A New PTS OFDM Scheme with Low Complexity for PAPR Reduction[1]

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Abstract

In this paper, we introduce a new partial transmit sequence (PTS) orthogonal frequency division multiplexing (OFDM) scheme with low computational complexity. In the proposed scheme, 2^n - point inverse fast Fourier transform (IFFT) is divided into two parts. An input symbol sequence is partially transformed using the first l stages of IFFT to generate an intermediate signal sequence and the intermediate signal sequence is partitioned into a number of intermediate signal subsequences. Then, the remaining $n-l$ stages of IFFT are applied to each of the intermediate signal subsequences and the resulting signal subsequences are summed after being multiplied by each member of a set of W rotating vectors to yield W distinct OFDM signal sequences. The one with the lowest peak to average power ratio (PAPR) among these OFDM signal sequences is selected for transmission. The new PTS OFDM scheme reduces the computational complexity while it shows almost the same performance of PAPR reduction as that of the conventional PTS OFDM scheme.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) system has been considered as one of the strong standard candidates for the next generation mobile radio communication system. By multiplexing a serial data symbol stream into a large number of orthogonal subchannels, the performance of OFDM system over frequency selective fading channels is better than that of the single carrier modulation system and the OFDM signals have efficient spectral bandwidth. One of the major drawbacks of OFDM system is that the OFDM signal can have high peak to average power ratio (PAPR). The high PAPR brings on the OFDM signal distortion in the nonlinear region of high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER).

Recently much work [1] – [4] has been done in developing a method to reduce the PAPR. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out-of-band radiation. Block coding [1], the encoding of an input data into a codeword with low PAPR is another well-known technique to reduce PAPR, but it incurs the rate decrease.

Selected mapping (SLM) and partial transmit sequence (PTS) [2] – [4] were proposed to lower the PAPR with a relatively small increase in redundancy but without any signal distortion. It is known that the PTS scheme is more advantageous than the SLM scheme if the amount of computational complexity is limited, but the redundancy of the PTS scheme is larger than that of the SLM scheme. As the number of subcarriers and the order of modulation are increased, reducing the computational complexity becomes more important than decreasing redundancy. The paper is organized as follows: in Section 2, OFDM system and PTS scheme are described. Section 3 introduces a new PTS OFDM scheme and discusses the computational complexity issue. The simulation results are shown in Section 4, and finally, the concluding remarks are given in Section 5.

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2. OFDM System and PTS Scheme

A. OFDM System

The OFDM signal sequence $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]^T$ using $N = 2^n$ subcarriers is expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{N} t}, \ \ 0 \le t \le N-1 \qquad (1)$$

where $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]^T$ is an input symbol sequence and t stands for a discrete time index. If we define $\mathbf{Q}_i^j = \mathbf{T}_j \mathbf{T}_{j-1} \cdots \mathbf{T}_{i+1} \mathbf{T}_i$, where $\mathbf{T}_i$ denotes $N \times N$ symmetric matrix representing $i-$th stage of IFFT, (1) can be written as

$$\mathbf{a} = \mathbf{Q}_1^n \mathbf{A}.$$

The PAPR of the OFDM signal sequence, defined as the ratio of the maximum divided by the average power of the signal, is expressed as

$$\text{PAPR}(\mathbf{a}) = \frac{\underset{0 \le t \le N-1}{\text{Max}} |a_t|^2}{\text{E}\left[|a_t|^2\right]}$$

where $\text{E}[\cdot]$ denotes the expected value [4].

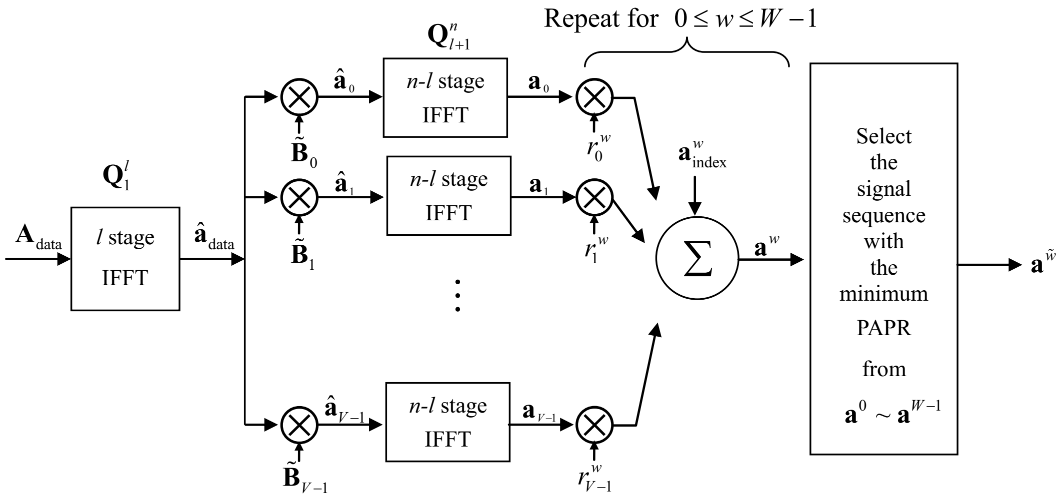

Figure 1: Block diagram of the new PTS OFDM scheme.

B. PTS OFDM Scheme

In PTS scheme, an input symbol sequence $\mathbf{A}$ is partitioned into V 'disjoint' symbol subsequences $\mathbf{A}_v = [A_{v,0}A_{v,1}\cdots A_{v,N-1}]^T,\ 0 \le v \le V-1$ as follows:

$$\mathbf{A} = \sum_{v=0}^{V-1} \mathbf{A}_v.$$

Here, the word 'disjoint' implies that for each given $k,\ 0 \le k \le N-1, A_{v,k} = 0$ except for at most a single v. The signal subsequence $\mathbf{a}_v = [a_{v,0}a_{v,1}\cdots a_{v,N-1}]^T$ is generated by applying inverse fast Fourier transform (IFFT) to each symbol subsequence $\mathbf{A}_v$, often called a subblock. Each signal subsequence $\mathbf{a}_v$ is then multiplied by a unit-magnitude constant r_v^w from a rotating vector $\mathbf{r}^w = [r_0^w r_1^w \cdots r_{N-1}^w]^T,\ 0 \le w \le W-1$ and summed to result in a PTS OFDM signal sequence $\mathbf{a}^w = [a_0^w a_1^w \cdots a_{N-1}^w]^T$ which can be expressed as

$$\mathbf{a}^w = \sum_{v=0}^{V-1} r_v^w \mathbf{a}_v.$$

The PAPR of $\mathbf{a}^{\mathbf{w}}$ is computed for each of W rotating vectors and compared. The one with the minimum PAPR is chosen for transmission. The index $\tilde{w}$ of the corresponding rotating vector $\mathbf{r}^{\tilde{w}}$ is expressed as

$$\tilde{w} = \underset{0 \le w \le W-1}{\operatorname{argmin}} \underset{0 \le t \le N-1}{\operatorname{Max}} \left| \sum_{v=0}^{V-1} r_v^w a_{v,t} \right|.$$

The subblock partitioning sequence is defined as a sequence $\mathbf{S} = [S_0 S_1 \cdots S_{N-1}]^T, S_k \in \{0, 1, \cdots V-1\}$ such that $S_k = v$ if $A_{v,k} = A_k$. In other words, $\mathbf{S}$ is used to allocate the k-th element A_k of an input symbol sequence $\mathbf{A}$ to v-th symbol subsequence $\mathbf{A}_v$ if $S_k = v$.

Let the v-th subblock index $\mathbf{B}_v = [B_{v,0}B_{v,1}\cdots B_{v,N-1}]^T,\ 0 \le v \le V-1$ sequence be generated as follows:

$$B_{v,k} = \begin{cases} 1, & S_k = v \\ 0, & S_k \ne v. \end{cases}$$

Then the v-th symbol subsequence $\mathbf{A}_v$ is expressed as

$$\mathbf{A}_v = \tilde{\mathbf{B}}_v \mathbf{A}$$

where $\tilde{\mathbf{B}}_v$ is an $N \times N$ diagonal matrix whose diagonal entries form the subblock index sequence $\mathbf{B}_v$. Then, the output signal sequence $\mathbf{a}^w$ is written as

$$\mathbf{a}^w = \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_1^n \tilde{\mathbf{B}}_v \mathbf{A}.$$

The known subblock partitioning methods can be classified into three categories. The first and simplest category is called an adjacent method which allocates N/V successive symbols to the same subblock. The second category is based on interleaving. In this method, the symbols with distance V are allocated to the same subblock. The last one is called a random partitioning method in which the input symbol sequence is partitioned randomly.

The PAPR reduction performance and the computational complexity of PTS scheme depend on the method of subblock partitioning. In other words, there is a trade-off between PAPR reduction performance and computational complexity in PTS scheme. The random partitioning is known to have the best performance in PAPR reduction. The interleaving method [3] can reduce the computational complexity of PTS scheme using Cooley-Tukey FFT algorithm, but the PAPR reduction performance is the worst.

3. New PTS OFDM Scheme

A. New PTS OFDM Scheme

Unlike the conventional PTS scheme where input symbol sequences are partitioned at the initial stage, in the

Table 1: Computational complexity reduction ratio over the conventional PTS OFDM scheme

$n-l$	CCRR(%)											
	$N=256(n=8)$			$N=1024(n=10)$			$N=2048(n=11)$			$N=8192(n=13)$		
	$V=4$	$V=8$	V=16	$V=4$	$V=8$	V=16	$V=4$	$V=8$	V=16	$V=4$	$V=8$	V=16
3	29	43	50	35	53	61	36	55	64	39	58	67
2	21	32	38	30	45	53	32	48	56	35	52	61
5	14	21	25	25	38	44	27	41	48	31	46	54
6	7	11	13	20	30	35	23	34	40	27	40	47

proposed scheme, the partially IFFT-ed input symbol sequences are partitioned at an intermediate stage. Let the partially IFFT-ed input symbol sequence be called an intermediate signal sequence.

Figure 1 shows the block diagram of the new PTS OFDM scheme. In this scheme, the 2^n - point IFFT based on decimation-in-time algorithm is divided into two parts. The first part is the first l stages of IFFT and the second part is the remaining $n-l$ stages. A set of V intermediate signal subsequences is generated using a subblock partitioning sequences. Compared to the conventional PTS scheme, the computational complexity of the new scheme is much relieved since the intermediate signal sequence $\hat{\mathbf{a}}_{\text{data}} = \mathbf{Q}_1^l \mathbf{A}_{\text{data}}$ is used in common for IFFT of V symbol subsequence.

The index of the rotating vector used for the transmitted signal sequence must be conveyed to the receiver in PTS scheme. Usually the index information is encoded for error detection and correction due to its importance. In the conventional PTS scheme, this information, represented as an index sequence of rotating vectors $\mathbf{A}_{\text{index}}$ is added to a data symbol sequence $\mathbf{A}_{\text{data}}$ to form the input symbol sequence $\mathbf{A}$, i.e., $\mathbf{A} = \mathbf{A}_{\text{data}} + \mathbf{A}_{\text{index}}$. But in our scheme, this summing operation (in fact, it is equivalent to augmentation) is not done at the symbol sequence stage but at the final stage after the IFFT operations as shown in Figure 1. Since the index signal sequences $\mathbf{a}^w_{\text{index}} = \mathbf{Q}_1^n \mathbf{A}^w_{\text{index}},\ 0 \le w \le W-1$ are used repeatedly, they can be stored in the memory and added to the IFFT of $\mathbf{A}_{\text{data}}$. Thus, the new PTS OFDM signal sequence $\mathbf{a}^w$ can be written as

$$\begin{aligned}\mathbf{a}^w &= \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_{l+1}^n \tilde{\mathbf{B}}_v \mathbf{Q}_1^l \mathbf{A}_{\text{data}} + \mathbf{Q}_1^n \mathbf{A}^w_{\text{index}} \\ &= \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_{l+1}^n \tilde{\mathbf{B}}_v \hat{\mathbf{a}}_{\text{data}} + \mathbf{a}^w_{\text{index}}.\end{aligned}$$

B. Subblock Partitioning Sequence

In this subsection, we suggest a simple but very promising subblock partitioning sequence for the case when the number of subblock is a power of 2. Let $\mathbf{M} = [M_0 M_1 \cdots M_{N-2}]^T$ be a binary m-sequence of length $N-1$, with the characteristic phase, i.e., satisfying that $M_k = M_{2k}$. For $V = 2^u$ subblocks, we propose a subblock partitioning sequence $\mathbf{S} = [S_0 S_1 \cdots S_{N-1}]^T$ given by

$$S_k = \begin{cases} 0, & k = 0 \\ \sum_{j=0}^{u-1} 2^j M_{k-1+j}, & 1 \le k \le N-1 \end{cases} \tag{2}$$

where the subscript of M is computed modulo $N-1$. Certainly from the run property of an m-sequence, the frequency of each symbol $v,\ 0 \le v \le V-1$, in $\mathbf{S}$ is exactly 2^{n-u}. Although not proven, this sequence is believed to have a good PAPR reduction performance due to the pseudo-random properties of an m-sequence. In fact, the numerical analysis shows that the sequence has the comparable performance as that offered by a random partitioning method.

C. Computational Complexity

When the number of subcarriers is $N = 2^n$, the numbers of complex multiplication n_{mul} and complex addition n_{add} of the conventional PTS OFDM scheme are given by $n_{\text{mul}} = 2^{n-1} nV$ and $n_{\text{add}} = 2^n nV$, where V is the number of subblocks. When the intermediate signal sequence is partitioned after the $l-$th stage of IFFT, it is clear that the numbers of complex computations of the new PTS OFDM scheme are given by $n_{\text{mul}} = 2^{n-1}n + 2^{n-1}(n-l)(V-1)$ and $n_{\text{add}} = 2^n n + 2^n (n-l)(V-1)$. Thus, the computational complexity reduction ratio (CCRR) of the new PTS OFDM scheme over the conventional PTS OFDM scheme is defined as

$$\begin{aligned}\text{CCRR} &= \left(1 - \frac{\text{Complexity of new PTS}}{\text{Complexity of conven. PTS}}\right) \times 100 \\ &= \left(1 - \frac{1}{V}\right) \frac{l}{n} \times 100\ [\%].\end{aligned}$$

Table 1 gives CCRR of the new PTS OFDM scheme over the conventional PTS OFDM scheme with typical values of V, l, and n.

4. Simulation Results

Simulations are performed for the OFDM system of the IEEE standard 802.16 for mobile wireless metropolitan area network (WMAN). The number of used subcarriers is 1702. Among the remaining 346 subcarriers, 345 subcarriers are set to zero to shape the power spectrum of the transmit signal and one subcarrier is used for DC. The 100,000

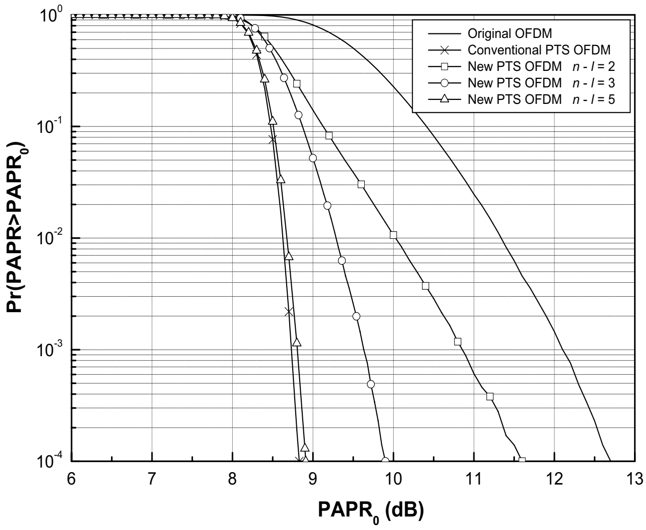

Figure 2: CCDF of the PAPR of new and conventional PTS OFDM scheme for various stages of multiplication when $N = 2048, V = 8, 16-$QAM constellation, and four times oversampling are used.

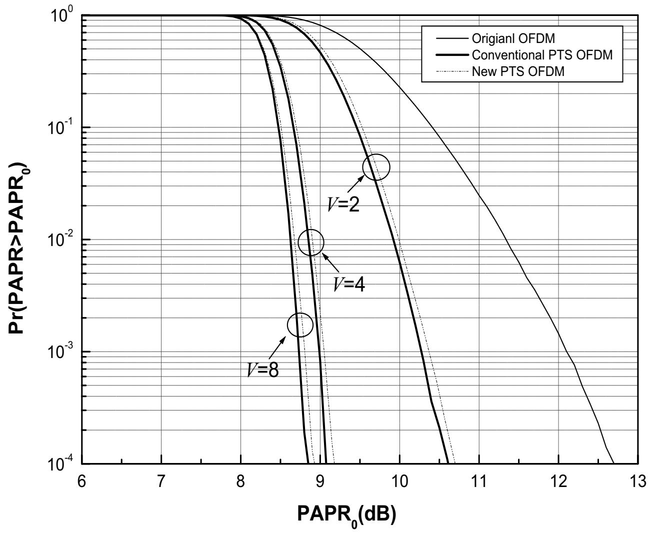

Figure 3: Performance comparison of the conventional PTS OFDM scheme and the new PTS OFDM scheme when $N = 2048$, $n - k = 5$, and $16 -$ QAM constellation, and four times oversampling are used.

input symbol sequences are generated randomly with uniform distribution. The OFDM signal is oversampled by a factor of four which is sufficient to represent the analog signal [5]. The symbols of the rotating factors are chosen from $\{\pm 1, \pm j\}$ for $V = 2, 4$ and from $\{\pm 1\}$ for $V = 8$.

Figures 2 and 3 illustrate the probability that the PAPR of the OFDM signal exceeds the given threshold. Figure 2 shows the simulation results as the stage of multiplication is varied for $n - l = 2, 3, 5$. The new PTS scheme with 2048 subcarriers has almost the same performance compared to the conventional PTS OFDM scheme when $n-l$ is 5. From the simulation results, we can say that the optimal value for $n - l$ does not depend on the number of subcarriers and it is around 5 when the number of subcarriers is between 256 and 8192.

Figure 3 shows a comparison of the performance between the conventional PTS OFDM scheme and the new PTS OFDM scheme with $n - l = 5$, $16 -$ QAM constellation and four times oversampling. As one can see, the new PTS OFDM scheme has almost the same performance of PAPR reduction as that of the conventional PTS OFDM scheme. In the case of $N = 2048$ and $n - l = 5$, the new PTS OFDM scheme reduces the computational complexity by $27\% \sim 48\%$ as the number of sequences V increases from 2 to 8.

5. Conclusion

There is a trade-off between the computational complexity and performance in the PAPR reduction method. A new PTS OFDM scheme has been proposed and its performance is analyzed in reference to the standard of $IEEE$ 802.16 for WMAN. The numerical analysis shows that the new PTS OFDM scheme with 2048 subcarriers reduces the computational complexity by 48% with the performance degradation under 0.2dB at 10^{-4} when an intermediate signal sequence is partitioned into 8 subblocks at the stage $l = 6$. Since the computational complexity reduction ratio increases as the number of subcarriers increases, the proposed scheme becomes more suitable for the high data rate OFDM systems such as a digital multimedia broadcasting system.

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