$z_3 = 0.4188443 - 0.6668735 i,$$

$$\mathcal{P} \approx -24.43607. \quad (4.42)$$

Symmetry: $\text{SO}(3)$, $\mathcal{N} = 0$. Orbits: $\langle 1, g_H, g_C, g_H g_C \rangle$.

Comment: coordinates are known algebraically, cf. appendix E.2.

4.2 Subtruncations

The locations of the critical points above suggest a number of further truncations to simpler subsectors. In particular we have the G_2 -invariant truncation, which in the parametrization used in [42] is obtained by setting

$$z_1 = -z_2 = -z_3 = z. \quad (4.43)$$

The superpotential and the Kähler potential reduce then to

$$\mathcal{W}_{G_2} = z^7 + 7z^4 + 7z^3 + 1, \quad \mathcal{K}_{G_2} = -7 \log(1 - z\bar{z}). \quad (4.44)$$

Within this truncation one finds 6 critical points: the $\text{SO}(8)$ point, S0600000, the $\text{SO}(7)^+$ point, S0668740, the $\text{SO}(7)^-$ points, S0698771 and $\bar{\text{S}}0698771$, and the G_2 points, S0719157 and $\bar{\text{S}}0719157$.

Other simple truncations to one complex scalar field are the $\text{SO}(4) \times \text{SO}(4)$ -invariant truncation obtained by setting

$$z_1 = z_3 = 0, \quad z_2 = z, \quad (4.45)$$

with

$$\mathcal{W}_{\text{SO}(4) \times \text{SO}(4)} = 1, \quad \mathcal{K}_{\text{SO}(4) \times \text{SO}(4)} = -\log(1 - z\bar{z}), \quad (4.46)$$

and the $\text{SU}(3) \times \text{U}(1)^2$ -invariant truncation

$$z_1 = z_2 = 0, \quad z_3 = -z, \quad (4.47)$$

with

$$\mathcal{W}_{\text{SU}(3) \times \text{U}(1)^2} = z^3 + 1, \quad \mathcal{K}_{\text{SU}(3) \times \text{U}(1)^2} = -3 \log(1 - z\bar{z}). \quad (4.48)$$

Although there are no critical points other than the maximally supersymmetric S0600000 one, those truncations admit nontrivial generalizations of the RG flows dual to three dimensional field theories with interfaces [42].

5 Numerical searches — an outline of the method

The TensorFlow code that was published alongside [1] is readily adapted to search for critical points not on the full 70-dimensional scalar manifold but on submanifolds that are invariant under some residual symmetry, such as the six-dimensional space studied here. As for the unconstrained problem, one starts from some random linear combination of the six $E_{7(7)}$ generators that is sufficiently close to the origin for the numerical value of the potential to still be reliable, and then numerically minimizes the violation of the (un-truncated) stationarity-condition.

This way, one manages to discover all the critical points on the scalar manifold listed in section 4.1 after about 10 000 such iterations. We observe that some tweaks to the code as published can improve search efficiency further. In particular, it turns out to be beneficial to not use a second order numerical optimization method (such as BFGS) directly, but to first perform a few hundred gradient descent steps per iteration before switching to such a more advanced method. Intuitively, if a second order optimization method gets to see from the start a sum of stationarity-violation contributions having very different scale, it will tend to be mostly sensitive to the most important contribution's second order approximation and hence in its first few steps move to very similar positions on the manifold, counteracting the need for good exploration.

Even with such tricks, the TensorFlow based search is fundamentally only a probabilistic method that converges to the various critical points with very uneven likelihood. So, it might be conceivable that, even with much computational effort, some critical points remain undiscovered. It hence makes sense to look for alternative approaches to the problem of finding critical points of algebraic functions (or, equivalently, intersections of algebraic varieties) on spaces of moderate dimension.

While simple techniques based on interval arithmetic or affine arithmetic appear too limited to conveniently study the restricted six-dimensional potential at hand, this problem is still well within reach of modern computational algebraic geometry.

For a task like this, one will typically want to first employ a modern computational algebraic geometry package such as [43] to reduce/factorize the problem, and then use an adaptive-precision homotopy continuation solver such as Bertini2 [44] that uses the algorithm described in [45] to systematically find solutions of the generalized problem with complex coordinates. These solutions then have to be filtered, discarding all those with non-real coordinates. Depending on the difficulty of the task, computations may take hours to days with Bertini2, and while this approach might hypothetically still miss some solutions, this is not observed to happen in practice.

These numerical algebraic geometry methods found the same list of critical points for our six-scalar model as the TensorFlow based search. We also note that for the eight-dimensional scalar manifold studied in [46], the same numerical methods manage to reproduce the list of critical points presented in that publication without uncovering additional ones.¹⁰

For all critical points listed in section 4, one can obtain algebraic expressions for their location, the potential, and other physical properties via inverse symbolic computation (i.e. employing the PSLQ algorithm). However, for S1424025 and S2443607, these expressions become rather lengthy.

6 Holographic RG flows

The truncation derived in section 2 makes it feasible to study explicitly supersymmetric holographic RG flows to the new $\mathcal{N} = 1$ critical point. For other supersymmetric critical points that lie within our truncation such flows have been constructed previously in [47–51].

¹⁰We thank Jonathan Hauenstein for performing these calculations.

Point	δ_α					
S0600000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
S0719157	3.44949	1.40825	1.40825	0.591752	0.591752	-1.44949
S0779422	3.56155	2.56155	1.33333	0.666667	-0.561553	-1.56155
S1384096	4.80254	3.71632	1.25126	0.748735	-1.71632	-2.80254

Table 1. Asymptotic exponents, δ_α , at the four supersymmetric critical points.

The RG flows we are interested in are given by domain wall solutions of the BPS equations in $\mathcal{N} = 8$ $d = 4$ supergravity with the metric of the form

$$ds^2 = e^{2A(r)} ds_{1,2}^2 + dr^2, \tag{6.1}$$

where $ds_{1,2}^2$ is the metric on the three-dimensional Minkowski space. Setting the supersymmetry variations of the supergravity fermion fields to zero, the standard analysis, see e.g. [51], yields the following system of BPS equations:

$$z'_a(r) = \mp \sqrt{2} g e^{\mathcal{K}/2} \mathcal{K}^{a\bar{b}} \frac{\mathcal{W}}{|\mathcal{W}|} \nabla_{\bar{b}} \overline{\mathcal{W}}, \quad \bar{z}'_{\bar{b}}(r) = \mp \sqrt{2} g e^{\mathcal{K}/2} \mathcal{K}^{a\bar{b}} \frac{\overline{\mathcal{W}}}{|\mathcal{W}|} \nabla_a \mathcal{W}, \tag{6.2}$$

$$A'(r) = \pm \sqrt{2} g |\mathcal{W}|, \tag{6.3}$$

for the dependence of the scalar fields, z_a and \bar{z}_a , and the metric function, A , on the radial coordinate.¹¹ In an $\mathcal{N} = 1$ theory these equations are completely determined by the Kähler potential and the superpotential of the truncated model as indeed they are in (6.2) and (6.3). The choice of sign in (6.2) reflects the freedom to choose the sign of the radial coordinate r in (6.1). In the calculations below we choose the upper sign in (6.2).

It follows from the discussion in sections 3 and 4 that the system of ODEs (6.2) has critical points at the SO(8) vacuum S0600000, the G_2 vacua S0719157, the $SU(3) \times U(1)$ vacuum S0779422, as well as the new $\mathcal{N} = 1$ SO(3) vacuum S1384096. The expectation is that there should be a web of domain wall solutions corresponding to RG flows between the superconformal fixed points of the dual ABJM theory. Indeed, in a simpler setting that included the first three points only, families of such flows were constructed explicitly in [51].

To study these domain wall solutions it proves convenient to split the complex scalar fields, z_a , into their real and imaginary parts, $z_a = x_a + i y_a$, $a = 1, 2, 3$. At each critical point, the real fields, (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , collectively denoted by ϕ_α , $\alpha = 1, \dots, 6$, have the asymptotic expansions

$$\phi_\alpha(r) = \sum_{\beta=1}^6 A_{\alpha\beta} e^{-\delta_\beta r/L} + \dots, \quad L^2 = -\frac{3}{\mathcal{P}}, \tag{6.4}$$

where L is the radius of the corresponding AdS_4 solution determined by the value of the potential \mathcal{P} at a given critical point. The exponents δ_α are related to the scaling dimensions,

¹¹As usual, a prime denotes a derivative with respect to r .

Δ_α , of the dual operators by

$$\delta_\alpha = \Delta_\alpha \quad \text{or} \quad \delta_\alpha = 3 - \Delta_\alpha . \quad (6.5)$$

The BPS equations (6.2) can be integrated numerically and, as expected, we find families of domain wall solutions interpolating between the SO(8) vacuum and the new SO(3) vacuum. Examples of such solutions are shown in the middle column in figure 1. There are also finely tuned solutions that realize holographically a “triangular RG flow” starting from the SO(8) vacuum in the UV, approaching one of the two G_2 vacua and then ultimately ending in the SO(3) vacuum in the deep IR. Plots of those flows are shown in the left and right columns in figure 1. Similarly, there are supersymmetric triangular RG flows, see figure 2, interpolating between the SO(8), the $SU(3) \times U(1)$, and the G_2 critical points. Those solutions were first studied in [51] and are also present within the consistent truncation here.

However, using a simple shooting method we were not able to find similar triangular RG flows involving the SO(8), the $SU(3) \times U(1)$, and the SO(3) points or for that matter all four supersymmetric points. Indeed, a more exhaustive numerical search using Machine Learning, to be discussed elsewhere, strongly suggests that such RG flows do not exist within our 6-scalar $SO(3) \times \mathbb{Z}_2$ -invariant truncation. Still, we suspect that those flows might exist within a larger truncation, perhaps the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ -invariant truncation discussed in section 7, whose remarkable properties make it a compelling candidate to look at.

To interpret this web of RG flows in the dual ABJM SCFT, it is convenient to employ the $\mathcal{N} = 1$ superspace language and, using the same notation as in [51], consider the following combinations of the eight chiral superfields,

$$\tilde{Z}_a = \Phi_{2a-1} + i\Phi_{2a}, \quad a = 1, 2, 3, 4. \quad (6.6)$$

Then the deformation of the ABJM superpotential,

$$\Delta W = \frac{1}{2}m_3(\tilde{Z}_1^2 + \tilde{Z}_2^2 + \tilde{Z}_3^2) + \frac{1}{2}m_7\Phi_7^2 + \frac{1}{2}m_8\Phi_8^2. \quad (6.7)$$

breaks the conformal invariance of the ABJM theory, but preserves $\mathcal{N} = 1$ supersymmetry. For general values of the mass parameters m_3 , m_7 and m_8 , the deformation preserves the SO(3) symmetry. The structure of the holographic RG flows above suggests that the IR dynamics of this model is controlled by a new interacting $\mathcal{N} = 1$ SCFT, which is the field theory dual of the new $\mathcal{N} = 1$ vacuum **S1384096**. Note that for $m_3 = 0$ we recover the RG flows to the G_2 and $SU(3) \times U(1)$ critical points discussed in [51].

7 A $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ -invariant truncation

A lesson one should draw from the construction of the critical point **S1384096** above, as well as from a similar construction in [14] of the $\mathcal{N} = 1$ supersymmetric critical point, **S1200000**, which is not captured by our truncation, is that discrete symmetries may lead to simple, explicit truncations of the $\mathcal{N} = 8$ supergravity that are accessible analytically. In both of these constructions, the scalar manifold of the truncated theory is simply a product of three Poincaré disks, $SU(1,1)/U(1)$. One may note that the same coset arises

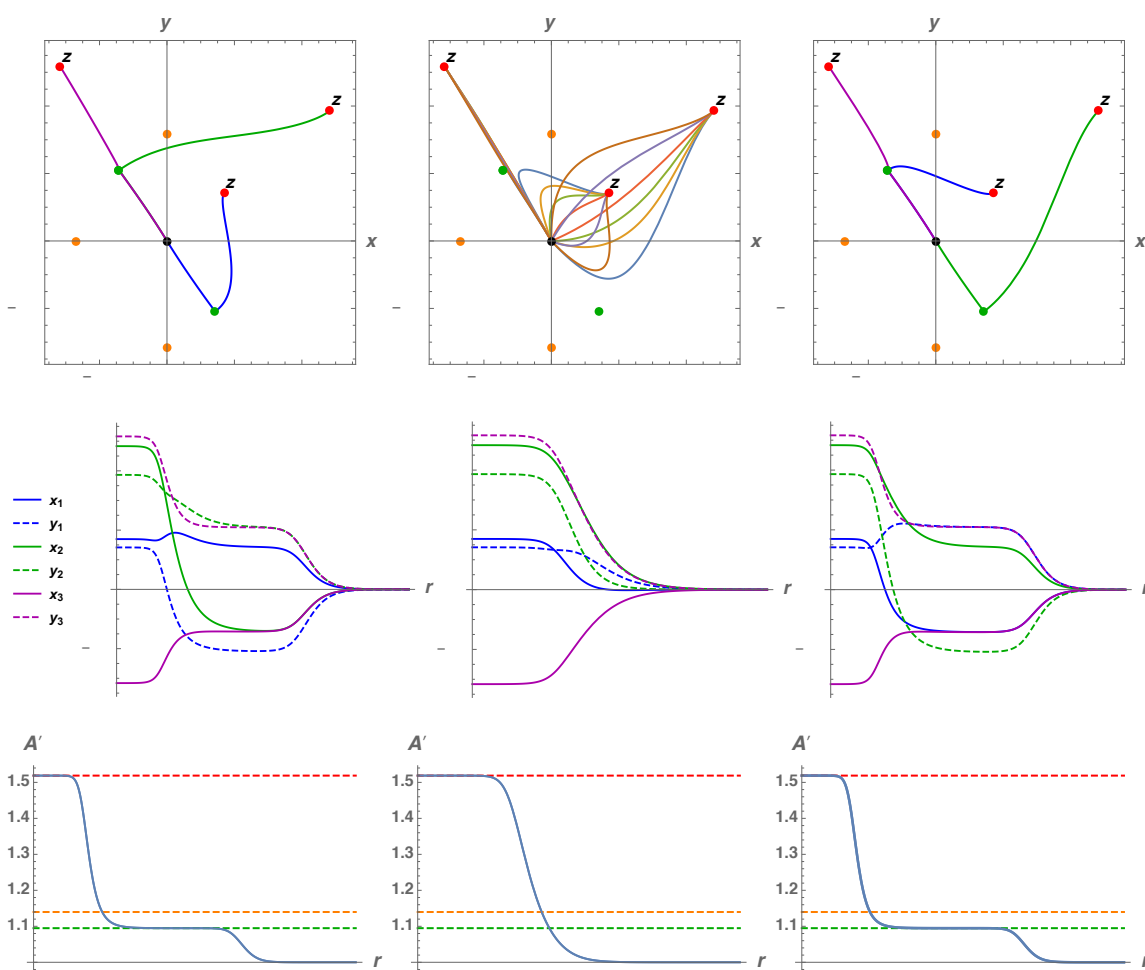


Figure 1. Numerical solutions to the BPS equations for RG flows from the $SO(8)$ point to the $SO(3)$ point. The generic flows (middle column) asymptote to flows through the G_2 point (side columns). The top row shows the flows in the superimposed z_1, z_2 and z_3 -planes. The colored dots represent the supersymmetric critical points: $SO(8)$ (black), G_2 (green), $SU(3) \times U(1)$ (orange), and $SO(3)$ (red). The middle row gives the radial dependence of the real scalars, x_1, \dots, y_3 . The bottom row gives A' along the flows, which asymptotes to a constant that depends on the radius of the AdS_4 vacuum.

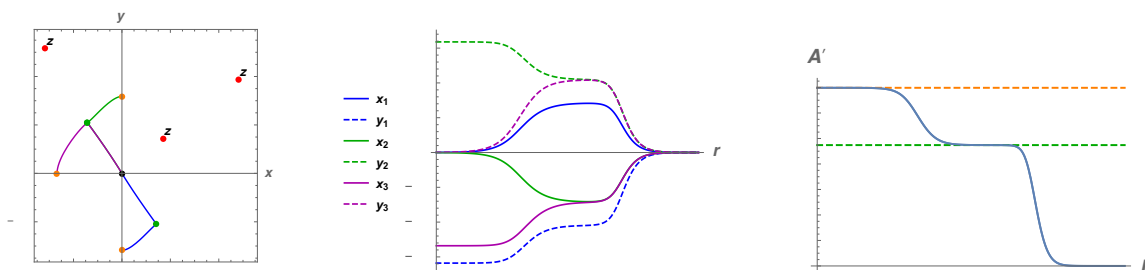


Figure 2. The RG-flow from the $SO(8)$ point to the $SU(3) \times U(1)$ point asymptotic to the G_2 point.

in the so-called STU-model with a particularly simple superpotential [52]. Observing that in our construction, two of the three $SU(1,1)$ factors are embedded non-regularly into $E_{7(7)}$, it seems suggestive to try interpreting the subspace studied here as originating from a collapsing of roots, starting from the maximal number of regularly embedded $SU(1,1)$'s. Both the enlargement of $SU(1,1) \times SU(1,1)$ to $G_{2(2)}$ when dropping the \mathbb{Z}_2 symmetry (which is obtained from $SO(8)$ by collapsing three roots) as well as the coefficients in the Kähler potential (2.15) suggest that one should look for a regular embedding of $SU(1,1)^{\times 7}$ into $E_{7(7)}$. The remarkable properties of this subgroup have been discussed in the context of qubit entanglement and black holes, cf. [53] and subsequent research, with a comprehensive review in [54], “curious supergravities” [55] and cosmology [56].

It turns out that an $\mathcal{N} = 1$ supersymmetric truncation with the scalar manifold given by the product of 7 Poincaré disks,

$$\left[\frac{SU(1,1)}{U(1)} \right]^7, \tag{7.1}$$

can be obtained using a discrete $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \subset SO(8)$ symmetry. It can be constructed explicitly as follows.

Consider the $S \equiv \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ group generated by the following $SO(8)$ rotations:

$$\begin{aligned} g_1 &= \text{diag}(1, 1, 1, 1, -1, -1, -1, -1), \\ g_2 &= \text{diag}(1, 1, -1, -1, 1, 1, -1, -1), \\ g_3 &= \text{diag}(1, -1, 1, -1, 1, -1, 1, -1), \end{aligned} \tag{7.2}$$

in \mathfrak{so}_v . The non-identity elements of this group are naturally labelled by the 7 points on the Fano plane (see, e.g., [57])

$$h_1 = g_1, \quad h_2 = g_2, \quad h_3 = g_3, \quad h_4 = g_1 g_2, \quad h_5 = g_2 g_3, \quad h_6 = g_3 g_1, \quad h_7 = g_1 g_2 g_3, \tag{7.3}$$

such that the product along each line is the identity. It is straightforward to verify explicitly that the Lie subalgebra of $\mathfrak{e}_{7(7)}$ invariant under S is precisely $\mathfrak{su}(1,1)^{\oplus 7}$, with the compact generators, \mathfrak{h}_a , of $\mathfrak{u}(1)^{\oplus 7}$ given by the matrices $\mathfrak{h}_a = i h_a$, $a = 1, \dots, 7$ in (7.3). Note that the generators \mathfrak{h}_j are orthogonal to $\mathfrak{so}(8)$ in $\mathfrak{su}(8)$. Since each $\mathfrak{su}(1,1)$ corresponds to a complex scalar field in the truncation, there is a natural identification of the resulting 7 complex scalars, ζ_a , with the points on the Fano plane,

$$\zeta_a \longleftrightarrow \mathfrak{h}_a = i h_a, \quad a = 1, \dots, 7. \tag{7.4}$$

As in section 2, the superpotential can be read off from the eigenvalue of the A_{ij}^1 tensor along the unbroken supersymmetry. A direct calculation yields the result

$$\begin{aligned} \mathcal{W}_{\mathbb{Z}_2^3} &= \zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_5 \zeta_6 \zeta_7 \\ &+ \zeta_1 \zeta_2 \zeta_3 \zeta_7 + \zeta_1 \zeta_2 \zeta_5 \zeta_6 + \zeta_1 \zeta_3 \zeta_4 \zeta_5 + \zeta_1 \zeta_4 \zeta_6 \zeta_7 + \zeta_2 \zeta_3 \zeta_4 \zeta_6 + \zeta_2 \zeta_4 \zeta_5 \zeta_7 + \zeta_3 \zeta_5 \zeta_6 \zeta_7 \\ &+ \zeta_1 \zeta_2 \zeta_4 + \zeta_1 \zeta_3 \zeta_6 + \zeta_1 \zeta_5 \zeta_7 + \zeta_2 \zeta_6 \zeta_7 + \zeta_2 \zeta_3 \zeta_5 + \zeta_3 \zeta_4 \zeta_7 + \zeta_4 \zeta_5 \zeta_6 + 1, \end{aligned} \tag{7.5}$$

where each cubic term, together with the complementary quartic term,¹² corresponds to one of the 7 lines in the Fano plane.

Even more remarkably, the terms in this polynomial match the 16 code words in the single-error-correcting (7, 4) Hamming code [58]. This particular binary code encodes each of the 16 possible 4-bit messages 0000, 0001, 0010, ..., 1111 as a different 7-bit code word in such a way that for each of the 7-bit code words, flipping an arbitrary single bit still always allows reliable reconstruction of the original message. Due to Fano plane symmetry, the group of bit-permutations that leaves the Hamming code invariant is the simple group $\text{PSL}(2, \mathbb{F}_7) \simeq \text{GL}(3, \mathbb{F}_2)$ of order 168.

The (7, 4) Hamming code is sometimes also called the (7, 4, 3) Hamming code, since changing one code word into another code word requires flipping at least three bits. This code is not self-dual. Its dual, i.e. the set of all code words of length seven that have zero inner product (in \mathbb{F}_2) with the (7, 4, 3) Hamming code, form the less well known (7, 3, 4) “little Hamming code” that (with our implicit conventions for labeling the Fano plane vertices) consists of the eight code words 0000000, 0010111, 0101101, 0111010, 1001011, 1011100, 1100110, 1110001. The set of points in \mathbb{Z}^7 whose coordinates are modulo-2 congruent to these eight code words form the E_7 lattice. Concretely, the first ‘shell’ of equidistant-from-origin points appears at distance-squared 4 and has 126 elements that correspond to the roots of E_7 . Performing the same construction with the (7, 4, 3) Hamming code rather than its dual (7, 3, 4) little Hamming code gives not the E_7 root lattice, but its dual E_7^* weight lattice: the first shell of equidistant-from-origin points here appears at distance-squared 3 and has 56 elements.

One notes that adding a parity bit to the (7, 4) Hamming code produces the self-dual (8, 4) Hamming code, which the above construction gives rise to the self-dual E_8 lattice. Considering that the maximal subgroup embedding $E_{7(7)} \times \text{SL}(2) \subset E_{8(8)}$ gives rise to an *eighth* hyperbolic disc, it seems natural to speculate that maximal gauged *three-dimensional* supergravity may have a corresponding (8, 4) Hamming code superpotential [59].

From the kinetic terms, or equivalently from the same embedding indices of $\mathfrak{su}(1, 1)$ ’s in $\mathfrak{e}_{7(7)}$, we find the canonical Kähler potential

$$\mathcal{K} = - \sum_{a=1}^7 \log(1 - \zeta_a \bar{\zeta}_a). \tag{7.6}$$

This completely specifies the truncation with the scalar potential in this sector given by (2.17).

One can check that all truncations of interest with fewer $\text{SU}(1, 1)/\text{U}(1)$ factors can be obtained by imposing additional continuous symmetry with respect to some subgroup of $\text{SO}(8)$. This amounts to setting some scalars equal (up to a sign) and/or setting them to zero. For example, the truncation discussed in this paper can be obtained by setting

$$\zeta_1 = -z_2, \quad \zeta_2 = \zeta_6 = \zeta_7 = -z_3, \quad \zeta_3 = \zeta_4 = \zeta_5 = z_1, \tag{7.7}$$

¹²Complementary terms are defined by their product given by $\zeta_1 \zeta_2 \dots \zeta_7$.

S0600000	S0668740	S0698771	S0719157	S0779422	S0800000
S0847213	S0869596	S0880733	S0983994	S1006758	S1039230
S1046017	S1067475	S1068971	S1075828	S1165685	S1200000
S1301601	S1362365	S1366864	S1384096	S1384135	S1400000
S1424025	S1470986	S1473607	S1477609	S1497038	S1571627
S1600000	S1637792	S1637802	S1650772	S1652212	S1805269
S1810641	S2096313	S2099077	S2140848	S2389433	S2443607
S2457396	S2519962	S2547536	S2702580	S2707528	S3305153

Table 2. The critical points in the \mathbb{Z}_2^3 -invariant sector.

upon which $\mathcal{W}_{\mathbb{Z}_2^3}$ reduces to (2.18) and the Kahler potential (7.6) to (2.15). Similarly, the $\text{SO}(2) \times \text{SO}(2) \times \mathbb{Z}_2 \times \mathbb{Z}_2$ -invariant truncation in [14] is obtained by setting

$$\zeta_1 = \zeta_3 = i \xi_1, \quad \zeta_2 = \zeta_7 = 0, \quad \zeta_4 = \zeta_5 = \xi_2, \quad \zeta_6 = \xi_0, \quad (7.8)$$

where ξ_i are the scalar fields in [14]. Finally, the superpotential of the STU model is obtained by keeping just one cubic term, for example by setting

$$\zeta_3 = \zeta_5 = \zeta_6 = \zeta_7 = 0. \quad (7.9)$$

A preliminary numerical search has revealed 48 critical points listed in table 2.¹³ As expected, those points include all 5 supersymmetric points: S0600000, S0719157, S0779422, S1200000, S1384096, and the non-supersymmetric stable point, S1400000. It is clear that this truncation should be the natural arena to study the holographic RG flows between the supersymmetric critical points and to look for the interplay between the structure of the critical points and the underlying octonion structure in the truncation. For further details we refer the reader to the follow up publication [60].

8 Conclusions

In this paper we presented an explicit construction of a new AdS_4 vacuum of $\mathcal{N} = 8$ supergravity which preserves $\mathcal{N} = 1$ supersymmetry. We also discussed the spectrum of all supergravity fields around this vacuum and its relation to operators in the holographically dual CFT. Moreover, we constructed numerical holographic RG flow solutions which interpolate between this new vacuum and other supersymmetric vacua of the $\mathcal{N} = 8$ supergravity.

One interesting outcome of combining an explicit analytic truncation with the numerical methods using TensorFlow or Bertini2 is a discovery of two additional non-supersymmetric and perturbatively unstable AdS_4 vacua that were not identified in the numerical search in [1]. A possible explanation is that the numerical algorithms in [1] were applied to the full 70-dimensional scalar manifold of the $\mathcal{N} = 8$ supergravity whereas here

¹³We thank Fridrik Freyr Gautason for an independent calculation of these points.

the search could be restricted to a simpler and explicitly known potential that depends on 6 scalars only. This also points to a possible strategy for refining the numerical search by restricting it to scalar submanifolds that are invariant under continuous and/or discrete symmetries of the points that had been found already.

Out of all known AdS₄ vacua in the $\mathcal{N} = 8$ supergravity there are only 6 that are perturbatively stable. The $SO(3) \times \mathbb{Z}_2$ supergravity truncation discussed in this paper contains 5 of these vacua. The one not included is the $U(1) \times U(1)$ $\mathcal{N} = 1$ vacuum studied in [14, 19]. The larger truncation with 14 scalar fields presented in section 7 contains all 6 perturbatively stable AdS₄ vacua and therefore is a natural starting point for the study of explicit holographic RG flows between them. In fact, by imposing additional $U(1)$ symmetry, one may further truncate to 10 scalar fields, while preserving all the interesting critical points. At the end such an analysis will elucidate the phase structure of the ABJM SCFT and will provide a rich testing ground for the “ μ -theorem” discussed in [61].

The results presented here and in [1] suggest that one should apply similar techniques to investigate the vacuum structure of other maximal supergravity theories using an amalgam of analytic and numerical methods. Two particularly interesting examples which can be embedded in string theory and have well-understood holographic duals are the $\mathcal{N} = 8$ ISO(7) gauged four-dimensional supergravity [23, 62, 63] and the maximal five-dimensional SO(6) gauged supergravity. We expect to report some preliminary results shortly [64].

Acknowledgments

N.B. and K.P. are grateful to Fridrik Freyr Gautason and Silviu Pufu for interesting discussions. T.F. would like to thank Jyrki Alakuijala and Rahul Sukthankar for feedback and encouragement on this work, Moritz Firsching and Sameer Agarwal for useful discussions on homotopy continuation methods, and Jonathan Hauenstein for confirming TensorFlow results with Bertini2. We also thank Moritz Firsching for providing minimal polynomials for the complex coordinates of the two algebraically most challenging critical points. The work of NB is supported in part by an Odysseus grant G0F9516N from the FWO and by the KU Leuven C1 grant ZKD1118 C16/16/005. KP is supported in part by DOE grant DE-SC0011687. NB and KP are grateful to the Mainz Institute for Theoretical Physics (MITP) of the DFG Cluster of Excellence PRISMA⁺ (Project ID 39083149), for its hospitality and its partial support during the initial stages of this project.

A Some group theory

We use the convention in which the gravitino, ψ_μ^i , transforms in $\mathbf{8}_v$, the spin-1/2 fermions are in the $\mathbf{56}_v$ while the scalars and the pseudoscalars in $\mathbf{35}^+ = \mathbf{35}_s$ and $\mathbf{35}^- = \mathbf{35}_c$ representations of $\mathfrak{so}(8)$. The metric is neutral under $\mathfrak{so}(8)$ and the gauge field is in the adjoint.

The commutant of the $\mathfrak{so}(3) \simeq \mathfrak{su}(2)$ symmetry algebra in $\mathfrak{so}(8)$ is $\mathfrak{u}(1) \times \mathfrak{u}(1)$. It arises from the following chain of maximal subalgebras:

$$\mathfrak{so}(8) \supset \mathfrak{su}(4) \times \mathfrak{u}(1)_2 \supset \mathfrak{su}(3) \times \mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2 \supset \mathfrak{su}(2) \times \mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2. \quad (\text{A.1})$$

The corresponding branchings of the $\mathfrak{so}(8)$ representations relevant for our analysis are as follows:¹⁴

$$\begin{aligned} \mathbf{8}_v &\longrightarrow \mathbf{4}_1 + \bar{\mathbf{4}}_{-1} \longrightarrow \mathbf{3}_{1,1} + \mathbf{1}_{-3,1} + \bar{\mathbf{3}}_{-1,-1} + \mathbf{1}_{3,-1} \\ &\longrightarrow \mathbf{3}_{1,1} + \mathbf{1}_{-3,1} + \bar{\mathbf{3}}_{-1,-1} + \mathbf{1}_{3,-1}. \end{aligned} \quad (\text{A.2})$$

For the vectors, the branching is

$$\begin{aligned} \mathbf{28} &\longrightarrow \mathbf{15}_0 + \mathbf{6}_2 + \mathbf{6}_{-2} + \mathbf{1}_0 \\ &\longrightarrow \mathbf{8}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} + \mathbf{1}_{0,0} + \mathbf{3}_{-2,2} + \bar{\mathbf{3}}_{2,2} + \mathbf{3}_{-2,-2} + \bar{\mathbf{3}}_{2,-2} + \mathbf{1}_{0,0} \\ &\longrightarrow \mathbf{5}_{0,0} + \mathbf{3}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} + \mathbf{1}_{0,0} + \mathbf{3}_{-2,2} + \bar{\mathbf{3}}_{2,2} + \mathbf{3}_{-2,-2} + \bar{\mathbf{3}}_{2,-2} + \mathbf{1}_{0,0}. \end{aligned} \quad (\text{A.3})$$

For the scalars we have

$$\begin{aligned} \mathbf{35}_s &\longrightarrow \mathbf{20}'_0 + \mathbf{6}_2 + \mathbf{6}_{-2} + \mathbf{1}_4 + \mathbf{1}_0 + \mathbf{1}_{-4} \\ &\longrightarrow \bar{\mathbf{6}}_{-4,0} + \mathbf{8}_{0,0} + \mathbf{6}_{4,0} + \mathbf{3}_{-2,2} + \bar{\mathbf{3}}_{2,2} + \mathbf{3}_{-2,-2} + \bar{\mathbf{3}}_{2,-2} + \mathbf{1}_{0,4} + \mathbf{1}_{0,0} + \mathbf{1}_{0,-4} \\ &\longrightarrow \mathbf{5}_{-4,0} + \mathbf{1}_{-4,0} + \mathbf{5}_{0,0} + \mathbf{3}_{0,0} + \mathbf{5}_{4,0} + \mathbf{1}_{4,0} + \mathbf{3}_{-2,2} + \bar{\mathbf{3}}_{2,2} \\ &\quad + \mathbf{3}_{-2,-2} + \bar{\mathbf{3}}_{2,-2} + \mathbf{1}_{0,4} + \mathbf{1}_{0,0} + \mathbf{1}_{0,-4}, \end{aligned} \quad (\text{A.4})$$

and for the pseudoscalars we find

$$\begin{aligned} \mathbf{35}_c &\longrightarrow \mathbf{10}_{-2} + \mathbf{15}_0 + \bar{\mathbf{10}}_2 \\ &\longrightarrow \bar{\mathbf{6}}_{2,-2} + \mathbf{3}_{-2,-2} + \mathbf{1}_{-6,-2} + \mathbf{8}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} + \mathbf{1}_{0,0} + \mathbf{6}_{-2,2} + \bar{\mathbf{3}}_{2,2} + \mathbf{1}_{6,2} \\ &\longrightarrow \mathbf{5}_{2,-2} + \mathbf{1}_{2,-2} + \mathbf{3}_{-2,-2} + \mathbf{1}_{-6,-2} + \mathbf{5}_{0,0} + \mathbf{3}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} \\ &\quad + \mathbf{1}_{0,0} + \mathbf{5}_{-2,2} + \mathbf{1}_{-2,2} + \mathbf{3}_{2,2} + \mathbf{1}_{6,2}. \end{aligned} \quad (\text{A.5})$$

To determine the commutant of $\mathfrak{so}(3)$ in $\mathfrak{e}_{7(7)}$ in (2.2), we also need

$$\begin{aligned} \mathbf{35}_v &\longrightarrow \mathbf{10}_2 + \mathbf{15}_0 + \bar{\mathbf{10}}_{-2} \\ &\longrightarrow \bar{\mathbf{6}}_{2,2} + \mathbf{3}_{-2,2} + \mathbf{1}_{-6,2} + \mathbf{8}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} + \mathbf{1}_{0,0} + \mathbf{6}_{-2,-2} + \bar{\mathbf{3}}_{2,-2} + \mathbf{1}_{6,-2} \\ &\longrightarrow \mathbf{5}_{2,2} + \mathbf{1}_{2,2} + \mathbf{3}_{-2,2} + \mathbf{1}_{-6,2} + \mathbf{5}_{0,0} + \mathbf{3}_{0,0} + \mathbf{3}_{4,0} + \bar{\mathbf{3}}_{-4,0} + \mathbf{1}_{0,0} \\ &\quad + \mathbf{5}_{-2,-2} + \mathbf{1}_{-2,-2} + \mathbf{3}_{2,-2} + \mathbf{1}_{6,-2}. \end{aligned} \quad (\text{A.6})$$

Finally for the spin-1/2 fields we have

$$\begin{aligned} \mathbf{56}_v &\longrightarrow \bar{\mathbf{20}}_{-1} + \mathbf{20}_1 + \mathbf{4}_1 + \bar{\mathbf{4}}_{-1} + \mathbf{4}_{-3} + \bar{\mathbf{4}}_3 \\ &\longrightarrow \bar{\mathbf{6}}_{-1,-1} + \mathbf{8}_{3,-1} + \mathbf{3}_{-5,-1} + \bar{\mathbf{3}}_{-1,-1} + \mathbf{6}_{1,1} + \mathbf{8}_{-3,1} + \bar{\mathbf{3}}_{5,1} + \mathbf{3}_{1,1} \\ &\quad + \mathbf{3}_{1,1} + \mathbf{1}_{-3,1} + \bar{\mathbf{3}}_{-1,-1} + \mathbf{1}_{3,-1} + \mathbf{3}_{1,-3} + \mathbf{1}_{-3,-3} + \bar{\mathbf{3}}_{-1,3} + \mathbf{1}_{3,3} \\ &\longrightarrow \mathbf{5}_{-1,-1} + \mathbf{1}_{-1,-1} + \mathbf{5}_{3,-1} + \mathbf{3}_{3,-1} + \mathbf{3}_{-5,-1} + \mathbf{3}_{-1,-1} + \mathbf{5}_{1,1} + \mathbf{1}_{1,1} + \mathbf{5}_{-3,1} + \mathbf{3}_{-3,1} \\ &\quad + \mathbf{3}_{5,1} + \mathbf{3}_{1,1} + \mathbf{3}_{1,1} + \mathbf{1}_{-3,1} + \mathbf{3}_{-1,-1} + \mathbf{1}_{3,-1} + \mathbf{3}_{1,-3} + \mathbf{1}_{-3,-3} + \mathbf{3}_{-1,3} + \mathbf{1}_{3,3}. \end{aligned} \quad (\text{A.7})$$

The singlets under $\mathfrak{so}(3)$ are the metric, two Abelian gauge fields, 2 spin-3/2 fields, 10 scalars, and 6 spin-1/2 fields. This is precisely the matter contents of a four-dimensional $\mathcal{N} = 2$ gauged supergravity coupled to 1 vector multiplet and 2 full hypermultiplets.

¹⁴We use the same group theory conventions as in [65].

B The full spectrum of $\mathcal{N} = 8$ supergravity

In this appendix we present the masses of all bosonic and fermionic fields of the four-dimensional $\mathcal{N} = 8$ supergravity around the $\text{SO}(3)$ invariant $\mathcal{N} = 1$ AdS_4 vacuum, S1384096, studied in the main text. The spectrum of the spin-0, spin-1/2, and spin-3/2 fields and their $\text{SO}(3)$ representations were already presented in [1] and is summarized in table 3, table 4, and table 6, respectively. The spin-2 graviton is of course massless and not charged under $\text{SO}(3)$ and the spectrum and $\text{SO}(3)$ representations of the spin-1 vector fields are presented in table 5. The latter masses are computed using the general mass formulae for spin-1 fields in [21].

Using the standard AdS/CFT dictionary, see [66] for a review, the spectrum of these fields is mapped to the spectrum of operators in the dual $\mathcal{N} = 1$ SCFT. The conformal dimensions of the dual operators of spin s can be computed using formulae in table 7. We use the dimensionless mass mL of the supergravity fields, where L is the AdS_4 scale. We have also indicated a reference where the derivation of each of the formulae can be found.

To understand how the spectrum of supergravity excitations maps to operators in the dual three-dimensional $\mathcal{N} = 1$ SCFT it is useful to recall some aspects of the representation theory of the $\mathcal{N} = 1$ superconformal algebra, see [67] for a recent discussion. Operators in the $\mathcal{N} = 1$ SCFT are labelled by their conformal dimension Δ and spin s and will be denoted by $|\Delta, s\rangle$.¹⁵ These operators belong to one of the superconformal multiplets summarized in table 8.

Using the information in tables 3–6 we can organize the spectrum of operators in the $\mathcal{N} = 1$ SCFT dual to the $\text{SO}(3)$ AdS_4 vacuum in the following superconformal multiplets:

- *Short spin-3/2.* This is simply the energy momentum multiplet which is neutral under $\mathfrak{so}(3)$ and contains the following operators

$$|\frac{5}{2}, \frac{3}{2}\rangle \quad |3, 2\rangle. \tag{B.1}$$

The supergravity modes corresponding to these operators are the spin-3/2 mode in the last line of table 6 and the metric.

- *Short spin-1/2.* This is the conserved $\mathfrak{so}(3)$ current and contains the following operators

$$|\frac{3}{2}, \frac{1}{2}\rangle \quad |2, 1\rangle. \tag{B.2}$$

The supergravity modes corresponding to these operators are in the $\mathbf{3}$ of $\mathfrak{so}(3)$ and correspond to the massless vector and spin-1/2 modes in table 5 and table 4, respectively.

¹⁵The authors of [67] label the operators with $j = 2s$.

#	$m^2 L^2$	SO(3) irreps	Δ
1*	16.26186	1	5.802541
1	16.09544	1	5.783158
1*	8.656777	1	4.802541
1	8.529126	1	4.783158
1*	8.094691	1	4.716316
3	5.322114	3	4.251747
3	5.182218	3	4.226210
5	3.817573	5	3.963244
1*	2.662058	1	3.716316
25	0	$5 \oplus 6 \times 3 \oplus 2 \times 1$	3
5	-0.108916	5	2.963244
5	-1.099493	5	2.572617
8	-1.396494	$5 \oplus 3$	0.5761462 or 2.423854
1*	-1.685601	1	0.7487353 or 2.251265
1*	-2.188131	1	1.251265 or 1.748735
3	-2.244202	3	1.423854 or 1.576146
5	-2.244727	5	1.427383 or 1.572617

Table 3. Masses of the 70 supergravity scalars at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators. The conformal dimensions are obtained using the standard AdS/CFT formula $m^2 L^2 = \Delta(\Delta - 3)$ and choosing the root of this quadratic equation which obeys the unitarity bound $\Delta \geq 1/2$. When $-9/4 \leq m^2 L^2 < 5/4$ one has a choice of alternate quantization, see [68], and both possible conformal dimensions are presented.

- *Long spin-1.* There are three such multiplets. We present all operators in them below and indicate also the $\mathfrak{so}(3)$ representations

$$\begin{aligned}
 &|2.704760, 1\rangle \quad |3.204759, \frac{1}{2}\rangle \quad |3.204759, \frac{3}{2}\rangle \quad |3.704760, 1\rangle, \quad \mathbf{3}, \\
 &|2.783158, 1\rangle \quad |3.283158, \frac{1}{2}\rangle \quad |3.283158, \frac{3}{2}\rangle \quad |3.783158, 1\rangle, \quad \mathbf{1}, \quad (\text{B.3}) \\
 &|2.847013, 1\rangle \quad |3.347013, \frac{1}{2}\rangle \quad |3.347013, \frac{3}{2}\rangle \quad |3.847013, 1\rangle, \quad \mathbf{3}.
 \end{aligned}$$

The supergravity modes corresponding to these operators are the spin- $\frac{1}{2}$, spin-1, and spin- $\frac{3}{2}$ modes of the corresponding dimension in table 4, table 5, and table 6, respectively.

#	$m^2 L^2$	SO(3) irreps	Δ
1	14.45932	1	5.302541
1	14.31228	1	5.283158
3	13.64583	3	5.194026
1	12.71861	1	5.066316
3	11.62481	3	4.909518
3	10.57386	3	4.751747
3	10.40843	3	4.726210
1	7.378374	1	4.216316
3	5.070367	3	3.751747
3	4.956009	3	3.726210
5	3.854328	5	3.463244
3	3.411457	3	3.347013
1	3.179652	1	3.283158
3	2.906203	3	3.204759
5	2.027360	5	2.923854
5	0.3278901	5	2.072617
8	0.1796520	5 \oplus 3	1.923854
1	0.0631340	1	1.751265
3	0	3	3/2

Table 4. Masses of the 56 spin-1/2 supergravity fermions at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators.

#	$m^2 L^2$	SO(3) irreps	Δ
3	7.322116	3	4.251748
3	7.182220	3	4.226210
3	5.258471	3	3.847013
1	4.962811	1	3.783158
3	4.610963	3	3.704760
3	1.564444	3	2.847013
1	1.396495	1	2.783158
3	1.201444	3	2.704760
5	0.603506	5	2.423854
3	0	3	2

Table 5. Masses of the 28 supergravity vector fields at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators.

#	$m^2 L^2$	SO(3) irreps	Δ
3	3.411457	3	3.347013
1	3.179652	1	3.283158
3	2.906203	3	3.204759
1*	1	1	5/2

Table 6. Masses of the 8 spin-3/2 supergravity fermions at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators.

Spin	Dimension	
0	$\Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2 L^2}$	[66]
$\frac{1}{2}$	$\Delta = \frac{3}{2} + mL $	[69]
1	$\Delta = \frac{3}{2} \pm \sqrt{\frac{1}{4} + m^2 L^2}$	[70]
$\frac{3}{2}$	$\Delta = \frac{3}{2} + mL $	[71, 72]

Table 7. Dimensions of operators dual to fields of spin, s , and mass, m .

Name	Primary	Descendants	Unitarity bound
Identity (B_1)	$ 0, 0\rangle$	-	$\Delta = 0$
Short scalar (A'_2)	$ \frac{1}{2}, 0\rangle$	$ \frac{1}{2}, \frac{1}{2}\rangle$	$\Delta = \frac{1}{2}$
Short spin (A_1)	$ s+1, s\rangle$	$ s+\frac{3}{2}, s+\frac{1}{2}\rangle$	$\Delta = s+1; s > 0$
Long scalar (L')	$ \Delta, 0\rangle$	$ \Delta+\frac{1}{2}, \frac{1}{2}\rangle; \Delta+1, 0\rangle$	$\Delta > \frac{1}{2}$
Long spin (L)	$ \Delta, s\rangle$	$ \Delta+\frac{1}{2}, s+\frac{1}{2}\rangle; \Delta+\frac{1}{2}, s-\frac{1}{2}\rangle; \Delta+1, s\rangle$	$\Delta > s+1; s > 0$

Table 8. The $\mathcal{N} = 1$ superconformal multiplets. The first column indicates also the notation for each multiplet used in [67].

- *Long spin-1/2.* There are three such multiplets. We present all operators in them below and indicate also the $\mathfrak{so}(3)$ representations

$$\begin{aligned}
 &|3.751747, \frac{1}{2}\rangle \quad |4.251748, 1\rangle \quad |4.251747, 0\rangle \quad |4.751747, \frac{1}{2}\rangle, \quad \mathbf{3}, \\
 &|3.726210, \frac{1}{2}\rangle \quad |4.226210, 1\rangle \quad |4.226210, 0\rangle \quad |4.726210, \frac{1}{2}\rangle, \quad \mathbf{3}, \quad (\text{B.4}) \\
 &|1.923854, \frac{1}{2}\rangle \quad |2.423854, 1\rangle \quad |2.423854, 0\rangle \quad |2.923854, \frac{1}{2}\rangle, \quad \mathbf{5}.
 \end{aligned}$$

The supergravity modes corresponding to these operators are the spin-0, spin- $\frac{1}{2}$, and spin-1 modes of the corresponding dimension in table 3, table 4, and table 5, respectively.

- *Long scalar.* There are seven such multiplets. We present all operators in them along with their $\mathfrak{so}(3)$ representations below

$$\begin{array}{llll}
 |4.802541, 0\rangle & |5.302541, \frac{1}{2}\rangle & |5.802541, 0\rangle, & \mathbf{1}, \\
 |4.783158, 0\rangle & |5.283158, \frac{1}{2}\rangle & |5.783158, 0\rangle, & \mathbf{1}, \\
 |3.716316, 0\rangle & |4.216316, \frac{1}{2}\rangle & |4.716316, 0\rangle, & \mathbf{1}, \\
 |2.963244, 0\rangle & |3.463244, \frac{1}{2}\rangle & |3.963244, 0\rangle, & \mathbf{5}, \\
 |1.572617, 0\rangle & |2.072617, \frac{1}{2}\rangle & |2.572617, 0\rangle, & \mathbf{5}, \\
 |1.423854, 0\rangle & |1.923854, \frac{1}{2}\rangle & |2.423854, 0\rangle, & \mathbf{3}, \\
 |1.251265, 0\rangle & |1.751265, \frac{1}{2}\rangle & |2.251265, 0\rangle, & \mathbf{1}.
 \end{array} \tag{B.5}$$

The supergravity modes corresponding to these operators are the spin-0 and spin- $\frac{1}{2}$ modes of the corresponding dimension in table 3 and table 4, respectively.

In addition to the modes discussed above it should be noted that due to the spontaneous breaking of the $\mathcal{N} = 8$ supersymmetry and the $\mathfrak{so}(8)$ gauge symmetry of the supergravity theory there are spin-0 and spin-1/2 modes that are “eaten” by the usual (super)Higgs mechanism. These are the spin-1/2 modes in the 3rd, 4th and 5th line of table 4 as well as the spin-0 mode in the 10th line of table 3.

It should also be noted that for spin-0 modes with mass in the range $-\frac{9}{4} \leq m^2 L^2 < -\frac{5}{4}$ there is an ambiguity in assigning a conformal dimension of the dual CFT operator. This happens because for both choices of sign in table 7 Δ obeys the unitarity bound. Invoking supersymmetry however uniquely fixes the choice of sign in table 7 and we have chosen the only possible sign that allows for organizing the bottom five entries in table 3 into $\mathcal{N} = 1$ superconformal multiplets.

C Discrete SO(8) rotations

Consider the SO(3) subgroup of SO(8) introduced in section 2. It is straightforward to check that any SO(8) rotation that commutes with this subgroup must be of the form

$$\left(\begin{array}{cccccccc}
 \cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 & 0 & 0 \\
 0 & \cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 & 0 \\
 0 & 0 & \cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 \\
 \mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 & 0 & 0 \\
 0 & \mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 & 0 \\
 0 & 0 & \mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \cos \beta & \sin \beta \\
 0 & 0 & 0 & 0 & 0 & 0 & \mp \sin \beta & \pm \cos \beta
 \end{array} \right). \tag{C.1}$$

Choosing the upper sign, we obtain the two parameter family of rotations, $g(\alpha, \beta)$, corresponding to the $U(1) \times U(1)$ gauge group of the $\mathcal{N} = 2$ supergravity of the $SO(3)$ -invariant truncation. For the lower sign, the rotations are given by

$$g(-\alpha, -\beta + \pi) g_S, \quad (\text{C.2})$$

where g_S is the generator (2.9) of the discrete \mathbb{Z}_2 symmetry that defines our truncation.

Since the scalar fields s_a , $a = 1, \dots, 6$, are by construction invariant under g_S , any residual nontrivial action of the $SO(8)$ subgroup given in (C.1) on the coset (2.11) must come from the $U(1) \times U(1)$ transformations that preserve (2.10). At a generic point in the coset, this leaves a discrete $\mathbb{Z}_2 \times \mathbb{Z}_4$ subgroup of $SO(8)$ generated by $g_H \equiv g(0, \pi)$ and $g_C \equiv g(\pi/2, \pi/2)$, which are elements of order 2 and 4, respectively. However, since $g_C^2 = g(\pi, \pi)$ acts trivially on the coset, we end up with only $\mathbb{Z}_2 \times \mathbb{Z}_2$ worth of $SO(8)$ rotations that preserve the scalar manifold in the $SO(3) \times \mathbb{Z}_2$ -invariant truncation. Those are generated by the transformations

$$\begin{aligned} g_H &: (z_1, z_2, z_3) \longrightarrow (-z_1, -z_2, z_3), \\ g_C &: (z_1, z_2, z_3) \longrightarrow (-\bar{z}_1, -\bar{z}_2, z_3). \end{aligned} \quad (\text{C.3})$$

Hence any point in the coset lies on an $SO(8)$ orbit obtained by acting on that point with 1, g_H , g_C and $g_H g_C$. When both z_1 and z_2 are either real or imaginary, the orbit degenerates to two points.

At special points of the coset there might be additional rotations, $g(\alpha, \beta)$, that map it onto another point on the coset. In particular, this happens for two critical points, S0880733 and S1039230, in section 4, with the special rotations given by $g_R \equiv g(-3\pi/4, \pi/4)$ and $g'_R \equiv g(-\pi/4, \pi/4)$, respectively.

While both rotations in (C.3) are obviously symmetries of the potential (2.17), only g_H preserves the superpotential (2.18). This is just the reflection of the fact that the second transformation acts nontrivially on the supersymmetry by mapping the $\mathcal{N} = 1$ supergravity given by the $SO(3) \times \mathbb{Z}_2$ -invariant truncation to an equivalent one obtained by a different discrete \mathbb{Z}_2 symmetry.

Finally, we note that 4-form $\Phi^{(3)}$ in (2.6) is invariant under the transformations (C.1). In particular, this implies that z_3 is invariant under the discrete symmetries above.

D New critical points S2096313 and S2443607

In this appendix we present numerical data for the two new critical points, S2096313 and S2443607, in the same format as in [1]. The parametrization of the scalar coset (2.11) in terms of s_a , $a = 1, \dots, 6$ introduced in section 2 is related to the one in [1] and in the tables

below by:

$$\begin{aligned}
 A &= \frac{1}{8}(2s_1 + s_5), & B &= \frac{1}{8}(s_1 - s_3 + 2s_5), & C &= \frac{1}{8}(s_5 - 2s_1), \\
 D &= \frac{1}{8}(s_1 - s_3 - 2s_5), & E &= -\frac{s_5}{8}, & F &= \frac{1}{8}(-s_1 - s_3), \\
 G &= \frac{1}{8}(s_2 + s_4 - s_6), & H &= \frac{1}{8}(-s_2 - s_4 - s_6), & I &= \frac{1}{8}(-3s_2 + s_4 + 3s_6), \\
 J &= \frac{1}{8}(3s_2 - s_4 + 3s_6),
 \end{aligned} \tag{D.1}$$

where the parameters A, \dots, J satisfy

$$A - B - E + F = 0, \quad B - D + 4E = 0, \quad C + D - E - F = 0, \quad 3G + 3H + I + J = 0. \tag{D.2}$$

D.1 Point S2096313

S2096313 : $\mathfrak{so}(8) \rightarrow \mathfrak{so}(3) + \mathfrak{u}(1)$

$$V/g^2 \approx -20.9631372891$$

$$\mathbf{8}_c \rightarrow 2 \times \mathbf{3} + \mathbf{1}^{++} + \mathbf{1}^{--}, \quad \mathbf{8}_{v,s} \rightarrow \mathbf{3}^+ + \mathbf{3}^- + \mathbf{1}^+ + \mathbf{1}^-,$$

$$\mathbf{28} \rightarrow \mathbf{5} + 2 \times \mathbf{3}^{++} + 3 \times \mathbf{3} + 2 \times \mathbf{3}^{--} + 2 \times \mathbf{1}$$

$$m^2/m_0^2[\psi] : 7.750_{\mathbf{1}^{++}\mathbf{1}^-}, 2.550_{\mathbf{3}^{++}\mathbf{3}^-}$$

$$\begin{aligned}
 m^2/m_0^2[\chi] : & 15.500_{\mathbf{1}^{++}\mathbf{1}^-}, 7.875_{\mathbf{3}^{++}\mathbf{3}^{--}}, 5.137_{\mathbf{3}^{++}\mathbf{3}^-}, 5.100_{\mathbf{3}^{++}\mathbf{3}^-}, \\
 & 4.596_{\mathbf{5}^{++}\mathbf{5}^-}, 2.475_{\mathbf{1}^{++}\mathbf{1}^{--}}, 0.875_{\mathbf{1}^{++}\mathbf{1}^-}, 0.754_{\mathbf{5}^{++}\mathbf{5}^-}, \\
 & 0.727_{\mathbf{3}^{++}\mathbf{3}^-}, 0.160_{\mathbf{3}^{++}\mathbf{3}^-}
 \end{aligned}$$

$$\begin{aligned}
 m^2/m_0^2[\phi] : & 18.400_{\mathbf{1}^{++++}\mathbf{1}^{----}}, 10.000_{\mathbf{1}^{++}\mathbf{1}^{--}}, 9.600_{\mathbf{5}^{++}\mathbf{5}^{--}}, 8.728_{\mathbf{1}^m}, \\
 & 8.596_{\mathbf{5}^m}, 0.400_{\mathbf{3}^m}, 0.000_{\mathbf{3}^{++}\mathbf{3}^{--}}, 0.000_{\mathbf{5}^{+2}\mathbf{3}^{++}\mathbf{2}\mathbf{3}^{--}\mathbf{1}^+}, \\
 & 0.000_{\mathbf{3}^m}, -0.596_{\mathbf{5}^m}, -0.612_{\mathbf{1}^m}, -1.200_{\mathbf{1}^{++}\mathbf{1}^{--}}, -2.000_{\mathbf{3}^m}, \\
 & -2.400_{\mathbf{5}^{m*}}, -2.516_{\mathbf{1}^{m*}}
 \end{aligned}$$

$$M_{\alpha\beta} = \text{diag}(-3A, -3A, A, A, A, A, A, A)$$

$$M_{\dot{\alpha}\dot{\beta}} = \text{diag}(C, D, B, B, C, D, D, C)$$

$$A \approx -0.1641598793, \quad B \approx -0.5046414602, \quad C \approx -0.0723920925,$$

$$D \approx 0.4088197326$$

D.2 Point S2443607

S2443607 : $\mathfrak{so}(8) \rightarrow \mathfrak{so}(3)$

$$V/g^2 \approx -24.4360747652$$

$$\mathbf{8}_{v,s,c} \rightarrow 2 \times \mathbf{3} + 2 \times \mathbf{1}, \quad \mathbf{28} \rightarrow \mathbf{5} + 7 \times \mathbf{3} + 2 \times \mathbf{1}$$

$$m^2/m_0^2[\psi] : 11.568_{\mathbf{1}}, 11.463_{\mathbf{1}}, 4.354_{\mathbf{3}}, 3.726_{\mathbf{3}}$$

$$\begin{aligned}
 m^2/m_0^2[\chi] : & 23.137_{\mathbf{1}}, 22.926_{\mathbf{1}}, 18.908_{\mathbf{3}}, 18.908_{\mathbf{3}}, 10.714_{\mathbf{1}}, 10.711_{\mathbf{1}}, 8.708_{\mathbf{3}}, \\
 & 7.451_{\mathbf{3}}, 6.299_{\mathbf{3}}, 6.141_{\mathbf{3}}, 5.757_{\mathbf{5}}, 5.744_{\mathbf{5}}, 2.122_{\mathbf{3}}, 2.064_{\mathbf{1}}, 1.624_{\mathbf{5}}, \\
 & 1.417_{\mathbf{3}}, 1.257_{\mathbf{5}}, 1.255_{\mathbf{1}}, 0.201_{\mathbf{3}}, 0.082_{\mathbf{3}}
 \end{aligned}$$

$$\begin{aligned}
 m^2/m_0^2[\phi] : & 55.474_{\mathbf{1}^m}, 55.474_{\mathbf{1}^m}, 20.600_{\mathbf{1}^m}, 20.559_{\mathbf{1}^m}, 19.887_{\mathbf{5}^m}, 19.876_{\mathbf{5}^m}, 8.040_{\mathbf{1}^m}, \\
 & 7.864_{\mathbf{3}^m}, 7.464_{\mathbf{3}^m}, 5.996_{\mathbf{5}^m}, 4.125_{\mathbf{1}^m}, 0.000_{\mathbf{3}^{+1}}, 0.000_{\mathbf{5}^{+5}\mathbf{3}^{+1}}, \\
 & -0.023_{\mathbf{3}^m}, -0.562_{\mathbf{1}^m}, -0.743_{\mathbf{5}^m}, -2.513_{\mathbf{3}^{m*}}, -2.836_{\mathbf{5}^{m*}}, -3.205_{\mathbf{1}^{m*}}
 \end{aligned}$$

$$M_{\alpha\beta} = \begin{pmatrix} A & 0 & 0 & 0 & 0 & B & 0 & 0 \\ 0 & E & 0 & 0 & C & 0 & 0 & 0 \\ 0 & 0 & F & 0 & 0 & 0 & 0 & D \\ 0 & 0 & 0 & F & 0 & 0 & -D & 0 \\ 0 & C & 0 & 0 & E & 0 & 0 & 0 \\ B & 0 & 0 & 0 & 0 & A & 0 & 0 \\ 0 & 0 & 0 & -D & 0 & 0 & F & 0 \\ 0 & 0 & D & 0 & 0 & 0 & 0 & F \end{pmatrix}$$

$$M_{\dot{\alpha}\dot{\beta}} = \text{diag}(G, G, G, H, H, H, I, J)$$

$$A \approx 0.2540142811, \quad B \approx 0.3965710947, \quad C \approx -0.1697655242,$$

$$D \approx 0.0009726589, \quad E \approx 0.0291540284, \quad F \approx -0.1415841547,$$

$$G \approx 0.4106803872, \quad H \approx 0.0401731985, \quad I \approx -0.6761535338,$$

$$J \approx -0.6764072234$$

D.3 Supplementary material

Numerical data for the position of the two new critical points, S2096313 and S2443607, in the same format as in [1], can be found in the supplementary material. Algebraic data on the locations of critical points S1424025 and S2443607, which give rise to formulae that are too complicated to be included in the text of this work, are also available as supplementary material attached to this paper.

E Minimal polynomials

The PSLQ algorithm used to obtain the minimal polynomials in sections 3 and 4 may also be used directly in the complex domain.¹⁶

As our coordinates map the hyperbolic plane to the unit disk, the relevant minimal polynomials for coordinates will be palindromic. If ζ is a zero, then ζ^{-1} will also be a zero, and this invariance under $\zeta \mapsto \zeta^{-1}$ makes the highest-order coefficient match the lowest-order coefficient, etc. It so turns out that for many critical points, a de-palindromizing substitution (via the inverse of a Zhukovsky transform, $\zeta \mapsto \zeta + \zeta^{-1}$), followed by a re-scaling, again leads to a palindromic polynomial.

E.1 Minimal polynomials for S1384096

The minimal polynomials for the coordinates of the new $\mathcal{N} = 1$ vacuum, $\mathcal{P}_{z_i}(\zeta)$, $i = 1, 2, 3$, turn out to be “doubly” palindromic, and this property can be exploited to simplify their presentation.

¹⁶We thank Moritz Firsching for pointing this out to us and computing all the minimal polynomials in this appendix.

Let

$$\begin{aligned}
 S_1(\zeta) &= \zeta^{12} + 2\zeta^{10} + 387\zeta^8 - 7276\zeta^6 + 59179\zeta^4 - 248970\zeta^2 + 416025, \\
 S_2(\zeta) &= 237169\zeta^{12} - 5533444\zeta^{10} + 54887568\zeta^8 - 295250296\zeta^6 + 905373664\zeta^4 \\
 &\quad - 1496099520\zeta^2 + 1038128400, \\
 S_3(\zeta) &= 3\zeta^6 + 30\zeta^5 + 191\zeta^4 + 690\zeta^3 + 1337\zeta^2 + 1314\zeta + 521.
 \end{aligned}
 \tag{E.1}$$

Define the polynomials, $M_i(\zeta)$, as follows

$$M_i(2\zeta) = (4\zeta)^{\text{ord}(S_i)} S_i(\zeta + \zeta^{-1}), \tag{E.2}$$

Then

$$\mathcal{P}_{z_i}(\zeta) = \zeta^{\text{ord}(M_i)} M_i(\zeta + \zeta^{-1}), \quad i = 1, 2, 3, \tag{E.3}$$

are the minimal polynomials with integer coefficients for S1384096.

E.2 Minimal polynomials for S1424025 and S2443607

The minimal polynomials for the coordinates of the critical points S1424025 and S2443607 also allow “double depalindromization”, and this property was exploited to obtain these expressions. Unfortunately, they are too complicated to be shown in the text, having degrees 208, 208, and 52 for S1424025, and degrees 464, 464, and 232 for S2443607. The supplementary material provides executable Python code that lists and verifies these polynomials.

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