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# A new subspace method for blind estimation of selective MIMO-STBC channels

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## Summary

In this paper, a new technique for the blind estimation of frequency and/or time-selective multiple-input multiple-output (MIMO) channels under space-time block coding (STBC) transmissions is presented. The proposed method relies on a basis expansion model (BEM) of the MIMO channel, which reduces the number of parameters to be estimated, and includes many practical STBC-based transmission scenarios, such as STBC-orthogonal frequency division multiplexing (OFDM), space-frequency block coding (SFBC), time-reversal STBC, and time-varying STBC encoded systems. Inspired by the unconstrained blind maximum likelihood (UML) decoder, the proposed criterion is a subspace method that efficiently exploits all the information provided by the STBC structure, as well as by the reduced-rank representation of the MIMO channel. The method, which is independent of the specific signal constellation, is able to blindly recover the MIMO channel within a small number of available blocks at the receiver side. In fact, for some particular cases of interest such as orthogonal STBC-OFDM schemes, the proposed technique blindly identifies the channel using just one data block. The complexity of the proposed approach reduces to the solution of a generalized eigenvalue (GEV) problem and its computational cost is linear in the number of sub-channels. An identifiability analysis and some numerical examples illustrating the performance of the proposed algorithm are also provided. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS: space-time block coding (STBC); orthogonal frequency division multiplexing (OFDM); space-frequency block coding (SFBC); time-reversal STBC; time-varying channels; blind channel estimation; second-order statistics (SOS)

## 1. Introduction

In the last 10 years, since the well-known work of Alamouti [1], and the later generalization by Tarokh *et al*. [2], several families of space-time block codes (STBCs) have been proposed to exploit the spatial diversity in multiple-input multiple-output (MIMO) systems. Some examples are the orthogonal STBCs (OSTBCs) [2], quasi orthogonal STBCs (QSTBCs) [3–5], trace-orthogonal codes (TOSTBC) [6,7], and perfect STBCs [8].

A common assumption for most of the STBCs is that perfect channel state information (CSI) is available at the receiver, which has motivated an increasing interest on blind channel estimation algorithms [9–19]. Blind techniques avoid the penalty in bandwidth

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efficiency or signal to noise ratio (SNR) associated, respectively, to training based approaches [20–22], or differential techniques [23–28]. Among blind channel estimation techniques, those solely based on second-order statistics (SOS) [12,14–16,18,19] are specially appealing due to their low computational complexity and their independence of the specific signal constellation.

Although the literature on blind and semiblind channel estimation under STBC transmissions is abundant, most of the research efforts have considered time-invariant flat-fading MIMO channels [10–19]. However, the number of techniques for more general settings such as time-varying [12,29–31] or frequency-selective channels [9,32–40] is more scarce. Specifically, the on-line algorithms in References [29–31] consider OSTBC transmissions over a time-varying flat-fading channel, and they can be seen as adaptive versions of the technique proposed in Reference [12]. On the other hand, the problem of blind estimation or equalization of frequency-selective MIMO channels has been addressed from two different points of view. Firstly, the techniques in References [9,32–35] apply standard blind channel estimation or equalization techniques, which do not completely exploit the structure induced by the STBC. Moreover, they require a relatively high number of available blocks at the receiver. Secondly, in References [36–40] the authors have proposed several subspace-based blind techniques, which require a large number of available blocks at the receiver side, and consequently long channel coherence times.

To our best knowledge, only a few techniques have considered the problem of blind decoding within a reduced number of blocks at the receiver. Specifically, for orthogonal codes the sources can be recovered by means of differential approaches [23–27] or the blind techniques proposed in References [41–43]. However, most of these techniques introduce some constraints in the signal constellation of the sources, which might not be satisfied if the signals have been linearly precoded [7]. On the other hand, the method proposed in Reference [43], which is independent of the specific symbol constellation, is based on a semidefinite-relaxation approach, which translates into a relatively high computational complexity.

In this paper, we propose a technique for the blind estimation of frequency and/or time-selective MIMO channels, which allows us to jointly address a wide class of STBC-based systems, to name a few: orthogonal frequency division (OFDM-STBC), space-frequency block coding (SFBC), time-reversal STBC, or STBC transmissions through a time-varying channel. Firstly, the frequency and/or time-varying MIMO channel is represented by means of a basis expansion model (BEM) [44,45], which limits the number of parameters to be estimated. Secondly, inspired by the unconstrained blind maximum-likelihood (UML) decoder, we propose a subspace-based blind channel estimation technique which reduces to the extraction of the main eigenvector of a generalized eigenvalue (GEV) problem. The proposed technique is solely based on the SOS of the observations, and therefore it can be directly applied even for linearly precoded sources. Furthermore, the technique is able to recover the channels within a reduced number of available blocks at the receiver, and unlike other approaches, its computational complexity is linear in the number of MIMO sub-channels.

The structure of the paper is as follows: The channel and STBC data models are introduced in Section 2. The proposed technique for the estimation of the channel parameters is presented in Section 3. In Section 4, we prove that, under mild assumptions, the theoretical solutions of the proposed method are those of the UML decoder. Section 5 summarizes the main properties of the proposed technique. Finally, the performance of the proposed method is evaluated by means of some numerical examples in Section 6, and the concluding remarks are pointed out in Section 7.

## 2. Channel and Data Model

### 2.1. Notation

#### 2.1.1. Vectors/matrices

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements $x_{i,j}$; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. Superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors, or scalars, the identity matrix of dimension $p$ will be denoted as $\mathbf{I}_p$, and $\mathbf{0}$ will denote the zero matrix of the required dimensions.

#### 2.1.2. Operators

The superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote transpose, Hermitian, and complex conjugate, respectively. The real and imaginary parts of a matrix $\mathbf{A}$ are denoted as $\Re(\mathbf{A})$ and $\Im(\mathbf{A})$. The trace, range (or column space), and Frobenius norm will be denoted as $\text{Tr}(\mathbf{A})$, $\text{range}(\mathbf{A})$ and $\|\mathbf{A}\|$, respectively. Finally, the



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column-wise vectorized version of matrix $\mathbf{A}$ will be denoted as $\text{vec}(\mathbf{A})$, and $\otimes$ will denote the Kronecker product.

## 2.2. MIMO Channel Model

Let us consider a set of $N_c$ flat fading MIMO channels. The $i$th MIMO channel is represented by the $n_T \times n_R$ complex channel matrix $\mathbf{H}_i$, where the element in the $k$th row and $l$th column of $\mathbf{H}_i$ denotes the response of the $i$th channel between the $k$th transmit and the $l$th receive antennas.

The correlation existing among the $N_c$ MIMO channels is represented by means of the following BEM: [44]

$$\mathbf{H}_i = \sum_{k=1}^{L_c} b_{i,k}\mathbf{\Theta}_k, \qquad i = 1, \ldots, N_c, \tag{1}$$

where $\mathbf{\Theta}_k \in \mathbb{C}^{n_T \times n_R}$ are the parameter matrices,

$$\mathbf{B} = \begin{bmatrix} b_{1,1} & \cdots & b_{1,L_c} \\ \vdots & \ddots & \vdots \\ b_{N_c,1} & \cdots & b_{N_c,L_c} \end{bmatrix} \tag{2}$$

is some orthogonal basis,[‡] and $L_c \leq N_c$ is the BEM order, which allows us to range from the case of perfectly correlated (i.e., identical) channels ($L_c = 1$), to the case of independent channels ($L_c = N_c$). Equation (1) can be rewritten in matrix form as

$$\underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{N_c} \end{bmatrix}}_{\mathbf{H}} = \underbrace{\left(\mathbf{B} \otimes \mathbf{I}_{n_T}\right)}_{\mathcal{B}} \underbrace{\begin{bmatrix} \mathbf{\Theta}_1 \\ \vdots \\ \mathbf{\Theta}_{L_c} \end{bmatrix}}_{\mathbf{\Theta}} \tag{3}$$

where $\mathbf{H} \in \mathbb{C}^{N_c n_T \times n_R}$, $\mathbf{\Theta} \in \mathbb{C}^{L_c n_T \times n_R}$, and $\mathcal{B} \in \mathbb{C}^{N_c n_T \times L_c n_T}$. Finally, the complex noise is considered independent for different channels, and it is assumed to be both spatially and temporally white with variance $\sigma^2$, $i = 1, \ldots, N_c$.

[‡]The orthogonality condition is not restrictive and all the results in the paper can be easily generalized for any full-column rank basis $\mathbf{B}$.

## 2.3. STBC Data Model

Let us consider a linear STBC transmitting $M$ symbols during $L$ uses of the $i$th MIMO channel $\mathbf{H}_i \in \mathbb{C}^{n_T \times n_R}$. The transmission rate is defined as $R = M/L$, and $M' = 2M$ is the number of real symbols transmitted in each block.[§]

For a STBC, the $n$th block of data can be expressed as

$$\mathbf{S}_i(\mathbf{s}_i[n]) = \sum_{k=1}^{M'} \mathbf{C}_{i,k} s_{i,k}[n] \quad i = 1, \ldots, N_c \tag{4}$$

where $\mathbf{s}_i[n] = [s_{i,1}[n], \ldots, s_{i,M'}[n]]^T$ contains the $M'$ real information symbols transmitted through the $i$th channel in the $n$th STBC block, and $\mathbf{C}_{i,k} \in \mathbb{C}^{L \times n_T}$, $k = 1, \ldots, M'$, are the code matrices.[‖]

The complex signal at the receive antennas can be written, for $i = 1, \ldots, N_c$, as

$$\begin{aligned} \mathbf{Y}_i[n] &= \mathbf{S}_i(\mathbf{s}_i[n])\mathbf{H}_i + \mathbf{N}_i[n] \\ &= \sum_{k=1}^{M'} \mathbf{W}_{i,k}(\mathbf{H}_i) s_{i,k}[n] + \mathbf{N}_i[n] \end{aligned} \tag{5}$$

where $\mathbf{N}_i[n] \in \mathbb{C}^{L \times n_R}$ represents the white complex noise with zero mean and variance $\sigma^2$, and

$$\mathbf{W}_{i,k}(\mathbf{H}_i) = \mathbf{C}_{i,k}\mathbf{H}_i, \quad k = 1, \ldots, M'. \tag{6}$$

Defining now $\mathbf{y}_i[n] = \text{vec}(\mathbf{Y}_i[n])$, $\mathbf{h}_i = \text{vec}(\mathbf{H}_i)$ and $\mathbf{n}_i[n] = \text{vec}(\mathbf{N}_i[n])$, Equation (5) can be rewritten as

$$\mathbf{y}_i[n] = \mathbf{W}_i(\mathbf{h}_i)\mathbf{s}_i[n] + \mathbf{n}_i[n], \quad i = 1, \ldots, N_c \tag{7}$$

where $\mathbf{W}_i(\mathbf{h}_i)$ can be seen as the $i$th complex equivalent channel, whose $k$th column is given by

$$\text{vec}(\mathbf{W}_{i,k}(\mathbf{h}_i)) = \mathbf{D}_{i,k}\mathbf{h}_i \tag{8}$$

with $\mathbf{D}_{i,k} = \mathbf{I}_{n_R} \otimes \mathbf{C}_{i,k}$, $k = 1, \ldots, M'$

Here, we must note that the data model in Equation (7) can be seen as a particular case of a complex system with a non-circular (improper) source [46,47], i.e., the real information symbols $\mathbf{s}_i[n]$ are observed through a complex equivalent channel given by $\mathbf{W}_i(\mathbf{h}_i)$. This

[§]In the particular case of real STBCs, we have $M' = M$ and real transmission and code matrices.

[‖]Usually, the STBC is common for all the channels, so we could drop the subindex $i$.

fact has been previously exploited in Reference [48] to equalize frequency-selective channels under STBC transmissions, and it has also been implicitly exploited by the blind OSTBC channel estimation techniques proposed in References [12,13,15,17–19]. In this paper, we exploit the impropriety of the sources by using the following real data model:

$$\tilde{\mathbf{y}}_i[n] = \tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\mathbf{s}_i[n] + \tilde{\mathbf{n}}_i[n], \quad i = 1, \ldots, N_\mathrm{c} \tag{9}$$

where $\tilde{\mathbf{y}}_i[n] = \left[\Re(\mathbf{y}_i^\mathrm{T}[n]), \Im(\mathbf{y}_i^\mathrm{T}[n])\right]^\mathrm{T}$, $\tilde{\mathbf{n}}_i[n] = \left[\Re(\mathbf{n}_i^\mathrm{T}[n]), \Im(\mathbf{n}_i^\mathrm{T}[n])\right]^\mathrm{T}$, $\tilde{\mathbf{h}}_i = \left[\Re(\mathbf{h}_i^\mathrm{T}), \Im(\mathbf{h}_i^\mathrm{T})\right]^\mathrm{T}$, and the $i$th real equivalent channel is

$$\begin{aligned} \underbrace{\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)}_{2Ln_\mathrm{R}\times M'} &= \left[\Re(\mathbf{W}_i^\mathrm{T}(\mathbf{h}_i)) \quad \Im(\mathbf{W}_i^\mathrm{T}(\mathbf{h}_i))\right]^\mathrm{T} \\ &= \left[\tilde{\mathbf{D}}_{i,1}\tilde{\mathbf{h}}_i \quad \tilde{\mathbf{D}}_{i,2}\tilde{\mathbf{h}}_i \quad \cdots \quad \tilde{\mathbf{D}}_{i,M'}\tilde{\mathbf{h}}_i\right] \end{aligned} \tag{10}$$

where

$$\tilde{\mathbf{D}}_{i,k} = \underbrace{\begin{bmatrix} \Re(\mathbf{D}_{i,k}) & -\Im(\mathbf{D}_{i,k}) \\ \Im(\mathbf{D}_{i,k}) & \Re(\mathbf{D}_{i,k}) \end{bmatrix}}_{2Ln_\mathrm{R}\times 2n_\mathrm{T}n_\mathrm{R}}, \quad k = 1, \ldots, M' \tag{11}$$

are the extended code matrices with real elements.

## 2.4. Linear Precoding of the Information Symbols

In general, STBCs are able to exploit the spatial diversity of the MIMO channel. However, in order to take advantage of the frequency and/or time diversity of the system, the information symbols have to be distributed among the different MIMO channel realizations. Fortunately, this can be easily done by means of linear precoding techniques [7,45]. Thus, we can assume without loss of generality that the transmitted symbols $\mathbf{s}_i[n]$ are obtained as

$$\underbrace{\begin{bmatrix} \mathbf{s}_1[n] \\ \vdots \\ \mathbf{s}_{N_\mathrm{c}}[n] \end{bmatrix}}_{\mathbf{s}[n]} = (\Re(\mathbf{G}) \otimes \mathbf{I}_{M'} + \Im(\mathbf{G}) \otimes \mathbf{J}_{M'}) \underbrace{\begin{bmatrix} \mathbf{d}_1[n] \\ \vdots \\ \mathbf{d}_{N_\mathrm{c}}[n] \end{bmatrix}}_{\mathbf{d}[n]} \tag{12}$$

where $\mathbf{d}_i[n] \in \mathbb{R}^{M'\times 1}$ is a vector containing the real and imaginary parts of the information symbols, which belong to some finite alphabet $\mathcal{S}$, $\mathbf{G} \in \mathbb{C}^{N_\mathrm{c}\times N_\mathrm{c}}$ is a unitary precoding matrix [7], and

$$\mathbf{J}_{M'} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_M \\ \mathbf{I}_M & \mathbf{0} \end{bmatrix}. \tag{13}$$

## 2.5. Some Particular Cases

The data and channel models introduced in this section are very general. Some particular cases of interest are summarized in Table I, where the matrix $\mathbf{F}_{N_\mathrm{c}\times P}(\delta)$ is defined as

$$\mathbf{F}_{N_\mathrm{c}\times P}(\delta) = \left[\mathbf{f}_{N_\mathrm{c}}(\delta)\ \mathbf{f}_{N_\mathrm{c}}\left(\tfrac{1}{N_\mathrm{c}} + \delta\right) \cdots \mathbf{f}_{N_\mathrm{c}}\left(\tfrac{P-1}{N_\mathrm{c}} + \delta\right)\right] \tag{14}$$

$\mathbf{f}_{N_\mathrm{c}}(f)$ is the Fourier vector of length $N_\mathrm{c}$ at normalized frequency $f$, and $\delta$ is a frequency offset in the FFT grid. Let us illustrate these equivalences in more detail

- *STBC-OFDM*: In this case, the sub-channels $\mathbf{H}_i$, $i = 1, \ldots, N_\mathrm{c}$, represent the frequency response of the MIMO channel in the $i$th subcarrier. The orthogonal basis $\mathbf{B}$ is given by the first $L_\mathrm{c}$ columns of the FFT matrix, and the parameters $\boldsymbol{\Theta}_k$ ($k = 1, \ldots, L_\mathrm{c}$) represent the finite impulse response of the MIMO channel. The STBC-OFDM scheme assumes that the frequency-selective MIMO channel remains constant during at least $L$ (the channel uses per STBC block) OFDM symbols, and it uses STBC transmission in each subcarrier.
- *SFBC*: SFBC can be seen as an alternative to STBC-OFDM systems based on only one OFDM symbol [49,50]. In this case, the temporal coherence requirement in STBC-OFDM systems is replaced by a constraint in the spectral coherence. In particular, the OFDM symbol is divided into groups of $L$ adjacent subcarriers, which see the same flat fading MIMO channel $\mathbf{H}_i$ and are used to transmit one STBC data block.
- *Time-reversal STBC*: Time-reversal orthogonal STBC was proposed in Reference [51] (see also References [52,53]) as a transmission technique to exploit the multipath diversity in MIMO systems with inter-symbol interference, and it was later generalized to non-orthogonal codes [54,55]. Interestingly, these schemes can be viewed as a particular case of a STBC-OFDM system with basis and precoding matrices



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Table I. Correspondence between the proposed data model and several well-known STBC-based communication systems.

| System | Basis $\mathbf{B}$ | Precoding $\mathbf{G}$ | Other parameters |
|---|---|---|---|
| STBC-OFDM | $\mathbf{B} = \mathbf{F}_{N_c \times L_c}(0)$ | Several options:<br>• No precoding: $\mathbf{G} = \mathbf{I}_{N_c}$<br>• Minimum MSE [7]: e.g., $\mathbf{G} = \mathbf{F}_{N_c \times N_c}(0)$ | • $L$ OFDM symbols<br>• $N_c$ subcarriers<br>• $L_c$ non-zero taps |
| SFBC | $\mathbf{B} = \mathbf{F}_{N_c \times L_c}(0)$ | Analogous to STBC-OFDM | • 1 OFDM symbol<br>• $N_c L$ subcarriers<br>• $L_c$ non-zero taps |
| Time-reversal STBC | $\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(\frac{1}{2N_c}\right)$ | $\mathbf{G} = \mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right)$ | • Blocks of length $N_c L$<br>• $L_c$ non-zero taps |
| Time-varying | Several options:<br>• Fourier basis<br>• Discrete prolate spheroidal sequences [56] | Analogous to STBC-OFDM | • $N_c L$ channel uses<br>• Maximum Doppler: $\frac{f_D}{f_s} = \frac{L_c - 1}{2LN_c}$ |
| Doubly-selective | $\mathbf{B}$ is the Kronecker product of the time and frequency basis | Several options:<br>• No precoding<br>• Precoding in time and/or frequency | This is a combination of the previous cases |

$$\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(\frac{1}{2N_c}\right) \quad \mathbf{G} = \mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right) \tag{15}$$

which satisfy the *time-reversal* property [52,53]

$$\mathbf{F}^T_{N_c \times N_c}\left(\frac{1}{2N_c}\right)\mathbf{F}_{N_c \times N_c}\left(\frac{1}{2N_c}\right) = \overleftarrow{\mathbf{I}}_{N_c} \tag{16}$$

where $\overleftarrow{\mathbf{I}}_{N_c}$ is obtained from $\mathbf{I}_{N_c}$ with its columns (or rows) in reverse order.

- *Time-varying Channels*: Let us consider a STBC transmission through a time-varying flat fading MIMO channel, which is considered static during the $L$ channel uses of a STBC block. Obviously, this assumption implies that the MIMO channel changes slowly, and therefore, it can be well approximated by a BEM [44]. For instance, the relationship between subsequent channel realizations could be modeled through the Fourier transform of the bandlimited time-varying channel response. Thus, $N_c$ consecutive realizations of the channel $\mathbf{H}_1, \ldots, \mathbf{H}_{N_c}$ can be represented by Equation (3), where the orthogonal basis is

$$\mathbf{B} = \mathbf{F}_{N_c \times L_c}\left(-\frac{L_c - 1}{2N_c}\right) \tag{17}$$

There exist other alternative basis for modeling the temporal variation of the channel, such as the discrete prolate spheroidal sequences (or finite Slepian sequences) [56], which avoid the spectral leakage problem associated to the Fourier basis. However, regardless of the particular basis selection, the number $L_c$ of parameters is directly related with the maximum Doppler frequency $f_D$ by means of

$$\frac{f_D}{f_s} = \frac{f_c}{f_s}\frac{v_{max}}{v_{light}} = \frac{L_c - 1}{2LN_c} \tag{18}$$

where $f_c$ and $f_s$ are the carrier and symbol frequencies, respectively, $v_{max}$ is the maximum relative speed between the transmitter and the receiver, and $v_{light}$ is the speed of light.

- *Doubly selective channels*: The above data model can be easily extended to the case of doubly-selective MIMO channels [44,45], for which the basis $\mathbf{B}$ can be interpreted as a Kronecker product between the time and frequency bases, whereas the parameter $L_c$ indicates the total number of degrees of freedom in the system, i.e., the product of the time and frequency diversities.

## 3. Blind Estimation of Selective MIMO Channels

In this section, we propose a general blind channel estimation technique inspired by the blind ML receiver. Unlike other approaches, the proposed scheme is able



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to recover the channel up to a real scalar from a reduced number of observations (STBC-OFDM or SFBC blocks). Let us start by introducing the joint ML estimator of the channel and information symbols.

### 3.1. Unconstrained Blind ML Receiver

In general, the blind maximum likelihood (ML) estimation of the channel and sources is a very difficult problem, which is due to the coupling among the channels $\mathbf{H}_i$, which depends on the parameters $\boldsymbol{\Theta}$, the coupling among the sources $\mathbf{s}_i[n]$, which depend on $\mathbf{d}[n]$, and the finite alphabet properties of the information symbols $\mathbf{d}[n]$. A direct simplification is obtained by relaxing the finite alphabet constraint, which decouples the signal estimates for different channels. Thus, assuming that for each channel $\mathbf{H}_i$, a set of $N$ STBC blocks is available at the receiver side, the unconstrained blind maximum likelihood decoder (UML) reduces to

$$\left\{\hat{\boldsymbol{\Theta}}^{\mathrm{UML}}, \hat{\mathbf{s}}_i^{\mathrm{UML}}[n]\right\} = \underset{\boldsymbol{\Theta},\mathbf{s}_i[n]}{\operatorname{argmin}} \sum_{i=1}^{N_c}\sum_{n=0}^{N-1} \left\|\tilde{\mathbf{y}}_i[n] - \tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\mathbf{s}_i[n]\right\|^2 \quad (19)$$

and solving for $\mathbf{s}_i[n]$ we obtain[¶]

$$\begin{aligned}\hat{\mathbf{s}}_i^{\mathrm{UML}}[n] &= \left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right)^{-1}\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n] \\ &= \tilde{\mathbf{V}}_i(\tilde{\mathbf{h}}_i)\tilde{\Sigma}_i^{-1}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{U}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n] \quad (20)\end{aligned}$$

where $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i) = \tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\tilde{\Sigma}_i(\tilde{\mathbf{h}}_i)\tilde{\mathbf{V}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)$ denotes the singular value decomposition (SVD) of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$. Therefore, combining Equations (19) and (20) the UML criterion can be rewritten as

$$\hat{\boldsymbol{\Theta}}^{\mathrm{UML}} = \underset{\boldsymbol{\Theta}}{\operatorname{argmax}} \sum_{i=1}^{N_c}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}_i^{\mathrm{T}}[n]\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\tilde{\mathbf{U}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{y}}_i[n] \quad (21)$$

or equivalently

$$\hat{\boldsymbol{\Theta}}^{\mathrm{UML}} = \underset{\boldsymbol{\Theta}}{\operatorname{argmax}} \sum_{i=1}^{N_{\mathrm{c}}} \operatorname{Tr}\left(\tilde{\mathbf{U}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{R}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)\right) \quad (22)$$

where

$$\mathbf{R}_{\tilde{\mathbf{y}}_i} = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}_i[n]\tilde{\mathbf{y}}_i^{\mathrm{T}}[n] \quad (23)$$

is the sample mean estimate of the correlation matrix for the observations of the $i$th channel.

### 3.2. Proposed Blind Channel Estimation Method

Although the relaxation of the finite alphabet constraint in the information symbols simplifies the blind channel estimation criterion, the channel estimates are still coupled through the basis expansion parameters $\boldsymbol{\Theta}$. On the one hand, this reduced-rank model allows us to take into account the correlation among consecutive channels. On the other hand, the coupling in the channel estimates and the non trivial dependency of $\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)$ w.r.t. the parameters $\boldsymbol{\Theta}$ preclude a direct solution of the criterion in Equation (22). In this subsection, we present a subspace-based blind channel estimation method which provides closed-form channel estimates.

The UML estimator in Equations (21) and (22) can be easily interpreted as a subspace technique, whose goal is to maximize the energy of the projections of the *observed signal subspaces*, obtained from $\tilde{\mathbf{y}}_i[n]$, onto the *parameter-dependent signal subspaces*, which are defined by the equivalent channel matrices $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ (or $\tilde{\mathbf{U}}_i(\tilde{\mathbf{h}}_i)$). Here, we propose an alternative criterion which consists in the maximization of the following weighted sum of energies

$$\hat{\boldsymbol{\Theta}} = \underset{\boldsymbol{\Theta}}{\operatorname{argmax}} \sum_{i=1}^{N_{\mathrm{c}}} E_i \operatorname{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \quad (24)$$

where[#]

$$\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i} = \tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}^{\mathrm{T}}, \quad i = 1, \ldots, N_{\mathrm{c}} \quad (25)$$

are the projection matrices onto the *observed signal subspaces*, $\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i} \in \mathbb{R}^{2Ln_R \times r}$ is a matrix containing the $r = \min(N, M')$ principal eigenvectors of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$, and $E_i$ denotes the signal energy in the $i$th channel, which is obtained as the sum of the $r$ largest eigenvalues of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.

[¶]We are assuming that the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ are full-column rank, which is a common assumption for all the STBCs.

[#]The prewhitening is not necessary in the OSTBC case. In other words, due to the orthogonality of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, $\boldsymbol{\Phi}_{\tilde{\mathbf{y}}_i}$ can be replaced by $\frac{r}{E_i}\mathbf{R}_{\tilde{\mathbf{y}}_i}$.



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The criterion in Equation (24) can be interpreted as follows: instead of maximizing the projections of $\tilde{\mathbf{y}}_i[n]$ onto the *parameter-dependent signal subspaces*, we maximize the projections of the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ onto the *observed signal subspaces*. This alternative criterion will allow us to obtain closed-form channel estimates. However, unlike Equation (22), the energy of the channels $\tilde{\mathbf{h}}_i$ (or equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$) in Equation (24) must be constrained to avoid trivial solutions. Although this could seem a minor problem, the selection of the constraint constitutes a key point in the derivation of the blind channel estimation criterion. Specifically, we propose the following constraint in the channel energies

$$\sum_{i=1}^{N_c} E_i \|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\|^2 = 1. \qquad (26)$$

As will be shown later, this constraint not only avoids trivial solutions, but also ensures that, under mild assumptions, the theoretical solutions of the overall channel estimation technique are those of the UML decoder.

Now, the dependency of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ with $\tilde{\mathbf{h}}_i$, given by Equation (10), allows us to write

$$\mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) = \sum_{k=1}^{M'} \tilde{\mathbf{h}}_i^{\mathrm{T}}\tilde{\mathbf{D}}_{i,k}^{\mathrm{T}}\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{D}}_{i,k}\tilde{\mathbf{h}}_i \qquad (27)$$

and

$$\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\|^2 = \sum_{k=1}^{M'} \tilde{\mathbf{h}}_i^{\mathrm{T}}\tilde{\mathbf{D}}_{i,k}^{\mathrm{T}}\tilde{\mathbf{D}}_{i,k}\tilde{\mathbf{h}}_i. \qquad (28)$$

Thus, the optimization problem given by Equations (24) and (26) can be reformulated as

$$\hat{\mathbf{\Theta}} = \underset{\mathbf{\Theta}}{\operatorname{argmax}} \sum_{i=1}^{N_c} \tilde{\mathbf{h}}_i^{\mathrm{T}}\mathbf{\Xi}_i\tilde{\mathbf{h}}_i, \quad \text{s.t.} \quad \sum_{i=1}^{N_c} \tilde{\mathbf{h}}_i^{\mathrm{T}}\mathbf{\Psi}_i\tilde{\mathbf{h}}_i = 1 \qquad (29)$$

where

$$\mathbf{\Xi}_i = E_i \sum_{k=1}^{M'} \tilde{\mathbf{D}}_{i,k}^{\mathrm{T}}\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{D}}_{i,k} \qquad (30)$$

and

$$\mathbf{\Psi}_i = E_i \sum_{k=1}^{M'} \tilde{\mathbf{D}}_{i,k}^{\mathrm{T}}\tilde{\mathbf{D}}_{i,k} \qquad (31)$$

The criterion proposed so far only exploits the structure imposed by the STBC. The additional structure provided by the time–frequency-selective behavior can be incorporated to the criterion through the reduced-rank BEM. In particular, defining the vectors $\boldsymbol{\theta}_k = \mathrm{vec}(\mathbf{\Theta}_k)$, the channel model in Equation (1) can be rewritten as

$$\mathbf{h}_i = \sum_{k=1}^{L_c} b_{i,k}\boldsymbol{\theta}_k, \quad i = 1, \ldots, N_c \qquad (32)$$

or equivalently, for $i = 1, \ldots, N_c$

$$\tilde{\mathbf{h}}_i = \sum_{k=1}^{L_c}\left(\begin{bmatrix} \Re(b_{i,k}) & -\Im(b_{i,k}) \\ \Im(b_{i,k}) & \Re(b_{i,k}) \end{bmatrix} \otimes \mathbf{I}_{n_T n_R}\right)\tilde{\boldsymbol{\theta}}_k \qquad (33)$$

where $\tilde{\boldsymbol{\theta}}_k = [\Re(\boldsymbol{\theta}_k^{\mathrm{T}}), \Im(\boldsymbol{\theta}_k^{\mathrm{T}})]^{\mathrm{T}}$. Therefore, the real vectorized channels are given by $\tilde{\mathbf{h}}_i = \mathbf{\Omega}_i\tilde{\boldsymbol{\theta}}$, where $\tilde{\boldsymbol{\theta}} = [\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}, \ldots, \tilde{\boldsymbol{\theta}}_{L_c}^{\mathrm{T}}]^{\mathrm{T}}$

$$\mathbf{\Omega}_i = \Re(\mathbf{b}_i^{\mathrm{T}}) \otimes \mathbf{I}_{2n_T n_R} + \Im(\mathbf{b}_i^{\mathrm{T}}) \otimes \begin{bmatrix} \mathbf{0} & -\mathbf{I}_{n_T n_R} \\ \mathbf{I}_{n_T n_R} & \mathbf{0} \end{bmatrix}, \qquad (34)$$

and $\mathbf{b}_i^{\mathrm{T}}$ is the $i$th row of the orthogonal basis $\mathbf{B}$. Thus, the combination of Equation (29) and $\tilde{\mathbf{h}}_i = \mathbf{\Omega}_i\tilde{\boldsymbol{\theta}}$ allows us to rewrite the proposed blind channel estimation criterion as a function of the channel expansion coefficients $\tilde{\boldsymbol{\theta}}$

$$\hat{\tilde{\boldsymbol{\theta}}} = \underset{\tilde{\boldsymbol{\theta}}}{\operatorname{argmax}}\ \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathbf{\Xi}\tilde{\boldsymbol{\theta}}, \quad \text{s.t.} \quad \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathbf{\Psi}\tilde{\boldsymbol{\theta}} \qquad (35)$$

where

$$\mathbf{\Xi} = \sum_{i=1}^{N_c} \mathbf{\Omega}_i^{\mathrm{T}}\mathbf{\Xi}_i\mathbf{\Omega}_i \qquad (36)$$

and

$$\mathbf{\Psi} = \sum_{i=1}^{N_c} \mathbf{\Omega}_i^{\mathrm{T}}\mathbf{\Psi}_i\mathbf{\Omega}_i. \qquad (37)$$

The solution of Equation (35) is obtained as the eigenvector $\hat{\tilde{\boldsymbol{\theta}}}$ associated to the largest eigenvalue $\beta$ of the following GEV:

$$\mathbf{\Xi}\hat{\tilde{\boldsymbol{\theta}}} = \beta\mathbf{\Psi}\hat{\tilde{\boldsymbol{\theta}}}. \qquad (38)$$

Finally, the overall blind channel estimation algorithm is summarized in Algorithm 1.



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**Algorithm 1** Summary of the proposed blind channel estimation algorithm.

Collect $N$ consecutive observation vectors $\tilde{\mathbf{y}}_i[n]$, for $i = 1, \ldots, N_c$ and $n = 0, \ldots, N-1$
Obtain the estimates of the correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}_i}$ with Equation (23).
Obtain the matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ and signal energies $E_i$ from the EV decomposition of $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.
Using the code matrices $\tilde{\mathbf{D}}_{i,k}$, obtain $\mathbf{\Xi}_i$ and $\mathbf{\Psi}_i$ with Equations (30) and (31).
Using the BEM, obtain $\mathbf{\Xi}$ and $\mathbf{\Psi}$ with Equations (36) and (37).
Obtain the channel estimate as the principal eigenvector of the GEV in Equation (38).

## 4. Identifiability Analysis

Although some intuitive necessary conditions can be easily obtained, the analysis of the blind channel identifiability from SOS under STBC transmissions is a difficult problem yet to be solved. In particular, several efforts have been made in the case of flat fading and time-invariant STBC systems [14,16,19], but the identifiability properties are only partially clear in the OSTBC case [57,58].

Here we show that, under mild assumptions, the theoretical solutions of the proposed criterion are those associated to the UML decoder. In other words, the channel estimates provided by the proposed technique are congruent with the data model.

Let us consider a noise-free scenario.[**] From Equations (21) and (22), it is easy to prove that the solutions $\hat{\tilde{\mathbf{h}}}_i$ of the UML criterion fulfill

$$\mathrm{range}\left(\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\right) \subseteq \mathrm{range}\left(\tilde{\mathbf{W}}_i(\hat{\tilde{\mathbf{h}}}_i)\right), \quad i = 1, \ldots, N_c \tag{39}$$

where the equality is satisfied iff the signal subspace is completely determined by the observations ($N \geq M'$). To continue the analysis, we must distinguish two different cases.

### 4.1. Case of $N \geq M'$

In this case, $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ ($i = 1, \ldots, N_c$) are the true projection matrices onto the signal subspaces. Therefore, the energy of the projections in Equation (24) is bounded by

$$\mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq \left\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right\|^2 \tag{40}$$

where the equality is satisfied iff $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ spans the true signal subspace. Finally, taking into account the energy constraint in Equation (26) it is clear that

$$\sum_{i=1}^{N_c} E_i \mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq 1 \tag{41}$$

and the equality is attained by the solutions of the UML decoder, which obviously include the true MIMO channel.

### 4.2. Case of $N < M'$

This is a more complicated situation in which the channels are not persistently excited by the sources, i.e., the observations do not completely characterize the signal subspace, and $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ is only a projection matrix onto a rank $N$ subspace belonging to the whole rank $M'$ signal subspace. Thus, we must distinguish between two different cases.

#### *4.2.1. Orthogonal STBCs (OSTBC)*

In this case, the channel can be unambiguously recovered by means of the proposed technique. In particular, the orthogonality property of OSTBCs is [57]

$$\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i) = \left\|\tilde{\mathbf{h}}_i\right\|^2\mathbf{I}_{M'}, \quad \forall\tilde{\mathbf{h}}_i \tag{42}$$

which ensures that, in the absence of noise

$$\mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq N\left\|\tilde{\mathbf{h}}_i\right\|^2 \tag{43}$$

where the equality is satisfied iff the observations $\tilde{\mathbf{y}}_i[n]$ belong to the subspace spanned by $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, or equivalently

$$\mathrm{range}\left(\tilde{\mathbf{U}}_{\tilde{\mathbf{y}}_i}\right) \subset \mathrm{range}\left(\tilde{\mathbf{W}}_i(\hat{\tilde{\mathbf{h}}}_i)\right), \quad i = 1, \ldots, N_c \tag{44}$$

i.e., if the estimates are congruent with the data model. Now, taking into account the orthogonality of the equivalent channels $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$, the energy constraint (26) can

[**]The same conclusions can be obtained by assuming perfect estimates ($N \to \infty$) of the correlation matrices $\mathbf{R}_{\tilde{\mathbf{y}}_i}$.



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be rewritten as

$$\sum_{i=1}^{N_c} E_i \left\|\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right\|^2 = M' \sum_{i=1}^{N_c} E_i \left\|\tilde{\mathbf{h}}_i\right\|^2 = 1 \quad (45)$$

which finally yields

$$\sum_{i=1}^{N_c} E_i \mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right) \leq \frac{N}{M'} \quad (46)$$

where the equality is attained by the solutions of the UML decoder.

#### 4.2.2. Non-orthogonal STBCs

In this case, the energies $\mathrm{Tr}\left(\tilde{\mathbf{W}}_i^{\mathrm{T}}(\tilde{\mathbf{h}}_i)\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)\right)$ are not necessarily maximized by the actual MIMO channels and then the channel can not be exactly recovered by means of the proposed technique. In other words, since the signal subspaces are not completely characterized by the projection matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$, the proposed technique might find spurious MIMO channels concentrating all the energy of $\tilde{\mathbf{W}}_i(\tilde{\mathbf{h}}_i)$ in the directions defined by $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$. However, we must note that the maximization has to be made simultaneously for the $N_c$ sub-channels, whereas the number of effective independent channels is given by $L_c$. Thus, when $N_c >> L_c$ there are not enough degrees of freedom to find spurious solutions, and the proposed technique will provide very accurate estimates.

## 5. Computational Cost and Comparison with Previous Works

The proposed blind technique has to solve two main steps. Firstly, the $N_c$ projection matrices $\mathbf{\Phi}_{\tilde{\mathbf{y}}_i}$ are obtained, which comes at a computational cost of order $\mathcal{O}(N_c L^3 n_R^3)$; and secondly, the channel parameters are recovered from the GEV in Equation (38), whose computational cost is $\mathcal{O}(n_{\mathrm{T}}^3 n_{\mathrm{R}}^3 L_c^3)$. Therefore, the computational complexity of the proposed blind channel estimation technique is $\mathcal{O}(n_{\mathrm{R}}^3(L^3 N_c + n_{\mathrm{T}}^3 L_c^3))$, i.e., it is linear in the number $N_c$ of channels, which corresponds with the number of subcarriers in multi-carrier systems, or the number of channel realizations in time-varying scenarios. This contrasts with previous applications of standard subspace techniques [36,37] (see also References [38–40]), which not only require a large number $N > M' N_c$ of STBC-OFDM blocks at the receiver, but also incur in a computational cost of $\mathcal{O}((L n_{\mathrm{R}} N_c)^3)$.

When compared with previous works, the proposed method solves the blind channel estimation problem for general STBC transmissions in a unified manner, which includes the case of time-varying channels, as well as the common STBC-OFDM, SFBC, and time-reversal systems. Moreover, the proposed method avoids the finite alphabet requirement associated to the semiblind algorithm in References [41,42], or the constant energy associated to differential approaches [23–27]. Therefore, it can be directly applied when the information symbols have been linearly precoded in order to exploit the multipath or temporal diversity of the channel. Finally, as we have shown in the previous section, in the absence of noise the channel parameters can be exactly recovered within only $N = M'$ (or $N = 1$ in the OSTBC case) blocks at the receiver side.

## 6. Simulation Results

In this section, the performance of the proposed technique is illustrated by means of some simulation examples. All the results have been obtained by averaging 1000 independent experiments. The MIMO channels $\mathbf{H}_i$ have been generated as a Rayleigh channel with unit-variance elements. The i.i.d. information symbols, which have not been linearly precoded ($\mathbf{G} = \mathbf{I}_{N_c}$), belong to a quadrature phase shift keying (QPSK) constellation. We have used MMSE receivers followed by a hard decision decoder, which in the case of OSTBC transmissions is equivalent to the ML receiver. The transmission schemes are based on two different STBCs for $n_{\mathrm{T}} = 4$ transmit antennas, namely, the OSTBC presented in Equation (7.4.10) of Reference [52], whose parameters are $M = 3$ and $L = 4$ ($R = 3/4$), and the quasi-orthogonal (QSTBC) proposed in Reference [3] ($M = L = 4$, $R = 1$).

The proposed method has been compared with the MMSE receiver with perfect CSI, which we refer to as clairvoyant MMSE, and with a training-based approach. In the particular case of OFDM-based transmissions, the training method is based on the use of $L_c$ equally spaced pilot subcarriers, and the channel estimate is obtained by means of the well-known least squares (LS) method. Finally, in order to avoid the ambiguities associated to the QSTBC blind channel estimation problem [16,57], we have applied the non-redundant precoding technique proposed in References [19], i.e., for each subchannel $\mathbf{H}_i$ we have used a rotated version of the QSTBC in Reference [3].



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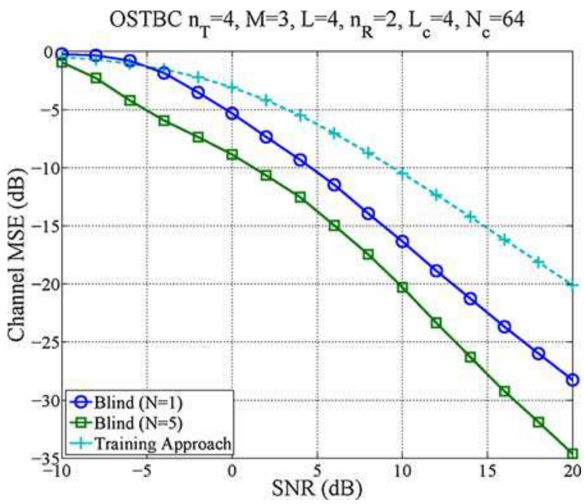


Fig. 1. MSE in the channel estimate for a $R = 3/4$ OSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

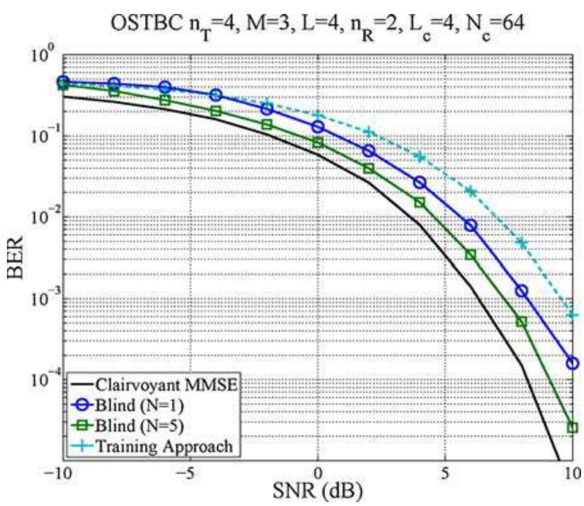


Fig. 2. BER after decoding for a $R = 3/4$ OSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

## 6.1. STBC-OFDM Systems

In this subsection, the proposed technique is evaluated in a STBC-OFDM MIMO system with $L_c = 4$ taps and different number $N_c$ of subcarriers.[††] In the first experiment, we consider the $R = 3/4$ OSTBC system with $n_R = 2$ receive antennas and $N_c = 64$ subcarriers. Figure 1 shows the mean square error (MSE) of the channel estimate for different numbers $N$ of available blocks at the receiver, whereas Figure 2 shows the bit error rate (BER) after decoding. As can be seen, the proposed method outperforms the training approach based on $L_c$ pilot carriers and, as it was expected, its accuracy increases with the number of available OSTBC-OFDM blocks. This point is also illustrated in Figure 3, which shows the evolution of the BER with the number $N$ of OSTBC-OFDM blocks. As can be seen, as $N$ increases, the proposed method achieves similar results to that of the receiver with perfect channel knowledge.

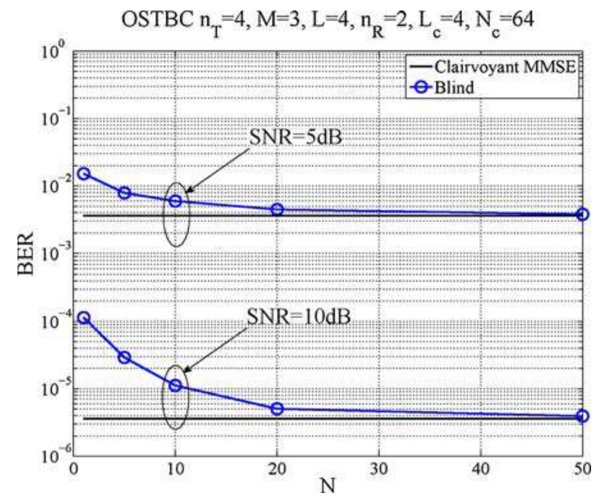


Fig. 3. BER after decoding for a $R = 3/4$ OSTBC. Effect of $N$ on the BER for two different SNR values. $N_c = 64$.

In the second set of examples, the performance of the proposed method for $N = 1$ and several numbers of subcarriers $N_c$ is evaluated. The results for the $R = 3/4$ OSTBC with $n_R = 2$ receive antennas are shown in Figures 4 and 5. As can be seen, for a fixed number

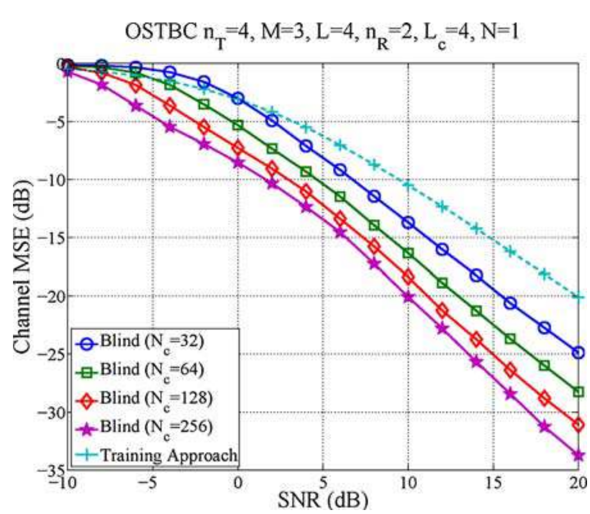


Fig. 4. MSE in the channel estimate for a $R = 3/4$ OSTBC. $N = 1$ and different numbers of subcarriers.

[††]Similar results have been obtained in the cases of SFBC or time-reversal STBC systems, but due to the lack of space we only present the STBC-OFDM case.

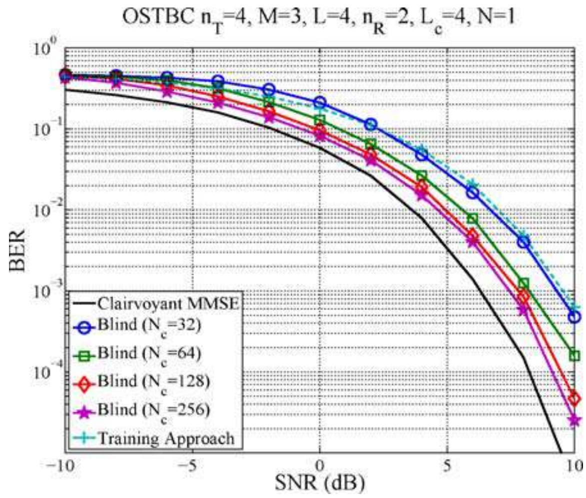


Fig. 5. BER after decoding for a $R = 3/4$ OSTBC. $N = 1$ and different numbers of subcarriers.

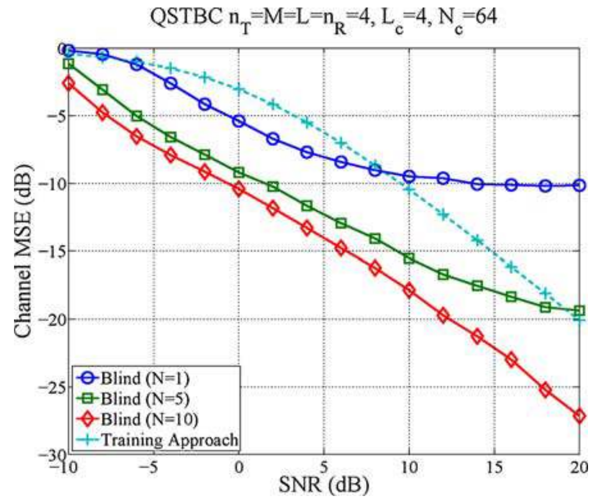


Fig. 6. MSE in the channel estimate for a $R = 1$ QSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

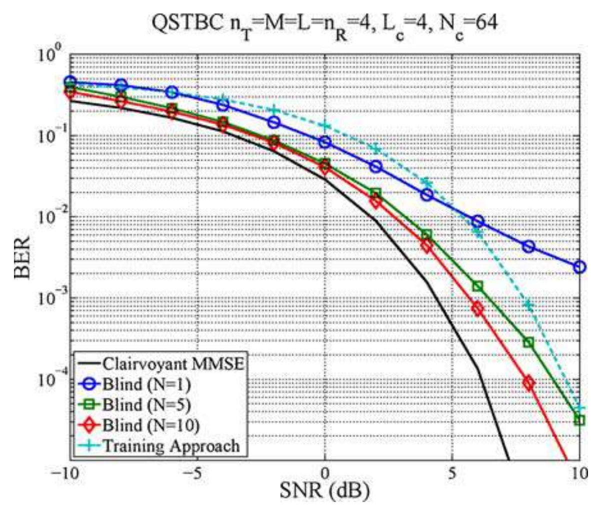


Fig. 7. BER after decoding for a $R = 1$ QSTBC. $N_c = 64$ and different numbers of available blocks at the receiver.

($L_c$) of parameters, the performance not only improves with the number of available OSTBC-OFDM blocks, but also with the number of subcarriers $N_c$. This can be seen as a direct consequence of the rank-reduced channel model, which is able to properly exploit the structure introduced by the channel. In other words, while the number of unknown parameters ($L_c n_T n_R$) remains constant, the available data to estimate the channel increases with $N_c$, which necessarily translates into better channel estimates.

The previous experiments have been repeated for the rate-one QSTBC with $n_T = n_R = M = L = 4$. Firstly, the results for $N_c = 64$ subcarriers and different numbers of available QSTBC-OFDM blocks are shown in Figures 6–8. As can be seen, the proposed technique is able to exactly recover the channel, in the absence of noise, when the number of available blocks is $N \geq M'$, i.e., when the signal subspaces are completely determined by the observations. As pointed out in Section 4.2.2, when this condition is not satisfied ($N < M'$), the proposed technique is not able to exactly recover the channel, which explains the noise–floor effect in Figures 6 and 7. However, as we can see in Figure 8, as $N$ increases, the results provided by the proposed blind channel estimation method become closer to those of the receiver with perfect channel knowledge.

Finally, the previous example has been repeated for only $N = 1$ QSTBC-OFDM block at the receiver side, $L_c = 4$ non-zero taps, and different numbers $N_c$ of subcarriers. The results are shown in Figures 9 and 10, where we can see that the noise floor in the channel estimate rapidly decreases with the number of subcarriers. Furthermore, we must point out that for practical SNR (or BER) values, the results provided by the proposed technique are not very far from those of the receiver with perfect channel knowledge. Actually, they are accurate enough to switch to a decision directed scheme or to provide a good starting point for an iterative implementation of the UML decoder.

## 6.2. Flat-fading Time-varying Channels

In this subsection, the performance of the proposed technique in time-varying MIMO channels is evaluated. Specifically, the time-varying channels are



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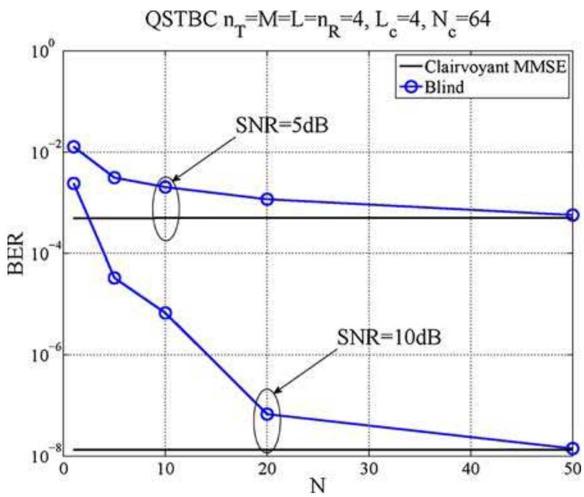


Fig. 8. BER after decoding for a $R = 1$ QSTBC. Effect of $N$ on the BER for two different SNR values. $N_c = 64$.

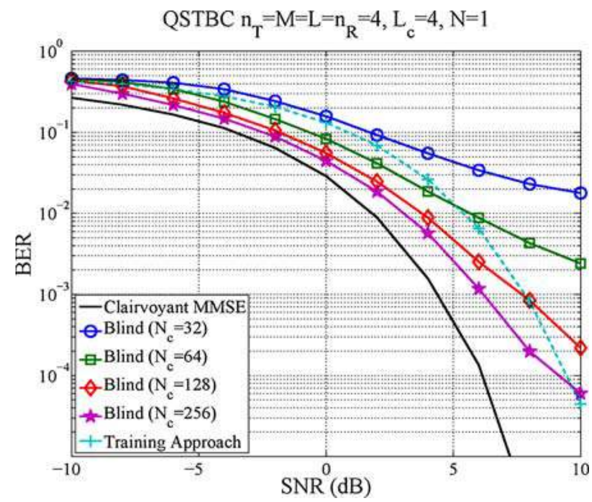


Fig. 10. BER after decoding for a $R = 1$ QSTBC. $N = 1$ and different numbers of subcarriers.

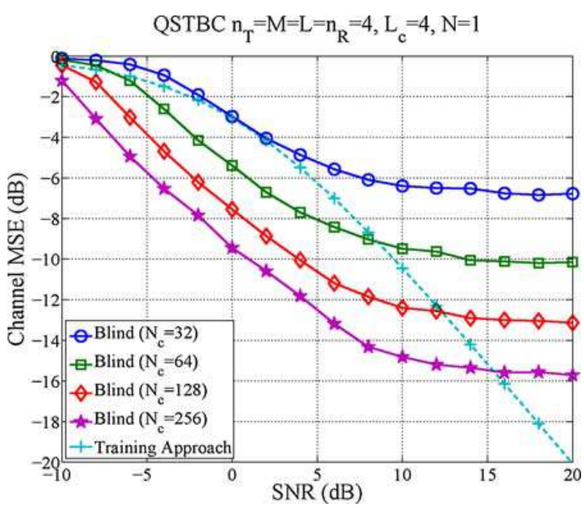


Fig. 9. MSE in the channel estimate for a $R = 1$ QSTBC. $N = 1$ and different numbers of subcarriers.

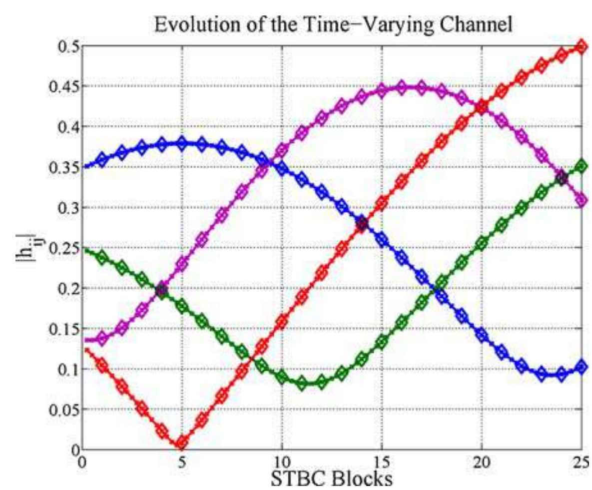


Fig. 11. Evolution of a time-varying channel during 100 symbol periods. $L_c = 5$, $N_c = 128$, $L = 4$, $f_D \simeq 3.9 \times 10^{-3}$.

modeled through a Fourier[‡‡] BEM with $L_c = 5$ parameters, which correspond to a maximum Doppler frequency of

$$f_D = \frac{1}{2N_c}. \tag{47}$$

Here, we must note that in the case of time-varying channels we always consider $N = 1$, i.e., only one STBC block is transmitted over each MIMO channel $\mathbf{H}_i$. Obviously, for lower Doppler frequencies the channel could be considered constant during the transmission of $N$ STBC blocks, and more accurate channel estimates would be obtained.

As an example, Figure 11 shows the evolution of four MIMO channel coefficients, for $N_c = 128$ ($f_D \simeq 3.9 \cdot 10^{-3}$), during 100 symbol periods, which corresponds to the transmission of $100/L = 25$ STBC blocks. As can be seen, the channel can be considered approximately constant during the transmission of one STBC block, which is a common requirement for all the STBC-based systems, but it rapidly changes during the whole transmission frame.

In the first experiment, we consider the $R = 3/4$ OSTBC with $n_R = 2$ receive antennas. The BER after

[‡‡] Similar results have been obtained in the case of discrete prolate spheroidal sequences.

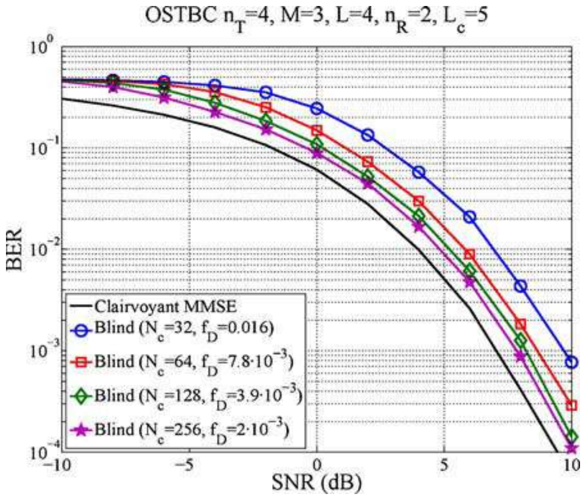


Fig. 12. BER after decoding for a $R = 3/4$ OSTBC. Time varying channels with different Doppler frequencies.

decoding for different Doppler frequencies is shown in Figure 12. As it was expected, the performance of the proposed technique improves with the temporal-coherence of the channel, which increases with $N_c$. Furthermore, for moderate Doppler frequencies, the performance of the blind technique is close to that of the clairvoyant receiver, and it avoids the 3-dB penalty associated to differential approaches.

Finally, the previous experiment has been repeated with the QSTBC code and $n_R = 4$ receive antennas. The results are shown in Figure 13, where we can observe the previously commented noise floor. However, for moderate Doppler frequencies, the blind technique still provides accurate results and its performance degradation with respect to the clairvoyant receiver is lower than the minimal loss (3-dB) associated to the QSTBC differential technique proposed in Reference [28].

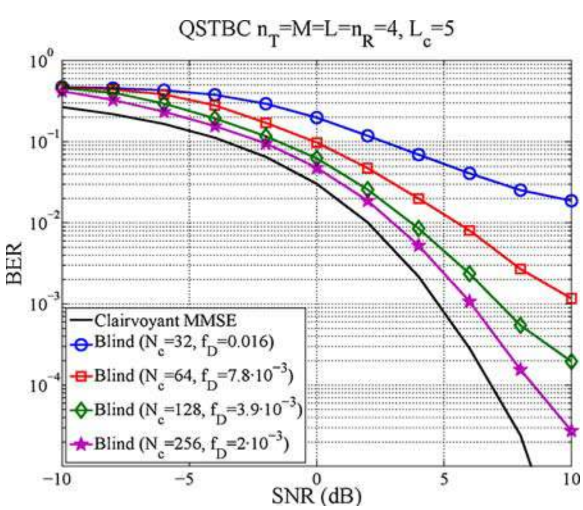


Fig. 13. BER after decoding for the $R = 1$ QSTBC. Time varying channels with different Doppler frequencies.

## 7. Conclusions

In this paper, a new technique for the blind estimation of frequency and/or time-selective MIMO channels, under STBC transmissions, has been presented. The proposed technique is based on a low-rank representation of the MIMO channel, which reduces the number of parameters to be estimated, and can be easily particularized to the cases of STBC-OFDM, SFBC, time-reversal STBC, and STBC transmissions over time-varying channels. The method, which is inspired by the UML decoder, reduces to the solution of a GEV problem, and its computational complexity is linear in the number of orthogonal channels (subcarriers in the particular case of STBC-OFDM systems). Furthermore, unlike other previously proposed approaches, in the absence of noise it is able to exactly recover the channels within a few data blocks at the receiver side. Finally, the proposed algorithm has been evaluated by means of numerical examples, showing that the overall system performance is close to that of the receiver with perfect channel knowledge.

## Acknowledgements

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