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Wieslaw Kubiak, Suresh Sethi,

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NOTES

A NOTE ON "LEVEL SCHEDULES FOR MIXED-MODEL ASSEMBLY LINES IN JUST-IN-TIME PRODUCTION SYSTEMS"*

WIESLAW KUBIAK AND SURESH SETHI
Faculty of Management, University of Toronto, Toronto, Ontario, Canada M5S 1V4

This note formulates an assignment problem for obtaining optimal level schedules for mixed-model assembly lines in JIT production systems. The problem was formulated as a quadratic integer programming problem in a recent paper by Miltenburg (1989) where, however, only enumerative algorithms and heuristics were proposed for its solution. Our assignment formulation can also be extended to more general objective functions than the one used by Miltenburg.
(PRODUCTION/SCHEDULING—MIXED-MODEL ASSEMBLY LINES, JUST-IN-TIME; PROGRAMMING—NONLINEAR INTEGER, ASSIGNMENT PROBLEM)

1. Introduction

Consider n products with demands $d_1, d_2, \ldots, d_n$ to be produced during a specified time horizon. Assume that each product takes a unit of time to be produced so that the specified time horizon can be considered to be $D_T = \sum_{i=1}^{n} d_i$. If $r_i = d_i/D_T$, then the scheduling objective for the assembly line is to keep the proportion of the cumulative production of product i to the total production as close to r_i as possible.

Let $x_{i,k}$ be the total cumulative production of product i in periods 1 through k. Miltenburg (1989) proposes the following nonlinear formulation of the problem:

$$\text{minimize} \quad \sum_{k=1}^{D_T} \sum_{i=1}^{n} (x_{i,k} - kr_i)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{i,k} = k, \qquad k = 1, 2, \ldots, D_T,$$

$$0 \le x_{i,k} - x_{i,k-1} \le 1, \qquad i = 1, 2, \ldots, n; \qquad k = 2, \ldots, D_T,$$

$$x_{i,k} \text{ is a nonnegative integer}, \qquad i = 1, 2, \ldots, n; \qquad k = 1, 2, \ldots, D_T. \qquad \text{(P1)}$$

Note that the constraints $x_{i,D_T} \le d_i$, $i = 1, 2, \ldots, n$ are not included as these would obviously be satisfied by any optimal solution. It should be noted that the problem is derived from considerations used in obtaining a balanced schedule for mixed-model assembly lines at Toyota; see Monden (1983) and Japan Management Association (1985).

2. The Equivalent Assignment Problem

Let us call $Z_j^{i*} = \lceil (2j-1)/2r_i \rceil$ as the *ideal position* for the jth unit of product i to be produced, $i = 1, 2, \ldots, n$, $j = 1, \ldots, d_i$. Let C_{jk}^i be the cost of assigning the jth unit of the product i to the kth period. If $k = Z_j^{i*}$, then the jth unit of product i has its ideal position and $C_{jk}^i = 0$. If the jth unit is produced too soon, i.e., $k < Z_j^{i*}$, then the excess inventory costs Ψ_{jl}^i are incurred in periods from $l = k$ to $l = Z_j^{i*} - 1$. On the other hand,

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if the jth unit is produced too late, i.e., $k > Z_j^{i*}$, then the excess shortage costs Ψ_{jl}^i are incurred in periods from $l = Z_j^{i*}$ to $l = k - 1$. Thus, we define

$$C_{jk}^i = \begin{cases} \sum_{l=k}^{Z_j^{i*}-1} \Psi_{jl}^i & \text{if} \quad k < Z_j^{i*}, \\ 0 & \text{if} \quad k = Z_j^{i*}, \\ \sum_{l=Z_j^{i*}}^{k-1} \Psi_{jl}^i & \text{if} \quad k > Z_j^{i*}, \end{cases}$$

where

$$\Psi_{jl}^i = |(j - lr_i)^2 - (j - 1 - lr_i)^2|,$$

$$(i, j) \in I = \{(i, j): i = 1, \ldots, n; j = 1, \ldots, d_i\}, \qquad l = 1, \ldots, D_T.$$

Let

$$x_{jk}^i = \begin{cases} 1 & \text{if } (i, j) \text{ is assigned to period } k, \\ 0 & \text{otherwise.} \end{cases}$$

Then, the appropriate assignment problem is

$$\begin{aligned} &\text{minimize} \quad \sum_{k=1}^{D_T} \sum_{(i,j)\in I} C_{jk}^i x_{jk}^i \\ &\text{s.t.} \quad \sum_{(i,j)\in I} x_{jk}^i = 1, \qquad k = 1, 2, \ldots, D_T, \\ &\sum_{k=1}^{D_T} x_{jk}^i = 1, \qquad (i, j) \in I, \\ &x_{jk}^i = 0 \text{ or } 1, \qquad k = 1, 2, \ldots, D_T; \qquad (i, j) \in I. \end{aligned} \tag{P2}$$

Using an induction argument, it is possible to show that an optimal solution for (P1) can be easily constructed from any optimal schedule of (P2); see Kubiak and Sethi (1989) for details.

3. Extensions

Objective function in (P1) can be generalized to

$$\sum_{k=1}^{D_T} \sum_{i=1}^{n} F^i(x_{i,k} - kr_i), \tag{1}$$

where $F^i(\cdot)$ is a nonnegative convex function satisfying $F^i(0) = 0$, $F^i(y) > 0$ for $y \neq 0$, $i = 1, 2, \ldots, n$. This generalization can also be reduced to an assignment problem (Kubiak and Sethi (1989)).

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