A NOISE-COMPENSATED LONG CORRELATION MATCHING METHOD FOR AR SPECTRAL ESTIMATION OF NOISY SIGNALS

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ABSTRACT

A noise-compensated long correlation matching (NCLCM) method is proposed for autoregressive (AR) spectral estimation of the noisy AR signals. This method first computes the AR parameters from the high-order Yule-Walker equations. Next, it employs these AR parameters and uses the low-order Yule-Walker equations to compensate the zeroth autocorrelation coefficient for the additive white noise. Finally, it solves the low- as well as high-order Yule-Walker equations in a least-squares sense to determine the AR parameters. It is shown that for the noisy AR signals the NCLCM method performs better than the conventional Burg method and the high-order Yule-Walker method.

INTRODUCTION

For signals following the autoregressive (AR) model, the conventional AR spectral estimation methods (such as the forward-backward linear prediction method and the Burg method) perform reasonably well [1]. But, when these signals are corrupted by the addition of white noise, the AR model is no more valid and, as a consequence, the performance of the conventional methods is poor for noisy signals [2].

It is well known that in the autocorrelation domain only the zeroth order autocorrelation coefficient is affected by the additive white noise, while the other autocorrelation coefficients remain unchanged [2]. This fact has been exploited by many authors [3-5] to develop AR spectral estimation methods for noisy AR signals. In these methods, the high-order Yule-Walker equations are used for estimating the parameters of the pth order AR model. Gersch [3] and Chan and Langford [4] have used an exact number of p high-order Yule-Walker equations to compute p AR parameters, while Cadzow [5] has employed an overdetermined set of q (>p) Yule-Walker equations for this purpose. It has been shown [5] that among these methods the Cadzow's method results in the best performance for the noisy signals. We shall refer this method hereafter as the high-order Yule-Walker (HOYW) method.

Recently, Bruzzone and Kaveh [6] have shown that these estimators based on high-order Yule-Walker equations are statistically inefficient. The loss of efficiency is more for the wide-band signals and less for the narrow-band signals. This loss of efficiency can be reduced significantly if the low-order Yule-Walker equations are also used alongwith the high-order Yule-Walker equations by the AR estimator. However, the use of low-order Yule-Walker equations has a problem that it involves the zeroth autocorrelation coefficient which is seriously affected by the presence of the additive white noise in the AR signal.

In the present paper, we propose a new method, called the noise-compensated long correlation matching (NCLCM) method, for AR spectral estimation of noisy signals. This method compensates the zeroth autocorrelation coefficient for the additive white noise and uses both low- and high-order Yule-Walker equations for estimating AR parameters. It is shown that this method is statistically more efficient than the method using the high-order Yule-Walker equations only.

THE NCLCM METHOD

The NCLCM method computes the parameters $\{a_i, i=1,2,---,p\}$ of the pth order AR model using the following four steps:

Step 1: Compute the (p+q) autocorrelation coefficients from the observed noisy signal $\{y_n, n=1,2,---,N\}$ as follows:

$$R(i) = \frac{1}{N-i}\sum_{n=1}^{N-i} y_n y_{n+i}, \qquad i=1,2,---,p+q,$$

where q>p.

Step 2: Compute (in a least-squares sense) the AR parameters, $\{a_i\}$, from the q high-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(i-k) = -R(i), \qquad i=p+1,p+2,---,p+q.$$

Step 3: Compute (in a least-squares sense) the zeroth autocorrelation coefficient, R(0), from the p low-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(|i-k|) = -R(i), \qquad i=1,2,---,p,$$

where the AR parameters, $\{a_i\}$, are already known

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from step 2.

Step 4: With the knowledge of (p+q+1) autocorrelation coefficients, {R(i), i=0,1,---,p+q}, compute (in a least-squares sense) the desired AR parameters, $\{a_i\}$, from the following (p+q) Yule-Walker equations which include both low- and high-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(|i-k|) = -R(i), \qquad i=1,2,---,p+q.$$

RESULTS

In this section, we study the NCLCM method for the noisy AR signals. In order to put this method in a proper perspective, we compare here its performance with that of the conventional Burg method and the HOYW method. We report here only preliminary results. Detailed results will be reported later on.

Statistical bias and variance in estimating the AR parameters are used here as criteria for performance evaluation. Since there is always a trade off involved between the bias and the variance, we combine the bias and the variance and use the root-mean-square (RMS) error as a measure of the overall performance of the spectral estimation method. The RMS error is defined as:

$$\text{RMS error} = [\text{Bias}^2 + \text{Variance}]^{1/2} .$$

Following Reddi [7], we also define the normalized total-mean-square (NTMS) error as a measure of spectral estimation performance. This measure combines the RMS errors of the individual AR parameters as follows:

$$\hat{E}\ (\text{in dB}) = 10 \log_{10} \left(\sum_{i=1}^{p} \Delta a_i^2 \Big/ \sum_{i=1}^{p} a_i^2 \right),$$

where Δa_i is the RMS error in estimating the ith AR parameter.

For illustration, we select here an example of the AR process which has been commonly used in the literature [2,4]. In this example, the signal is generated from a fourth-order AR system (with parameters $a_1=-2.7607$, $a_2=3.8106$, $a_3=-2.6535$ and $a_4=0.9238$), driven by zero-mean unit-variance white Gaussian noise process. Here the AR signal consists of two narrow-band peaks in its spectrum.

We generate 100 independent realizations of the fourth-order AR process and use N=40 data points during steady-state to compute AR parameters for each of these realizations. Different spectral estimation methods are used here with p=4 and p+q=N/2. Statistical bias, variance and root-mean-square (RMS) error in estimating AR parameters are computed by ensemble-averaging over these 100 realizations. Results are listed in Table I for the Burg, the HOYW and the NCLCM methods. It can be seen from this table that the NCLCM method results in better performance than the Burg and the HOYW methods. In terms of NTMS error, it shows an advantage of 7.8 dB over the Burg method and 0.9 dB over the HOYW method. Also, it can be seen that the Burg method results in smaller variance than the HOYW and the NCLCM methods, but it has too much of bias and, hence, its overall performance is significantly poorer than the other two methods.

In order to see the performance of these methods for long data records, we study them for the noisy AR signal (with SNR=20 dB) for N=140 data points. Results are shown in Table II and Fig. 1. We show in Fig. 1(a), (b) and (c) the power spectral estimates for the first 20 realizations resulting from the three methods and in Fig. 1(d) the average of these estimates along with the true power spectrum of the AR process. It can be seen from Table II and Fig. 1 that the NCLCM method is only marginally superior to the HOYW method, but it is significantly better than

Table I. Statistical performance of different methods in estimating the AR parameters $\{a_1, a_2, a_3, a_4\}$ for the noisy AR signal at SNR=20 dB with N=40 data points.

	a_1	a_2	a_3	a_4
True values	-2.7607	3.8106	-2.6535	0.9238
Bias-Burg	1.6418	-3.2937	2.9936	-0.9751
Bias-HOYW	0.5130	-1.0876	1.0173	-0.3662
Bias-NCLCM	0.4371	-0.9417	0.8908	-0.3284
Variance-Burg	0.0387	0.0745	0.0599	0.0243
Variance-HOYW	0.2430	0.9823	0.8683	0.1239
Variance-NCLCM	0.2066	0.8679	0.7865	0.1143
RMS error-Burg	1.6535	3.3050	3.0036	0.9874
RMS error-HOYW	0.7114	1.4714	1.3795	0.5080
RMS error-NCLCM	0.6306	1.3247	1.2570	0.4713

Table II. Statistical performance of different methods in estimating the AR parameters $\{a_1, a_2, a_3, a_4\}$ for the noisy AR signal at SNR=20 dB with N=140 data points.

	a_1	a_2	a_3	a_4
True values	-2.7607	3.8106	-2.6535	0.9238
Bias-Burg	1.6273	-3.3154	3.0419	-1.0190
Bias-HOYW	0.0441	-0.1063	0.1078	-0.0450
Bias-NCLCM	0.0422	-0.1032	0.1047	-0.0441
Variance-Burg	0.0081	0.0179	0.0120	0.0078
Variance-HOYW	0.0064	0.0287	0.0265	0.0043
Variance-NCLCM	0.0062	0.0277	0.0253	0.0040
RMS error-Burg	1.6298	3.3181	3.0439	1.0228
RMS error-HOYW	0.0915	0.2000	0.1951	0.0796
RMS error-NCLCM	0.0893	0.1958	0.1905	0.0773

the Burg method. Also, the variance in estimating the AR parameters is comparable for the three methods, while the bias from the Burg method is significantly more than that from the other two methods.

Next, we study the effect of data-record length on the performance of the three methods (Burg, HOYW and NCLCM). For this, we take SNR=20 dB and compute the value of the NTMS error for different data-record lengths. Results are shown in Fig. 2. It can be observed from this figure that the HOYW and the NCLCM methods show improvement with record length, while the Burg method shows no such improvement. We also notice that the NCLCM method results in better performance than the HOYW method for all record lengths (and more so for short records).

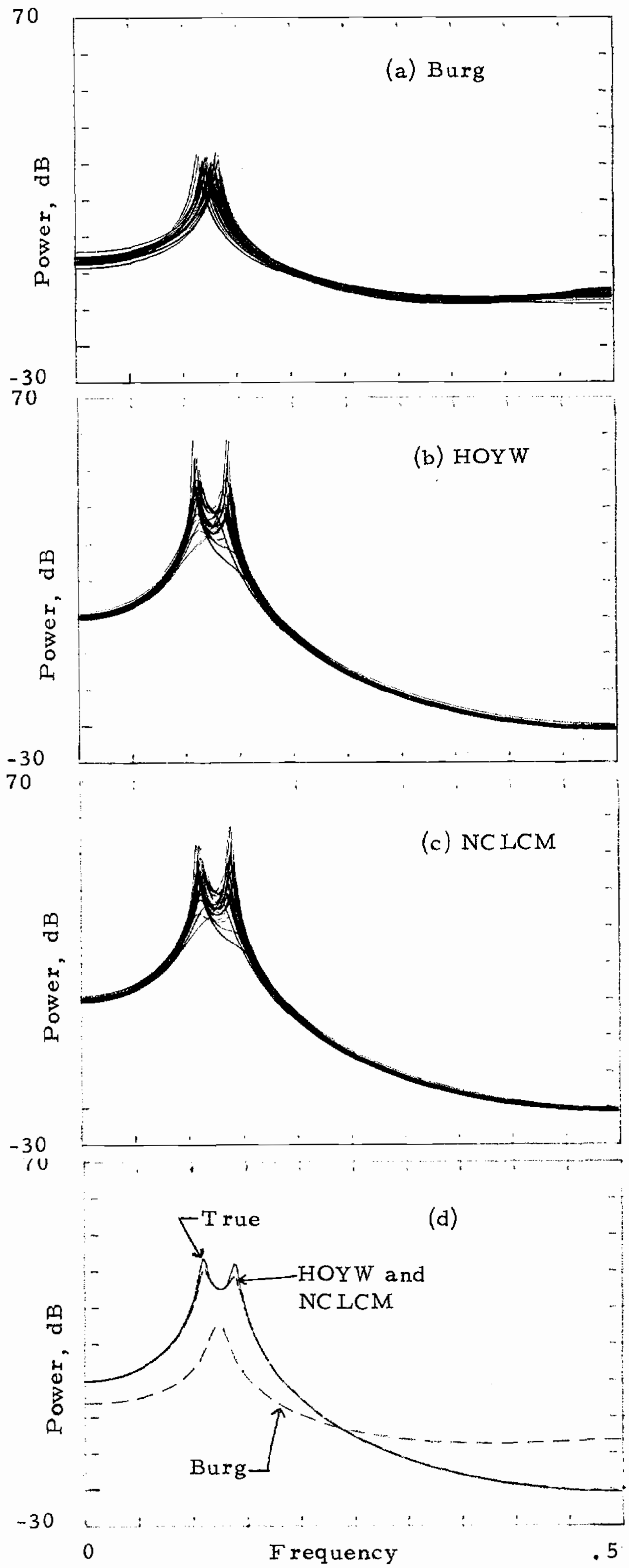

Fig. 1. Spectral estimation performance of the Burg, HOYW and NCLCM methods for the noisy AR signal with SNR=20 dB and N=140.

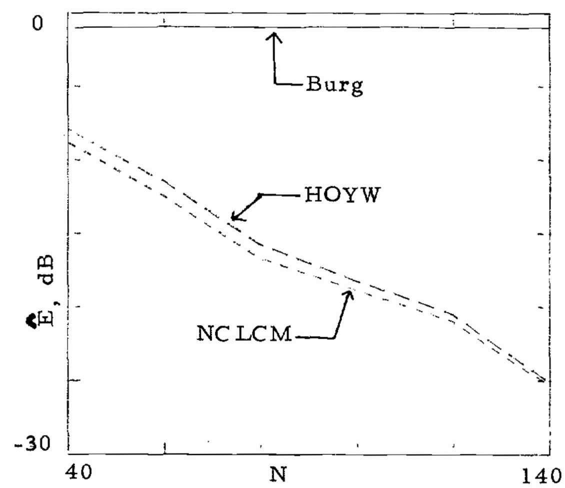

Fig. 2. Normalized total-mean-square error as a function of record length. SNR=20 dB.

So far, we have studied the Burg, the HOYW and the NCLCM methods for the noisy signals having a fixed SNR value. Now, we study these methods for different SNR values. For this, we take N=40 and study the NTMS error as a function of SNR. Results are shown in Fig. 3. We can see from this figure that the performance of each of the three methods improves with SNR. For low SNR (0-10 dB) and high SNR (40-50 dB) values, the three methods are comparable in performance. For medium SNR values (20-30 dB), the NCLCM method shows advantage over the other two methods.

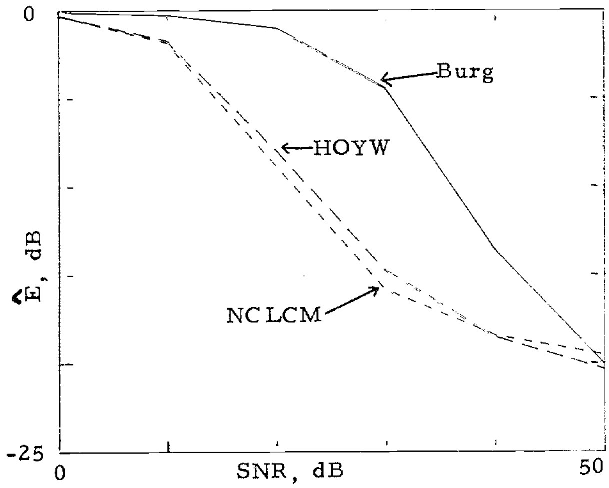

Fig. 3. Normalized total-mean-square error as a function of SNR. N=40.

CONCLUSIONS

In the present paper, the NCLCM method has been proposed for AR spectral estimation of the noisy signals. We have studied the performance of this method for different data-record lengths and different SNR values. Preliminary results have shown that for medium SNR values (20 to 30 dB) the NCLCM method performs better than the Burg and the HOYW methods for all record lengths. But, for low SNR (0 to 10 dB) and high SNR (40 to 50 dB) values the three methods are comparable in performance.

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