A New Type of Timing Attack: Application to GPS

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Main Result

- « Hamming Weight Cryptanalysis » of GPS

Hamming weight of several
(ephemeral) secret exponents

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Long term secret = Private key

Outline

- Introduction
- GPS Identification Scheme
- Hamming Weight Cryptanalysis
- Timing Attack on GPS
- Countermeasures
- Conclusion

Introduction

- GPS: Identification Scheme, [Girault 91]
- Modification of [Schnorr 89]
- Designed for smart cards
- Efficient for the prover: $z = y + cx$
- Security proof [Poupard, Stern 98]: Statistical Zero-Knowledge
- Selected by Nessie in 2003

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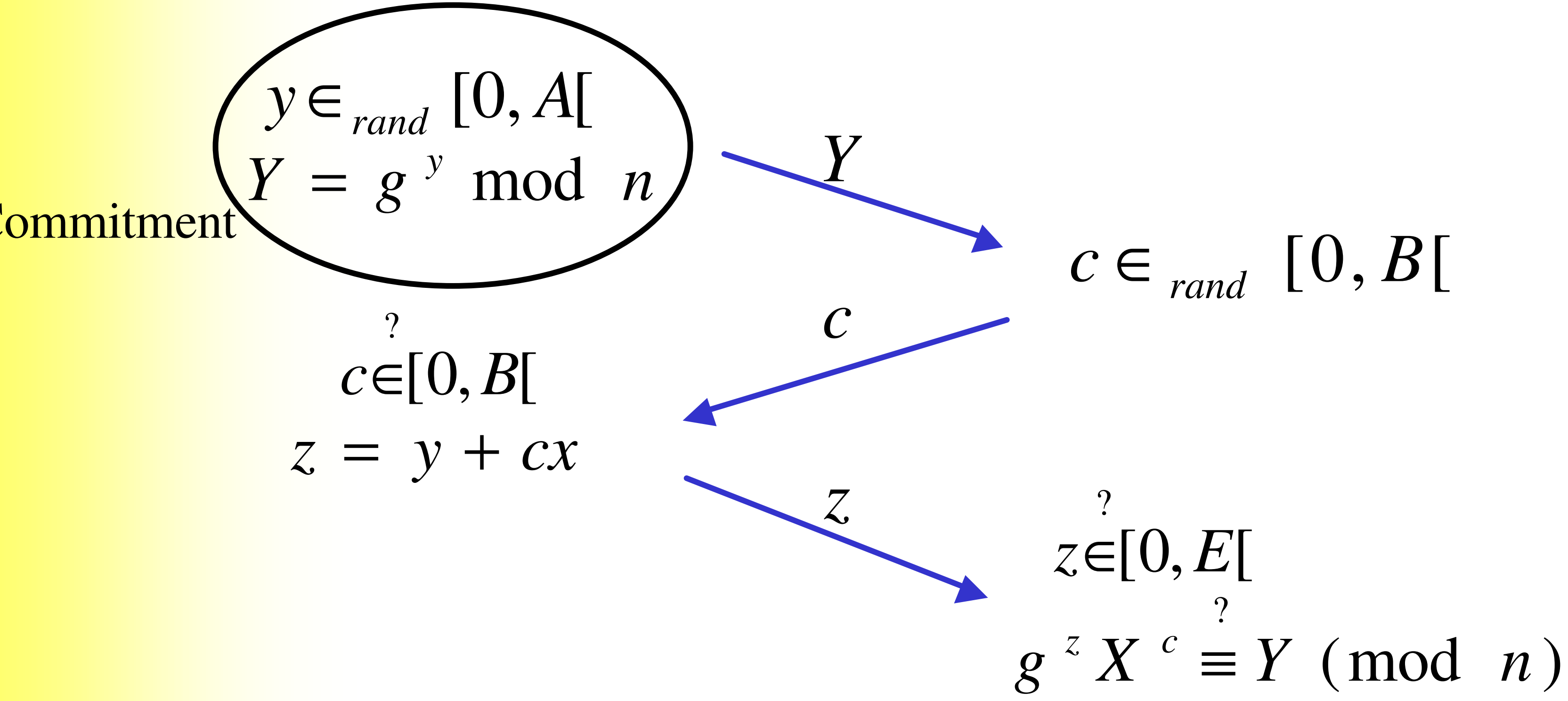
Basic GPS Parameters

- A modulus $n = pq$
- Integers A, B, S such that $A >> BS$
- An integer g ($g = 2$)
- Prover's private key: $x \in [0, S[$
- Prover's public key: $X = g^{-x} \bmod n$
- $E = A + (B-1)(S-1)$

Now $|A| = 240, |B| = 16, |S| = 160$

A Round of GPS

$X = g^{-x} \bmod n$



$y \in_{rand} [0, A[$

$Y = g^{y} \bmod n$

Commitment

Y

$c \in_{rand} [0, B[$

c

$c \overset{?}{\in} [0, B[$

$z = y + cx$

z

$z \overset{?}{\in} [0, E[$

$g^{z} X^{c} \overset{?}{\equiv} Y \pmod{n}$

$$\boxed{g^{z} X^{c} \equiv g^{y+cx} (g^{-x})^{c} \equiv g^{y} \equiv Y \pmod{n}}$$

The Commitment Step

- The commitment pairs $(y, Y = g^{y} \bmod n)$ can be computed:
  - Outside the card
    - Efficient
    - Limited number of identifications
  - Inside the card before the identification
    - Requires power
  - Inside the card during the identification
    - Requires a crypto-processor

Off-line variant

On-line variant

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HWC Principle

- Input: a list $(Hw(y^{(i)}), z^{(i)} = y^{(i)} + x)_{i=1,😐,k}$

  where $y^{(i)} \in_{rand} [0, A[$

- Output: a candidate value $\tilde{x}$ that is close to the key (i.e. such that $Hd(\tilde{x}, x)$ is small)

Information on the lsb

- We have $z = y + x$:

x_{159} 😐 $x_1 x_0$ → $P(x_0 = 1)$

⬆

- y_{239} 😐 $y_{160} y_{159}$ 😐 $y_1 y_0$ → $P(y_0 = 1) = \frac{Hw(y^{(i)})}{240}$

z_{239} 😐 $z_{160} z_{159}$ 😐 $z_1 z_0$ → known

$$y_0^{(i)} \oplus x_0^{(i)} = z_0^{(i)}$$

Each $w^{(i)}, z^{(i)}$ couple leads to an estimation of $P(x_0 = 1)$

Combining these estimations

- M_0 = mean of the k estimations of $P(x_0 = 1)$
- If $M_0 > \frac{1}{2}$ then $\tilde{x}_0 = 1$
  else $\tilde{x}_0 = 0$
- Assuming that $\tilde{x}_0 = x_0$, compute $(y_0{}^{(i)})_{i=1,\text{😐},k}$
- Update $(Hw(y^{(i)}))_{i=1,\text{😐},k}$
- Guess the carries $carry_1{}^{(i)}{}_{i=1,\text{😐}\,k}$
- Now ready to guess x_1

$$y_1{}^{(i)} \oplus x_1{}^{(i)} \oplus carry_1{}^{(i)} = z_1{}^{(i)}$$

Guessing the next bit

- We have $z = y + x$:

$$
\begin{array}{r}
x_{159} \;😐\; x_1 x_0 \\
+\; y_{239} \;😐\; y_{160} y_{159} \;😐\; y_1 y_0 \\
\hline
z_{239} \;😐\; z_{160} z_{159} \;😐\; z_1 z_0
\end{array}
$$

$$y_1^{(i)} \oplus x_1^{(i)} \oplus carry_1^{(i)} = z_1^{(i)}$$

$$y_0^{(i)} \oplus x_0^{(i)} = z_0^{(i)}$$

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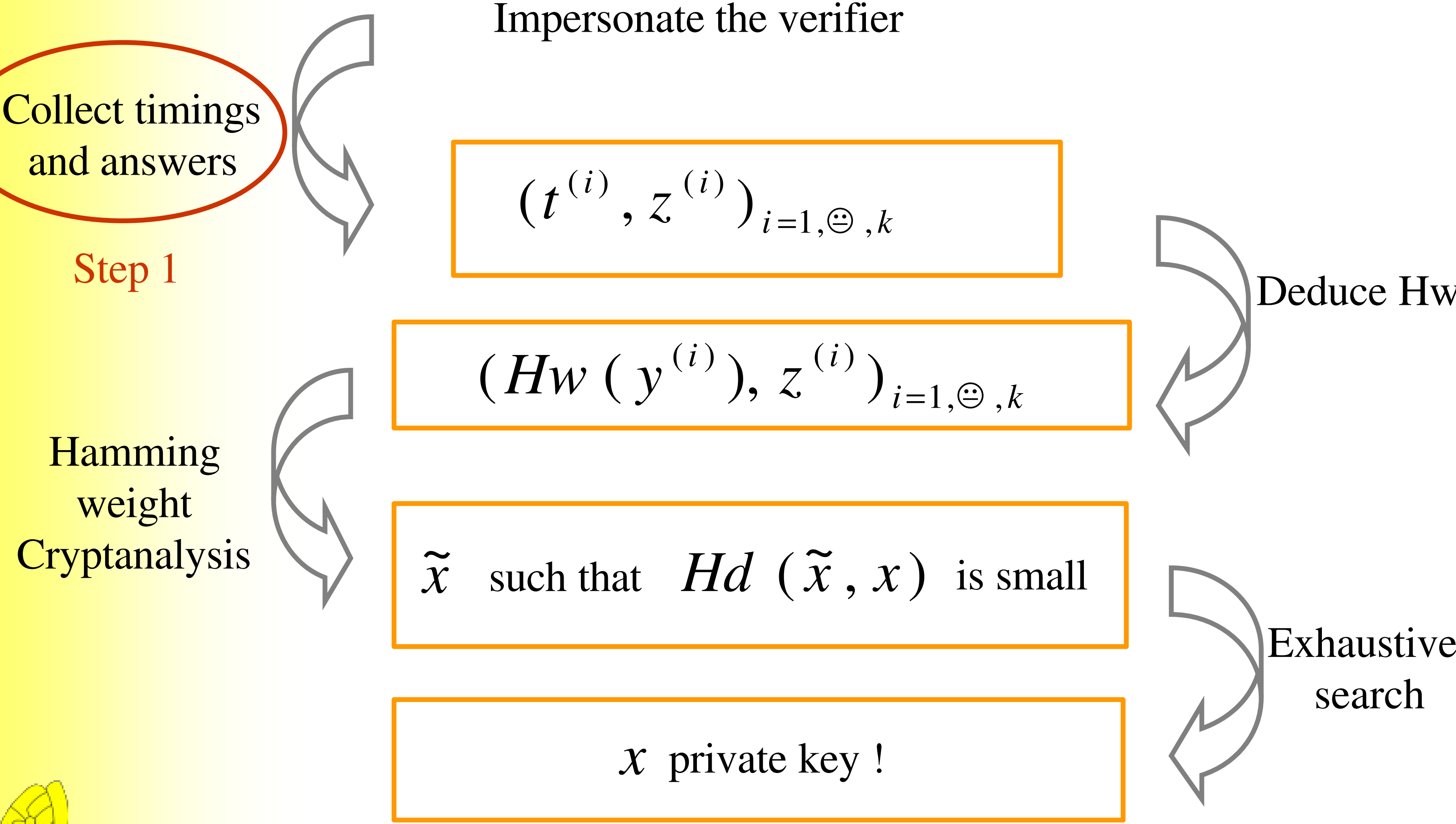
Conditions of success

- For HWC to work, the attacker must:
  - Impersonate a honest verifier
  - Get Hamming weights
- A natural way to do it: Timing Attack
  - Commitment computed *on-line*
  - Square and Multiply algorithm (or similar)

Attack Summary



Impersonate the verifier

Collect timings and answers

Step 1

$(t^{(i)}, z^{(i)})_{i=1,\text{😐},k}$

Deduce Hw

$(Hw(y^{(i)}), z^{(i)})_{i=1,\text{😐},k}$

Hamming weight Cryptanalysis

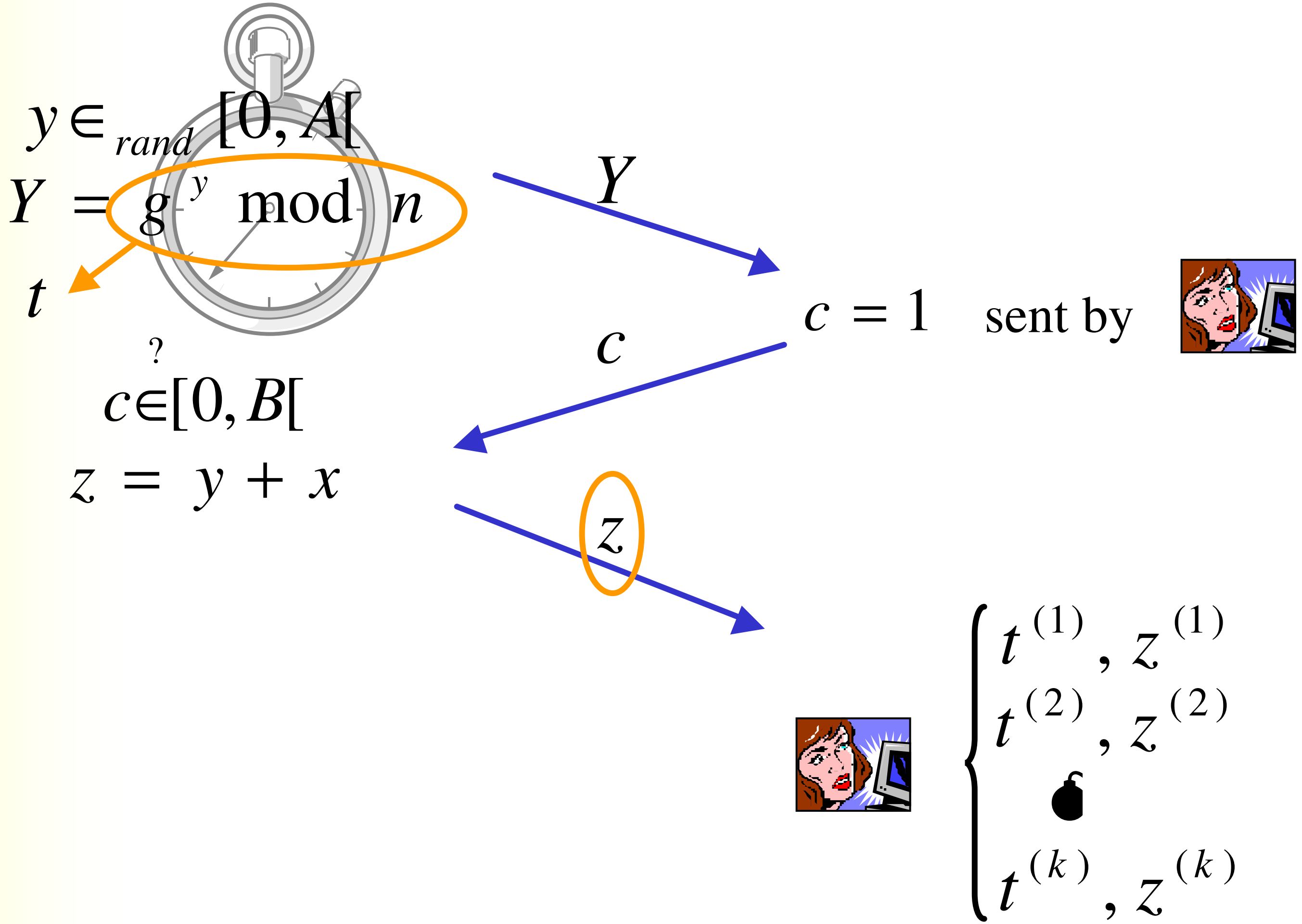
$\tilde{x}$ such that $Hd(\tilde{x}, x)$ is small

Exhaustive search

x private key !

Step 1

$X = g^{-x} \bmod n$



$y \in_{rand} [0, A[$

$Y = g^{y} \bmod n$

t

Y

$c = 1$ sent by

c

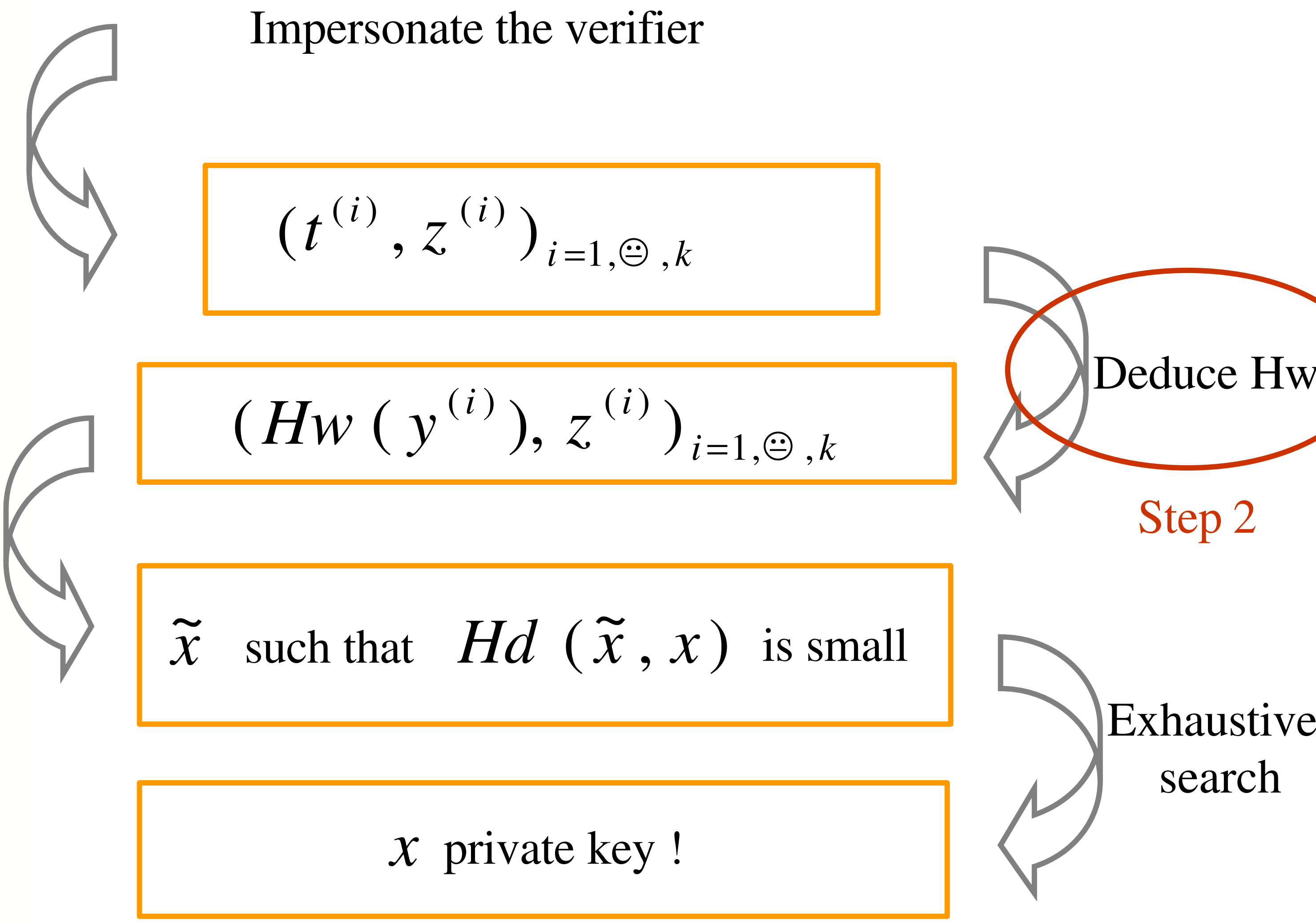
$c \overset{?}{\in} [0, B[$

$z = y + x$

z

$$\begin{cases} t^{(1)}, z^{(1)} \\ t^{(2)}, z^{(2)} \\ \vdots \\ t^{(k)}, z^{(k)} \end{cases}$$

Attack Summary



Impersonate the verifier

Collect timings and answers

$(t^{(i)}, z^{(i)})_{i=1,\dots,k}$

Deduce Hw

Step 2

$(Hw(y^{(i)}), z^{(i)})_{i=1,\dots,k}$

Hamming weight Cryptanalysis

$\tilde{x}$ such that $Hd(\tilde{x}, x)$ is small

Exhaustive search

x private key !

Step 2

- When $g^{y} \bmod n$ is computed with Square and Multiply then

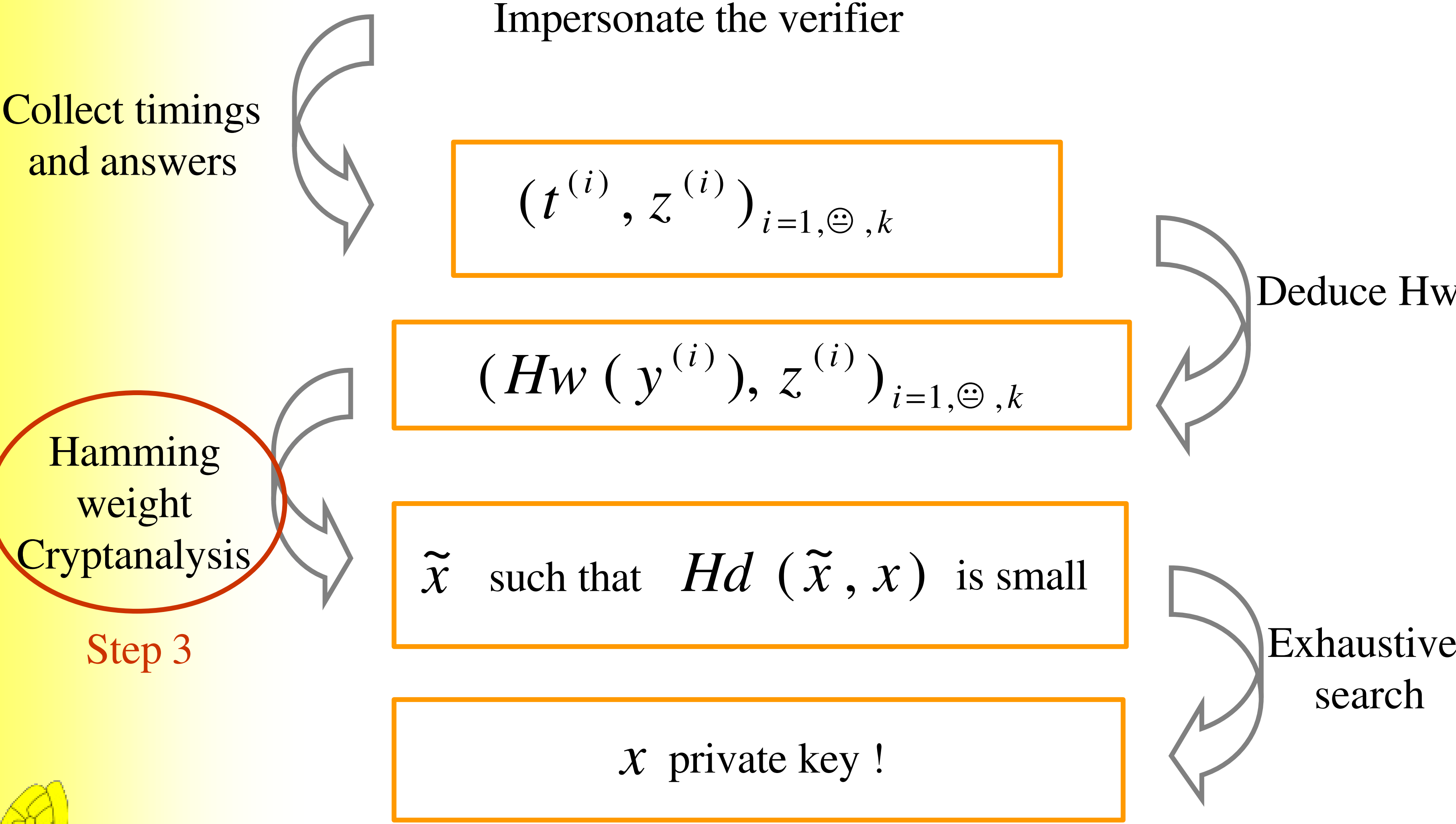
$$\begin{tikzcd}
Hw(y) \arrow[r, "linear"] \arrow[dr, "linear"'] & \text{Number of multiplications} \arrow[d, "linear"] \\
 & t \ \text{Computation time}
\end{tikzcd}$$

- With a linear regression

$$t^{(1)},😐,t^{(k)} \longrightarrow Hw^{(1)},😐,Hw^{(k)}$$

- Works whether CRT is used or not

Attack Summary



Impersonate the verifier

Collect timings and answers

$$(t^{(i)}, z^{(i)})_{i=1,\dots,k}$$

Deduce Hw

$$(Hw(y^{(i)}), z^{(i)})_{i=1,\dots,k}$$

Hamming weight Cryptanalysis

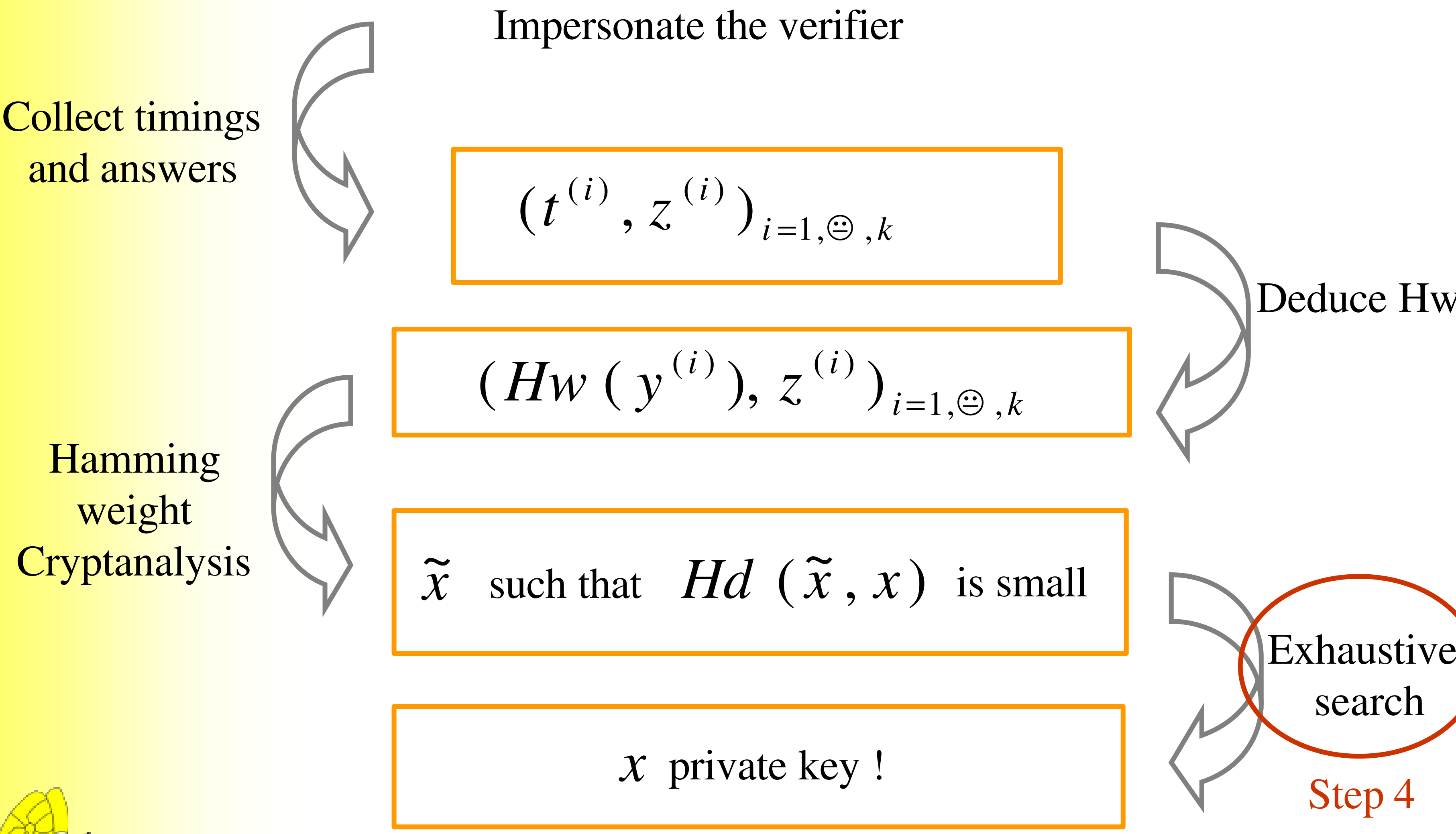
Step 3

$\tilde{x}$ such that $Hd(\tilde{x}, x)$ is small

Exhaustive search

x private key !

Attack Summary



Impersonate the verifier

Collect timings and answers

$(t^{(i)}, z^{(i)})_{i=1,\text{😐},k}$

Deduce Hw

$(Hw(y^{(i)}), z^{(i)})_{i=1,\text{😐},k}$

Hamming weight Cryptanalysis

$\tilde{x}$ such that $Hd(\tilde{x}, x)$ is small

Exhaustive search

Step 4

x private key !

Step 4: Experimental Results

k (number of samples)	200	400	600	800	1000
Immediate keys $\tilde{x} = x$	0%	2%	21%	52%	72%
seconds $Hd(\tilde{x}, x) \leq 2$	0%	3%	54%	80%	89%
hours $Hd(\tilde{x}, x) \leq 4$	0%	6%	72%	94%	97%
days $Hd(\tilde{x}, x) \leq 5$	0%	10%	77%	96%	98%
avg. distance $Hd(\tilde{x}, x)$	46	16.1	3.9	1.4	0.7

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Countermeasures

- Message blinding (Kocher)
- Tweak Montgomery multiplication (Dhem, Walter)

} Unappropriate

- Exponent blinding

} Efficiency !!!

Exponent blinding

- Before blinding: $g^{y} \bmod n$

  $$\text{where } |y| = 240 \text{ to } 300$$

- After blinding: $g^{y+t\times\varphi(n)} \bmod n$

  $$\text{where } |y + t\times\varphi(n)| = |n|$$

- It hides $Hw(y)$ but it’s not efficient

Countermeasures

- Message blinding (Kocher)
- Tweak Montgomery multiplication (Dhem, Walter)

Unappropriate

- Exponent blinding

Efficiency !!!

- Square & Multiply always

33% overhead

- Division Chains (MIST)
- Use pre-computed commitments

OK

Conclusion

- Hamming Weight Cryptanalysis is feasible
  - – Short list of Hamming weights → 160 bit key !
  - – A fast algorithm
  - – Works with approximations of $Hw(y^{(i)})$
- Application of HWC: Timing Attack
  - – An efficient side-channel attack on GPS

What if CRT is used ?

- Instead of $g^{y} \bmod n$, the prover computes $g^{y \bmod p-1} \bmod p$ then $g^{y \bmod q-1} \bmod q$
- Since $y \ll p, q$, we have $y \bmod p-1 = y$ and $y \bmod q-1 = y$
- The attack still works