

A New Optimal Nonlinear Approach to Half Car Active Suspension Control

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Abstract—In this paper, a nonlinear optimal control law based on a quadratic cost function is developed, and applied on a half-car model for the control of active suspension systems. Nonlinear model of half-car is constructed using the nonlinear dynamics of the electro hydraulic actuator and dynamic characteristics of the dampings and springs. The states of half car model are first estimated by Extended Kalman Filter (EKF) and then the estimated states predicted by Taylor series expansion and finally a control law is introduced by minimizing the local differences between the predicted and desired states. The derived control law has an analytical form which is easy to apply and also it is not required online numerical computations in optimization. Performance of the nonlinear optimal controller is compared to the existing passive suspension system and the proportional integral sliding mode controller. The obtained results demonstrate that use of the proposed nonlinear optimal control technique improves the tradeoff between ride quality and suspension travel compared to the passive suspension system and the proportional integral sliding mode method.

Index Terms—nonlinear optimal control, active suspension, nonlinear model, Extended Kalman Filter, proportional integral sliding mode.

I. INTRODUCTION

The important functionality of the vehicle suspension system is to support the vehicle body as well as to provide the riding comfort to the passengers by rejecting the unpleasant vibratory motion induced from the irregular road inputs. Also, the suspension should maintain adequate vertical load to provide the vehicle stability when the car turns, brakes or accelerates [1]. The vehicle stability and riding comfort has mutually adverse effects, therefore, simply the passive damper, which is widely used in the usual vehicle, could not satisfy the riding comfort as well as driving stability simultaneously. To overcome this problem many researchers have proposed to use active suspensions [2]. Unlike passive systems which can only store or dissipate energy, active suspensions can continuously change the energy flow to or from the system when required. Furthermore, characteristics of active suspensions can adapt to instantaneous changes in driving conditions detected by sensors. As a result active suspensions can improve both riding comfort and handling performance to satisfactory levels.

However, as the recent trends of the vehicle industry is to be luxurious and driver comfort is more required, the electrical controlled suspension system is now installed and widely utilized [3]. Especially, the active damper is widely adopted for the luxurious vehicle because it can overcome the shortage of the passive damper by providing the ride comfort and vehicle stability at the same time. In the active suspension system, the sprung mass can vary according to the variation of passenger numbers and cargo as well as fuel loading condition. Also the actuator itself has the strong non-linear characteristics and the fluctuations of the hydraulic system's parameters are not small, i.e. fluid compressibility, fluid leakage and electrical servo components.

Various control strategies such as optimal control [4], nonlinear control [5], robust control [6], adaptive control [7] and intelligent control [8] have been proposed in the past years to control the active suspension system. Most control methods have been used for active suspension systems are optimal control method [9], [10]. In these optimal control methods a linear model for automotive proposed and different optimal control approaches such as linear quadratic regulator and linear quadratic gaussian have been used. But real model of automotive is nonlinear. Therefore for a good design, it is necessary that an actual model for automotive used. In this paper, according to system requirements, an optimal nonlinear approach [11-13] is applied. Similar to this method has been used for control of yaw dynamics to improve vehicle lateral stability [14], [15]. The proposed controller has two distinguished features: firstly, it is based on continuous nonlinear model and can handle the model nonlinearity successfully. Secondly, the optimality of the control law provides the possibility of using lower control energy for achievement of the desired performance. In this paper, a new optimal predictive approach is utilized to design a non-linear controller. This method, which employs a nonlinear continuous-time dynamic model, leads to an analytical closed-form control law which is suitable to implement. The rest of the paper is organized as follows:

The model of a half-car suspension system and dynamics of it is initially investigated in section two. Then in the section three, an optimization-based nonlinear control law is developed. The main properties of the proposed controller and its advantages than the other conventional control methods are discussed. Some comparative simulation results including a passive suspension and active suspension system which is based on proportional integral sliding mode control (PISMC) theory and our resulting active suspension system are

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illustrated in section four. Section five contains the conclusions of the paper.

II. SUSPENSION MODEL

The well-known rigid half-car vehicle model, which is shown in Figure 1, is widely used for active suspension design. The model comprises three parts: the sprung mass and two unsprung masses. Let the sprung and unsprung masses be denoted by M , m_f , m_r , respectively. The half-car model is a four degrees-of-freedom system. The sprung mass is assumed to be a rigid body and has freedoms of motion in the vertical and pitch direction. The Z_c denotes the vertical displacement at the center of gravity and θ is the pitch angle of the sprung mass. The front and rear displacements of the sprung and the unsprung masses are denoted by Z_f , Z_r and Z_1 , Z_2 . In the model, the disturbances, Z_{01} , Z_{02} are caused by road irregularities. The control signals, $u_f(t)$, $u_r(t)$ are generated by the actuators.

Using Newton's second law the equations of motion can be given as:

$$\begin{aligned} M\ddot{z}_c &= f_f + f_r \\ I\ddot{\theta} &= f_f a - f_r b \\ m_f \ddot{z}_1 &= -k_{f2}(z_1 - z_{01}) - f_f \\ m_r \ddot{z}_2 &= -k_{r2}(z_2 - z_{02}) - f_r \end{aligned} \quad (1)$$

$$f_f = F_{k_{f1}} + F_{b_f} + u_f$$

$$f_r = F_{k_{r1}} + F_{b_r} + u_r$$

$F_{k_{i1}}$ and F_{b_i} ($i \in \{f, r\}$) are as follows:

$$F_{k_{f1}} = k_{f1}^l (z_1 - z_c - a\theta) + k_{f1}^{nl} (z_1 - z_c - a\theta)^3 \quad (2)$$

$$F_{k_{r1}} = k_{r1}^l (z_2 - z_c + b\theta) + k_{r1}^{nl} (z_2 - z_c + b\theta)^3 \quad (3)$$

$$\begin{aligned} F_{b_f} &= b_f^l (\dot{z}_1 - \dot{z}_c - a\dot{\theta}) \\ &+ b_f^{nl} \left(\sqrt{\left| \dot{z}_1 - \dot{z}_c - a\dot{\theta} \right|} \text{sign}(\dot{z}_1 - \dot{z}_c - a\dot{\theta}) - \left| \dot{z}_1 - \dot{z}_c - a\dot{\theta} \right| \right) \end{aligned} \quad (4)$$

$$\begin{aligned} F_{b_r} &= b_r^l (\dot{z}_2 - \dot{z}_c + b\dot{\theta}) \\ &+ b_r^{nl} \left(\sqrt{\left| \dot{z}_2 - \dot{z}_c + b\dot{\theta} \right|} \text{sign}(\dot{z}_2 - \dot{z}_c + b\dot{\theta}) - \left| \dot{z}_2 - \dot{z}_c + b\dot{\theta} \right| \right) \end{aligned} \quad (5)$$

In these equations k_{f1} , k_{r1} and k_{f2} , k_{r2} are front and rear nonlinear suspension stiffness and the front and rear linear tire stiffness, respectively. The front and rear nonlinear suspension dampings are denoted by b_f , b_r . The suspension spring forces at the front and rear and suspension damping forces at front and rear are denoted by $F_{k_{f1}}$, $F_{k_{r1}}$ and F_{b_f} , F_{b_r} respectively. The nonlinear nonlinear suspension stiffness k_{i1} ($i \in \{f, r\}$) are combination of a linear coefficient k_{i1}^l and a nonlinear k_{i1}^{nl} and also the nonlinear suspension damping b_i ($i \in \{f, r\}$) are made of b_i^l and b_i^{nl} .

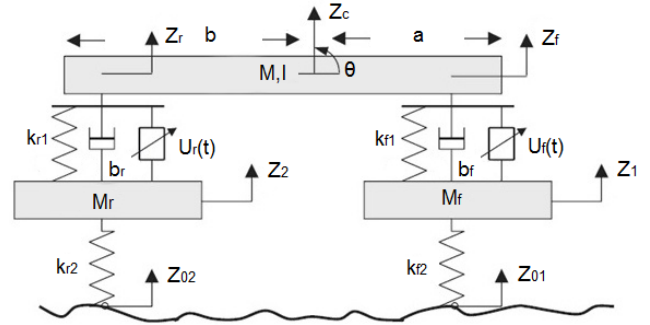


Fig. 1 Half-car suspension vehicle model.

Hydraulic actuators are used for suspension systems to generate the pushing force between the sprung and unsprung masses. Let the hydraulic actuator used for suspension model be a four-way valve-piston system, in which the force u_i is

$$u_i(t) = AP_{Li} \quad (6)$$

where A is the area of piston and P_{Li} is the pressure drop across the piston with respect to the front and rear suspensions ($i \in \{f, r\}$) [16]. The derivative of P_{Li} is given by Eqs. (7) and (8).

$$\frac{V_i}{4\beta_e} \dot{P}_{Lf} = Q_f - C_{ip} P_{Lf} - A(\dot{z}_c + a\dot{\theta} - \dot{z}_1) \quad (7)$$

$$\frac{V_i}{4\beta_e} \dot{P}_{Lr} = Q_r - C_{ip} P_{Lr} - A(\dot{z}_c - b\dot{\theta} - \dot{z}_2) \quad (8)$$

where V_i is the total actuator volume, β_e is the effective bulk modulus of system, Q_i ($i \in \{f, r\}$) is the hydraulic load flow, and C_{ip} is the total leakage coefficient of piston.

The relationship between the spool valve displacement and the hydraulic load flow is given as:

$$Q_i = \text{sign}[P_s - \text{sign}(x_{vi})P_{Li}] C_d \omega x_{vi} \sqrt{\frac{1}{\rho} |P_s - \text{sign}(x_{vi})P_{Li}|} \quad (9)$$

where C_d is the discharge coefficient, ω is the spool valve area gradient, x_{vi} is the displacement of spool valve, ρ is the hydraulic fluid density, and P_s is the supply pressure. The x_{vi} is controlled by the input to the servo-valve v_i . The valve dynamics are approximated by a linear filter with time constant τ :

$$\dot{x}_{vi} = \frac{1}{\tau} (-x_{vi} + v_i) \quad (10)$$

The nonlinear model of the hydraulic actuator is as follows:

$$\dot{P}_{Lf} = \alpha Q_f - \beta P_{Lf} - \alpha A(\dot{z}_c + a\dot{\theta} - \dot{z}_1) \quad (11)$$

$$\dot{P}_{Lr} = \alpha Q_r - \beta P_{Lr} - \alpha A(\dot{z}_c - b\dot{\theta} - \dot{z}_2) \quad (12)$$

$$\alpha Q_i = \gamma \text{sign}[P_s - \text{sign}(x_{vi})P_{Li}] x_{vi} \sqrt{|P_s - \text{sign}(x_{vi})P_{Li}|} \quad (13)$$

$$\dot{x}_{vi} = \frac{1}{\tau} (-x_{vi} + v_i) \quad (14)$$

where $\alpha = \frac{4\beta_e}{V_i}$, $\beta = \alpha C_{ip}$, $\gamma = \frac{C_d \omega}{\sqrt{\rho}} \alpha$

If defined the state variable, X , the control input, V , the disturbance input, W as:

$$\begin{aligned}x_1 &= z_c + a\theta - z_1 & x_2 &= \dot{z}_c + a\dot{\theta} & x_3 &= z_c - b\theta - z_2 \\x_4 &= \dot{z}_c - b\dot{\theta} & x_5 &= z_1 - z_{01} & x_6 &= \dot{z}_1 & x_7 &= z_2 - z_{02} \\x_8 &= \dot{z}_2 & x_9 &= \mu P_{L_f} & x_{10} &= x_{v_f} & x_{11} &= \mu P_{L_r} & x_{12} &= x_{v_r}\end{aligned}$$

$$V = [v_1, v_2]^T$$

$$W = [w_1, w_2]^T$$

$\mu = 10^{-6}$ is used to rescale the pressure drop, P_{L_i} ($i \in \{f, r\}$), to improve numerical accuracy. The components of the input disturbance (the rate of change in the height of road irregularities) for front and rear roads are $w_1(t) = \dot{Z}_{01}(t)$ and $w_2(t) = w_1(t - t_d)$, where t_d represents the time delay between front and rear tire axis. When nonlinear equations and nonlinear model of the hydraulic actuator use, the nonlinear suspension system has the following from:

$$\dot{X} = f + BV + DW \quad (15)$$

$$f = \begin{pmatrix} x_2 - x_6 \\ a_1 f_f + a_2 f_r \\ x_4 - x_8 \\ a_2 f_f + a_3 f_r \\ x_6 \\ -\frac{1}{M_f} f_f - \frac{K_{f2}}{M_f} x_5 \\ x_8 \\ -\frac{1}{M_r} f_r - \frac{K_{r2}}{M_r} x_7 \\ -\mu\alpha A(x_2 - x_6) - \beta x_9 + \mu\gamma x_{10} \left(\text{sign} \left(P_s - \text{sign}(x_{10}) \frac{x_9}{\mu} \right) \sqrt{P_s - \text{sign}(x_{10}) \frac{x_9}{\mu}} \right) \\ \frac{1}{\tau} (-x_{10}) \\ -\mu\alpha A(x_4 - x_8) - \beta x_{11} + \mu\gamma x_{12} \left(\text{sign} \left(P_s - \text{sign}(x_{12}) \frac{x_{11}}{\mu} \right) \sqrt{P_s - \text{sign}(x_{12}) \frac{x_{11}}{\mu}} \right) \\ \frac{1}{\tau} (-x_{12}) \end{pmatrix}$$

where

$$f_f = -k_{f1}^l x_1 - k_{f1}^n x_1^3 + b_f^l (x_6 - x_2) + b_f^n \left(\sqrt{|(x_6 - x_2)|} \text{sign}(x_6 - x_2) - |(x_6 - x_2)| \right)$$

$$+ \frac{A}{\mu} x_9$$

$$f_r = -k_{r1}^l x_3 - k_{r1}^n x_3^3 - b_r^l (x_8 - x_4) + b_r^n \left(\sqrt{|(x_8 - x_4)|} \text{sign}(x_8 - x_4) - |(x_8 - x_4)| \right)$$

$$+ \frac{A}{\mu} x_{11}$$

$$a_1 = \frac{1}{M} + \frac{a^2}{I}, \quad a_2 = \frac{1}{M} - \frac{ab}{I}, \quad a_3 = \frac{1}{M} + \frac{b^2}{I}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

III. CONTROL SYSTEM DESIGN

In this section firstly the states of the system estimated by the Extended Kalman Filter (EKF). The Kalman Filter (KF) is one of the most widely used methods for tracking and estimation due to its simplicity, optimality, tractability and robustness [17]. However, the application of the KF to nonlinear systems can be difficult. The most common

approach is to use the EKF which simply linearizes all nonlinear models so that the traditional linear Kalman Filter can be applied [18]. The EKF is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms. This dynamic system can be disturbed by some noise, mostly assumed as white noise. To improve the estimated state the EKF uses measurements that are related to the state disturbed as well [19]. Thus the EKF consist of two steps:

- 1) The prediction
- 2) The correction

In the first step the state is predicted with the dynamic model. In the second step it is corrected with the observation model, so that the error covariance of the estimator is minimized. In this sense it is an optimal estimator. The dynamic model represented by

$$\dot{X}(t) = f(X(t), m(t)) \quad (16)$$

where $m(t)$ is the noise vector. In the linear case this can easily be rewritten as

$$\dot{X}(t) = F.X(t) + n(t) \quad (17)$$

where F is the dynamic matrix and is constant, $X(t)$ is the state vector and $n(t)$ is the dynamic noise which is usually assumed as white noise and has the covariance matrix $Q(t)$.

The observation model represents the relationship between the state and the measurements. Usually the observations are made at discrete time steps t_i

$$l(t_i) = h(X(t_i), v(t_i)) \quad (18)$$

where $v(t_i)$ is the noise of the measurement process at the epoch t_i . In the linear case the measurements can be described by a system of linear equations, which depend on the state variables. The vector from of this system is

$$l(t_i) = H.X(t_i) + w(t_i) \quad (19)$$

where $l(t_i)$ is the vector of the observations at the epoch t_i , H is the observation matrix and $w(t_i)$ is the noise of the measurement process with the covariance matrix $R(t_i)$. Like the dynamic matrix, in a linear system the observation matrix H is a constant matrix as well.

Like mentioned before, the prediction is the first step of the EKF. The predicted state, or better the a priori state is calculated by neglecting the dynamic noise and solving the differential equation the describe the dynamic model

$$\dot{X}^-(t_i) = f(X^-(t_i)) \quad (20)$$

By representing this equation by a Taylor series with respect to X at the predicted state $X^-(t_i)$ and assuming that the higher order terms can be neglected, Eq. (20) can be rewritten as

$$\dot{X}^-(t_i) = F(t_i).X^-(t_i) \quad (21)$$

where

$$F(t_i) = \left. \frac{\partial f(X)}{\partial X} \right|_{X=X^-(t_i)} \quad (22)$$

The state vector at time t_i can be expressed by Taylor series with respect to an approximate state $X^-(t_0)$.

$$X^-(t_i) = X^-(t_0) + \dot{X}^-(t_0)(t_i - t_0) + \frac{1}{2} \ddot{X}^-(t_0)(t_i - t_0)^2 + \dots \quad (23)$$

by using Eq. (21) this can be rewritten as

$$X^-(t_i) = X^-(t_0) + F(t_i)X^-(t_0)(t_i - t_0) + \frac{1}{2} F^2(t_i)X^-(t_0)(t_i - t_0)^2 + \dots \quad (24)$$

Thus the solution $X^-(t_i)$ of the differential equations, in other words the actual predicted state is a linear combination of the initial state $X^-(t_0)$

$$X^-(t_i) = \phi_0^{t_i} \cdot X^-(t_0) \quad (25)$$

$\phi_0^{t_i}$ is called the state transition matrix, which transforms any initial state $X(t_0)$ to its corresponding state $X(t_i)$ at time t_i

from the Eqs. (21) and (25)

$$\dot{X}^-(t_i) = F(t_i) \cdot X^-(t_i) = F(t_i) \cdot \phi_0^{t_i} \cdot X^-(t_0) \quad (26)$$

and by using (26) again, one can see

$$\dot{X}^-(t_i) = \frac{d}{dt} X^-(t_i) = \frac{d}{dt} (\phi_0^{t_i} \cdot X^-(t_0)) = \frac{d}{dt} (\phi_0^{t_i}) \cdot X^-(t_0) \quad (27)$$

by comparing (26) and (27) it follows that:

$$\frac{d}{dt} (\phi_0^{t_i}) = F(t_i) \cdot \phi_0^{t_i} \quad (28)$$

with the initial matrix $\phi_0^0 = I$, because $X(t_0) = I \cdot X(t_0)$.

And now the covariance matrix $P^-(t_i)$ of the predicted state vector is obtained with the law of error propagation

$$P^-(t_i) = \phi_{t_{i-1}}^{t_i} \cdot P(t_{i-1}) \cdot (\phi_{t_{i-1}}^{t_i})^T + Q \quad (29)$$

In the more generalized form, where also the covariance matrix of the noise Q is a function of time, the covariance matrix is:

$$P^-(t_i) = \phi_{t_{i-1}}^{t_i} \cdot P(t_{i-1}) \cdot (\phi_{t_{i-1}}^{t_i})^T + \int_{t_{i-1}}^{t_i} Q(t) dt \quad (30)$$

In the correction step the predicted state vector $X^-(t_i)$ is improved with observations made at the epoch t_i , thus the a posteriori state has the form

$$X^+(t_i) = X^-(t_i) + \Delta X(t_i) \quad (31)$$

with the covariance matrix

$$P^+(t_i) = P^-(t_i) + \Delta P(t_i) \quad (32)$$

As said before the EKF is an optimal filter, this means that the state variances in the state covariance matrix P^+ are minimized.

As P^- is already known from the prediction step it follows that ΔP is minimized.

$$\Delta P(t_i) = E \left[\Delta X(t_i) \Delta X(t_i)^T \right] \quad (33)$$

This condition is complied with

$$\Delta X(t_i) = P^-(t_i) H^T(t_i) (H(t_i) P^-(t_i) H^T(t_i) + R(t_i))^{-1} \cdot (l(t_i) - H(t_i) X^-(t_i)) \quad (34)$$

$$\Delta X(t_i) = K(t_i) \cdot (l(t_i) - H(t_i) X^-(t_i))$$

with

$$K(t_i) = P^-(t_i) H^T(t_i) (H(t_i) P^-(t_i) H^T(t_i) + R(t_i))^{-1} \quad (35)$$

$K(t_i)$ is called the gain matrix. The difference

$(l(t_i) - l^-(t_i))$ is called the measurement residual. It reflects the discrepancy between the predicted measurement $l^-(t_i) = H X^-(t_i)$ and the actual measurement $l(t_i)$.

Finally the corrected state is obtained by

$$X^+(t_i) = X^-(t_i) + K(t_i) \cdot (l(t_i) - l^-(t_i)) \quad (36)$$

In this equation the estimated state and the measurements are weighted and combined to calculate the corrected state. That means, if the measurement covariance is much smaller than that of the predicted state, the measurements weight will be high and the predicted states will be low. And so the uncertainty can be reduced. The covariance matrix of the a posteriori state is given with the law of error propagation by $P^+(t_i) = P^-(t_i) - K(t_i) H(t_i) P^-(t_i) = (I - K(t_i) H(t_i)) P^-(t_i)$ (37)

In this paper distance of front and rear of automobile to surface of road are considered as a measurable output for numerical calculation. Now, we are ready for to design of the controller. The main goal of the control system is to make the estimated states x_n ($n=1,2,\dots,12$) to follow the desired state x_{nd} . Briefly, the estimated states for the next time interval, $x_n(t+h)$, is first predicted by Taylor series expansion and then the current control v_m ($m=1,2$) will be found based on continuous minimization of predicted tracking error. Note that h denotes to the predictive period and is a real positive number. Let us first approximate $x_n(t+h)$ by a k th-order Taylor series at t :

$$x_n(t+h) = x_n(t) + h \dot{x}_n(t) + \frac{h^2}{2!} \ddot{x}_n(t) + \dots + \frac{h^k}{k!} x_n^{(k)}(t) \quad (38)$$

$$x_n(t+h) = \zeta(X(t), \dot{V}(t), \dots, V^{k-1}(t), \dot{W}(t), \dots, W^{k-1}(t))$$

Now, the key issue is to choose the order k in a way which is suitable for the purposes of controller design on the basis of predictions. The expansion order k is determined as the lowest order of the derivative of state x_n in which the input V first appears explicitly. Hence, state vector $X(t+h)$ is as follows:

$$X(t+h) = L_1(t, X(t), W(t), \dot{W}(t), \dots, W^{k-1}(t)) + L_2(t, X(t)) V(t) \quad (39)$$

Note that the arguments of functions may be frequently dropped through the rest of paper for simplicity of notations. Now, we consider a performance index that penalizes the next instant tracking error and the current control expenditure in the following form:

$$J[V(t)] = \frac{1}{2} (X(t+h) - X_d(t+h))^T W_1 (X(t+h) - X_d(t+h)) + \frac{1}{2} V(t)^T W_2 V(t) \quad (40)$$

where W_1 and W_2 are symmetric positive semi-definite and symmetric positive definite weighting matrices, respectively. Minimization of the performance index must be sought in order to improve the tracking accuracy of states at the next instant and consequently obtain the optimum behavior of the vehicle. Now, the expanded performance index can be obtained as a function of control input by substituting Eq. (39) into (40) as:

$$J[V(t)] = \frac{1}{2} (L_1 + L_2 V - X_d)^T W_1 (L_1 + L_2 V - X_d) + \frac{1}{2} V(t)^T W_2 V(t) \quad (41)$$

The necessary condition for optimality is

$$\frac{\partial J}{\partial V} = 0 = L_2^T W_1 (L_1 + L_2 V - X_d) + W_2 V \quad (42)$$

which, leads to

$$V = -(L_2^T W_1 L_2 + W_2)^{-1} L_2^T W_1 (L_1 - X_d) \quad (43)$$

It is considered that the analytically defined predictive control law, Eq. (43), is a closed form which depends on the states of the estimated system and disturbance input. Generally, the proposed control law has two free parameters: the predictive time h and the weighting matrices W_1 and W_2 . The dynamic performance of the controller is extremely sensitive to the values of these parameters. In the derived control law, the predictive period h is treated as a controller parameter rather than the integration step size. This feature is not shared by discrete-time systems [20]. Also, it can be established that a certain degree of robustness in the presence of some modeling uncertainties is achievable through small values of h . We see that the proposed tracking controller technique naturally leads to a special case of feedback linearization. But the current control law (43) has some important advantages over the input/output linearization control. It can be established that the predictive controller is robust in the presence of a class of modeling uncertainties and doesn't need the exact knowledge of the system nonlinearity unlike the feedback linearization. Optimal property of the proposed control law is another important advantage that provides the possibility of limiting the control by regulation of weighting matrices.

IV. SIMULATION AND DISCUSSION

Computer simulations are carried out to verify the effectiveness of the designed nonlinear optimal control system. The vehicle parameters for a compact sedan that are used in simulation are listed in Table 1. Matrices W_1 , W_2 and the predictive period h of the Nonlinear Optimal control and other parameters are accurately regulate for the computer simulations. Let the set of typical road disturbance be in the form of

$$Z_{01}(t) = \begin{cases} a(1 - \cos(8\pi t)) & 0.5 \leq t \leq 0.75 \\ b(1 - \cos(8\pi t)) & 3 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases} \quad (44)$$

where a and b denote the bump amplitude (Fig. 2). This type of road disturbance has been used by [21], [22] in their studies. Furthermore, the maximum travel distance of the suspension travel is $\pm 8cm$ as suggested in [22]. For comparison purposes, the performance of the designed nonlinear optimal control is compared to the proportional integral sliding mode control (PISMC) approach. For the system in Eq. (15), we utilized the PI sliding surface defined as

$$\text{follows: } \sigma(t) = CX(t) + \int_0^t (CE)X(\eta)d\eta \quad (45)$$

where C (dimension $[C] = m \times n$) and E (dimension $[E] = n \times n$) are constant matrices. It is well known that if the system is able to enter the sliding mode, hence $\sigma(t) = 0$.

Therefore the equivalent control, $V_{eq}(t)$ can thus be obtained

by letting $\dot{\sigma}(t) = 0$ [23].

Table 1: Parameters of half car model

Symbol	Value	Symbol	Value
M, I	580 kg, 910 kg.m ²	b_f^l, b_f^nl	1000 Ns/M, 500 Ns/M
m_f, m_r	40 kg, 30 kg	b_r^l, b_r^nl	1000 Ns/M, 500 Ns/M
k_{f1}^l, k_{f1}^{nl}	10000 N/M, 10000 N/M	a, b	1.25 m, 1.45 m
k_{r1}^l, k_{r1}^{nl}	10000 N/M, 10000 N/M	α, β	$4.515 \cdot 10^{13} \text{ N/m}^5, 1s^{-1}$
k_{f2}, k_{r2}	100000 N/M, 100000 N/M	γ	$1.545 \cdot 10^9 \text{ N/m}^{5/2} \text{ kg}^{1/2}$
t_d, τ	0.25 s, $\frac{1}{30}$ s	A	$3.35 \cdot 10^{-4} \text{ m}^2$
		P_s	10342500 pa

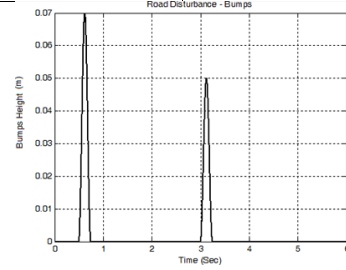


Fig. 2 Typical road disturbance.

We design the control scheme that drives the state trajectories of the system in Eq. (15) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter. For the system in Eq. (15), the following control law is proposed:

$$V(t) = -(CB)^{-1} C(f + DW) - (CB)^{-1} (CE)X(t) - (CB)^{-1} \phi \sigma(t) - (CB)^{-1} \varepsilon \cdot \text{sign}(\sigma(t)) \quad (46)$$

where $\phi \in \mathfrak{R}^{m \times m}$ is a positive symmetric design matrix and ε is the positive constant.

In order to fulfill the objective of designing an active suspension system, i.e., to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and the wheel deflection.

Fig. 3 shows the suspension travels of both controllers for an active suspension system and a passive suspension system for comparison purposes. The result shows that the suspension travel within the travel limit, i.e., $\pm 8cm$ and the result also shows that the active suspension utilizing the proposed controller, performs better as compared to the others.

Fig. 4 illustrates clearly how the proposed controller, can effectively absorb the vehicle vibration in comparisons to the PISMC method and the passive system. The body acceleration in the proposed control design system is reduced significantly, which guarantee better ride comfort.

In this paper, the peak values of the vertical acceleration are also presented. These values indicate the maximum magnitudes of the related acceleration experienced by the vehicle body or passenger. The peak values are calculated as:

$$\|X\|_{\infty} = \max(|x_i|) \quad i = 1, \dots, n \quad (47)$$

Here, $\|\cdot\|_{\infty}$ is the ∞ -norm. Peak values for the vertical acceleration are depicted in Fig. 5 and Fig. 6. As seen, the peak values are substantially decreased by the proposed controller. Fig. 7 shows that the wheel deflection is also smaller using the proposed controller. Therefore it is concluded that the active suspension system with the proposed control improves the

ride comfort while retaining the road handling characteristics, as compared to the PISMIC and the passive suspension system.

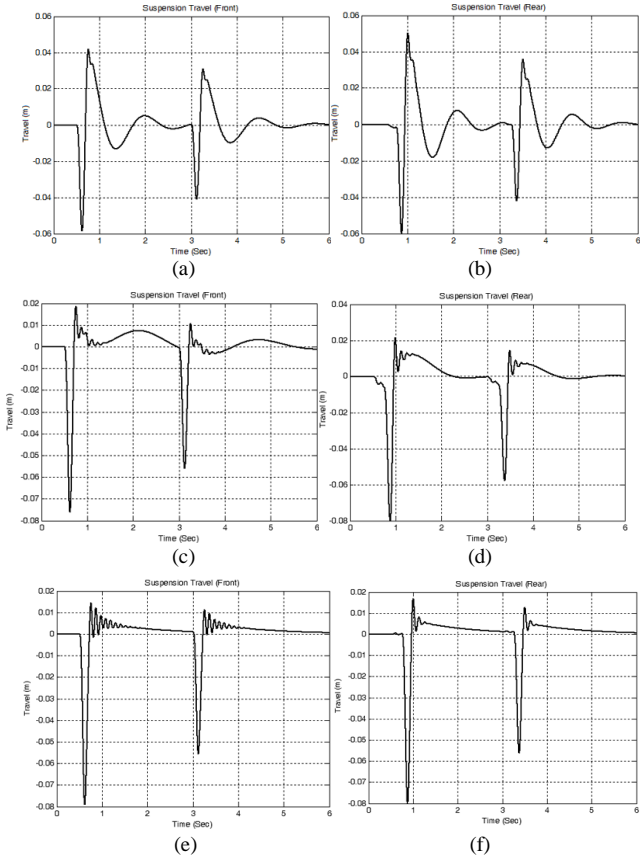


Fig. 3 The suspension travels: (a) Front passive suspension, (b) Rear passive suspension, (c) Front suspension by the PISMIC, (d) Rear suspension by the PISMIC, (e) Front suspension by the nonlinear optimal controller and (f) Rear suspension by the nonlinear optimal controller.

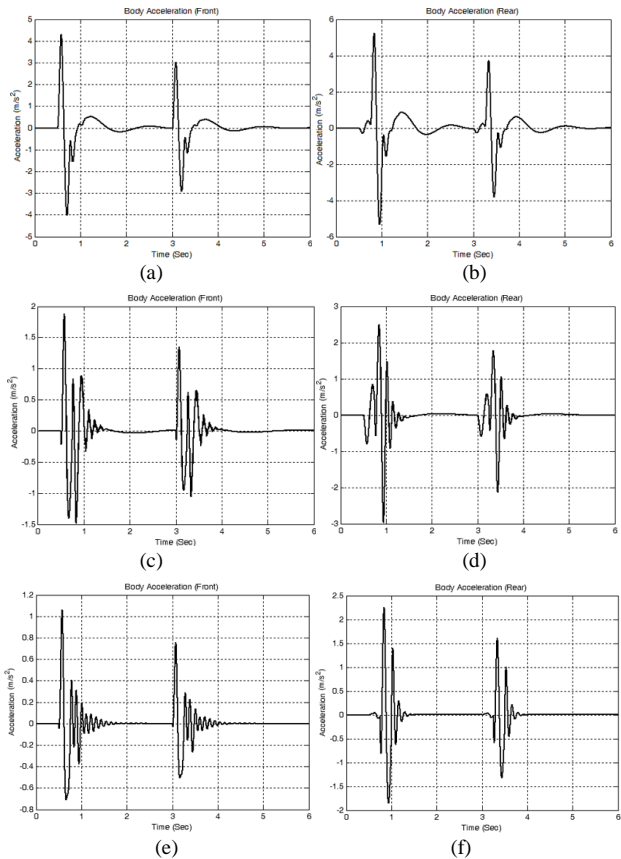


Fig. 4 The body accelerations: (a) Front passive suspension, (b) Rear passive suspension, (c) Front suspension by the PISMIC, (d) Rear suspension by the PISMIC, (e) Front suspension by the nonlinear optimal controller and (f) Rear suspension by the nonlinear optimal controller.

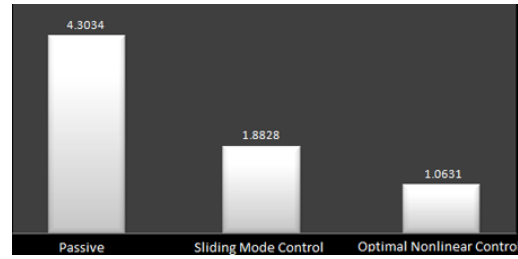


Fig. 5 The peak values for vertical accelerations of front suspension.

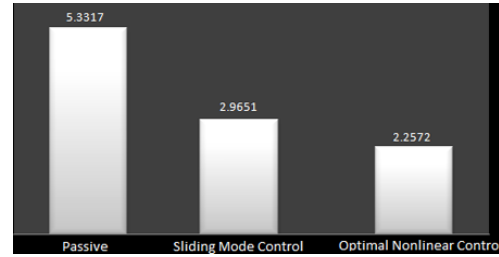


Fig. 6 The peak values for vertical accelerations of rear suspension.

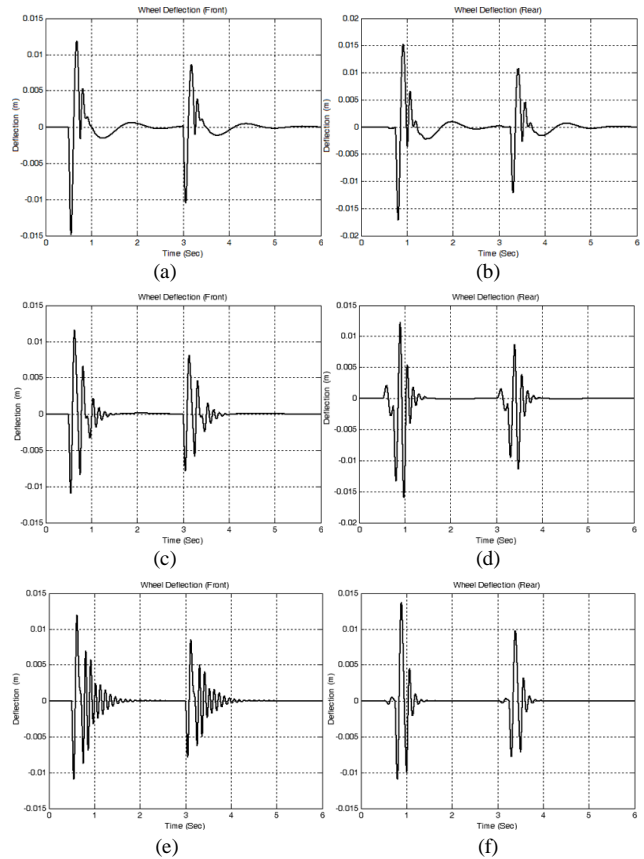


Fig. 7 The wheel deflections: (a) Front passive suspension, (b) Rear passive suspension, (c) Front suspension by the PISMIC, (d) Rear suspension by the PISMIC, (e) Front suspension by the nonlinear optimal controller and (f) Rear suspension by the nonlinear optimal controller.

V. CONCLUSION

Active suspension designs must resolve the inherent tradeoffs between ride quality, handling, suspension travel and power consumption. According to these aims, an optimization law is developed for suspension system control

based on the states estimation and states prediction of a non-linear half car model. The proposed control law minimized the states tracking errors and led to a special case of feedback linearization. The proposed optimal non-linear control law is given in an analytical closed form that is easy to solve and online optimization is not necessary. Our proposed controller applied on sedan half car model and compared with passive suspension system and active suspension system by the PISMIC method. The obtained results demonstrated that the body acceleration and wheel deflection of our proposed controller was better than passive suspension system and active suspension system by the PISMIC method. Also control signals of nonlinear optimal controller were less than the PISMIC method. The results demonstrated that our controller indeed proved to be effective in the inherent tradeoff between ride quality and suspension travel as compared to the PISMIC method and the passive suspension system.

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