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ADDENDUM TO "A NEW PROOF OF THE EXISTENCE OF (q^2-q+1)-ARCS IN $PG(2,q^2)$"

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We use the same notation as in [1]. G.L. Ebert [2] pointed out to us a mistake in the proof of Theorem 2 of [1]. As a matter of fact, it not true that the Hermitian curve in $PG(2,\mathcal{F})$ with equation

$$\bar{\mathcal{H}} : YZ^q + ZX^q + XY^q = 0$$

is fixed by the collineation group G. However, the proof still works if G fixes at least one non-degenerate Hermitian curve in $PG(2,\bar{\mathcal{F}})$, since the argument does not depend on the canonical equation of $\bar{\mathcal{H}}$. To determine such a Hermitian curve in $PG(2,\bar{\mathcal{F}})$, we show how to obtain a linear collineation τ in $PG(2,\bar{\mathcal{F}})$ mapping $\bar{\mathcal{H}}$ to a non-degenerate Hermitian curve $\hat{\mathcal{H}}$ fixed by G. To do this, we notice at first that the linear collineation group fixing $\bar{\mathcal{H}}$ is isomorphic to $PGU(3,q^2)$ and hence it has a cyclic subgroup Γ of order q^2-q+1. Actually, Γ is a subgroup of a Singer cyclic group Σ of $PG(2,q^2)$. We may choose a generator σ of Σ with the matrix representation

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{pmatrix}$$

where $x^3 - cx^2 - bx - a$ is an irreducible polynomial over $GF(q^2)$. Let τ be the linear collineation in $PG(2,q^6)$ associated with the matrix

$$T = \begin{pmatrix} 1 & 1 & 1 \\ \zeta & \zeta^{q^2} & \zeta^{q^4} \\ \zeta^2 & \zeta^{2q^2} & \zeta^{2q^4} \end{pmatrix}$$

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where ζ is a primitive element in $GF(q^6)$. It is easy to check that $T^{-1}CT$ is the diagonal matrix $\mathcal{D}$ =diag(ζ,ζ^{q^2},ζ^{q^4}). By putting $\omega = \sigma^{q^2+q+1}$, we see that $\tau^{-1}\omega\tau$ is represented by the matrix $\mathcal{D}^{q^2+q+1}$ which turns out to be diag(ζ^{q^2+q+1},$\zeta^{q^4+q^3+q^2}$,$\zeta^{q^5+q^4+1}$). If we multiply this last matrix by $\zeta^{-(q^2+q+1)}$ and put $b = \zeta^{q^5+q^4-q^2-q}$, this matrix may be written in the form diag(1,b^{-q^2},b). On the other hand, the matrix diag(1,b^{-q^2},b) is a generator of the group G. As a consequence we have that G fixes the Hermitian curve $\hat{\mathcal{H}}$ in $PG(2,\bar{\mathcal{F}})$ which is the image curve of $\bar{\mathcal{H}}$ under the linear collineation τ. For the rest of the proof of Theorem 2, it is sufficient to replace $\bar{\mathcal{H}}$ by $\hat{\mathcal{H}}$.

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