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# A new optimisation procedure to design minibus services: an application for the Lisbon Metropolitan Area 

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#### Abstract

This paper focuses on the formulation of a Mixed Integer Linear Program (MILP) that was used in the design of a new alternative intermediate transport mode and examines the viability of its implementation on the Lisbon Metropolitan Area. Due to the high combinatorial nature of the problem, the modelling process was divided into four phases: estimation of the potential demand, definition of stops, determination of the service $\mathrm{O} / \mathrm{D}$ matrix and establishment of the most profitable routes. The developed algorithm is able to generate solutions for large areas, although limited by the number of stops considered in the routing phase.


Keywords: Minibus services, linear optimization, P-median problem, Vehicle Routing Problem.

## 1 Introduction

The Vehicle Routing Problem (VRP) has always been an important topic in Operations Research. This problem has been thoroughly explored, being the base for a wide range of logistic and transportation optimisation applications (Bodin, Golden, Assad, \& Ball, 1983). Many authors have tried different specifications in order to develop more efficient algorithms that are able to deal with the NP-Completeness of this problem (Dell'Amico, Monaci, Pagani, \& Vigo, 2007).

Among this set of problems, the Vehicle Routing and Scheduling Problem with Time-Windows constraints (VRSPTW) formulation tries to establish the optimal configuration of the fleet and also their routes, with fixed starting and finishing points, where it has to cover an a priori set of stops that have to be visited within a certain time window. This problem is highly NP-Complete and several heuristics have been developed: one possible approach is to try to compute several routes at the same time (parallel procedure); another option is to adopt a more greedy approach and establish routes in a sequential procedure (Solomon, 1987).

[^0]The School Bus Routing Problem (SBRP) presents a similar formulation as the VRSPTW. Yet, the definition of the concentration points for children pick-up/drop-off is part of the optimisation problem. Some authors have formulated the problem in several stages (e.g. location of stops, children assignment, bus routing and scheduling) (Park \& Kim, 2010), while others have formulated it in a single stage using a location-distribution problem (Nagy \& Salhi, 2007). Typically, these algorithms consider that a bus can only serve a single depot, although in some other formulations the same vehicle can serve several schools, increasing highly the complexity of the problem.(Park \& Kim, 2010).

Another typical VRP based problem is the Team Orienteering Problem (TOP). Here, the location of stops is also defined but not all of them have to be visited. The objective is to cover the most "profitable" stops within a limited time interval (Chao, Golden, \& Wasil, 1996).

The formulation presented in this paper falls within the same family of the previous problems. Nonetheless, some specificities of our new transport service under analysis required considerable adaptations to the base problem. In our system the set of stops that are going to be served are not established and depend on the benefit that they generate. Also, the trips are constrained to an initial and end time limit that cannot be exceeded.

We intend to develop a new modelling tool having as test bed the Lisbon Metropolitan Area (LMA).
This paper is organised into 5 main sections: after this initial literature review we specify our service; afterwards, we present the general framework of the developed methodology; and a detailed description of all the different algorithms used in each phase; followed by the outputs of a small application; and a discussion about the added value and next developments to this formulation.

## 2 The Express Minibus service specification

The transport service that was tested in this model presents the following characteristics:

- Makes use of low capacity buses ( 8,16 and 24 seats);
- Fixed routes and schedules;
- High commercial speed, similar to the private car travel time, considering a maximum increase relatively to the private vehicle real travel time (Table 1);
- Number of stops in each route is limited by the total travel time that cannot exceed the limits explained above;
- The structural costs of this service include all the usual items of a transportation service (drivers, backoffice staff, rolling stock, maintenance facilities, office space, etc.).

Table 1 Travel time tolerances of the Minibus

| Estimated Travel Time by Private Vehicle | Minibus' Travel Time (TT) maximum tolerance |
| :---: | :---: |
| Less than 10 minutes | +5 minutes |
| Between 10 and 45 minutes | $5+$ TTReal x $(1 / 7)[$ minutes $]$ |
| Over 45 minutes | $10+$ TTReal x $(1 / 9)[$ minutes $]$ |

The revenues of this service are only based on the tariff charged to the customers. Two different tariff specifications were tested:

- A fixed tariff system where the users is charged for a single ticket for the all trip;
- A tariff system similar to the traditional taxi, where the user has a base fee to which is added a variable cost dependent on the number of kilometres travelled.


## 3 Problem formulation

As already stated, we have developed a methodology that enables not only the estimation of the potential demand of this new service but also the design of the service's routes (stops and schedules).

This problem is a NP-complete problem as it presents a high complexity and a large set of decision variables. Therefore, we were not able to follow a holistic approach and we had to divide it into four different phases:

- An initial phase which assessed the behaviour of potential users in the study area, using a decision tree model to estimate the willingness of travellers to use this new service, based on attributes of their current trip chain configuration and lifestyle. This phase was designated as Demand Estimation.
- After this phase, we defined the potential location of the stops of the system, based on the estimated potential demand of the different places in the study area and demand periods. This phase was designated as Stops Location.
- The following phase of the model estimated the potential demand of each link between the defined Minibus stops, and set the system potential O/D matrix. This phase was designated as Minibus Link Load Estimation.
- The last phase of the model, computed the most profitable Minibus routes for the given O/D matrix, and defined the path of each vehicle and its occupation during the analysis time period. This phase was designated as Minibus Routing.
The model considers as possible locations for stops all the census block centroids ( 32,763 in the LMA) and a discretisation of time in blocks of 15 minutes.

Our methodology will not consider trips, but the probability of a trip to be performed in the new transport mode proposed for a given demand scenario, therefore resulting in real and not integer demand estimates.

## 4 Demand Estimation

The demand data used in the model was generated through a synthetic travel simulation model developed and calibrated for the Lisbon Metropolitan Area (LMA), which generated 4,827,642 daily trips inside the LMA (José M. Viegas \& Martinez, 2010). This dataset was then filtered to reduce it to a subset of trips to which this new service may fit. As it intends to be self-sustainable, we searched for high demand periods. Therefore, we only used trips that were done in peak periods, where, for this assessment, we selected the morning peak (7:00 am to 10:00 am). We also had to select $\mathrm{O} / \mathrm{D}$ pairs of parishes that had a minimal number of trips ( 1,000 in this case), non walking trips and trips longer than 2,000 meters (considered too short for the Minibus service). This filter led to a reduction of $91 \%$ of the total daily LMA trips, where $51 \%$ of them are done in private vehicle.

As the proposed service was not present in the original dataset, we estimated its potential demand using a behavioural model. The best option would be to use a stated preferences survey, including this mode as an option, to calibrate a discrete choice model, which was not available. Though, we followed a simplified methodology that tried to measure the impacts of some attributes over the mode choice selection. This methodology encompassed two stages: in the first stage we identified the main variables that could have more influence in the willingness to change to this new service (number of daily trips, public pass ownership, daily distance travelled, car availability, daily activity time and the existence, or not, of a non commute subway trip); in the second stage we used a simplified Delphi method to estimate the weight that each selected attribute would have on the choice, considering a likert scale from $A$ to $E$ (high to low willingness to change).

The willingness to change distribution of the entire data base is summarised in Table 2 where we can observe a low percentage of trips that would easily shift to this new service ( $\mathrm{A}+\mathrm{B}$ less than $30 \%$ ) and a high number of trips with very low probability to change.

Table 2 Percentage of the willingness to change to the Minibus service

| Classes | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Percentage [\%] | 2.05 | 26.94 | 27.32 | 14.24 | 29.45 |

Depending on the probability associated to each level of the likert scale, we were able to test different scenarios. We will just present one demand scenario that considers a linear willingness variation between $90 \%$ and $10 \%$, within the scale.

The questionnaire presented at the Delphi Method, apart from the personal aggregated mobility characteristics, did also assess the willingness of a person to migrate to the Minibus service based on the current trip mode used and trip purpose (see Table 3 and Table 4). These probabilities were then considered independent of the other attributes and introduced as correction factors to the above estimates.

The application of all this reduction resulted in an initial set of trips of $81,652.96$ ( $18.94 \%$ of the initial trips).

Table 3 Probability of change according to the transport mode

| Mode | Reduction |
| :--- | :---: |
| Ferry | 0.900 |
| Other Transport | 0.420 |
| Private car | 0.750 |
| Suburban Bus | 0.750 |
| Subway | 0.300 |
| Taxi | 0.100 |
| Train | 0.600 |
| Urban Bus | 0.380 |

Table 4 Probability of change according to the trip purpose

| Purpose | Reduction |
| :--- | :---: |
| Commuting (Leave home) | 1.000 |
| Commuting (Return home) | 1.000 |
| In service | 0.000 |
| Meal | 0.075 |
| Other | 0.500 |
| Personal matters | 0.400 |
| Pick/Drop familiar | 0.050 |
| Shopping/Leisure | 0.200 |

## 5 Location of Stops

In this phase, we intended to estimate the location of the potential Minibus' stops, where the centroids of the census blocks were used as the potential stops locations. Due to the high complexity and combinatorial scale of the problem, given the high number of census blocks, we opted to use a "Divide and Conquer" heuristic. In the divide step we developed a clustering algorithm using the Euclidean distance and in the conquer step we used the formulation of a location problem.

### 5.1 Divide step

The goal of this step was to divide the original area into small sub-areas, where the stops' location problem was computable. Our clustering algorithm was constrained by the following:

1. The number of members (city blocks) in each group could not exceed 200 elements;
2. The distance from each object to the centre of the cluster had to be less than 2000 meters;
3. The highest distance between members of a cluster had to be lower than 2500 meters.

To compute the distances between the elements of the cluster, we used the Euclidean distance constrained by the existence of physical barriers which could prevent a direct connection, in space, between two census blocks. There might be census blocks that physically are close, but blocked by physical geographic separators placed on the territory as railways, rivers, motorways, etc., as well as common urban mobility barriers, which may force walking around obstacles. For this purpose, we considered, as barriers, the existing highways, motorways and railroads.

In our formulation, a barrier is formed by a set of linear segments that are linked together and only allow the passage of people on the end points of the barrier. Our algorithm uses a shortest path algorithm formulation (Dijkstra algorithm), inspired in the work of J. M. Viegas \& Hansen (1985), where the graph is not an input but it is formed during the run of the algorithm. When calculating the distance between two elements, the algorithm starts by trying a direct connection. If it succeeds, it will return the Euclidean distance; otherwise, if the connection intersects one or more barriers, the algorithm will introduces in the graph, the ends of each barrier it has intersected, and introduce them in a "labelled" nodes array. Then, the algorithm tries to reach the first labelled node in the graph and two things might happen: either it is able to reach it and evaluates if the distance to the origin node is less than the current estimate, and the reached node is added to a "scanned" array; or, there is one or more barriers between these new pair of nodes and so, the algorithm adds more nodes to the graph that become new possible destinations of the current origin and labels them as possible targets.

This process is repeated until all possible nodes have been "labelled". Afterwards, the algorithm advances to the first possible "scanned" node and repeats the all process again. The algorithm stops when the number of "labelled" nodes is equal to the number of the "scanned" nodes already used in the algorithm. After this procedure, it is possible to know the shortest distance between the two initial nodes.

### 5.2 Conquer phase

The selection of the potential stops of the service was based on an adaptation of the p-median algorithm, commonly used in location problems (Melkote \& Daskin, 2001), where the p value was not set, and a cost of stops' formation was added to the objective function. Instead of minimising the distance to the p nodes, the algorithm
maximised the "potential demand" that could be generated by the creation of a stop, measured by the number of people that could potentially use it.

The problem was formulated considering a discrete definition of time ( 12 periods of 15 minutes), and a maximum walking acceptability by customers of 1250 meters (approximately a 15 minutes' walk). It was also defined that the stops would have to be separated by a minimum distance, to avoid the creation of too many close stops, which would degrade the performance of the service that intends to be faster than a regular bus service.

The mathematical formulation of the problem is described below:
Sets: $N=\{1, \ldots, C\}$ set of all the available nodes where a Minibus Stop can exist and also represents the location of the origins and destinations of the flows, where $C$ is the maximum number of nodes; $A=\{1, \ldots, K\}$ set of all the available arcs between nodes where K is the maximum number of arcs; $\mathrm{T}=\{1 \ldots, 12\}$ set of all the considered time steps.
Decision variables: $Y_{i_{t}}$ : binary variable for the existence of a stop in node $i$, at time step $t$, where $i_{t} \in \mathrm{~N} ; X_{i_{i} y_{t}}$ : continuous variable that represents the percentage of people who come from node $i$, at time step $t$, and use the stop $y$ for their trip, where $i_{t}, y_{t} \in \mathrm{~N} ; T_{i}$ : binary variable responsible for the existence of a stop in the node $i$, in any time step, where $i \in \mathrm{~N}$.
Data: $D_{i j}$ : matrix that represents the walking distance between each pair of network nodes where $i, j \in \mathrm{~N} ; L_{a_{i_{t}}{ }^{\prime}}$ : matrix that represents the demand in arc $a$ with the starting point of the trip in the node $i$, at time step $t$, where $a_{t}$ $\epsilon \mathrm{A}$ and $i_{t} \in \mathrm{~N} ; P_{i_{t}}$ : vector that represents the number of arcs, at time step $t$, that have its origin in the node $i_{t}$ where $i_{t} \in \mathrm{~N} ;$ Tot $_{i}$ : vector that represents the number of arcs that leave from node $i$, regardless of the time step, where $i \in \mathrm{~N}$.
Constants: $d_{\max }$ : maximum walking distance that is acceptable between a trip starting/end point and the origin/destination stop location.

With this notation, the objective function is described by the following expression:

$$
\begin{equation*}
\operatorname{Max}(P)=\sum_{i_{i}, y_{t} \in N, a_{t} \in A} L_{a_{i_{t}}} X_{i_{4} y_{t}}-\sum_{i \in N} T_{i} \times N S P \times \operatorname{Tot}_{i}-\sum_{i_{t} \in N} Y_{i_{t}} \times N S T \times P_{i_{t}} \tag{1}
\end{equation*}
$$

NSP and NST are parameters of the model, where NSP represents the average number of passengers that have to exist in order to justify the creation of a new stop and NST stands for the minimal number of passengers, in a fixed time step, that justify the creation of an additional stop.

This function maximises the potential demand generated in an area with the least stops possible in all the existing time steps and takes into consideration three components:

1. The number of people that use the defined stops;
2. A deduction considering the cost of creating a stop;
3. A penalty for using a different set of stops in different time steps, aiming to homogenise the location of the stops in each area across the period of operation.
The solution space is constrained by the following equations:

$$
\begin{equation*}
\sum_{y_{t} \in N} X_{i_{t} y_{t}} \leq 1 \forall i_{t} \in N, \forall t \in T \tag{2}
\end{equation*}
$$

Ensures that the demand is not exceeded;

$$
\begin{equation*}
X_{i, y_{t}} \leq Y_{y_{t}} \forall i_{t}, y_{t} \in N, \forall t \in T \tag{3}
\end{equation*}
$$

Guarantees that the demand is only allocated to existing stops;

$$
\begin{equation*}
T_{i} \geq \frac{1}{12} \times \sum_{t \in T} Y_{i_{t}} \forall i \in N \tag{4}
\end{equation*}
$$

Ensures that the variable $T_{i}$ exists if the node $i$ is used as a stop at any time step;

$$
\begin{equation*}
\left(T_{i}+T_{j}\right) \times\left(D_{\mathrm{ij}}-D_{\min }\right) \geq D_{\min } \times\left(T_{i}+T_{j}-2\right) \forall i, j \in N \text { and } i \neq j, \forall t \in T \tag{5}
\end{equation*}
$$

Assures that the existing stops are separated, at least, by $D_{\min }$ meters (a parameter to be adjusted).

## 6 Minibus Link Load Estimation

After having defined the potential demand of our service and its concentration points, we were able to calculate the potential O/D matrix of our service. In this step, we took into consideration that the probability of a person using a stop is inversely proportional to its distance. To model this distance decay effect, we used an inverse logistic function taking values between land 0 , where the references for calibration was a reduction of 0.9 , if a person had to walk 5 minutes until the stop, and a reduction of 0.1 if the stop was at a 15 minutes walking distance, based on the calibration of these parameters for public transport accessibility studies previously performed (Martínez \& Viegas, 2009). The demand degradation was considered both at the origin and destination stop that might also not be the final destination of the trip. Prior to the assignment of the demand to the Minibus' stops $\mathrm{O} / \mathrm{D}$ pair, the algorithm assesses the estimated travel time between the stops in order to verify that both the origin and destination stops exist on compatible time intervals, taking into account the access time to the stop, plus travelling time and walking time to the final destination, using a discrete time specification (from 1 to 12 time intervals). The walking distance was again computed considering the existence of physical barriers.

As a person might be accessible to several stops, the demand was proportionally distributed among the possible O/D pairs.

## 7 Minibus routing

The Minibus routing problem was formulated as a variation of the Vehicle Routing problem (VRP), where, due to the lack of computer's memory in the branch and bound procedure, the route and schedule of each vehicle was computed independently following a greedy approach over the potential demand, stopping when the first non profitable route was generated.

The aggregation of the several greedy estimates may not lead to the global optimum of the system. Yet, the result may be appropriate for a critical analysis of the potential benefit of such a service in the LMA.

As the potential demand was initially considered inelastic to the price of the service and the model will test several tariff specifications, we introduced a demand-price elasticity model. This elasticity was estimated, once again, through an inverse logistic function. This function was calibrated using, as reference, the current daily trip cost of a public transport pass holder as a lower bound, with a $98 \%$ of price acceptance, and the equivalent taxi trip price as the upper bound, with a $2 \%$ of price acceptance. This relation is dependent on the trip's nature, due to the distance and time dependence on taxi's fare, which, in the LMA, adds a component to the fare when travelling under $30 \mathrm{~km} / \mathrm{h}$. In order to establish an association between the inverse logistic function parameters and the travel distance, we estimated a linear relation between travel distance and the inverse logistic function parameters based on: a regression between the trip distance and the average speed of the taxi and a regression between the average speed and the percentage of time under $30 \mathrm{~km} / \mathrm{h}$.

The model presents the following formulation:
Sets: $\mathrm{N}=\{1, \ldots, \mathrm{C}\}$ set of all the available nodes where a Minibus Stop may exist and also represents the location of the origins and destinations of the flows, where $C$ is the maximum number of nodes; $A=\{1, \ldots, K\}$ set of all the available arcs between nodes, where K is the maximum number of arcs; $\mathrm{T}=\{1, \ldots, 12\}$ set of all the considered time steps;
Decision variables: $M B_{a_{t}}$ : continuous variable that represents the number of people that are travelling in arc $a$, at time step $t$, inside the Minibus, where $a_{t} \in \mathrm{~A} ; E_{a_{t}}$ : binary variable responsible for the existence of a trip in the arc $a$, in each time step $t$, by the Minibus, where $a_{t} \in \mathrm{~A} ; P_{a_{t}}$ : continuous variable that quantifies the percentage of the total arc's demand assigned, at time step $t$, inside the Minibus, where $a_{t} \in A ; P_{a_{t}}^{\prime}$ : binary variable that represents the existence, or not, of demand in arc $a$, in time step $t$, where $a_{t} \in \mathrm{~A}$.
Data: $D_{i j}$ : matrix that represents the distance between each pair of network nodes where $i, j \in \mathrm{~N} ; T T_{a}$ : vector that represents the number of quarters of hour that arc $a$ takes to travel, where $a \in \mathrm{~A} ; \operatorname{TTReal}_{a_{t}}$ : vector that represents
the real travel time of arc $a$, where $a \in \mathrm{~A} ;:$ vector that represents the potential demand in the arc $a$, at time step $t$ where $a_{t} \in \mathrm{~A}$; Elast ${ }_{a}$ : continuous variable, between 0 and 1, that represents the percentage of clients of arc $a$ that are willing to use the service for the given tariff (dependent of $D_{i j}$ as presented above on the price demand elasticity index), where $a \in \mathrm{~A} ; C_{a}$ : vector that represents the maximum additional time accepted by clients when comparing the service with the car travel time, where $a \in \mathrm{~A}$;
Constants: Cap $_{\text {bus }}$ : maximal capacity of a single Minibus; Tariff ${ }_{\text {bus }}$ : fixed component of the ticket price charged to the passengers for a single trip; Tarifv ${ }_{\text {bus }}$ : variable fee charged to the user by kilometre travelled; Costf $f_{\text {bus }}$ : fixed cost of operation of the Minibus; Costv bus : variable cost of each kilometre travelled by each Minibus.

With this notation, the objective function is described by the following expression:

$$
\begin{align*}
& \operatorname{Max}(\text { Balance })=\text { Tariff }_{\text {bus }} \times \sum_{a_{t} \in A, t \in T-1} P_{a_{t}} \times \text { Elast }_{a} \times \text { Proc }_{a_{t}}+ \\
& + \text { Tarifv }_{\text {bus }} \times \sum_{a_{t} \in A, t \in T-1, i, j \in \mathrm{~N}} D_{i j} \times P_{a_{t}} \times \text { Elast }_{a} \times \text { Proc }_{a_{t}}-  \tag{6}\\
& -\sum_{a_{t} \in A, t \in T-1, \mathrm{i}, \mathrm{j} \in \mathrm{~N}}\left(\operatorname{Costv}_{\text {bus }} \times D_{i j} \times E_{a_{t}}+\text { Costf }_{\text {bus }}\right)
\end{align*}
$$

Where $i$ and $j$ are, respectively, the origin and destination of arc $a$.
This equation maximizes the total profit in all the existing time steps and takes into consideration two parts:

1. The profit obtained per user, where two possible fare systems can be tested: a fixed fare component and a distance dependent tariff, where, while running the fixed fare plan price, the variable part is not considered;
2. The cost of using a Minibus.

This solution space is subject to the following constraints:

$$
\begin{equation*}
M B_{a_{t}} \leq E_{a_{t}} \times \operatorname{Cap}_{\text {bus }} \forall a_{t} \in A, \forall t \in T \tag{7}
\end{equation*}
$$

Ensures that the capacity of a Minibus is not exceeded;

$$
\begin{equation*}
\sum_{a_{t} \in A} E_{a_{t}} \leq 1 \forall t \in T \tag{8}
\end{equation*}
$$

Guarantees that one Minibus at a time step is in a single arc. This constraint introduces a limitation to the system preventing that one Minibus may make more than one stop in the same time step ( 15 minutes). This simplification was not considered to be relevant because of the nature of the system, which favours routes with few stops;

$$
\begin{equation*}
P_{a_{t}} \leq 1 \forall a_{t} \in A, \forall t \in T-1 \tag{9}
\end{equation*}
$$

Assures that the real demand for every arc is not exceeded;

$$
\begin{equation*}
M B_{a_{t}} \geq \text { Elast }_{a} \times \text { Proc }_{a_{t}} \times P_{a_{t}} \forall a_{t} \in A, \forall t \in T \tag{10}
\end{equation*}
$$

Ensures that the passengers at a stop that are travelling to one of the Minibus' destinations get in;

$$
\begin{equation*}
E_{a_{t}} \leq \sum_{b_{t} \in A, t \in t+T T_{\mathrm{a}} \leq 13} E_{b_{t+1 T_{a}}} \forall t \in T, \forall a_{t} \in A \tag{11}
\end{equation*}
$$

where the destination of arc $a_{t}$ is the same as the origin of arc $b_{t}$ in the respective time step. This function warrants that the passengers that enter in one stop are the same, or less, that exist in the Minibus' destination at the time step plus the travel time between stops. The time limit corresponds to 13 intervals to give the possibility of a Minibus finishing his route despite it is outside the defined period. We considered that concluding the route a maximum of 15 minutes after the defined hour would be acceptable;
where the destination of arc $a_{t}$ and $b_{t}$ are the same in the respective time steps and where $-1+\operatorname{round}\left(\left(\frac{C_{a}}{60}+T_{\text {RReal }}^{a} a\right) \times 4\right)$ represents, respectively, the upper limit of the 15 minutes intervals. This function assures that the passengers travelling only exist if the path exists.

$$
\begin{equation*}
P_{a_{t}^{\prime}}^{\prime} \leq \sum_{\left.b_{h} \in A, \text { het.t.1-1+round }\left(\frac{C_{0}}{60} 7 T \text { Reala }\right)_{a} \times 4\right) \leq 13} E_{b_{n}} \quad \forall t \in T, \forall a_{t} \in A \tag{13}
\end{equation*}
$$

where the destination of arc $a_{t}$ and $b_{t}$ are the same in the respective time steps and where $-1+\operatorname{round}\left(\left(\frac{C_{a}}{60}+\right.\right.$ TTReal $\left.\left._{a}\right) \times 4\right)$ represents, respectively, the upper limit of the 15 minutes intervals. This constraint is not a service constraint but a model workflow constraint which warrants that whenever demand is assigned to an arc, the travel time tolerance will be measured.

$$
\begin{equation*}
\sum_{b_{h} \in A, \text { h } \in \text { t.t } \mathrm{t}+\operatorname{round}\left(\left(\frac{C_{a}}{60}+\mathrm{TTReal}_{\mathrm{a}}\right) \times 4\right) \leq 13} E_{b_{h}} \times \operatorname{TTReal}_{b} \leq \operatorname{TTReal}_{a}+\frac{C_{a}}{60}+M \times\left(1-P_{a_{t}}^{\prime}\right) \forall t \in T, \forall a_{t} \in A \tag{14}
\end{equation*}
$$

Ensures that the passenger's travel time is below the tolerable limits. The value of $M$ was set to a high value $(1,000)$, used as a bonus value to ensure that only ares with demand will respect the constraint;

## 8 An application for the Lisbon Metropolitan Area

In this section we present an application of the model to the LMA, focusing on the outputs of the main components of the model: the location of stops and the routing phases. The purpose of this application is to show the different outputs that can be obtained by the model, and its ability to deal with a large study area.

The results from the location of stops phase, showed the formation of 531 clusters or sub-areas of analysis to run in the stop location problem. The average size of the clusters obtained was 471.02 ha. and an average distance to the cluster's centroids of 587.24 m .

After this divide phase, we computed the stop location model for the obtained sub-areas using as model parameters NSP $=0.301$, NST $=0.362$ (cost of stop formation in the objective function) and $\mathrm{d}_{\min }=500 \mathrm{~m}$ (minimum distance between formed stops). The aggregate results produced 1,365 stops for the LMA. The spatial distribution of the results is presented in Figure 1.

After the definition of the location of the potential stops for the service, we computed the O/D matrix. Due to the high spatial combinatorial nature of our problem, after the run of this part of the algorithm, our potential demand was reduced to $6.16 \%$.

Prior to the calculation of the last phase of the model, we observed that the total number of potential stops was too large to be computed by the routing algorithm. In order to test the viability of the model we developed a reduced stops' configuration, selecting 22 stops that were located in high demand concentration areas and sufficiently apart from each other. This selection was performed using a ranking index for the stops, which took into consideration the potential of a stop aggregating all the potential demand in the neighbouring area. The used set of stops is also presented in Figure 1.

The model was then run for the two designed fare systems: the fixed tariff and the taxi tariff. The reference values used to estimate them, for each O/D pair, were obtained through cost data provided by private transport operators in the LMA. The prices of the fixed tariff system for the Minibus with 8,16 and 24 seats were $2.94 €$, $1.74 €$ and $1.21 €$, respectively. In the taxi tariff system, the fixed component of the Minibus fare was $0.77 €, 0.45 €$ and $0.31 €$, and the variable component was $0.109 € / \mathrm{km}, 0.066 € / \mathrm{km}$ and $0.046 € / \mathrm{km}$, respectively.

In the fixed tariff scheme, the algorithm generated 11 routes (Figure 2 presents an example), the first one with a 24 seats Minibus and the remaining 10 with 16 seats. The total morning peak period profit was $552.97 €$, serving 908.90 passengers.

In the taxi tariff scheme, it was generated 28 different Minibuses routes (Figure 3 presents an example), mainly with 8 seats but some of them with 16 , and the total profit was $552.68 €$, serving 1437.29 passengers.


Figure 1 Spatial distribution of the stops generated by the model and the selected set for the presented application.


Figure 2 Example of a fixed tariff route.


Figure 3 Example of a taxi tariff scheme route.

## 9 Results discussion and future developments

This paper illustrates the development of a new modelling tool to assess the viability of the implementation of a new transport solution in an exploratory phase. The results obtained suggest that the proposed service might be selfsustainable. To withdraw more robust conclusions prior to the system deployment, a stated preferences survey should be developed to obtain a more accurate estimation of the potential demand.

The developed model was based on traditional Operations Research models, which were reformulated and adapted to the current context and linked under a common framework. The presented modelling formulation goes significantly beyond the current literature and practice on the design of a new public transport service, which barely use optimisation tools in their planning process. The model presents a high complexity level, with a large number of decision variables that range from the definition of the stops' location to the routing process. Nevertheless, the current model presents some limitations in terms of computational capability, which led to several simplifications and sub-optimal results. Further investigation in developing more efficient algorithms and heuristics should be addressed in the future.

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