A NORMALIZED EQUATION OF AXIALLY LOADED PILES IN ELASTO-PLASTIC SOIL

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ABSTRACT

Based on the elasto-plastic soil model, a set of normalized equations of axially loaded piles are established. It was found that the curves of normalized load-characteristic value of piles or normalized settlement-characteristic value of piles are distributed in a range and bounded by two lines. The upper bound is the curve of high end bearing pile and the lower bound is the curve of the pure friction pile. Due to the character of axially loaded piles, the normalized equations present a simple and effective approach to evaluate the load and settlement of piles.

Key words: Elasto-plastic soil, axially loaded pile, normalized equation.

1. INTRODUCTION

The pile is widely used in the geotechnical engineering, especially the axially loaded piles, because piles will increase the bearing capacity and decrease the settlement of foundation effectively in soft layers. In general, the allowable settlement of structure controls the design of foundation. Hence, the load-settlement relation of piles is most concerned by the geotechinical engineers.

The relationship of shear stress and displacement along the pile of real soil is nonlinear and varies with the depth as shown in Fig. 1. Hence, the close form solution is unavailable in the usual case. For example, if the relation of shear stress and displacement is a hyperbolic function, the solution must be expressed by an elliptic function. If the relation of shear stress and displacement of soil is constant but linearly varies with depth, the solution must be expressed by the Bessel function (O'Neil, 1991). Both the elliptic function and Bessel function are very complicated. These solutions must be resort to the numerical analysis.

The load transfer method (*t-z* curve) presented by Coyle and Reese (1966) is a very popular numerical method used in the pile engineering. Although this method is very effective and simple, the computer work is unavoidable. The purpose of this paper is to present a convenient equation for the axially loaded piles in elasto-plastic soil in normalized form for geotechnical engineers. This normalized equation will provide us an easy way to predict the pile behavior. The solutions can be calculated without difficulty using a calculator.

2. FORMULATION

Since it is only possible to obtain exact solutions for relatively simple problems, the elasto-plastic soil model to be con
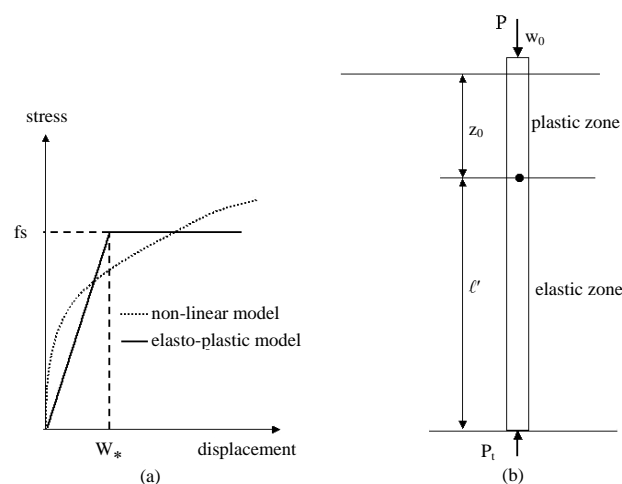
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Fig. 1 Axially loaded pile and soil model

sidered is shown in Fig. 1(a). Before the soil displacement reaching the yielding displacement w_*, the soil behaves elastically. In the elastic range, the shear stress increases linearly with displacement. Once the displacement of soil attains the yielding displacement w_*, the shear stress keeps constant as long as the displacement increases. The ratio of maximum shear stress f_s to the yielding displacement w_* is the coefficient of subgrade reaction in shear stress k_s.

2.1 Elastic Condition

In this analysis, a circular solid pile of length l and outer diameter d is considered. The elastic modulus of pile shaft is E. For the Winkler model, the surrounding soil layer and end bearing layer is assumed to have a constant subgrade reaction of shear stress k_s and subgrade reaction of vertical stress k_t, respectively. The strength of the end bearing layer is usually equal to or higher than that of the surrounding soil layer. If a load P is applied to the pile top as shown in Fig. 1(b), the settlement of the pile (w) along the depth z is controlled by the following differential equation (Scott, 1981)

$$\frac{d^2 w}{dz^2} - \lambda^2 w = 0 \tag{1}$$

$$\lambda = \sqrt{\frac{\pi d k_s}{EA}} \tag{2}$$

in which A = the cross area of pile and λ = the characteristic length of pile. For Eq. (1), the general solution includes two constants as follows,

$$w = C_1 \, e^{-\lambda z} + C_2 \, e^{\lambda z} \tag{3}$$

The two boundary conditions considered are: at the pile top $z = 0$, $F = P$; at the pile tip $z = l$, $F = P_t = A w_l k_t$. The variable w_l is the displacement of the pile tip. Based on the general solution, the constant C_1 and C_2 are solved. Thus, the pile settlement at the pile top is

$$w_0 = \frac{P}{\lambda EA} \left(\frac{1 + \dfrac{k_t}{\lambda E} \tanh(\lambda l)}{\tanh(\lambda l) + \dfrac{k_t}{\lambda E}} \right) \tag{4}$$

From Eq. (4), the force at the pile top is

$$P = \lambda EA w_0 \left(\frac{\tanh(\lambda l) + \dfrac{k_t}{\lambda E}}{1 + \tanh(\lambda l) \dfrac{k_t}{\lambda E}} \right) \tag{5}$$

Because of the special form of the equation above, we can define a function of hyperbolic tangent of η which equals the value of $k_t / \lambda E$. The variable η is the characteristic value of the end bearing capacity of the pile.

$$\tanh(\eta) = \frac{k_t}{\lambda E} \tag{6}$$

The hyperbolic tangent of η is in proportion to the subgrade reaction of vertical stress k_t and is inverse to λE. We will see later that $\tanh(\eta)$ is in terms of the ratio of the elastic modulus of soil at the pile tip to the elastic modulus of the pile shaft. Then, it is interesting to note that the relationship between the applied force and settlement involves a hyperbolic tangent function:

$$P = \lambda EA w_0 \tanh(\lambda l + \eta) \tag{7}$$

in which λl is the characteristic value of the pile. Both λl and η are nondimensional value. In the elastic condition, a linear relation exists between the settlement and load at the pile top. It is seen apparently in this compacted form that the contribution of the values of η and λl to the pile is in the hyperbolic tangent function. If the value of η equals zero, a pure friction pile, the value of λl will dominate the behavior.

2.2 Elasto-Plastic Condition

As the load on the pile top increases, the soil surrounding the pile begins to yield downward. As the soil yields to a depth of z_0, the pile can be divided into two parts in the analysis as shown in Fig. 1(b). In the upper part, the soil is yielding to the depth of z_0. In the lower part, the soil is still in an elastic condition to the range of length l'. Since the shear stress keeps constant in the plastic zone, the differential equation is similar to Eq. (1) and

may be expressed as

$$\frac{d^2 w}{dz^2} - \frac{\pi \, df_s}{EA} = 0 \tag{8}$$

In which $f_s = k_s \, w_*$. There are two constants to be determined in the general solution. The first condition considered at the pile top is: $z = 0$; $F = P$ or $w = w_0$. The connected point C provides the second condition. For the elastic zone, at the connected point C, the settlement is w_* and the force is P_{z0}. From the elastic solution in Eq. (7), $P_{z0} = \lambda Ea w_* \tanh(\lambda l' + \eta)$. Hence, the load applied to the pile top may be expressed as

$$P = \lambda EA \, w_* \tanh(\lambda l' + \eta) + \pi \, df_s \, z_0 \tag{9}$$

the settlement at the pile top is therefore

$$w_0 = w_* + \frac{1}{2} w_* \left(\frac{P}{\lambda EA w_*} \right)^2 - \frac{1}{2} w_* \tanh^2(\lambda l' + \eta) \tag{10}$$

Corresponding to the change of l' or $(l - z_0)$, Eqs. (9) and (10) give the load and settlement at the pile top. The ultimate load is the load as the friction along the pile is fully mobilized, *i.e.*, $l' = 0$. In the derivation above, the soil of end bearing capacity is assumed in the elastic condition. Basically, both the load and settlement of pile are controlled by the characteristic value λl and η as in the elasto-plastic condition. If two different pile-soil systems constitute the same λl and η values, they will display the same behavior.

From Eq. (9), as the settlement at the pile top just equals w_*, the corresponding load is very useful in the application and is denoted as the normalized load factor P_c

$$P_c = \lambda \, EA \, w_* \tanh(\lambda l + \eta) \tag{11}$$

3. SOIL PARAMETERS

In the elastic theory, the parameters are the soil modulus E_s and Poisson ratio v. Since the variation of the value of Poisson ratio is in a narrow range and has little influence on the calculated result, a constant value of 0.35 or 0.4 usually will be used in the analysis. These parameters can be obtained conveniently from laboratory or field test. In the Winkler model, the soil parameters are the coefficient of subgrade reactions k_s for side friction and k_t for end bearing of the pile. These parameters are usually obtained from experiences or back analyses via in-situ pile tests.

Based on the elastic theory, Scott (1981) derived the relations of the coefficient of subgrade reactions k_t and k_s, respectively, as follows:

$$k_t = \frac{E_s}{d(1 - v^2)} \tag{12}$$

$$k_s = \frac{E_s}{4d(1 - v^2)} \tag{13}$$

In which d is the diameter of the pile. The value of k_t is four times of k_s. The value of some typical elastic modulus of soil for

engineering purpose has been compiled by Das (1999) and is listed in Table 1. The corresponding values of k_t and k_s using Eqs. (12) and (13) as diameter equal 1 m are listed in Table 1 also for reference. Although the Eqs. (12) and (13) are very simple, they are very useful for the preliminary estimation.

For the in-situ concrete piles, the elastic modulus of pile shaft depends on the yielding strength of concrete. The value is about in the range of $2.0 \sim 2.4 \times 10^7$ kN/m². The ratio of pile length to diameter (l/d) is in the range of $20 \sim 50$. The less the ratio, the more rigid the pile. Conversely, the larger the ratio, the more compressible the pile.

Substituting Eq. (13) into Eq. (2), we obtain

$$\lambda l = \sqrt{\frac{E_s}{E(1-\nu^2)}\left(\frac{l}{d}\right)} \tag{14}$$

If the influence of Poisson ratio is ignored, the value of λl will underestimate about 6%, then the approximate relation is

$$\lambda l \approx \sqrt{\frac{E_s}{E}\left(\frac{l}{d}\right)} \tag{15}$$

The characteristic value of the pile is in terms of the ratio of length to diameter and the ratio of elastic modulus of soil to that of the pile shaft. According to the ratio suggested by Poulos and Davis (1980), the values of λl for various types of soil are calculated using Eq. (15) and listed in Table 2. Based on the value of λl, the pile can be divided into two categories. For a rigid pile, the value of λl is equal to or less than 0.5. For the compressible pile, the value of λl is larger than 0.5. It will be seen in an equation later, the amount of normalized settlement is in proportion to the square of λl.

Substituting Eqs. (12) and (15) into Eq. (6), we obtain

$$\tanh(\eta) = \sqrt{\frac{E_s}{E(1-\nu^2)}} = \sqrt{\frac{E_b}{E(1-\nu^2)}} \tag{16}$$

If the influence of Poisson ratio is ignored, the value of $\tanh(\eta)$ will underestimate about 6%. Then the approximate relation is

$$\tanh(\eta) \approx \sqrt{\frac{E_b}{E}} \tag{17}$$

The characteristic value of the end bearing capacity of the pile is in terms of the ratio of elastic modulus of soil at the pile tip to the elastic modulus of the pile shaft. The end bearing layer of the pile is usually stiff or dense soil or bed rock. Hence, the symbol E_b is used instead of E_s. Deere and Miller (1966) presented a classification graph for intact rock specimens based on the ratio of the elastic modulus to the unconfined compressive strength, together with the absolute value of the latter. For most rocks, the ratio E/q_u lies in the range of 200 to 500. Hence, three classes are defined as follows: For the higher class, the ratio is larger than 500. For the medium class, the ratio is between 200 and 500. For the lower class, the ratio is less than 200. Some relevant values of rocks and the calculated values of η are listed in Table 3.

Based on the value of η, the pile can be divided into four categories. For the pure friction pile, the value of η is zero. For

Table 1 Soil parameters

Soil type	E_s (MN/m²)[*]	k_s (MN/m²)[#]	k_t (MN/m²)[+]
Soft clay	4 ~ 21	1.14 ~ 5.99	4.56 ~ 23.94
Medium clay	21 ~ 41	5.99 ~ 11.69	23.94 ~ 46.74
Stiff clay	41 ~ 96	11.69 ~ 27.36	46.74 ~ 109.44
Loose sand	10 ~ 24	2.85 ~ 6.84	11.40 ~ 27.36
Medium sand	17 ~ 28	4.85 ~ 7.98	19.38 ~ 31.92
Dense sand	35 ~ 55	9.98 ~ 15.68	39.90 ~ 62.70

*   Das (1999)
\#   By Eq. (12) for d = 1m, ν = 0.35.
\+   By Eq. (11) for d = 1m, ν = 0.35.

Table 2 Value of λl

Soil type	E/E_s[*]	l/d	λl
Soft clay	6000	20	0.26
Medium clay	2000	35	0.78
Stiff clay	300	50	2.89
Loose sand	1500	20	0.52
Medium sand	1000	35	1.11
Dense sand	500	50	2.24

* Poulos (1980)

Table 3 Value of η

Rock type	q_u (kN/m²)	E^*/q_u	E_b	E_b/E	tanh (η)	η
Stiff clay	2000	150	3.0×10^4	0.0135	0.116	0.12
Soft rock	5000	200	2.0×10^5	0.045	0.23	0.22
Medium rock	20000	300	6.0×10^5	0.27	0.52	0.58
Medium rock	25000	400	1.0×10^6	0.45	0.67	0.81
Hard rock	40000	500	2.0×10^7	0.91	0.95	1.80
Hard rock	40000	520	2.08×10^7	0.95	0.97	2.10

* $E = 2.2\times10^7$ kN/m²

the stiff clay or soft rock, the lower end bearing capacity, the value of η is between 0.01 and 0.5. For the medium end bearing capacity, the value of η is between 0.5 and 1.5.

For the hard rock, the end bearing capacity, the value of η is between 1.5 and 3.0. Since the limiting value of hyperbolic tangent function is one and the value of tanh(3.0) is 0.99, it is reasonable to adopt η = 3.0 to be the upper bound for end bearing.

Regarding the yielding displacement of soil, it is hard to determine from the elastic theory. The value of the yielding displacement depends on the soil properties and pile dimensions. Vijayergiya (1977) has compiled some information of yielding displacement from in-situ pile tests, and this information is listed in Table 4 for reference.

4. NORMALIZED EQUATION OF LOAD AND SETTLEMENT

The equation of load or settlement derived above can be written in a normalized form when the normalized load factor P_c (in Eq. (11)) and normalized displacement factor w_* are used.

Table 4   Yielding displacement of soil, w_* (Vijayergiya, 1977)

In clay		In sand	
Pile size (mm)	w_* (mm)	Pile size (mm)	w_* (mm)
152	3.05	323	6.35
406	2.54 ~ 7.62	406	5.08 ~ 10.16
610	7.62 ~ 10.16	457	12.70 ~ 25.40
762	10.16 ~ 20.32	610	10.16

For the solution of elastic condition, the normalized equation obtained from Eq. (7) is

$$\frac{P}{P_c} = \frac{w_0}{w_*} \tag{18}$$

It is a linear relation. It means that the settlement will be in proportion to yielding displacement as the applied load is less or equal to the normalized load factor P_c.

For the elasto-plastic condition, the normalized form of the Eqs. (9) and (10) are

$$\frac{P}{P_c} = \frac{(\lambda l - \lambda l') + \tanh(\lambda l' + \eta)}{\tanh(\lambda l + \eta)} \tag{19}$$

$$\frac{w_0}{w_*} = 1 + \frac{1}{2}\left((\lambda l - \lambda l')^2 + 2(\lambda l - \lambda l')\tanh(\lambda l' + \eta)\right) \tag{20}$$

Both the normalized load equation and normalized settlement equation are parametric functions of λl, $\lambda l'$ and η. The parameter λl stands for the property of the pile-soil system. The parameter $\lambda l'$ and η stand for the amount of load mobilized along the pile and end bearing capacity, respectively. The value of η is related to the characteristic length λ of pile as shown in Eq. (6). The higher the value of η, the large the end bearing. As $\eta = 0$, it is a pure friction pile. Therefore, the solution implies that the pure friction pile is a special case of the end bearing pile.

As the shear stress along the pile is fully mobilized, *i.e.*, $l' = 0$, the Eqs. (19) and (20) will be simplified to the following equations:

$$\frac{P_u}{P_c} = \frac{(\lambda l) + \tanh(\eta)}{\tanh(\lambda l + \eta)} \tag{21}$$

$$\frac{w_0}{w_*} = 1 + \frac{1}{2}\left((\lambda l)^2 + 2(\lambda l)\tanh(\eta)\right) \tag{22}$$

These relations provide us a simple way to evaluate the ultimate load or settlement in terms of the normalized load factor P_c and normalized displacement factor w_* as the friction is fully mobilized along the pile.

For comparison, the curves of the normalized equations are plotted in Figs. 2 and 3 with varying characteristic values of λl and η. The value of η is specially assigned, *i.e.*, $\eta = 0.0, 0.5, 1.5$ and 3.0. It is seen that these curves are more and more close together as the value of η increases. It is clear to see that these curves will approach a limiting value as $\eta \geq 3.0$.

4.1 Lower Bound Curve

For the pure friction pile, *i.e.*, $\eta = 0$, the Eqs. (21) and (22) will reduce to

$$\frac{P_u}{P_c} = \frac{(\lambda l)}{\tanh(\lambda l)} \tag{23}$$

$$\frac{w_0}{w_*} = 1 + \frac{1}{2}(\lambda l)^2 \tag{24}$$

Equation (23) is a transcendental function and gives the ultimate load of pile. Equation (24) is a parabolic function and gives the settlement of the pure friction pile. There is a single point corresponding to each characteristic value of the pile. The points for $\eta = 0$ are connected using the solid line as shown in Figs. 2 and 3. This curve represents the lower bound for a family of curves of the normalized load or normalized settlement.

4.2 Upper Bound Curve

If $\eta \geq 3.0$, as the pile is installed on the very hard bed rock, $\tanh(\eta) \cong 1$ and $\tanh(\lambda l + \eta) \cong 1$. For this high end bearing pile, Eqs. (21) and (22) will reduce to

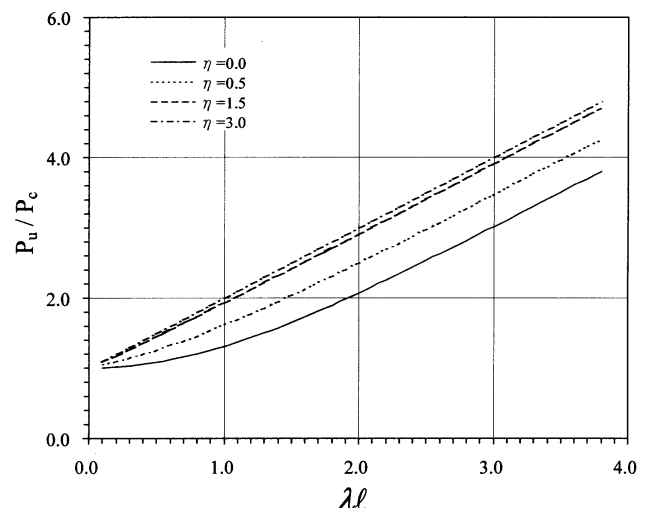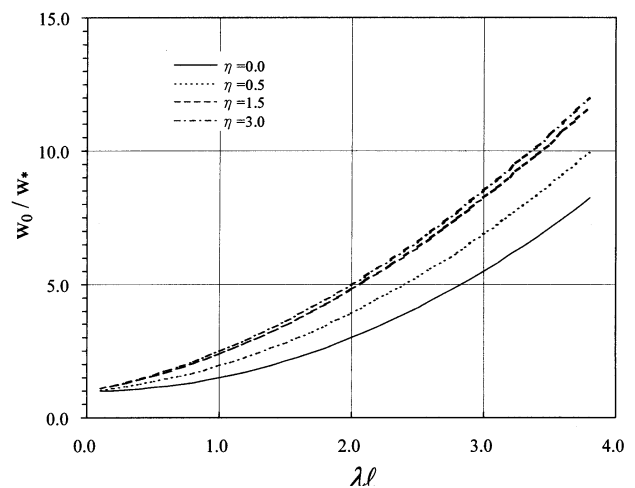$$\frac{P_u}{P_c} = 1 + \lambda l \tag{25}$$

Fig. 2   Normalized load-λl curve

Fig. 3   Normalized settlement-λl curve

$$\frac{w_0}{w_*} = \frac{1}{2} + \frac{1}{2}(1+\lambda l)^2 \qquad (26)$$

Equation (25) is a linear function and gives the ultimate load of pile. Equation (26) is a parabolic function and gives the settlement of pile. These two equations are plotted in Figs. 2 and 3 using the long dashed line to represent the upper bound of the normalized load or normalized settlement curves. In the figure, there are another two curves for $\eta = 0.5$ and $\eta = 1.5$ bounded by the upper and lower curves.

Due to the hyperbolic tangent function existing in Eq. (23), it seems complicated at first glance. However, as the value of λl increases, the value of $\tanh(\lambda l)$ will approach one. Therefore, the term on the right side of Eq. (23) will become λl. Comparison between Eqs. (23) and (25) shows that the difference is one. It is not hard to see in Fig. 2, the lines representing these functions are parallel as λl is larger than 2.0.

It is interesting to note that both Eqs. (24) and (26) are parabolic functions. It means that the amount of the normalized settlement is in proportion to the square of λl. The difference of these two equations is λl. It is evident to see in Fig. 3 that the distance between the upper and lower bounds increases with the increase of λl.

5. APPLICATION

Although the pile considered in the previous formulation belongs to in-situ concrete piles, these equations are still available for driven piles with hollow sections. For pipe piles, the area of the pile shaft in the calculation is the cross section area instead of the total area. In practice, the equations involve two different conditions as follows:

1. If the applied load is equal to or less than the normalized load factor P_c, the soil is in the elastic condition, and Eq. (18) is used.
2. If the applied load is larger than the normalized load factor P_c, part of the soil has arrived to yielding condition, and Eqs. (19) and (20) are used.

5.1 Example

A bored pile was installed in the medium clay of length 50 m and diameter 1 m. From soil tests, the elastic modulus of soil is found to be $E_s = 30000$ kN/m^2, and the yielding displacement of soil is $w_* = 0.5$ cm. In the analysis, elastic modulus of pile shaft $E = 2.2 \times 10^7$ kN/m^2 is used. Compute the settlement and load on the pile top as the pile is fully mobilized under the two conditions; A: For the pure friction pile, B: For the pile with end bearing capacity.

Solution:

From Eq. (13), for $d = 1$ m and $\nu = 0.35$ then $k_s = 8550$ kN/m^3

From Eq. (12), for $d = 1$ m and $\nu = 0.35$ then $k_t = 34200$ kN/m^3

For condition A: $\eta = 0$.

From Eq. (2), $\lambda = 0.039$ and then $\lambda l = 1.95$

From Eq. (11), $P_c = \lambda EA w_* \tanh(\lambda l + \eta) = 3233$ kN

From Eq. (23), $\dfrac{P_u}{P_c} = \dfrac{(\lambda l)}{\tanh(\lambda l)} = 2.03$, $P_u = 6563$ kN

From Eq. (24), $\dfrac{w_0}{w_*} = 1 + \dfrac{1}{2}(\lambda l)^2 = 2.9$,

$$w_0 = 2.90 \times 0.5 = 1.45 \text{ cm}$$

For condition B:

From Eq. (6), $\tanh(\eta) = 0.04$ then $\eta = 0.04$

From Eq. (11), $P_c = \lambda EA w_* \tanh(\lambda l + \eta) = 3243$ kN

From Eq. (21), $\dfrac{P_u}{P_c} = \dfrac{(\lambda l) + \tanh(\eta)}{\tanh(\lambda l + \eta)} = 2.07$, $P_u = 6713$ kN

From Eq. (22), $\dfrac{w_0}{w_*} = 1 + \dfrac{1}{2}\left((\lambda l)^2 + 2(\lambda l)\tanh(\eta)\right) = 2.98$,

$$w_0 = 2.98 \times 0.5 = 1.49 \text{ cm}$$

It is seen that the equation obtained above provides a simple and effective way for both the pure friction and end bearing pile. The curves in Figs. 2 and 3 present an easy chart to obtain the answer. For condition A in the example, it is a pure friction pile. The lower bound curve in Figs. 2 and 3 gives the answer. For condition *B* in the example, it is an end bearing pile with $\eta = 0.04$. The value of η is small. Then the curve of $\eta = 0$ in Figs. 2 and 3 can provide the answer with enough accuracy. For the other conditions which are not included in Figs. 2 and 3, an approximate answer will be obtained through interpolation.

5.2 Case Study

A bored pile was installed in the medium silt clay and the end bearing layer is sandstone. The length of pile is 45 m, and the diameter is 1 m. In the field, the load test procedure follows the conventional static load test. Since it belongs to a proof test, there is no instrument on the pile. The maximum load of the load test is 6000 kN.

Based on the record of the field test shown on the paper of Hsiung and Hung (2004), the elastic modulus of pile shaft E is 2.2×10^7 kN/m^2, the yielding displacement of soil w_* is 2.6 mm, k_s is 12000 kN/m^3 and k_t is 684000 kN/m^3. What is the load and settlement of this pile using the normalized equations offered in this paper?

Solution:

From Eq. (2), $\lambda = 0.0466$ then $\lambda l = 2.1$

From Eq. (6), $\tanh(\eta) = 0.668$ then $\eta = 0.81$

The value of λl is more than 0.5, the pile belongs to compressible piles. The value of η is more than 0.5, the pile belongs to medium end bearing piles. The first point to be considered is that the settlement at pile top just equal to the yielding displacement of soil.

From Eq. (11), $P_c = 2072$ kN and $w_0 = 2.6$ mm.

As the load on the top of pile increases, the mobilized friction of pile goes gradually downward. For example, $\lambda l'$ equals λl in the beginning, then $\lambda l'$ equals $0.8\lambda l$, $0.6\lambda l$, …, and $0.2\lambda l$. The final point to be considered is under the condition of friction fully mobilized, $\lambda l' = 0.0\lambda l$.

From Eq. (21), $\dfrac{P_u}{P_c} = 2.79$, the applied load is $P_u = 5773$ kN.

From Eq. (22), $\dfrac{w_0}{w_*} = 4.61$, the settlement is $w_0 = 11.99$ mm.

Table 5   Calculated loads and settlements of the pile in the case study

$\lambda l'$	P_0 / P_c	w_0 / w_*	P (kN)	w (mm)
2.10	1.00	1.00	2072	2.60
1.68	1.41	1.50	2931	3.91
1.26	1.82	2.17	3770	5.63
0.84	2.20	2.96	4562	7.71
0.42	2.54	3.83	5258	9.95
0.00	2.79	4.61	5773	11.99

The load and settlement of these points are calculated using Eqs. (19) and (20) and listed in Table 5. For comparison, the calculated points and the field test data are plotted together in Fig. 4. Since the elasto-plastic soil model is considered in the equations derived, the load-settlement relation displays nonlinear behavior when the load is more than the normalized load factor P_c.

Actually, the ultimate load of this test pile should be more than 5773 kN. Because the soil of end bearing capacity is assumed in the elastic condition in the equation derived, the ultimate capacity of bearing layer is not mobilized fully. This assumption will induce that the estimated bearing capacity is on the conservative side. This is the limitation of equations developed in this presentation. However, the coincidence or the tendency between the test record and calculated value is quite well. It is evident that the normalized equations provide a simple and effective way to evaluate the load or settlement of the pile.

6. CONCLUSIONS

From the equations derived, data analysis and comparison of case study described in the paper, the following conclusions are drawn:

1. Based on the elasto-plastic soil model, a set of normalized equations of axially loaded pile have been established. If the applied load is equal to or less than the normalized load factor P_c, a linear relationship exists between the normalized load and settlement. If the applied load is larger than the normalized load factor P_c, two parametric equations of λl and η present the relationship of the normalized load and settlement, respectively. These equations provide a simple and effective way to evaluate the load or settlement of the pile, and the solutions can be calculated using a calculator.
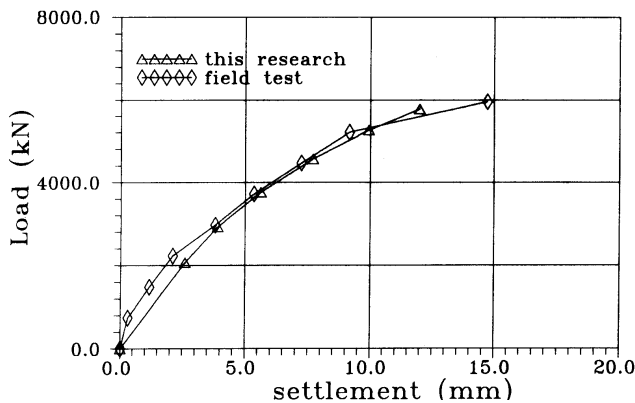

Fig. 4   Load-settlement curve of case study

2. The behavior of axially loaded piles is determined by two nondimensional variables, λl and η. The variable λl is the characteristic value of the pile and the variable η is the characteristic value of the end bearing capacity of the pile. Based on the value of λl, the pile can be divided into two categories: the rigid and compressible piles. Based on the value of η, the pile can be divided into four categories: pure friction, low, medium, and high end bearing capacity.

3. Basically, the dominant parameters in the equations are the yielding displacement, coefficient of subgrade reactions of shear stress k_s and vertical stress k_t. These parameters are usually obtained from experiences or back analyses via in-situ pile tests. The relations suggested by Scott (1981) are useful for the preliminary estimation.

4. The curves of the normalized load-λl and normalized settlement-λl for various value of η are distributed in a range and bounded by two curves. The upper bound is the curve for the high end bearing pile of $\eta = 3$. The lower bound is the curve for pure friction pile of $\eta = 0$. The mathematical functions of these curves are simple. These curves present a chart to determine the load or settlement of pile easily and effectively.

5. Because the soil of end bearing capacity is assumed in the elastic condition, the ultimate capacity of bearing layer is not mobilized fully. This assumption will induce that the estimated bearing capacity of pile is on the conservative side. This is the limitation of equations developed in this presentation.

NOTATIONS

The following symbols are used in this paper.

A = cross section area of pile

d = pile diameter

E = elastic modulus of pile shaft

E_b = elastic modulus of end bearing soil

E_s = elastic modulus of pile shaft soil

I = moment inertia of pile shaft

f_s = maximum shear stress of soil

k_h = coefficient of subgrade reactions of lateral stress

k_s = coefficient of subgrade reactions of shear stress

k_t = coefficient of subgrade reactions of vertical stress

l = pile length

P = pile load

P_c = normalized load factor

P_u = ultimate load

w_0 = settlement at pile top

w_l = settlement at pile tip

w_* = yielding displacement

w = settlement of pile

λ = characteristic length

λl = characteristic value of pile

η = characteristic value of end bearing

ν = Poisson ratio of soil

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