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A note on measurement of contingency between two binary variables in judgment tasks

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Varied measures of contingency have appeared in the psychological judgment literature concerned with binary variables. These measures are examined, and the inappropriateness of some are noted. As well, it is argued that accurate judgments about related variables should not be used to infer that the judgments are based on the appropriate information.

A number of studies in the psychological literature have been concerned with judgments of contingency or correlation between two binary variables (Allan & Jenkins, in press; Gray, 1976; Green, Jurd, & Seggie, 1979; Inhelder & Piaget, 1958; Jenkins & Ward, 1965; Seggie, 1975; Seggie & Endersby, 1972; Smedslund, 1963; Ward & Jenkins, 1965). One purpose of the present note is to evaluate the various measures of contingency that have been used in these studies. A second purpose is to argue that accurate judgments about two variables that are related should not be used to infer that the judgments are based on the appropriate information.

MEASUREMENT OF CONTINGENCY OR CORRELATION

Consider the 2 by 2 matrix in Figure 1. There are two values of the variable A (A_1 and A_2) and two values of the variable B (B_1 and B_2). The letters (a, b, c, and d) in the cells represent the joint frequency of one value of A and one value of B. Two frequently used summary numbers in the statistical literature to reflect the degree of contingency or correlation between two binary variables are χ^2 and ϕ, where

$$\chi^2 = N(ad - bc)^2 / [(a + b)(c + d)(a + c)(b + d)] \quad (1)$$

and

$$\phi = \sqrt{\chi^2/N}. \quad (2)$$

Both χ^2 and ϕ reflect the dependence of variable A on variable B and the dependence of variable B on variable A.

There has been a surprising lack of agreement in the psychological judgment literature about an appropriate measure of the relationship between the two variables of interest. Smedslund (1963) states, "The concept of correlation in its elementary logical form is the ratio of the sum of two diagonal cell frequencies in a fourfold table, and the sum of the other two diagonal cell frequencies, or the total sum" (p. 165). Using the notation of Figure 1, Smedslund's measures of correlation, referred to as Δr_1 and Δr_2 in this paper, are

$$\Delta r_1 = (a + d)/(b + c)$$
and
$$\Delta r_2 = (a + d)/N. \quad (3)$$

Rather than using the ratio of the sums of the two diagonal cell frequencies, Inhelder and Piaget (1958) took the difference as their measure of correlation. Using the notation of Figure 1, the Inhelder and Piaget measure of correlation, referred to as Δd in this paper, is

$$\Delta d = (a + d) - (b + c). \quad (4)$$

Allan and Jenkins (in press), Jenkins and Ward (1965), and Ward and Jenkins (1965) argued that an appropriate measure of the dependency of one variable on another variable is ΔP, which is the difference between two

	B_1	B_2	
A_1	a	b	a+b
A_2	c	d	c+d
	a+c	b+d	N=a+b+c+d

Figure 1. A 2 by 2 matrix representing the events in a binary judgment task.

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independent conditional probabilities. A measure of the dependency of variable B on variable A is

$$\Delta P_R = P(B_1 | A_1) - P(B_1 | A_2)$$
$$= a/(a + b) - c/(c + d)$$
$$= (ad - bc)/[(a + b)(c + d)], \quad (5a)$$

and that of variable A on variable B is

$$\Delta P_C = P(A_1 | B_1) - P(A_1 | B_2)$$
$$= a/(a + c) - b(b + d)$$
$$= (ad - bc)/[(a + c)(b + d)]. \quad (5b)$$

While Δr and Δd are both based on comparing the sums of the two diagonal cell frequencies, ΔP is based on comparing the products of the two diagonal cell frequencies.

A comparison of Equation 1 and Equation 5 reveals that

$$\chi^2 = N\Delta P_R \Delta P_C. \quad (6)$$

That is, χ^2 reflects a two-way dependency, and ΔP_R and ΔP_C each reflect a one-way dependency.

A comparison of Equation 3 with Equations 1 and 5 indicates that no simple relationship exists between Δr and χ^2 or ΔP. An increase in the value of (a + d) must result in a larger value of Δr, but not necessarily of χ^2 or ΔP.

Jenkins and Ward (1965) and Ward and Jenkins (1965) pointed out that when $\Delta P = 0$, $\Delta d = 0$ only if the two marginal row frequencies and/or the two marginal column frequencies are equal, that is, when $a + b = c + d$ and/or $a + c = b + d$. If one of these conditions does not hold, then even when there is no relationship between the two binary variables (e.g., $\chi^2 = 0$), Δd could be nonzero, and an invalid conclusion about the existence of a relationship could be reached.

In general,

$$\Delta d = N\Delta P_R = [4\Delta P_C(a + c)(b + d)]/N \quad (8a)$$

for $(a + b) = (c + d)$, and

$$\Delta d = N\Delta P_C = [4\Delta P_R(a + b)(c + d)]/N \quad (8b)$$

for $(a + c) = (b + d)$. It is clear from Equation 8a that for equal row frequencies there is a simple relationship between Δd and ΔP_R, but not between Δd and ΔP_C. For example, for constant values of ΔP_R and N, Δd will be constant even though ΔP_C will vary with changes in (a + c) and (b + d). Similar comments can be made about Equation 8b. It is important to realize that, while equal marginal frequencies for the row (column) variable make Δd equivalent to $\Delta P_R(\Delta P_C)$ as a measure of dependency, it does not establish the equivalence of Δd and $\Delta P_C(\Delta P_R)$.

The inappropriateness of Δd as a measure of dependency can also be seen by considering the relationship between Δd and χ^2:

$$\Delta d = \sqrt{[4(a + c)(b + d)\chi^2]/N} \quad (9a)$$

for $a + b = c + d$,

$$\Delta d = \sqrt{[4(a + b)(c + d)\chi^2]/N} \quad (9b)$$

for $a + c = b + d$, and

$$\Delta d = \sqrt{N\chi^2} \quad (9c)$$

for $a + b = c + d = a + c = b + d$. For $\chi^2 \neq 0$, a simple relationship exists only when all the marginal frequencies are equal.

Green et al. (1979) made use of two types of problems that were identical as far as the matrix entries were concerned, but which differed in the task required of the subject. In the one-way problem the subject was asked to judge the dependency of one variable on the other variable. The matrix was described to indicate that, if a relationship existed between the two variables, it was a causal one. In the two-way problem the subject was asked to judge the overall dependency of each variable upon the other. The matrix was described to indicate that, if a relationship existed, it was a noncausal one. Green et al. (1979) correctly argued that, while ΔP_R or ΔP_C is an appropriate measure for the one-way problem, neither measure, in isolation, allows a precise calculation of association for the two-way problem. Further, they acknowledged the warning of Jenkins and Ward (1965) and Ward and Jenkins (1965) that Δd should not be used if neither pair of marginal frequencies is equal. However, they incorrectly concluded that when one pair of marginal frequencies is equal, Δd is an appropriate measure for the two-way problem. From Equation 9 it is clear that for $\chi^2 \neq 0$, Δd provides an accurate assessment of the two-way problem only when all the marginal frequencies are equal. The two matrices presented in Figure 2 provide a concrete example. The row frequencies are equal in each matrix

(A)

	B_1	B_2	
A_1	30	20	50
A_2	5	45	50
	35	65	100

(B)

	B_1	B_2	
A_1	50	0	50
A_2	25	25	50
	75	25	100

Figure 2. Two matrices that are identical in Δd and ΔP_R but differ in ΔP_C and χ^2.

and identical in the two matrices; the column frequencies are unequal in each matrix and differ between the two matrices. For both matrices, $\Delta d = 50$ and $\Delta P_R = .50$. For Matrix A, $\Delta P_C = .55$ and $\chi^2 = 27.50$; for Matrix B, $\Delta P_C = .67$ and $\chi^2 = 33.33$. For the two-way problem, these two matrices illustrate different degrees of relationship between the two variables. If a subject based his judgment on Δd, he would judge the two matrices as equivalent, and he would be in error.

INFERENCES FROM ACCURATE JUDGMENTS

Seggie (1975) asked his subjects to make a decision regarding hospitalization vs. nonhospitalization for a patient with a disease on the basis of past recovery rates of other patients. He used six different problems; in three of the problems the recovery was contingent on hospitalization, and in the other three the relationship was noncontingent. Decisions about the three contingent problems were fairly accurate, but subjects had difficulty with the three noncontingent problems. Seggie (1975) concluded that "the ability to make an empirical utilization of the relationships between binary variables was a function of the nature of the relationship" (p. 41). According to Seggie (1975), the subject is able to discriminate between contingent and noncontingent problems. For contingent problems, judgments are based on the appropriate information; for noncontingent, they are not.

Recent data reported by Allan and Jenkins (in press) clearly demonstrate that judgments about contingent problems that appear accurate do not necessarily indicate that the judgment is based upon the appropriate information. In the 2R condition of their experiment, they made use of a two-response/two-outcome judgment task. On each of a series of trials, the subject was required to make one of two alternative responses, which was then followed by the presentation of one of the two outcome events. At the end of a series of 100 response/outcome pairings, some subjects judged the degree of influence the response choice exerted over the outcome presentation, and others judged the degree of connection between response choice and outcome presentation. Each subject made an influence or connection judgment about 10 different problems, 5 contingent and 5 noncontingent. Allan and Jenkins (in press) found that judged influence (connection) was greater for some of the noncontingent problems than for the contingent problems and that judgments appeared to be related to ΔP for the contingent problems but not for the noncontingent problems. However, they demonstrated that the best overall description of all 10 problems was in terms of "a." For both contingent and noncontingent problems, as "a" increased, so did judged influence (connection). Judgments appeared to be related to ΔP for contingent problems because "a" and ΔP were positively correlated. The description of both types of problems in terms of one variable avoids the awkward assumption that subjects can distinguish between contingent and noncontingent problems but then treat noncontingent problems as though they represented a contingent relation.

It is not being argued that Seggie's (1975) subjects were basing their judgments on "a." Rather, it is being suggested that when judgments of contingent problems and of noncontingent problems appear to be influenced by different aspects of the available information, a closer examination of the data is warranted. It could be, as Allan and Jenkins (in press) found, that subjects base their judgments on the same information.

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