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A NEW TRAJECTORY CORRECTION TECHNIQUE FOR LINACS*

T. O. RAUBENHEIMER AND R. D. RUTH
Stanford Linear Accelerator Center, Stanford University, Stanford, CA, 94309

ABSTRACT

In this paper, we describe a new trajectory correction technique for high energy linear accelerators. Current correction techniques force the beam trajectory to follow misalignments of the Beam Position Monitors (BPMs). Since the particle bunch has a finite energy spread and particles with different energies are deflected differently, this causes "chromatic" dilution of the transverse beam emittance. The algorithm, which we describe in this paper, reduces the chromatic error by minimizing the energy dependence of the trajectory. To test the method we compare the effectiveness of our algorithm with a standard correction technique in simulations on a design linac for a Next Linear Collider (NLC).[1] The simulations indicate that chromatic dilution would be debilitating in a future linear collider because of the very small beam sizes required to achieve the necessary luminosity. Thus, we feel that this technique will prove essential for future linear colliders.

INTRODUCTION

In a linear collider there are many effects which dilute the beam emittance. This dilution then causes a reduction in the collider's luminosity. In this paper, we describe a new trajectory correction technique for linear accelerators which reduces the "chromatic" emittance dilution. This technique is described in greater detail in Ref. 2.

The trajectory in a linac is corrected with dipole correctors. Usually, the correction algorithms attempt to zero the Beam Position Monitors (BPMs) which measure the trajectory in the linac. For example, in the Stanford Linear Collider (SLC) linac, a "one-to-one" algorithm is used to implement the correction. Here, a single dipole corrector is used to zero a single (downstream) BPM. Using this algorithm, each of the matched BPMs can be zeroed within the limitations of the corrector strength and the BPM precision, i.e., the intrinsic noise in the BPM measurements.

The problem with this technique is that the BPMs are typically misaligned, both electronically and mechanically. Thus, the corrected trajectory is kicked from side to side, following the BPM misalignments. Chromatic errors occur when the beam trajectory is deflected since the deflections differ for particles with different energies. With one-to-one correction, the chromatic dilution tends to grow with the square root of the number of BPMs.[3] For example, we have found that in the NLC[1] using the one-to-one correction technique with 10 μm BPM misalignments leads to a 25% vertical emittance dilution. The 10 μm alignment *is* about one order-of-magnitude better than can be achieved with techniques now in practice.

In this paper, we discuss an algorithm which is less dependent on the BPM misalignment errors than is the one-to-one algorithm. Our approach is to measure two trajectories where we change the linac focusing structure between measurements. By subtracting the two trajectories, the resulting difference orbit is independent of the BPM alignment errors. In theory, the quadrupole misalignments could now be found. Unfortunately, the difference orbit still has errors due to the finite BPM precision and additional unknown deflections. Rather than trying to solve for the individual misalignments, we simply correct the trajectory to minimize the difference orbit; this will then minimize the chromatic error.

THEORY

Chromatic Errors and Dilution

In this paper, we use the term chromatic error to describe the variation of the central trajectory with energy; it is a single particle effect. Chromatic errors arise because particles with different energies are deflected differently. We define the "chromatic error" of a particle with an energy $(1+\delta(s))E(s)$ as the difference between its trajectory and the trajectory of a particle with the design energy E(s). Thus, the chromatic error for this, particle, at the end of a linac, is

$$\Delta x(s_f) = \sum_i \theta_i \left[R_{12}(s_i, s_f) - \frac{1}{(1+\delta_i)} R_{12}(\delta; s_i, s_f) \right] \quad , \quad (1)$$

where we have set the initial conditions x_0 and x'_0 to zero. The parameter θ_i is the deflection at longitudinal position s_i and includes kicks from both correctors and errors, δ_i is the relative energy deviation from the design energy: $\delta_i \equiv \Delta E(s_i)/E(s_i)$, and the matrix element $R_{12}(s_i, s_f)$ transforms a deflection at s_i to a transverse position at s_f; the coefficient $R_{12}(\delta)$ is the R_{12} matrix element for a particle with energy deviation 6(s).

A particle beam consists of particles distributed in six-dimensional phase-space. Chromatic dilution of the transverse phase-spaces occurs for two reasons: first, the chromatic errors will cause each constant energy slice of the beam distribution to have a different centroid; they will follow different central trajectories. Second, the beam ellipses, i.e., the second-order moments, of the constant energy slices will differ since particles with different energy experience different focusing.

In this paper, we discuss correcting only the first contribution to the chromatic dilution. If the focusing structure of the accelerator is properly "matched," the second effect will typically be small. For example, in NLC linac this effect dilutes the emittance by less than 0.1%. Of course, when the linac is not properly matched this second effect can become large, but matching the linac is a separate issue and is beyond the scope of this report.

Trajectory Correction

To reduce the chromatic emittance dilution, we correct the energy dependence of the central trajectory. To do this, we vary the *effective* beam energy and then correct the *difference* of the resulting trajectory and the original trajectory. In a linac, there are two methods of changing the effective energy: changing the beam energy or, equivalently, changing the magnet strengths.

In principle, we could use the difference orbit to solve for the quadrupole misalignments and the initial conditions exactly, *provided* that there are (N_q + 2) BPMs which do not have precision errors and all the additional deflections are negligible. Obviously, this is not realistic. When the additional errors are included, the difference orbit is not a function of just (N_q + 2) unknowns. Thus, we cannot estimate the individual quadrupole misalignments and the initial conditions accurately. We have found that the best approach is to perform a least-squares solution for the unknowns using both the original trajectory and the difference orbit with the appropriate weighting. Thus, we solve for the dipole corrector strengths which minimize the sum

$$\sum_{j=1}^{N_q} \frac{(m_j + X_j)^2}{\sigma_{\rm prec}^2 + \sigma_{\rm BPM}^2} + \frac{(\Delta m_j + \Delta X_j)^2}{2\sigma_{\rm prec}^2} \quad . \quad (2)$$

Here, $\sigma_{\rm prec}$ is the RMS of the BPM precision errors and $\sigma_{\rm BPM}$ is

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an estimate of the RMS of the BPM misalignments relative to the linac centerline. In addition, m_j and X_j are the measured and predicted trajectories at the jth BPM, and Δm_j and ΔX_j are the measured and predicted difference orbits. We will subsequently refer to this algorithm as Dispersion-Free (DF) correction.

Error Effects

The algorithm corrects the chromatic errors by correcting a measured difference orbit which is created by changing the effective beam energy. Thus, the algorithm relies upon the *resemblance* between this measured difference orbit and the chromatic error. We can divide any errors into two categories: errors which cause the measured difference orbit to differ from the actual difference orbit (measurement errors) and errors which cause the difference orbit to differ from the chromatic error of a particle within the beam. Errors in neither category will not degrade the correction of the chromatic dilution and thus can be ignored.

In contrast, errors in either of the two categories will cause the algorithm to converge to an incorrect solution. BPM precision and beam jitter errors are examples of errors from the first category. RF deflections, magnet scaling errors, and effects due to the nonlinearity of the chromatic error are examples of errors in the second category. These effects are estimated in ref. 2 and are found not to cause serious degradation in the performance of the DF algorithm.

EXAMPLES

We have written a computer program to test the DF correction technique against the one-to-one correction algorithm on a preliminary design of the NLC linac. The program simulates random transverse misalignments of the quadrupoles and BPMs, and BPM precision errors.

The NLC main linac will accelerate bunches from 16.5 GeV to 250 GeV. Our lattice is composed of 210 simple FODO cells with phase advances of 90 degrees.[3] The bunch is assumed to have an RMS relative energy spread σ_ϵ of 1.0% at the beginning of the linac; this then decreases inversely with the beam energy as the bunch is accelerated. We have not included the energy spread induced by the longitudinal wakefields or BNS damping since wakefields will be small in the NLC by design. Finally, the beam emittances in the NLC are $\gamma\epsilon_x = 3 \times 10^{-6}$ m-rad and $\gamma\epsilon_y = 3 \times 10^{-8}$ m-rad, and the beam size at the end of the linac is roughly 10 μm x 1 μm.

To simulate correcting the orbit in the NLC, we use twenty different sets of random errors. The errors are found from gaussian distributions which have been cut off at two sigma. The quadrupoles are misaligned with an RMS of 70 μm relative to the linac centerline and the BPMs are misaligned 70 μm *relative* to the quadrupoles. Furthermore, we have included BPM precision errors of 2 μm, assuming that a measurement precision the order of the beam size will be achieved in the NLC.

Table 1. Correction in the NLC.

	1–to–1	DF
Orbit RMS	$89 \pm 3\,\mu$m	$54 \pm 3\,\mu$m
BPM RMS	$3 \pm 0.5\,\mu$m	$80 \pm 3\,\mu$m
Magnification of ϵ_y	$7.20 \pm 3.2\,\epsilon_{y0}$	$1.02 \pm 0.02\,\epsilon_{y0}$

Results from correcting the twenty sets of errors with the two correction schemes are listed in Table 1; the error on the data is equal to one standard deviation. The Orbit RMS data is the RMS of the trajectory relative to the linac centerline, while the BPM RMS data is the RMS of the BPM measurements. Notice that the one-to-one algorithm zeros the BPM readings (within the BPM precision) while the actual trajectory is relatively large. In contrast, our method corrects both the actual trajectory and

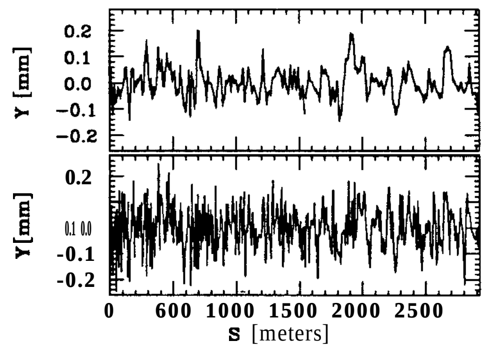

Figure 1: Trajectory after correction in the NLC.

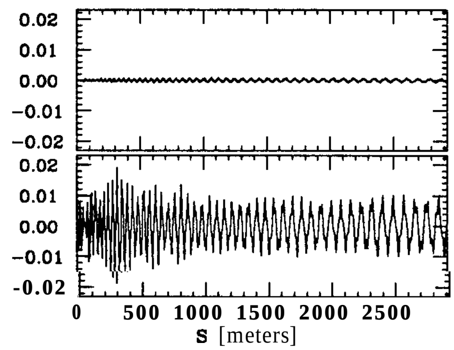

Figure 2: Difference orbit after correction in the NLC.

the measured BPM readings. In fact, the DF correction algorithm does better at correcting the actual trajectory than does the one-to-one method.

Of course, we are not only interested in correction of the trajectory. The dilution of the vertical emittance due to the chromaticerror is listed in the bottom row of Table 1. Obviously, the one-to-one correction technique leads to a large (factor of seven) increase in the vertical emittance. In contrast, the new technique performs very well, virtually eliminating the chromatic dilution.

The difference between correction techniques is illustrated in Figs. 14. Figure 1 compares the trajectory after DF correction (upper plot) with the result of one-to-one correction (lower plot) and Fig. 2 compares, in the same manner, the *difference* between the trajectory of an on-energy particle and a particle whose energy differs from the design, the energy difference being equal to the RMS energy spread. One can see that both techniques are comparable when correcting the trajectory, but the chromatic error, i.e., the difference orbit, is much smaller after DF correction.

Figure 3 is a plot of the Y-Y' phase-space at the end of the NLC linac after correction with the one-to-one algorithm. The curve plots the endpoints of particle trajectories having energies between +a, and $-\sigma_\epsilon$. Also, for reference, the RMS beam size, excluding the chromatic errors, is plotted about the design energy trajectory. Obviously, there is a large chromatic dilution in Fig. 3; the emittance magnification is roughly 9.1. For comparison, we plot, in Fig. 4, the Y-Y' phase-space at the end of the NLC after DF correction. This is the same phase-space, although with different scales, as Fig. 3. After DF correction, the emittance magnification is quite small, roughly a factor of 1.011.

It is evident from Table 1 and Figs. 14 that the DF correction technique performs substantially better that the one-to-one method. In all of the data shown, the effective energy change used by the DF algorithm *was* $\Delta E/E$ = 10%. In Fig. 5 we plot results of the DF correction technique, again found from the correction of twenty sets of random errors, versus the change in effective energy $\Delta E/E$. There are three curves: the dotted is the emittance magnification which has a scale on the right, the solid

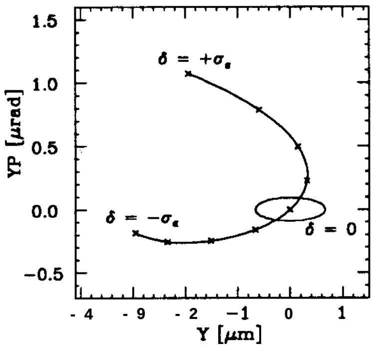

Figure 3: Y-Y' phase-space at the end of the NLC after l-to-l correction.

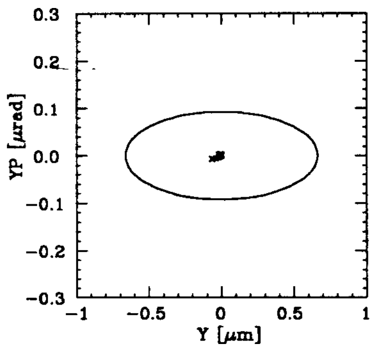

Figure 4: Y-Y' phase-space at the end of the NLC after DF correction.

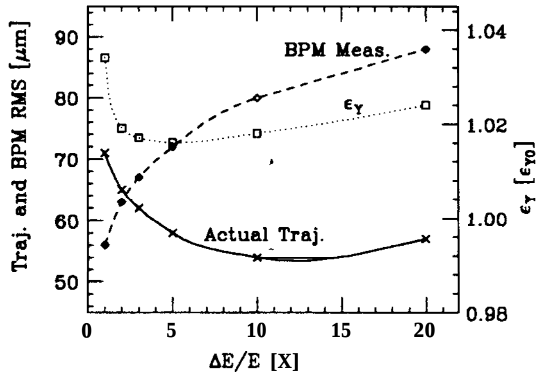

Figure 5: DF correction vs. the energy change $\Delta E/E$.

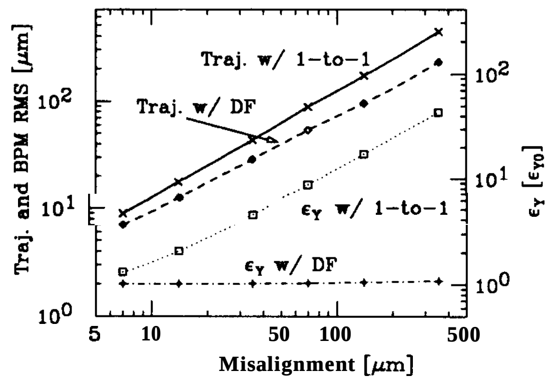

Figure 6: l-to-l and DF correction vs. the misalignments.

is the RMS of the trajectory, and the dashed curve is the RMS of the BPM measurement of the trajectory. Notice that both the emittance magnification and the RMS of the trajectory have broad minimums. The increase which occurs as $\Delta E/E$ increases is due to the nonlinearity of the chromatic error. In contrast, as $\Delta E/E$ decreases, the effectiveness of the algorithm is reduced since the difference orbit becomes lost in the noise of the BPM precision errors.

Finally, the dependence of the trajectory correction techniques on the misalignment amplitude is illustrated in Fig. 6. Here, we have varied the RMS magnitude of the vertical BPM and quadrupole misalignments. The points plotted were found from the average of correcting twenty sets of random errors. The solid and dashed lines, at the top of the plot, are the RMSs of the actual trajectory after correction with the one-to-one and DF techniques, respectively; these curves have scales on the left side of the figure. Although the DF technique is slightly better at correcting the actual trajectory, the two are very similar.

The two other curves, the dotted and the dot-dash lines, are the emittance dilution after correction with the one-to-one and DF techniques. The dilution after one-to-one correction is strongly dependent upon the misalignment magnitude. Here, the dilution varies from roughly 25% to 4000% as the misalignments increase. In contrast, the dilution after DF correction is only weakly dependent upon the misalignment magnitude; it increases slowly from roughly 1% to 6% of the emittance. Thus, when using DF correction, the chromatic dilution is effectively uncoupled from the magnitude of the misalignments.

SUMMARY

In this paper, we have described a new trajectory correction algorithm for linear accelerators that reduces the chromatic dilution of the transverse emittance while correcting the trajectory. The chromatic dilution arises because the beam is deflected due to stray fields and misalignments and it scales, roughly, with the size of the misalignments relative to the beam size. Future linear collider designs tend to have very small beams to achieve the necessary luminosity, and thus, if uncorrected, chromatic dilution would impose *extremely* tight alignment tolerances in these future machines.

We have demonstrated the effectiveness of our algorithm in simulations of the NLC linac while comparing with a standard technique, the one-to-one algorithm. In all cases, the DF correction algorithm reduced the chromatic dilution substantially while correcting the trajectory as well as the one-to-one algorithm. From simulation, we found that with 70 μm misalignments in the NLC, the one-to-one algorithm causes roughly a 700% increase in the vertical emittance. In contrast, the DF correction algorithm reduced the chromatic dilution to a few percent.

To conclude, we believe that our algorithm can effectively control the chromatic dilution in a linear accelerator while correcting the trajectory. It is important to note that with our method the chromatic dilution is roughly independent of the magnitude of the misalignments; the dilution depends upon the BPM precision and the magnitude of the scaling and RF errors (not discussed in this paper). This will be especially important for future linear colliders where it will be difficult to achieve extremely tight transverse alignment tolerances in a multikilometer linac.

REFERENCES

1. The NLC is a 250 GeV on 250 GeV linear collider being studied at SLAC. Some information can be found in Ref. 3. More detailed parameters can be found in both: ***Proceedings of the DPF Summer Study, Snowmass '88,*** and, ***Proceedings of the Int. Workshop on Next-Generation Linear Colliders,*** SLAC-335 (1988).
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3. R. D. Ruth, "Beam Dynamics in Linear Colliders," ***Proceedings of the XIV Int. Conf. on High Energy Acc.***, Tsukuba, Japan, 1989; SLAC-PUB-5091 (1989).