A NOTE ON THE MIXING OF ANGULAR MOMENTUM IN A NEUTRALLY BUOYANT FLUID

By M. J. MANTON*

[Manuscript received 1 February 1973]

Abstract

The effect of turbulent mixing in a neutrally buoyant rotating fluid is considered in a region where the mean flow is axisymmetric. The separate actions of molecular and turbulent mixing are distinguished. It is shown that a rotating turbulent flow capable of mixing angular momentum must vary in the axial direction. In particular, it would seem that a secondary circulation is required for a flow to support turbulence over its whole volume. The relative roles of turbulence and secondary circulations in the mixing of angular momentum are discussed.

I. INTRODUCTION

It has been proposed by Scorer (1965, 1966) that neutrally buoyant turbulence in a rotating flow should produce a concentration of vorticity at the centre of the turbulent region. The argument was based firstly on the assumption that the mean angular momentum of the fluid about some axis was governed by a transport equation, and secondly on the assumption that the turbulence acted viscously such that an eddy viscosity could be defined; whence, the mean angular momentum of the fluid tended to become constant and a concentrated vortex formed about the axis. Bretherton and Turner (1968) put forward the same argument, together with some analysis, and also tried unsuccessfully to create a concentrated vortex by stirring a rotating fluid. As the Rossby number of their rotating turbulence was less than about 3, turbulent mixing could not dominate the fluid motion. A demonstration of angular momentum mixing by Gough and Lynden-Bell (1968) has been shown to be due to thermally driven mean circulations and not to turbulent mixing (Strittmatter *et al.* 1970).

Recently, McEwan (1973) has produced concentrated vortex motions in a neutrally buoyant rotating fluid. The fluid was contained in a circular cylinder which rotated about its axis of symmetry. The turbulence was generated by breaking inertial waves which were produced by oscillating the top surface of the container.

In the present work, the effect of turbulent mixing is considered in a region where the mean motion is essentially axisymmetric. Firstly, the separate roles of molecular and turbulent mixing are discussed in order to emphasize their distinct physical actions. The ability of some simple rotating flows to maintain steady vorticity concentrations is then examined, independently of any simplification of the turbulent

* Department of Mathematics, Monash University, Clayton, Vic. 3168.

stress terms (e.g. the mixing length hypothesis) in the equations of motion. Restriction of the study to neutrally buoyant axisymmetric flows rather limits the possible sources of turbulent energy, but if turbulence alone can mix angular momentum then it ought to be independent of buoyancy forces. Also, the annular flow between differentially rotating cylinders is not considered because this case is equivalent to a linear Couette flow to the extent that the vorticity is concentrated in the boundary layers near the walls of the container. Only angular momentum concentrations away from any side boundary of the fluid are discussed here. It is argued that, although turbulence alone might mix angular momentum, a secondary circulation is required to maintain the turbulence and the subsequent vortex flow. Hence, it is not clear whether a steady concentration of vorticity is maintained by the turbulence or by the secondary circulation.

II. Equations of Motion

The equations describing the conservation of volume and momentum for a viscous incompressible fluid are

$$\frac{\partial(ru)}{\partial r}+\frac{\partial v}{\partial \theta}+\frac{\partial(rw)}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=\nu\left(\nabla^2 u-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}\right), \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+w\frac{\partial v}{\partial z}+\frac{uv}{r}+\frac{1}{\rho r}\frac{\partial p}{\partial \theta}=\nu\left(\nabla^2 v-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial \theta}\right), \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=\nu\nabla^2 w, \tag{4}$$

where

$$\nabla^2 f=\frac{\partial^2 f}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}.$$

Here (r,θ,z) form a cylindrical polar coordinate system with corresponding velocity components (u,v,w), p is the pressure, and ν and ρ are the fluid viscosity and density respectively.

For turbulent motion we can decompose the velocity such that

$$(u,v,w)=(U,V,W)+(u',v',w'), \tag{5}$$

where $U=\langle u\rangle$ etc. are the mean velocity components, the angle brackets denoting an ensemble average. Putting the expansion (5) into equation (3), using equation (1), and averaging, we find that the equation for the conservation of mean azimuthal momentum becomes

$$\frac{\partial(rV)}{\partial t}+U\frac{\partial(rV)}{\partial r}+\frac{V}{r}\frac{\partial(rV)}{\partial\theta}+W\frac{\partial(rV)}{\partial z}+\frac{1}{r}\frac{\partial(r^2\langle u'v'\rangle)}{\partial r}+\frac{\partial\langle v'^2\rangle}{\partial\theta}+\frac{\partial(r\langle w'v'\rangle)}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial\theta}$$

$$=\nu\left\{r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2(rV)}{\partial\theta^2}+\frac{\partial^2(rV)}{\partial z^2}+\frac{2}{r}\frac{\partial U}{\partial\theta}\right\}, \quad (6)$$

where $p = P+p'$ and $\langle p'\rangle = 0$. In this form, equation (6) expresses the conservation of angular momentum about the z axis.

We now consider a region in which the mean angular momentum rV about the z axis is mixed in some manner. This implies that rV is a dominant dependent variable in the region, i.e. the magnitude of V is at least comparable with that of the other velocity components and rV depends upon r, θ, and z in a simple manner. Thus, although background motions produced by forces external to the mixing region may exist, there must be a predominant circulation about the z axis. In a neutrally buoyant fluid, pressure forces are the source of such background motions, and in equation (6) they are represented by the $\partial P/\partial\theta$ term. Hence, for the angular momentum about the z axis to be relatively unaffected by the background motions, azimuthal pressure gradients within the mixing region must be weak. Then the mean angular momentum will satisfy a transport equation: the mean advection of rV will be balanced by viscous and turbulent transport processes only. Further, for the angular momentum within a region to be mixed, it would seem that the mean streamlines must be closed so that angular momentum is not advected away from the region. Also, since generally the only preferred direction in the mixing region is given by the z axis, the closed streamline motion must tend to be axisymmetric.

For statistically steady axisymmetric flows, the equations for the conservation of volume and azimuthal momentum become

$$\frac{\partial(rU)}{\partial r}+\frac{\partial(rW)}{\partial z}=0 \quad (7)$$

and

$$U\frac{\partial(rV)}{\partial r}+W\frac{\partial(rV)}{\partial z}$$

$$=\nu\left\{r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)+\frac{\partial^2(rV)}{\partial z^2}\right\}-\frac{1}{r}\frac{\partial(r^2\langle u'v'\rangle)}{\partial r}-\frac{\partial(r\langle w'v'\rangle)}{\partial z}. \quad (8)$$

The relation for the turbulent energy is found from equations (1)–(4) and (7) and (8) to be

$$\left(U\frac{\partial}{\partial r}+W\frac{\partial}{\partial z}\right)\langle q^2\rangle+\frac{1}{r}\frac{\partial(r\langle u'q^2\rangle)}{\partial r}+\frac{\partial\langle w'q^2\rangle}{\partial z}+\frac{1}{r}\frac{\partial r\langle u'p'\rangle}{\partial r}+\frac{\partial\langle w'p'\rangle}{\partial z}$$

$$-\nu\left(\langle u'\,\nabla^2u'\rangle+\langle v'\,\nabla^2v'\rangle+\langle w'\,\nabla^2w'\rangle-\frac{\langle u'^2\rangle+\langle v'^2\rangle}{r^2}+\frac{4}{r^2}\left\langle v'\frac{\partial u'}{\partial\theta}\right\rangle\right)$$

$$=-\langle u'^2\rangle\frac{\partial U}{\partial r}-\langle w'^2\rangle\frac{\partial W}{\partial z}-\langle v'^2\rangle\frac{U}{r}-r\langle u'v'\rangle\frac{\partial(V/r)}{\partial r}-\langle v'w'\rangle\frac{\partial V}{\partial z}-\langle u'w'\rangle\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial r}\right), \quad (9)$$

where $q^2 = \frac{1}{2}(u'^2+v'^2+w'^2)$. We shall be particularly interested in the terms on the right-hand side of equation (9). These terms represent the production of turbulent energy through the interaction of the Reynolds stresses with the mean rate of strain field. The remaining terms in (9) describe either the viscous dissipation of energy or processes which only transfer energy from one part of the fluid to another.

III. Viscous and Turbulent Mixing

Having considered the assumption that rV satisfies a transport equation, we may now proceed to Scorer's (1965) second hypothesis. That is, we shall assume that turbulent transport acts to mix rV in a manner given by the mixing length theory. Further, if there is no mean radial velocity and no mean axial gradients then equation (8) becomes

$$\nu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)+K\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)=0, \tag{10}$$

where K is a constant turbulent diffusivity. (Clearly, turbulent mixing can be described by more complex diffusivities, but the purpose here is to discuss the general physical nature of such a mixing process. It is not suggested that a formulation like (10) necessarily describes accurately the action of turbulence, although such a model has been used successfully in other situations.) Bretherton and Turner (1968) pose the apparent paradox that, since turbulent mixing can produce uniformity of rV, perhaps molecular viscous mixing ought to yield the same result, thus contradicting the second law of thermodynamics. They resolve the problem by demonstrating that random isotropic mixing by the fluid molecules produces solid body rotation. However, a further comment on the essential difference between turbulent and viscous mixing may be worth while.

Although the two mixing processes are modelled by the same functional relation in (10), they are basically different. This is apparent from the boundary condition associated with each case:

(1) In the absence of turbulence, $V(r)$ is the solution of the Dirichlet problem

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)=0 \qquad \text{for} \qquad r<R,$$

with $V/r = \Omega$ on $r = R$ and with V/r regular as $r \to 0$. Thus viscosity ensures that the local angular momentum (vorticity) of the fluid matches that of the boundary, and it requires the vorticity to be finite everywhere. The viscous solution is therefore $V = r\Omega$, that is, the local angular momentum of the fluid is uniformly distributed.

(2) On the other hand, the turbulent part of equation (10) is not valid near the boundaries. This is because turbulent mixing is a conservative process. The terms of equation (8), modelled in (10), are $\langle \boldsymbol{u}'.\nabla(rv')\rangle$ which describe the advection of fluctuating angular momentum by the turbulent velocity components. In the absence of viscous forces, such a turbulent transport cannot transfer the angular momentum to the boundary, and hence $V(r)$ is the solution of the Neumann problem

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right)=0 \qquad \text{for} \qquad r<R,$$

with $\partial(rV)/\partial r = 0$ on $r = R$, that is, there is no flux of rV through the boundary. The turbulent solution is therefore $rV = \text{const.}$, and thus the mean angular momentum about the z axis is uniformly distributed.

It is apparent that while viscous effects cause local mixing in the neighbourhood of a point, turbulence produces a global stirring of the fluid. Each mechanism corresponds to one of the two independent solutions of the second-order equation of motion.

IV. Simple Rotating Flows

(a) *Fully Developed Flow*

The maintenance of a region of concentrated vorticity as suggested by Scorer (1965) clearly requires a steady source of turbulence. We now seek simple rotating flows that can generate statistically steady turbulence and hence can maintain a concentrated vortex. Regions that do not produce turbulent energy cannot cause strong vorticity concentrations because the mixing time scale of a turbulent motion is comparable with its decay time, i.e. the turbulence decays before it effectively stirs a region.

We ask first whether a fully developed rotating axisymmetric flow with no mean axial gradients can support turbulence. The motion away from the ends of a long rotating cylinder might be expected to approach this state. With $U = 0 = W$, equation (8) reduces to

$$\frac{\mathrm{d}(r^2\langle u'v'\rangle)}{\mathrm{d}r} = \nu r^2 \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}\frac{\mathrm{d}(rV)}{\mathrm{d}r}\right).$$

This relation can be integrated to yield

$$\nu r\frac{\mathrm{d}(V/r)}{\mathrm{d}r} - \langle u'v'\rangle = 0, \tag{11}$$

where the constant of integration is found to be zero because the stress (11) is regular at $r = 0$. Equation (11) states that there is no net stress acting on the fluid, independently of the boundary condition at the wall of the container.

The turbulent energy equation (9) for a fully developed flow is reduced such that the rate of production of turbulence is given by

$$T = -r\langle u'v'\rangle\frac{\mathrm{d}(V/r)}{\mathrm{d}r}. \tag{12}$$

This is the only source term, as the remaining terms in (9) represent the redistribution of turbulent energy throughout the flow and the viscous dissipation of energy. Substituting the stress (11) into equation (12), we see that

$$T = -\langle u'v'\rangle^2/\nu \leqslant 0,$$

that is, interaction between the mean rate of strain field and any turbulent stress tends to decrease the turbulent energy. We conclude that a fully developed rotating flow cannot support a steady turbulent motion because the net stress on the fluid is zero.

(b) Flow with Uniform Axial Pressure Gradient

The simplest extension of the fully developed flow in (*a*) above is perhaps a flow which is driven by a uniform axial pressure gradient. In this case, the equation (4) for the mean axial momentum reduces to

$$\frac{1}{r}\frac{\mathrm{d}(r\langle u'w'\rangle)}{\mathrm{d}r} - \Pi = \nu\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}W}{\mathrm{d}r}\right),$$

where $\Pi = -\rho^{-1}\,\partial P/\partial z$ is the axial pressure gradient. Applying the condition that $\langle u'w'\rangle$ and W are regular at $r = 0$, we find that the pressure gradient supports a net stress in the flow, and in particular

$$\nu\,\mathrm{d}W/\mathrm{d}r = \langle u'w'\rangle - \tfrac{1}{2}r\Pi\,. \tag{13}$$

The azimuthal momentum equation is identical with that for a fully developed flow, namely

$$\nu r\frac{\mathrm{d}(V/r)}{\mathrm{d}r} = \langle u'v'\rangle\,. \tag{14}$$

It is seen from equation (9) that the rate of production of turbulent energy is given by

$$T = -r\langle u'v'\rangle\frac{\mathrm{d}(V/r)}{\mathrm{d}r} - \langle u'w'\rangle\frac{\mathrm{d}W}{\mathrm{d}r}\,. \tag{15}$$

Hence equations (13)–(15) imply that

$$T = r\langle u'w'\rangle\Pi/2\nu - (\langle u'v'\rangle^2 + \langle u'w'\rangle^2)/\nu\,. \tag{16}$$

Clearly, a necessary condition for the production of turbulence is that the shear stress $\langle u'w'\rangle$ be positive, such that momentum is transferred away from the axis of symmetry and towards the wall of the container. Equations (13) and (16) also show that T is positive only if $\mathrm{d}W/\mathrm{d}r$ is negative. Thus, if the mean axial velocity decreases away from the axis then turbulence can be maintained by drawing its energy from the mean pressure gradient.

Although the flow can be turbulent, equation (14) implies that the mean angular momentum cannot be uniform unless the stress $\langle u'v'\rangle$ is negative. However, because there is no net (r,θ) component of shear stress any nonzero $\langle u'v'\rangle$ tends to transfer energy from the turbulence to the mean flow. The equation for the conservation of $\langle u'v'\rangle$ can be derived from equations (2) and (3), and it is found that the rate of production of $\langle u'v'\rangle$ by interaction with the mean flow is

$$S = \frac{2\langle v'^2\rangle V}{r} - \frac{\langle u'^2\rangle}{r}\frac{\mathrm{d}(rV)}{\mathrm{d}r}\,. \tag{17}$$

Therefore, if rV is constant then the rate of production of $\langle u'v'\rangle$ is positive, whereas $\langle u'v'\rangle$ itself is negative. Although such a state is not impossible (negative $\langle u'v'\rangle$ may be produced by pressure–velocity correlations, say), it must be considered as improbable.

From the above considerations, the most probable turbulent configuration for a rotating flow with a uniform axial pressure gradient would seem to be an axial flow with $\mathrm{d}W/\mathrm{d}r < 0$ and an azimuthal flow corresponding to solid body rotation, i.e. to V/r constant. Then $\langle u'v'\rangle$ is zero and the production of $\langle u'v'\rangle$ is, from equation (17), equal to $2(\langle v'^2\rangle - \langle u'^2\rangle)V/r$, which is zero when the azimuthal stress matches the radial stress.

(c) Flow with Axial Stress Gradients

It would seem from subsections *(a)* and *(b)* that a rotating turbulent flow capable of mixing angular momentum must have some variation in the axial direction. Thus, we consider a flow with axial stress gradients but no radial or axial mean velocity; that is, there is no mean circulation and the only transport is turbulent. Such a situation might occur in McEwan's (1973) experiment, in which the oscillating top surface of the cylindrical container may be considered as a local source of turbulent stress and energy. It may also be taken as a first approximation to any configuration (such as that of Bretherton and Turner 1968) in which the turbulence is generated by *in situ* mechanical mixing.

Now for this flow the azimuthal momentum equation (8) reduces to

$$\frac{\partial(r^2\langle u'v'\rangle)}{\partial r} + \frac{\partial(r^2\langle w'v'\rangle)}{\partial z} - v\left\{r^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV)}{\partial r}\right) + \frac{\partial^2(r^2V)}{\partial z^2}\right\} = 0,$$

and this may be integrated to yield

$$r^2\langle u'v'\rangle - vr^3\frac{\partial(V/r)}{\partial r} + \frac{\partial}{\partial z}\left\{\int_0^r\left(\langle w'v'\rangle - v\frac{\partial V}{\partial z}\right)r^2\,\mathrm{d}r\right\} = 0. \tag{18}$$

Hence there is a net (r,θ) component of stress balanced by the axial gradient of the (θ,z) component of stress. The rate of production of turbulent energy is, from equation (9),

$$T = -r\langle u'v'\rangle\frac{\partial(V/r)}{\partial r} - \langle v'w'\rangle\frac{\partial V}{\partial z}. \tag{19}$$

Substituting equation (18) into (19), we see that

$$T = -vr^2\left(\frac{\partial(V/r)}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial(V/r)}{\partial r}\frac{\partial}{\partial z}\left\{\int_0^r\left(\langle w'v'\rangle - v\frac{\partial V}{\partial z}\right)r^2\,\mathrm{d}r\right\} - \langle w'v'\rangle\frac{\partial V}{\partial z}.$$

The total rate of production of turbulence at a given axial position is

$$2\pi\int_0^R \mathrm{d}r\,r\,T(r,z) = -2\pi v\int_0^R \mathrm{d}r\,r\left\{r^2\left(\frac{\partial(V/r)}{\partial r}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right\}$$
$$+2\pi\frac{\partial}{\partial z}\left\{\int_0^R \mathrm{d}r\,r\left(r\Omega - V\right)\left(\langle w'v'\rangle - v\frac{\partial V}{\partial z}\right)\right\}, \tag{20}$$

where $V/r = \Omega$ at $r = R$, that is, the container of the fluid is rotating with angular velocity Ω. Thus the total production of turbulent energy is maintained by the product of the axial gradient of the (θ, z) stress component and the azimuthal velocity relative to coordinates rotating with the container.

If equation (20) is applied to the experiment of McEwan (1973) then the appropriate boundary conditions at the ends of the container would seem to be

$$V = r\Omega, \qquad \langle w'v' \rangle = 0 \qquad \text{at } z = 0;$$

and

$$V = r\Omega, \qquad \langle w'v' \rangle = f(r) \qquad \text{at } z = H;$$

that is, the vertical oscillations of the top of the container may produce a mean shear stress, although the mean azimuthal velocity corresponds essentially to that for solid body rotation. The total rate of production of turbulent energy over the body of the fluid is then

$$2\pi \int_0^H \mathrm{d}z \int_0^R \mathrm{d}r\, r\, T(r,z) = -2\pi\nu \int_0^H \mathrm{d}z \int_0^R \mathrm{d}r\, r\left\{r^2\left(\frac{\partial(V/r)}{\partial r}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right\} \leqslant 0.$$

Thus in this configuration the motion itself generates no net turbulent energy. The turbulence at any point within the fluid arises by turbulent diffusion away from the oscillating top surface. This turbulence might mix rV and so create a concentration of vorticity, the consequences of which are considered in subsection (*d*) below. However, the effect is localized to the neighbourhood of the container end and there is no large-scale turbulent mixing over the whole volume. It may be argued that the oscillating end generates inertial waves which propagate throughout the fluid, and hence large-scale mixing can be achieved as these waves break. Indeed, this may happen in the initial instance when the waves first become steep enough to break. However, the turbulence so produced cannot maintain itself by interaction with the mean flow, and it will decay in regions far from the oscillating end or source.

As the motion approaches a statistically steady state, waves generated at the source do not propagate far before breaking. Thus, there exists a localized region of turbulence near the source, with weak (non-turbulent) inertial waves radiating throughout the fluid. The inertial waves produce normal Reynolds stresses but they cannot stir the fluid effectively.

(*d*) *Flow with Secondary Circulation*

It now appears that a flow which is to support turbulence over its whole volume must have a secondary circulation. We note also that if turbulent mixing alone is sufficient to initiate a vorticity concentration then the vortex itself subsequently sets up a secondary circulation due to its pressure field. Thus, consideration of steady vorticity concentrations ought to include the effect of mixing by the secondary mean motion, in addition to the effect of turbulent mixing.

When all the mean velocity components are present, equation (7) implies that a streamfunction $\psi(r,z)$ for the secondary motion may be introduced such that

$$rU = \partial\psi/\partial z \qquad \text{and} \qquad rW = -\partial\psi/\partial r. \tag{21}$$

By assuming that the secondary motion is at least as strong as the turbulence and that the Reynolds number of the azimuthal flow is large, a first approximation to equation (8) is

$$U\,\partial(rV)/\partial r + W\,\partial(rV)/\partial z = 0\,, \tag{22}$$

that is, viscous and turbulent diffusion are neglected. Using equations (21), we find that the solution of (22) is

$$rV = F(\psi)\,,$$

which shows that the angular momentum rV is constant along a streamline of the secondary motion. For a finite container at least, the secondary circulation must form closed streamlines and hence the secondary motion tends to mix the angular momentum, independently of the turbulence. It we assume that the turbulence acts as a mixing agent, as described in Section III, then rV ought definitely to be uniformly distributed over the volume of the fluid. This follows from Batchelor's (1956) discussion of diffusion within regions bounded by closed streamlines.

V. Vorticity Concentration in Nature

There are three primary sources of naturally occurring turbulence:

(1) Shear flow instability. This is unlikely to produce mixing of angular momentum because its mean motion is not usually axisymmetric, i.e. the angular momentum is not governed by a simple transport equation.

(2) Buoyancy instability. This is probably the most abundant source of turbulence in nature. However, such a situation invariably has mean circulations associated with it and, as these convection currents can mix angular momentum, any mixing by turbulence is only in addition to the large-scale stirring.

(3) Breaking of waves (inertial or internal). A single group of breaking waves cannot produce any significant concentration of vorticity because the time scale associated with the turbulent mixing of the region is comparable with the decay time of the turbulence. A steady state vorticity concentration requires the energy source to be vigorous enough to maintain the secondary circulation which accompanies any vortex-like flow. Thus, even if we assume that turbulence generated by breaking waves can initially mix the angular momentum of a region to form a central vortex, the subsequent motion must include a mean circulation which tends to mix angular momentum, independently of the turbulence. On the other hand, it is not clear that the turbulence alone creates the vorticity concentration. The breakdown of an internal or inertial wave is a somewhat organized phenomenon over a region comparable in scale with its wavelength at least.

It seems that any naturally occurring turbulence which could mix angular momentum has secondary motions associated with it. Thus the mixing of angular momentum by turbulence alone is unlikely to occur.

VI. Acknowledgment

I should like to thank Professor B. R. Morton for discussion of this work.

VII. References

Batchelor, G. K. (1956).—*J. Fluid Mech.* **1**, 177.
Bretherton, F. P., and Turner, J. S. (1968).—*J. Fluid Mech.* **32**, 449.
Gough, D. O., and Lynden-Bell, D. (1968).—*J. Fluid Mech.* **32**, 437.
McEwan, A. D. (1973).—A laboratory demonstration of angular momentum mixing. *Geophys. Fluid Dynamics* (in press).
Scorer, R. S. (1965).—Vorticity in nature. Rep. Dept. Maths., Imperial College, London.
Scorer, R. S. (1966).—*Sci. J.* **2**, 46.
Strittmatter, P. A., Illingworth, G., and Freeman, K. C. (1970).—*J. Fluid Mech.* **43**, 539.