$W(z)$ is determined using the estimate of $B(z)$ obtained in the previous iteration as described in Section III-B. The estimate of the coefficients of $B(z)$ always converged after 6 iterations. Table III gives the simulation results as prefilter order q is varied. Generally, the best performance is achieved for a value of q somewhere in the interval $N/2 \leq p + q \leq 3N/4$. This is not unlike SVD methods [8], [9]. Simulations for different SNR values but with q fixed at 13 are tabulated in Table IV. The estimates of the frequencies are appreciably worse than the CR bound (threshold) when SNR is less than 7 dB. We note that the estimation accuracy is identical to that of SVD method [8], [9]. In fact, the threshold SNR is also identical to that of SVD method [8], [9], for this example, i.e., 7 dB. Further results are available in [3].

VI. Conclusions

In this correspondence we have shown how FIR prefiltering can provide an effective and simple means of improving the performance of Prony's method. However, any prefiltering should be such that it does not violate the basic property of Prony's method. This can be ensured by restricting the transient effects of the prefilter from lingering too long. We have described two prefiltering schemes. In the first method the prefilter is synthesized by using prior information regarding the approximate regions in the frequency domain where the signals are known to lie. For the second method no prior knowledge is necessary. The prefilter is computed from the data samples themselves.

The simulation results given in Section VI show that the performance of methods 1 and 2 are equal to or better than (if prior knowledge is used) those predicted by the CR bounds. The iterations of method 2 converged in all the trials we have attempted. But proof of convergence is not provided.

Acknowledgment

The first author thanks L. Jackson for pointing out [6].

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A New Technique for Velocity Estimation of Large Moving Objects

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***Abstract*—A new technique for motion estimation of large moving objects is presented. This technique is based on analyzing the Hartley transform spectrum of the image sequence directly, instead of using it to compute other transforms. This method is faster than other techniques based on the Fourier transform.**

I. Introduction

The estimation of velocity of large moving objects is needed in many applications, such as biomedical cell motion analysis, tracking dust storms and clouds, and in industrial and military applications. Some researchers [1]-[5] used spatiotemporal-frequency techniques in motion analysis employing the fast Fourier transform (FFT) in their algorithms. This correspondence presents a new approach based on the analysis of the fast Hartley transform (FHT) of the image sequence for motion estimation. Researchers [6]-[8] have shown that computing the Fourier, cosine, and sine transforms from the FHT is faster than computing it from their fast transforms. Hence, the direct application of the FHT is faster than other transforms, even when computed from the FHT. The presented modification to the algorithm of [4] allows it to run faster while giving the same accuracy and results, with the additional advantage of validating the results.

In this technique, the FHT is applied to the image sequence followed by a peak detection procedure. The location of the peak is related to the velocity of the moving object. Dividing the temporal frequency f_p corresponding to the detected peak by the corresponding spatial frequency k_p gives the velocity of the moving object. The Fourier spectrum for a spatial frequency of k_p is then computed. This is followed by a peak detection of the Fourier spectrum to validate the previous results and find the direction of the velocity of the moving object.

The organization of the paper is as follows. The analytical model and formulations for a large moving object in a time sequence are presented in Section II. Section III covers an algorithm for motion estimation, and Section IV presents the simulation results. Finally, concluding remarks are given in Section V.

II. Analytical Formulation

The analytical formulation will be presented for the one-dimensional time sequence. Reference may be made to [4] for the details

Manuscript received June 8, 1989; revised June 12, 1990.
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IEEE Log Number 9041607.

of transforming a two-dimensional image sequence to two one-dimensional sequences.

A model for a large moving object in one-dimensional time sequence is given by

$$o[n, m] = \sum_{i=1}^{i=r} A_i \, \delta[n - L_i - mV_i] \quad (1).$$

where n, m are the pixel coordinates at data point n and frame m; A_i, L_i, V_i, r are the subobject amplitudes, initial position (from the first data point of the first frame), the time frames at which the subobjects enter the sequence, the velocity of the moving object, and the size of the moving object in pixels, respectively; with $\delta[\ ]$ as the dirac-delta function.

In the above model, the large moving object is treated as a number of one-pixel objects that are adjacent and moving with the same velocity (i.e., $V_i = V_j$; $i, j = 1, r$, and $i \neq j$).

The discrete Hartley transform (DHT) of (1) is given by

$$H[k,f] = \sum_{i=1}^{i=r} \sum_{n=0}^{n=N-1} \sum_{m=0}^{m=N-1} A_i \delta[n - L_i - mV_i] \cdot \text{cas}\,[2\pi fm/N]\, \text{cas}\,[2\pi kn/N] \quad (2)$$

where N is the number of pixels and frames in the image, and

$$\text{cas}\,\phi = \cos\phi + \sin\phi.$$

Assume that subobject i appears in the sequence in M_i frames (i.e., from frame m_i to $m_i + M_i - 1$). Let

$$kV_i = f_i \qquad \text{then } V_i = f_i/k \quad (3)$$

Substituting (3) in (2), applying the limits, and summing with respect to n, we have

$$\begin{aligned} H[k,f] = \sum_{i=1}^{i=r} \sum_{m=m_i}^{m=m_i+M_i-1} \frac{1}{2} A_i \Big\{ &\exp\left(j2\pi[m(f_i - f) + kL_i]/N\right) \\ &+ \exp\left(-j2\pi[m(f_i - f) + kL_i]/N\right) \\ &+ \frac{1}{j}\Big[\exp\left(j2\pi[m(f_i + f) + kL_i]/N\right) \\ &- \exp\left(-j2\pi[m(f_i + f) + kL_i]/N\right)\Big]\Big\}. \end{aligned}$$

Summing with respect to m and combining terms, we have

$$\begin{aligned} H[k,f] = \sum_{i=1}^{i=r} M_i A_i \Big\{ &\frac{\text{sinc}\,(f_i - f)M_i/N}{\text{sinc}\,(f_i - f)/N} \\ &\cdot \cos\left(\pi[kL_i + (f_i - f)]\,[2m_i + (M_i - 1)/N]\right) \\ &+ \frac{\text{sinc}\,(f_i + f)M_i/N}{\text{sinc}\,(f_i + f)/N} \sin\left(\pi[kL_i + (f_i + f)]\right. \\ &\left.\cdot\,[2m_i + (M_i - 1)/N]\right)\Big\}. \end{aligned} \quad (4)$$

The peaks of (4) are at frequencies $\pm f_i$. The velocity of the moving object is related to f_i by (3).

A simple peak detection algorithm is used to detect the peaks in the Hartley domain. The temporal frequency corresponding to the detected peak is found. The velocity of the moving object is obtained by dividing the temporal frequency f_p by the respective spatial frequency k_p.

The algorithm proposed in this note is faster than other techniques based on other transforms (viz., Fourier, cosine, and sine transforms). However, it has one limitation—the direction of the velocity is not unique. One way to overcome this problem is to estimate the direction of motion by [4]. A faster implementation of [4] is done by computing the Fourier spectrum for the spatial frequency k_p found above instead of computing the whole Fourier spectrum. A peak is detected in the Fourier spectrum. The location of the peak gives the direction (i.e., if the peak is located in the negative frequency range then the velocity is positive, otherwise it is negative). This modification makes the proposed algorithm faster than other algorithms even in applications where the direction of velocity is unknown.

III. Algorithm for Motion Estimation

The algorithm for motion estimation using the presented technique described in Section II is as follows.

1) Apply the Hartley transform to the time sequence in order to obtain $H[k, f]$.

2) Set the selected spatial frequency to k_s.

3) Detect a peak in the Hartley spectrum for a spatial frequency of K_s (i.e., in $H[k_s, f]$) and get the corresponding temporal frequency, f_p. The velocity of the moving object is given by $V_h = f_p/k_s$.

4) Compute the Fourier spectrum from $H[k, f]$ for a spatial frequency, k_s (i.e., compute $F[k_s, f]$).

5) Detect a peak in $F[k_s, f]$ and get the corresponding temporal frequency $\hat{f}_p$. The velocity of the moving object is given by $V_f = \hat{f}_p/k_s$ and is opposite to the sign of $\hat{f}_p$.

6) Compare V_h with V_f, if not equal, then increment k_s and go to step 3, else done.

IV. Simulations

The simulations were carried out on an IBM PS/2 model 80 computer using Turbo C. The graphs were produced using Surfer Access System software of Golden Software Inc.

Fig. 1(a) shows a sequence with a large moving object. The Hartley transform is applied to the time-sequence of Fig. 1(a). Fig. 1(b) shows the Hartley spectrum of the sequence. It is clear from the figure that there are different peaks at the different values of the spatial frequency k as expected from the mathematical formulations. Fig. 1(c) shows the Fourier spectrum for the same sequence. It is clear from Figs. 1(b) and (c) that the peaks in the Fourier spectrum correspond to negative temporal frequencies while the peaks in the Hartley spectrum corrrespond to both positive and negative temporal frequencies as expected from [4, eqs. (4) and (5)]. Fig. 1(d) shows the spectrum at a spatial frequency of 2. The detected peak is at a temporal frequency of -4. Hence, the velocity of the moving object is $4/2 = 2$ pixels per frame (PPF). The Fourier spectrum at a spatial frequency of 2 is computed. Since the peak in this spectrum is in the negative temporal frequency range, the moving object's velocity is positive (i.e., 2 PPF from left to right).

V. Conclusions

We have described a simple and computationally efficient algorithm for motion detection of large moving objects. The technique is faster than other motion detection methods even when computed from the FHT. However, it has one limitation. Although it estimates the amplitude of the velocity correctly, the direction is not unique. In some applications the direction is known. However, on those applications, when the direction is not known, the suggested modification to technique [4] can be used. The applicability of this technique has been demonstrated by a new mathematical formulation and simulation results obtained on the IBM PS/2.

Acknowledgment

The author would like to thank the referees for their comments on the preliminary manuscript and for bringing [5], [6], and [8] to his attention. The incorporation of their suggestions and comments significantly improved the clarity of the final manuscript.

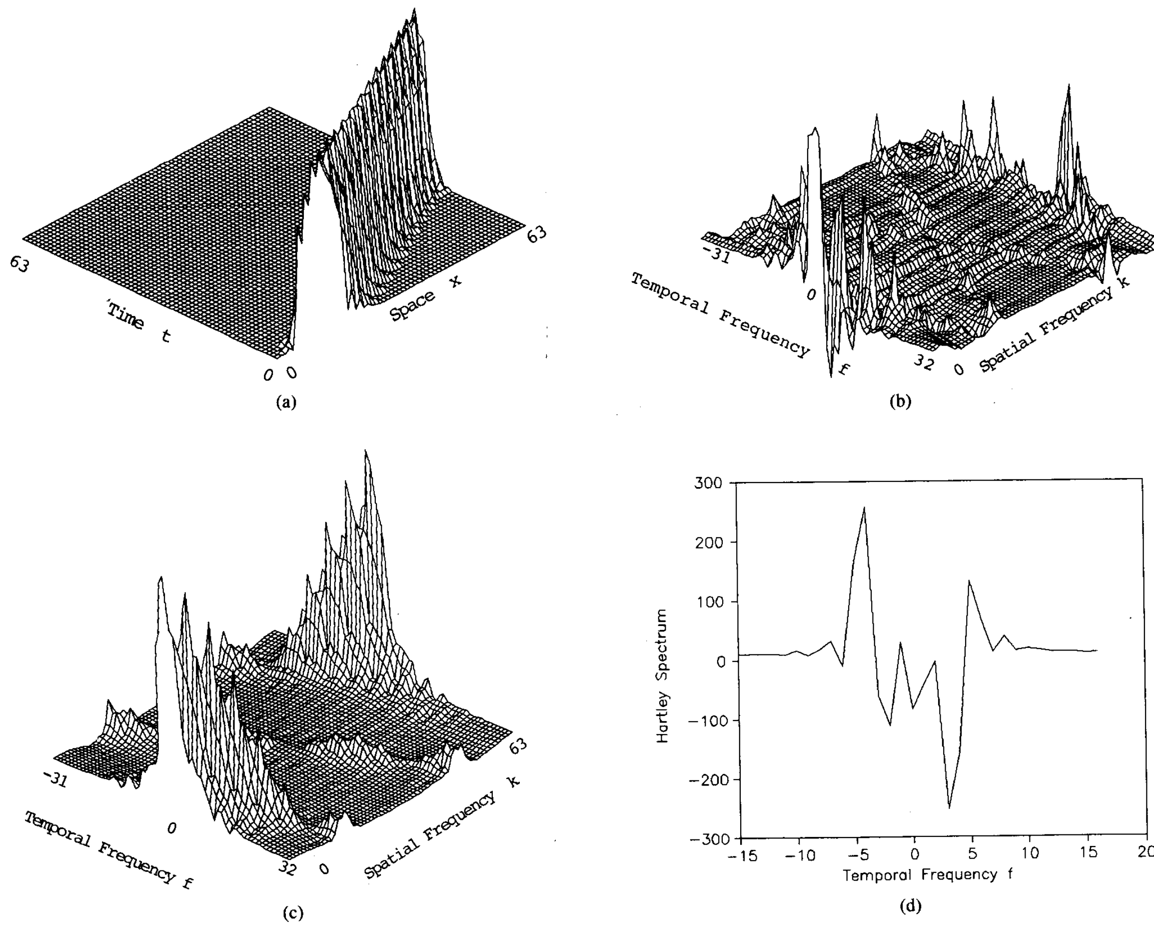

Fig. 1. (a) A sequence of large moving objects. (b) The Hartley spectrum of the time sequence. (c) The Fourier spectrum of the time sequence. (d) The Hartley spectrum at a spatial frequency of 2.

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Toeplitz Determinants and Positive Semidefiniteness

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Abstract—**We explore the role that the determinants of real, symmetric, Toeplitz matrices play in testing for their positive semidefiniteness. We show that the "leading principal minor" test used to test for positive definiteness is not sufficient in general to test for positive semidefiniteness of Toeplitz matrices, except in certain cases. We derive several properties and show in which cases the leading principal minor test is indeed sufficient. We then present a simple method for testing the positive semidefiniteness of all symmetric Toeplitz matrices.**

I. INTRODUCTION

It is well known that the eigenvalues of a positive definite (abbreviated p.d.) matrix are all positive and those of a positive

Manuscript received June 14, 1990; revised September 12, 1990.
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IEEE Log Number 9041604.

1053-587X/91/0300-0743$01.00 © 1991 IEEE