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A NOTE ON ACHROMATIC COLORING OF STAR GRAPH FAMILIES

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Abstract

In this paper, we find the achromatic number of central graph, middle graph and total graph of star graph, denoted by $C(K_{1,n}), M(K_{1,n})$ and $T(K_{1,n})$ respectively.

1 Introduction

For a given graph $G = (V, E)$ we do an operation on G, by subdividing each edge exactly once and joining all the non adjacent vertices of G. The graph obtained by this process is called central graph [14] of G denoted by $C(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph [4] of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G)\cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one the following holds: (i) x, y are in $E(G)$ and x, y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [4, 5] of G, denoted by $T(G)$ is defined as follows. The vertex set of $T(G)$ is $V(G)\cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one the following holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y are adjacent in G. (iii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

An achromatic coloring [1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15] of a graph G is a proper vertex coloring of G in which every pair of colors appears on at least one pair of adjacent vertices. The achromatic number of G denoted $\chi_c(G)$, is the greatest number of colors in an achromatic coloring of G.

The achromatic number was introduced by Harary, Hedetniemi and Prins [6]. They considered homomorphisms from a graph G onto a complete graph K_n. A homomorphism from a graph G to a graph G' is a function $\phi : V(G) \to V(G')$

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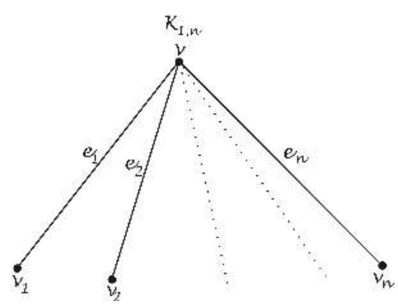
such that whenever u and v are adjacent in G, $u\phi$ and $v\phi$ are adjacent in G'. They show that, for every (complete) n-coloring τ of a graph G there exists a (complete) homomorphism ϕ of G onto K_n and conversely. They noted that the smallest n for which such a complete homomorphism exists is just the chromatic number $\chi = \chi(G)$ of G. They considered the largest n for which such a homomorphism exists. This was later named as the achromatic number $\psi(G)$ by Harary and Hedetniemi [6]. In the first paper [6] it is shown that there is a complete homomorphism from G onto K_n if and if only $\chi(G) \leq n \leq \psi(G)$.

2 Achromatic coloring of central, middle and total graph of star graphs

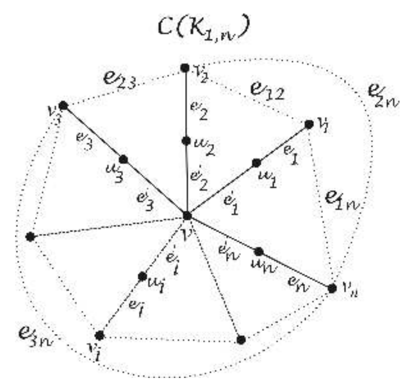
Theorem 2.1. *For any star graph $K_{1,n}$, the achromatic number,*

$$\chi_c[C(K_{1,n})] = n + 1.$$

Proof. Let $v_1, v_2, \cdots, v_n$ be the pendant vertices of $K_{1,n}$ and let v be the vertex of $K_{1,n}$ adjacent to $v_i (1 \leq i \leq n)$. Obviously, $deg(v) = n$. Let the edge vv_i be subdivided by the vertex $u_i (1 \leq i \leq n)$ in $C(K_{1,n})$, and let $V = \{v_1, v_2, \cdots v_n\}, V' = \{u_1, u_2, \cdots u_n\}$. Clearly $V[C(K_{1,n})] = V \cup V' \cup \{v\}$. The number of edges in $C(K_{1,n})$ is $\binom{n+1}{2} + n = \frac{n^2 + 3n}{2} < \binom{n+2}{2}$. Hence $\chi_c[C(K_{1,n})] \leq n + 1$. Note that in $C(K_{1,n})$, the induced subgraph $\langle v_1, v_2, \cdots v_n \rangle$ is complete, and $\{u_1, u_2, \cdots u_n\}$ is independent set.



Star graph $K_{1,n}$
Figure 1(a)

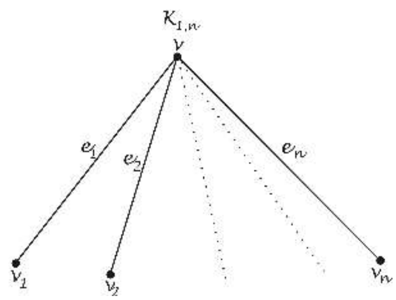


Central graph of Star graph $K_{1,n}$
Figure 1(b)

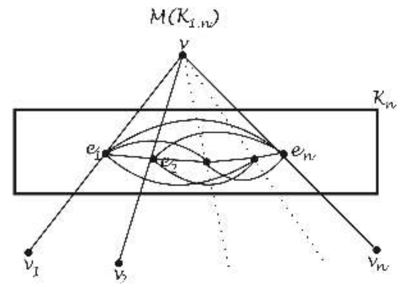
The following $(n+1)$-coloring for $C(K_{1,n})$ is achromatic: For $(1 \leq i \leq n)$, assign the color c_i for v_i. Assign color c_{n+1} for all $u_i (1 \leq i \leq n)$. Assign color c_1 for v. Thus we have $\chi_c[C(K_{1,n})] = n + 1$. □

Theorem 2.2. *For any star graph $K_{1,n}$ the achromatic number,*

$$\chi_c[M(K_{1,n})] = n + 1.$$



Star graph $K_{1,n}$
Figure 2(a)

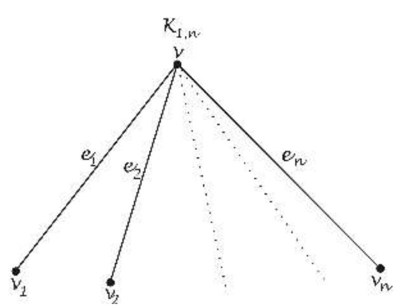


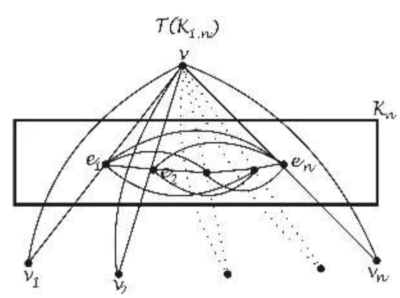
Middle graph of Star graph $K_{1,n}$
Figure 2(b)

Proof. Let $V(K_{1,n}) = \{v, v_1, v_2. \cdots, v_n\}$. By the definition of middle graph, each edge of $vv_i, (1 \leq i \leq n)$ of $K_{1,n}$ is subdivided by the vertex e_i in $M(K_{1,n})$ and the vertices $v, e_1, e_2, \cdots, e_n$ induce a clique of order $(n+1)$ in $M(K_{1,n})$.i.e.,$V[M(K_{1,n})] = \{v\} \cup \{v_i/1 \leq i \leq n\} \cup \{e_i/1 \leq i \leq n\}$. Now consider the color class $C = \{c_1, c_2, \cdots c_n, c_{n+1}\}$, and assign the achromatic coloring to $M(K_{1,n})$ as follows: For $(1 \leq i \leq n)$, assign the color c_i for e_i and assign color c_{n+1} to v. For $(2 \leq i \leq n-1)$, assign color c_1 for v_i and assign color c_n to v_1. Thus we have $\chi_c[M(K_{1,n})] \geq n+1$. As the number of edges in $M(K_{1,n}) = \dfrac{n^2+3n}{2} < \dbinom{n+2}{2}$. Therefore $\chi_c[M(K_{1,n})] \leq n+1$. Hence $\chi_c[M(K_{1,n})] = n+1$. □

Theorem 2.3. *For any star graph $K_{1,n}$ the achromatic number,*

$$\chi_c[T(K_{1,n})] = n + 2.$$





Star graph $K_{1,n}$
Figure 3(a)

Total graph of Star graph $K_{1,n}$
Figure 3(b)

Proof. Let $V(K_{1,n}) = \{v, v_1, v_2, \cdots, v_n\}$ and $E(K_{1,n}) = \{e_1, e_2, \cdots, e_n\}$. By the definition of total graph, we have $V[T(K_{1,n})] = \{v\} \cup \{e_i/1 \leq i \leq n\} \cup \{v_i/1 \leq i \leq n\}$, in which the vertices $v, e_1, e_2, \cdots, e_n$ induce a clique of order $(n+1)$. As the number of edges in $T(K_{1,n}) = \dfrac{n^2+5n}{2} < \dbinom{n+3}{2}$. Hence $\chi_c[T(K_{1,n})] \leq n+2$. The following $(n+2)$-coloring for $T(K_{1,n})$ is achromatic: For $(1 \leq i \leq n)$, assign the color c_i for e_i and assign color c_{n+1} to v. For $(1 \leq i \leq n)$, assign color c_{n+2} for v_i. Thus we have $\chi_c[T(K_{1,n})] = n+2$. □

Theorem 2.4. *For any star graph $K_{1,n}$, $\chi_c[C(K_{1,n})] = \chi_c[M(K_{1,n})] = \chi[M(K_{1,n})] = \chi[T(K_{1,n})] = n+1$.*

3 Observations

We observe that the achromatic number of middle graph of cycles and paths are as follows.

(i) The achromatic number of middle graph of cycle C_n, $\chi_c[M(C_n)] \geq n$.

(ii) The achromatic number of middle graph of path P_n, $\chi_c[M(P_n)] \geq n$.

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