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A Note on Characterization of Prime Ideals of Γ -Semigroups in terms of Fuzzy Subsets

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Abstract

In this paper the notion of fuzzy prime ideal in Γ-semigroups has been introduced and studied. Relationship between prime ideals of a Γ-semigroup and that of its operator semigroups have been obtained which are used to revisit analogous results on ideals of Γ-semigroups and its operator semigroups.

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1 Introduction

Γ-semigroup was introduced by Sen and Saha[8] as a generalization of semigroup and ternary semigroup. Many results of semigroups have been extended to Γ-semigroups directly and via operator semigroups[1, 2, 3] of a Γ-semigroup. Fuzzy semigroups have been introduced by Kuroki[4] as a generalization of classical semigroups, using the concept of fuzzy set introduced by Zadeh[9]. Since then many authors have studied semigroups in terms of fuzzy sets. Motivated by Kuroki[4], Mustafa et all[5] we have initiated the study of Γ-semigroups in terms of fuzzy sets[7]. This paper is a continuation of our study of Γ-semigroups in terms of fuzzy sets. We introduce here the notion of fuzzy prime ideals in Γ-semigroups. They are found to satisfy characteristic function criterion and level subset criterion. As we did for fuzzy ideals of a Γ-semigroup in [7], in order to make operator semigroups to work, we establish here various relationships between fuzzy prime ideals of a Γ-semigroup and that of its operator semigroups . Among other results we obtain an inclusion preserving bijection between the set of all prime ideals of a Γ-semigroup(not necessarily with unities) and that of its operator semigroups. As an immediate application of this we obtain a new proof of an important result of Γ-semigroup.

2 Preliminaries

We recall the following definitions and results which will be used in the sequel.

Definition 2.1 [9] *A fuzzy subset of a non-empty set* X *is a function* $\mu : X \to [0, 1]$.

Definition 2.2 [3] *Let* S *and* Γ *be two non-empty sets.* S *is called a* Γ*-semigroup if there exist mappings from* $S \times \Gamma \times S$ *to* S*, written as* $(a, \alpha, b) \longrightarrow a\alpha b$*, and from* $\Gamma \times S \times \Gamma$ *to* Γ*, written as* $(\alpha, a, \beta) \longrightarrow \alpha a\beta$ *satisfying the following associative laws* $(a\alpha b)\beta c = a(\alpha b\beta)c = a\alpha(b\beta c)$ *and* $\alpha(a\beta b)\gamma = (\alpha a\beta)b\gamma = \alpha a(\beta b\gamma)$ *for all* $a, b, c \in S$ *and for all* $\alpha, \beta, \gamma \in \Gamma$.

Definition 2.3 [7] *A non-empty fuzzy subset* μ *of a* Γ*-semigroup* S *is called a fuzzy left ideal(right ideal) of* S *if* $\mu(x\gamma y) \geq \mu(y)$*(resp.* $\mu(x\gamma y) \geq \mu(x)$*)* $\forall x, y \in S$*,* $\forall \gamma \in \Gamma$.

Definition 2.4 [7] *A non-empty fuzzy subset* μ *of a* Γ*-semigroup* S *is called a fuzzy ideal of* S *if it is both fuzzy left ideal and fuzzy right ideal of* S.

Definition 2.5 [7] *Let* μ *be a fuzzy subset of a set* S*. Then for* $t \in [0, 1]$ *the set* $\mu_t = \{x \in S : \mu(x) \geq t\}$ *is called* t*-level subset or simply level subset of* μ.

Proposition 2.6 [7] *Let I be a non-empty subset of a Γ-semigroup S and μ_I be the characteristic function of I, then I is a left ideal(resp. right ideal, ideal) of S if and only if μ_I is a fuzzy left ideal(resp. fuzzy right ideal, fuzzy ideal) of S.*

Proposition 2.7 [7] *A non-empty fuzzy subset μ in a Γ-semigroup S is a fuzzy ideal iff for any $t \in [0,1]$, the t-level subset of μ (if non-empty), is an ideal of S.*

Definition 2.8 [2] *Let S be a Γ-semigroup. An ideal P of S is said to be prime if, for any two ideals A and B of S, $A\Gamma B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.*

Theorem 2.9 *Let I be an ideal of a Γ-semigroup S. Then the following are equivalent.*

(i) *I is prime.*

(ii) *For $x, y \in S$, $x\Gamma S\Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$.*

(iii) *For $x, y \in S$, $x\Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$.*

Proof. By Theorem 3.4[2] $(i) \Rightarrow (ii)$.

$(ii) \Rightarrow (iii)$

Let us suppose that (ii) holds and $x\Gamma y \subseteq I$. Then $x\Gamma s\Gamma y \subseteq I$ as $x\Gamma s\Gamma y \subseteq x\Gamma y$ Hence by (ii), $x \in I$ or $y \in I$.

$(iii) \Rightarrow (i)$

Let us suppose that (iii) holds. Let for two ideals A, B of S, $A\Gamma B \subseteq I$. If possible, suppose $A \nsubseteq I$ or $B \nsubseteq I$. Then $x \in A$ and $y \in B$ such that $x \notin I$ and $y \notin I$. This implies that $x\Gamma y \subseteq I$ with $x \notin I$, $y \notin I$. This is a contradiction to (iii). Hence either $A \subseteq I$ or $B \subseteq I$. Consequently I is a prime ideal of S. ■

3 Fuzzy Prime Ideal

Definition 3.1 *A fuzzy ideal μ of a Γ-semigroup S is called fuzzy prime ideal if $\inf_{\gamma\in\Gamma} \mu(x\gamma y) = \max\{\mu(x), \mu(y)\}$ $\forall x, y \in S$.*

Example: Let S be the set of all 1×2 matrices over GF_2(the finite field with two elements) and Γ be the set of all 2×1 matrices over GF_2. Then S is a Γ-semigroup where $a\alpha b$ and $\alpha a\beta(a, b \in S$ and $\alpha, \beta \in \Gamma)$ denote the usual matrix product. Let $\mu : S \rightarrow$[0,1] be defined by

$$\mu(x) = \begin{cases} 0.3 & \text{if } a\text{=(0,0)} \\ 0.2 & \text{otherwise} \end{cases}$$

Then μ is a fuzzy prime ideal of S.

Theorem 3.2 *Let S be a Γ-semigroup and $\emptyset \neq I \subseteq S$. Then the following are equivalent.*
(i)I is a prime ideal of S.
(ii) The characteristic function μ_I of I is a fuzzy prime ideal of S.

Proof. $(i) \Rightarrow (ii)$
Let I be a prime ideal of S and μ_I be the characteristic function of I. Since $I \neq \phi$, μ_I is non-empty. Let $x, y \in S$. Suppose $x\Gamma y \subseteq I$. Then $\mu_I(x\gamma y) = 1$ for $\gamma \in \Gamma$. Hence $\inf\limits_{\gamma\in\Gamma}\mu_I(x\gamma y)$ =1. Now I being prime, $x \in I$ or $y \in I$ (cf. Theorem 2.9). Hence $\mu_I(x) = 1$ or $\mu_I(y) = 1$ which gives $\max\{\mu_I(x), \mu_I(y)\} = 1$. Thus we see that $\inf\limits_{\gamma\in\Gamma} \mu_I(x\gamma y) = \max\{\mu_I(x), \mu_I(y)\}$. Now suppose that $x\Gamma y \nsubseteq I$. Then for $\gamma \in \Gamma$, $x\gamma y \nsubseteq I$ which means that $\mu_I(x\gamma y) = 0$. Consequently, $\inf\limits_{\gamma\in\Gamma} \mu_I(x\gamma y) = 0$. Now since I is a prime ideal of S, $x \notin I$ and $y \notin I$. Hence $\mu_I(x) = 0$ and $\mu_I(y) = 0$. Consequently, $\max\{\mu_I(x), \mu_I(y)\} = 0$. Thus we see that in this case also $\inf\limits_{\gamma\in\Gamma} \mu(x\gamma y) = \max\{\mu_I(x), \mu_I(y)\}$.
$(ii) \Rightarrow (i)$
Let μ_I be a fuzzy prime ideal of S. Then μ_I is a fuzzy ideal of S. So by Proposition 2.6, I is an ideal of S. Let $x, y \in S$ such that $x\Gamma y \subseteq I$. Then $\mu_I(x\gamma y) = 1$. Hence $\inf\limits_{\gamma\in\Gamma}\mu_I(x\gamma y) = 1$. Let $x \notin I$ and $y \notin I$. Then $\mu_I(x) = 0 = \mu_I(y)$, which means $\max\{\mu_I(x), \mu_I(y)\} = 0$. This implies that $\inf\limits_{\gamma\in\Gamma}\mu_I(x\gamma y) = 0$. Thus we get a contradiction. Hence $x \in I$ or $y \in I$. Thus we see that I is a prime ideal of S(cf. Theorem 2.9). ■

Theorem 3.3 *Let S be a Γ-semigroup and μ be a non-empty fuzzy subset of S. Then the following are equivalent.*
(i) μ is fuzzy prime ideal of S
(ii) For any $t \in [0, 1]$ the t-level subset of μ(if it is non-empty) is a prime ideal of S.

Proof. $(i) \Rightarrow (ii)$
Let μ be a fuzzy prime ideal of S. Let $t \in [0, 1]$ such that μ_t is non-empty. Let for $x, y \in S$, $x\Gamma y \subseteq \mu_t$. Then $\mu(x\gamma y) \geq t$ $\forall\gamma \in \Gamma$. So $\inf\limits_{\gamma\in\Gamma}\mu(x\gamma y) \geq t$. Since μ is a fuzzy prime ideal, it follows that $\max\{\mu(x), \mu(y)\} \geq t$. So $\mu(x) \geq t$ or $\mu(y) \geq t$. Hence $x \in \mu_t$ or $y \in \mu_t$. So μ_t is a prime ideal of S(cf. Theorem 2.9).
$(ii) \Rightarrow (i)$
Let every non-empty level subset μ_t of μ be a prime ideal of S. Let $x, y \in S$. Let $\inf\limits_{\gamma\in\Gamma} \mu(x\gamma y) = t$ (we note here that since $\mu(x\gamma y) \in [0, 1]$ $\forall\gamma \in \Gamma$, $\inf\limits_{\gamma\in\Gamma} \mu(x\gamma y)$ exists). Then $\mu(x\gamma y) \geq t$ $\forall\gamma \in \Gamma$. So $x\gamma y \in \mu_t$ $\forall\gamma \in \Gamma$. So μ_t is non-empty and $x\Gamma y \subseteq \mu_t$. Since μ_t is a prime ideal of S, $x \in \mu_t$ or $y \in \mu_t$(cf. Theorem 2.9). So $\mu(x) \geq t$ or $\mu(y) \geq t$. So $\max\{\mu(x), \mu(y)\} \geq t$, i.e., $\max\{\mu(x), \mu(y)\} \geq$

$\inf\limits_{\gamma\in\Gamma} \mu(x\gamma y)$.......(1). Again by Proposition 2.7, μ is a fuzzy ideal of S. So $\forall\gamma \in \Gamma$, $\mu(x\gamma y) \geq \mu(x)$ and $\mu(x\gamma y) \geq \mu(y)$. So $\mu(x\gamma y) \geq \max\{\mu(x), \mu(y)\}$ $\forall\gamma \in \Gamma$. Hence $\inf\limits_{\gamma\in\Gamma}\mu(x\gamma y) \geq \max\{\mu(x), \mu(y)\}$.......(2). Combining (1) and (2), thus $\inf\limits_{\gamma\in\Gamma}\mu(x\gamma y) = \max\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy prime ideal of S. ■

4 Corresponding Fuzzy Prime Ideals

Unless otherwise stated, throughout this section S denotes a Γ-semigroup and L, R its left and right operator semigroups respectively.

Definition 4.1 [2] *Let S be a Γ-semigroup. Let us define a relation ρ on $S\times\Gamma$ as : $(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α). Let $L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. This semigroup L is called the left operator semigroup of the Γ-semigroup S.*

Dually the right operator semigroup R of Γ-semigroup S is defined where the multiplication is defined by $[\alpha, a][\beta, b] = [\alpha a\beta, b]$.

Definition 4.2 *For a fuzzy subset μ of R we define a fuzzy subset μ^* of S by $\mu^*(a) = \inf\limits_{\gamma\in\Gamma} \mu([\gamma, a])$, where $a \in S$. For any subset σ of S we define a fuzzy subset $\sigma^{*'}$ of R by $\sigma^{*'}([\alpha, a]) = \inf\limits_{s\in S} \sigma(s\alpha a)$, where $[\alpha, a] \in R$. For a fuzzy subset δ of L, we define a fuzzy subset δ^+ of S by $\delta^+(a) = \inf\limits_{\gamma\in\Gamma} \delta([a, \gamma])$ where $a \in S$. For any fuzzy subset η of S we define a fuzzy subset $\eta^{+'}$ of L by $\eta^{+'}([a, \alpha]) = \inf\limits_{s\in S}$ $\eta(a\alpha s)$, where $[a, \alpha] \in L$.*

Lemma 4.3 [7] *If μ is a fuzzy subset of R, then $(\mu_t)^* = (\mu^*)_t$ where $t \in Im(\mu)$, provided the sets are non-empty.*

Lemma 4.4 [7] *If σ is a fuzzy subset of S, then $(\sigma_t)^{*'} = (\sigma^{*'})_t$ where $t \in Im(\sigma)$, provided the sets are non-empty.*

Proposition 4.5 *Suppose μ is a fuzzy prime ideal of R. Then μ^* is a fuzzy prime ideal of S.*

Proof. Since μ is a fuzzy prime ideal of R, μ_t is a prime ideal of R[6] $\forall t \in Im(\mu)$. Hence $(\mu_t)^*$ is a prime ideal of S[2]. Now $(\mu_t)^*$ and $(\mu^*)_t$ are non-empty. Hence $(\mu_t)^* = (\mu^*)_t$($cf.$ Lemma 4.3). This gives $(\mu^*)_t$ is a prime ideal of S for all $t \in Im(\mu)$. Hence μ^* is a fuzzy prime ideal of S($cf.$ Theorem 3.3). ■

Proposition 4.6 *Suppose σ is a fuzzy prime ideal of S. Then $\sigma^{*'}$ is a fuzzy prime ideal of R.*

Proof. Since σ is a fuzzy prime ideal of S, σ_t is a prime ideal of S $\forall t \in Im(\sigma)(cf.$ Theorem 3.3). Hence $(\sigma_t)^{*'}$ is a prime ideal of R[2]. Also $(\sigma_t)^{*'}$ and $(\sigma^*)_t$ are non-empty . So $(\sigma_t)^{*'} = (\sigma^{*'})_t(cf.$ Lemma 4.4), $(\sigma^{*'})_t$ is a prime ideal of R for all $t \in Im(\mu)$. Consequently $\sigma^{*'}$ is a fuzzy prime ideal of R[6]. ■

Remark: The left operator analogous of the above two propositions are also true.

Theorem 4.7 *Let S be a Γ-semigroup and R be its right operator semigroup. Then there exist an inclusion preserving bijection $\mu \mapsto \mu^{*'}$ between the set of all fuzzy prime ideals of R and set of all fuzzy prime ideals of S, where μ is a fuzzy prime ideal of R.*

Proof. Let $x \in S$. Then $(\mu^{*'})^*(x)=\inf_{\alpha\in\Gamma}\mu^{*'}[\alpha, x]=\inf_{\alpha\in\Gamma}\inf_{s\in S}\mu(s\alpha x) \geq \mu(x)$(since μ is a fuzzy ideal). Again for $x \in S$, $(\mu^{*'})^*(x)= \inf_{\alpha\in\Gamma}\inf_{s\in S}\mu(s\alpha x) = \inf_{s\in S}\inf_{\alpha\in\Gamma}\mu(s\alpha x) = \inf_{s\in S}(\max\{\mu(s), \mu(x)\})$(since μ is a fuzzy prime ideal) $\leq \max\{\mu(x), \mu(x)\}=\mu(x)$. Thus we see that $(\mu^{*'})^* = \mu$. Hence the mapping is one-one. Now for $[\alpha, x] \in R$, $(\mu^*)^{*'}[\alpha, x] = \inf_{s\in S}\mu^*(s\alpha x)= \inf_{s\in S}\inf_{\beta\in\Gamma}\mu([\beta, s\alpha x])= \inf_{s\in S}\inf_{\beta\in\Gamma}\mu([\beta, s][\alpha, x]) \geq \mu[\alpha, x]$. Hence $\mu \subseteq (\mu^*)^{*'}$. Since μ is a fuzzy prime ideal, $\mu([\alpha, x], [\beta, s]) = \max((\mu[\alpha, x], [\beta, s]))\forall s \in S$ and $\forall \beta \in \Gamma$. Hence for $s = x$ and $\beta = \alpha$, $\mu([\alpha, x], [\beta, s]) = \mu[\alpha, x]$. This together with the relation $(\mu^*)^{*'}[\alpha, x] = \inf_{s\in S}\inf_{\beta\in\Gamma}\mu([\alpha, x], [\beta, s])$ gives $(\mu^*)^{*'}[\alpha, x] \leq \mu[\alpha, x]$ for all $[\alpha, x] \in R$. This means $(\mu^*)^{*'} \subseteq \mu$. Thus we see that $\mu = (\mu^*)^{*'}$. This proves that the mapping is onto. Now let $\mu_1, \mu_2 \in FI(S)$ be such that $\mu_1 \subseteq \mu_2$. Then for all $[\alpha, x] \in R, \mu_1^{*'}([\alpha, x]) = \inf_{s\in S}\mu_1(s\alpha x) \leq \inf_{s\in S}\mu_2(s\alpha x) = \mu_2^{*'}([\alpha, x])$. Thus $\mu_1^{*'} \subseteq \mu_2^{*'}$. Similarly we can show that if $\mu_1 \subseteq \mu_2$ where $\mu_1, \mu_2 \in FI(R)(FLI(R))$ then $\mu_1^* \subseteq \mu_2^*$. Hence $\mu \mapsto \mu^{*'}$ is an inclusion preserving bijection. ■

Remark: Similar result holds for the Γ-semigroup S and the left operator semigroup L of S.

Now we establish the following two lemmas to deduce the inclusion preserving bijections between the set of all prime ideals of a Γ-semigroup and that of its operator semigroups with the fuzzy notions of Γ-semigroups.

Lemma 4.8 *Let I be an ideal, μ a fuzzy ideal of a Γ-semigroup S and R the right operator semigroup of S. Then $(\mu_I)^{*'} = \mu_{I^{*'}}$, where μ_I is the characteristic function of I.*

Proof. Let $[\beta, y] \in R$. Then $(\mu_I)^{*'}([\beta, y]) = \inf\limits_{s\in S}\mu(s\beta y)$. Suppose $[\beta, y] \in I^{*'}$. Then $s\beta y \in I$ for all $s \in S$. Hence $\mu_I(s\beta y) = 1$ for all $s \in S$. This gives $\inf\limits_{s\in S}\mu(s\beta y) = 1$ whence $(\mu_I)^{*'}([\beta, y]) = 1$. Also $\mu_{I^{*\prime}}([\beta, y]) = 1$. Hence $(\mu_I)^{*'}([\beta, y]) = \mu_{I^{*\prime}}([\beta, y])$. Suppose $[\beta, y] \notin I^{*\prime}$. Then for some $t \in S, t\beta y \notin I$. So $\mu_I(t\beta y) = 0$. This gives $\inf\limits_{s\in S}\mu_I(s\beta y) = 0$ i.e., $(\mu_I)^{*'}([\beta, y]) = 0$. Again $\mu_{I^{*\prime}}([\beta, y]) = 0$. Thus in this case also $(\mu_I)^{*'}([\beta, y]) = \mu_{I^{*\prime}}([\beta, y])$. Hence $(\mu_I)^{*'} = \mu_{I^{*'}}$. ■

Similar is the proof of the following lemma.

Lemma 4.9 *Let I be a (left) ideal of the right operator semigroup R of a Γ-semigroup S. Then $(\lambda_I)^* = \lambda_{I^*}$, where λ_I denotes the characteristic function of I.*

Remark 4.10 *Dually we can deduce for left operator semigroup L of the Γ-semigroup $S, (i)(\lambda_I)^{+'} = \lambda_{I^{+'}}, (ii)(\lambda_I)^+ = \lambda_{I^+}$, where λ_I denotes the characteristic function of I.*

Theorem 4.11 [2] *Let S be a Γ-semigroup(not necessarily with unities). Then there exists an inclusion preserving bijection between the set of all prime ideals of S and that of its right operator semigroup R via the mapping $I \to I^{*'}$.*

Proof. Let us denote the mapping $I \to I^{*'}$ by ϕ. This is actually a mapping follows from dual of Lemma 3.12[2]. Now let $\phi(I_1) = \phi(I_2)$. Then $I_1^{*'} = I_2^{*'}$. This implies that $\lambda_{I_1^{*'}} = \lambda_{I_2^{*'}}$. (where λ_I is the characteristic function I.) Hence by Lemma 4.8, $(\lambda_{I_1})^{*'} = (\lambda_{I_2})^{*'}$. This together with Theorem 4.7 gives $\lambda_{I_1} = \lambda_{I_2}$ whence $I_1 = I_2$. Consequently ϕ is one-one. Let I be a prime ideal of R. Then its characteristic function λ_I is a fuzzy prime ideal of R. Hence by Theorem 4.7, $((\lambda_I)^*)^{*'} = \lambda_I$. This implies that $\lambda_{(I^*)^{*'}} = \lambda_I(cf.$ Lemmas 4.8 and 4.9). Hence $(I^*)^{*'}$=I i.e., $\phi(I^*) = I$. Now since I^* is a prime ideal of S ([2]), it follows that ϕ is onto. Let I_1, I_2 be two prime ideals of S with $I_1 \subseteq I_2$. Then $\lambda_{I_1} \subseteq \lambda_{I_2}$. Hence by Theorem 4.7, we see that $(\lambda_{I_1})^{*'} \subseteq (\lambda_{I_2})^{*'}$ i.e., $\lambda_{I_1^{*'}} \subseteq \lambda_{I_2^{*'}}$ ($cf.$ Lemma 4.8)which gives $I_1^{*'} \subseteq I_2^{*'}$. ■

Remark: The result similar to the above for the left operator semigroup L of the Γ-semigroup S can be deduced by the Remark 4.10 and Theorem 4.7.

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