



A Note on the Paper "A Fixed Point Theorems in S_b -Metric Spaces"

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Abstract. Very recently, N. Souayan and N. Mlaiki [Nazir Souayan and Nabil Mlaiki, A fixed point theorem in S_b -metric spaces, *J. Math. Comput. Sci.* 16 (2016), 131-139] and S. Sedghi et al. [S. Sedghi, A. Gholidahneb, T. Došenović, J. Esfahani, S. Radenović, Common fixed point of four maps in S_b -metric spaces, to appear in *J. Linear Topol. Algebra*], introduced the concept of S_b -metric space as a generalization of S -metric space. In this paper, we modified the definition of S_b -metric introduced by Souayan and Mlaiki and prove some coupled common fixed point theorems in S_b -metric space. We also present an example to confirm our theoretical results.

To the memory of Professor Lj. Ćirić (1935–2016)

1. Introduction and Preliminaries

S. Sedghi, N. Shobe and A. Aliouche [28] introduced the concept of S -metric space by modifying D -metric and G -metric spaces. The definition of S -metric space is as follows.

Definition 1.1. Let X be a nonempty set. An S -metric on X is a function $S : X^3 \rightarrow [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,

1. $S(x, y, z) \geq 0$,
2. $S(x, y, z) = 0$ if and only if $x = y = z$
3. $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$

The pair (X, S) is called S -metric space.

Sedghi et. al. [28] gave following remarks.

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It is easy to see that every D^* -metric is S -metric, but in general the converse is not true.

In [28] it is also shown the following lemma

Lemma 1.2. *In an S -metric space, we have*

$$S(x, x, y) = S(y, y, x)$$

S. Sedghi and Nguyen van Dung [27] remarked that every S -metric space is topologically equivalent to a metric space.

Bakhtin [3] introduced the concept of b -metric space as a generalization of metric space. S. Czerwick [6, 7] extended the Banach contraction principle in b -metric space.

The definition of b -metric is given below

Definition 1.3. [3] *Let X be a nonempty set. A b -metric on X is a function $d : X^2 \rightarrow [0, \infty)$ if there exists a real number $s \geq 1$ such that the following conditions hold for all $x, y \in X$*

- (i) $d(x, y) = 0$ if and only if $x = y$,
- (ii) $d(x, y) = d(y, x)$
- (iii) $d(x, z) \leq s[d(x, y) + d(y, z)]$

The pair (X, d) is called a b -metric space.

Inspired by the works of Bakhtin [3] and Sedghi et. al. [28], Nizar and Nabil [35] introduced the concept of S_b -metric space.

Definition 1.4. [35] *Let X be a nonempty set and let $s \geq 1$ be a given number. A function $S_b : X^3 \rightarrow [0, \infty)$ is said to be S_b -metric if and only if for all $x, y, z, t \in X$, the following conditions hold:*

- (i) $S_b(x, y, z) = 0$ if and only if $x = y = z$,
- (ii) $S_b(x, x, y) = S_b(y, y, x)$ for all $x, y \in X$,
- (iii) $S_b(x, y, z) \leq s[S_b(x, x, t) + S_b(y, y, t) + S_b(z, z, t)]$

The pair (X, S_b) is called an S_b -metric space. See also ([29], Definition 1.7).

Remark 1.5. [35] *Note that the class of S_b -metric spaces is larger than the class of S -metric spaces. Indeed, every S -metric space is an S_b -metric space with $s = 1$. However, the converse is not always true.*

Example 1.6. [35] *Let X be a nonempty set and $\text{card}(X) \geq 5$. Suppose $X = X_1 \cup X_2$ a partition of X such that $\text{card}(X_1) \geq 4$. Let $s \geq 1$. Then*

$$S_b(x, y, z) = \begin{cases} 0, & \text{if } x = y = z = 0; \\ 3s, & \text{if } (x, y, z) \in X_1^3; \\ 1, & \text{if } (x, y, z) \notin X_1^3. \end{cases}$$

for all $x, y, z \in X$. S_b is a S_b -metric on X with coefficient $s \geq 1$.

Definition 1.7. [35] *Let (X, S_b) be an S_b -metric space and $\{x_n\}$ be a sequence in X . Then*

- (i) *A sequence $\{x_n\}$ is called convergent if and only if there exists $z \in X$ such that $S_b(x_n, x_n, z) \rightarrow 0$ as $n \rightarrow \infty$. In this case we write $\lim_{n \rightarrow \infty} x_n = z$.*

(ii) A sequence $\{x_n\}$ is called Cauchy sequence if and only if $S_b(x_n, x_n, x_m) \rightarrow 0$ as $n, m \rightarrow \infty$.

(iii) (X, S_b) is said to be a complete S_b -metric space if every Cauchy sequence $\{x_n\}$ converges to a point $x \in X$ such that

$$\lim_{n,m \rightarrow \infty} S_b(x_n, x_n, x_m) = \lim_{n \rightarrow \infty} S_b(x_n, x_n, x) = S_b(x, x, x)$$

(iv) Define the diameter of a subset Y of X by

$$\text{diam}(Y) := \sup\{S_b(x, y, z) \mid x, y, z \in Y\}.$$

Sedghi et.al. [28] claimed that S -metric space is a generalization of a G -metric, that is, every G -metric is an S -metric. But Dung et.al. [8] explained with following examples that the assertion of Sedghi et.al [28] is not correct. Dung et.al. [8] further claimed that the class of all S -metric and the class of all G -metric are all distinct.

In definition 1.4 condition (ii) is not true in general. In order to make definition 1.4 a general one, we modify as follows.

Definition 1.8. Let X be a nonempty set and let $s \geq 1$ be a given number. A function $S_b : X^3 \rightarrow [0, \infty)$ is said to be S_b -metric if and only if for all $x, y, z, t \in X$, the following conditions holds:

- (i) $S_b(x, y, z) = 0$ if and only if $x = y = z$,
- (ii) $S_b(x, y, z) \leq s[S_b(x, x, t) + S_b(y, y, t) + S_b(z, z, t)]$

the pair (X, S_b) is called an S_b -metric space.

Definition 1.9. A S_b -metric S_b is said to be symmetric if

$$S_b(x, x, y) = S_b(y, y, x) \text{ for all } x, y \in X.$$

Now we recall one important definitions.

Definition 1.10. Let X be a non-empty set and let $f, g : X \times X \rightarrow X$ be two mappings. A point $(x, y) \in X \times X$ is called coupled coincidence point of f and g if $f(x, y) = g(x, y)$ and $f(y, x) = g(y, x)$; then $(g(x, y), g(y, x))$ is called a coupled point of coincidence; a common coupled fixed point of f and g if $f(x, y) = g(x, y) = x$ and $f(y, x) = g(y, x) = y$, ([1], [2], [4], [5], [9],[11],[13]-[21], [23]-[25], [30], [31], [34]).

2. Main Results

Now we prove our first common coupled fixed point result in the framework of complete symmetric S_b metric spaces:

Theorem 2.1. Let (X, S_b) be a complete symmetric S_b -metric space with parameter $s \geq 1$ and let the mappings $f, g : X^2 \rightarrow X$ satisfying

$$\begin{aligned} S_b(f(x, y), f(x, y), g(u, v)) \leq & a_1 \frac{S_b(x, x, u) + S_b(y, y, v)}{2} \\ & + a_2 \frac{S_b(f(x, y), f(x, y), g(u, v))S(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_3 \frac{S_b(f(x, y), f(x, y), g(u, v))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_4 \frac{S_b(x, x, f(x, y))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} \end{aligned}$$

$$\begin{aligned}
 & +a_5 \frac{S_b(x, x, f(x, y))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\
 & +a_6 \frac{S_b(u, u, g(u, v))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\
 & +a_7 \frac{S_b(u, u, g(u, v))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)}
 \end{aligned} \tag{1}$$

for all $x, y, u, v \in X$ and $a_1, a_2, \dots, a_7 \geq 0$ with $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$ and $s < \frac{1-a_2-a_3-a_6-a_7}{a_1+a_4+a_5}$. Then f and g have a unique common coupled fixed point in X .

Proof. Let $x_0, y_0 \in X$ be arbitrary points. Define

$$\begin{aligned}
 x_{2k+1} &= f(x_{2k}, y_{2k}) \quad , \quad y_{2k+1} = f(y_{2k}, x_{2k}) \\
 x_{2k+2} &= g(x_{2k+1}, y_{2k+1}) \quad , \quad y_{2k+2} = g(y_{2k+1}, x_{2k+1})
 \end{aligned}$$

for $k = 0, 1, 2, \dots$. Then

$$\begin{aligned}
 S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) &= S_b(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1})) \\
 &\leq a_1 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}{2} \\
 &\quad +a_2 \frac{S_b(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1}))S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_3 \frac{S_b(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1}))S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_4 \frac{S_b(x_{2k}, x_{2k}, f(x_{2k}, y_{2k}))S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_5 \frac{S_b(x_{2k}, x_{2k}, f(x_{2k}, y_{2k}))S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_6 \frac{S_b(x_{2k+1}, x_{2k+1}, g(x_{2k+1}, y_{2k+1}))S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_7 \frac{S_b(x_{2k+1}, x_{2k+1}, g(x_{2k+1}, y_{2k+1}))S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &= a_1 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}{2} \\
 &\quad +a_2 \frac{S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_3 \frac{S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_4 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1})S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_5 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1})S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_6 \frac{S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})S_b(x_{2k}, x_{2k}, x_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})} \\
 &\quad +a_7 \frac{S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})S_b(y_{2k}, y_{2k}, y_{2k+1})}{1 + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}
 \end{aligned}$$

$$\leq a_1 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}{2} + (a_2 + a_3)S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + (a_4 + a_5)S_b(x_{2k}, x_{2k}, x_{2k+1}) + (a_6 + a_7)S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

So,

$$(1 - a_2 - a_3 - a_6 - a_7)S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \leq \left(\frac{a_1}{2} + a_4 + a_5\right)S_b(x_{2k}, x_{2k}, x_{2k+1}) + \frac{a_1}{2}S_b(y_{2k}, y_{2k}, y_{2k+1})$$

and therefore,

$$S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \leq \frac{\frac{a_1}{2} + a_4 + a_5}{1 - a_2 - a_3 - a_6 - a_7} S_b(x_{2k}, x_{2k}, x_{2k+1}) + \frac{\frac{a_1}{2}}{1 - a_2 - a_3 - a_6 - a_7} S_b(y_{2k}, y_{2k}, y_{2k+1}). \tag{2}$$

Proceeding similarly one can prove that

$$S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \leq \frac{\frac{a_1}{2} + a_4 + a_5}{1 - a_2 - a_3 - a_6 - a_7} S_b(y_{2k}, y_{2k}, y_{2k+1}) + \frac{\frac{a_1}{2}}{1 - a_2 - a_3 - a_6 - a_7} S_b(x_{2k}, x_{2k}, x_{2k+1}) \tag{3}$$

Adding (2) and (3) we have

$$S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \leq \frac{a_1 + a_4 + a_5}{1 - a_2 - a_3 - a_6 - a_7} [S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]$$

Therefore,

$$S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \leq h[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]$$

where $h = \frac{a_1 + a_4 + a_5}{1 - a_2 - a_3 - a_6 - a_7} < 1$.

Also, we can show that

$$S_b(x_{2k+2}, x_{2k+2}, x_{2k+3}) + S_b(y_{2k+2}, y_{2k+2}, y_{2k+3}) \leq h[S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2})] \leq h^2[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]$$

Continuing this way, we have

$$S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1}) \leq h[S_b(x_{n-1}, x_{n-1}, x_n) + S_b(y_{n-1}, y_{n-1}, y_n)] \leq h^2[S_b(x_{n-2}, x_{n-2}, x_{n-1}) + S_b(y_{n-2}, y_{n-2}, y_{n-1})] \leq \dots \leq h^n[S_b(x_0, x_0, x_1) + S_b(y_0, y_0, y_1)]$$

If $S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1}) = S_{b_n}$, then

$$S_{b_n} \leq hS_{b_{n-1}} \leq h^2S_{b_{n-2}} \leq \dots \leq h^nS_{b_0}.$$

So for $m > n$,

$$\begin{aligned}
 S_b(x_n, x_n, x_m) + S_b(y_n, y_n, y_m) &\leq s[2S_b(x_n, x_n, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, x_m) \\
 &\quad + 2S_b(y_n, y_n, y_{n+1}) + S_b(y_{n+1}, y_{n+1}, y_m)] \\
 &= 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\
 &\quad + s[S_b(x_{n+1}, x_{n+1}, x_m) + S_b(y_{n+1}, y_{n+1}, y_m)] \\
 &\leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\
 &\quad + s^2[2S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(x_{n+2}, x_{n+2}, x_m) \\
 &\quad + 2S_b(y_{n+1}, y_{n+1}, y_{n+2}) + S_b(y_{n+2}, y_{n+2}, y_m)] \\
 &= 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\
 &\quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\
 &\quad + s^2[S_b(x_{n+2}, x_{n+2}, x_m) + S_b(y_{n+2}, y_{n+2}, y_m)] \\
 &\leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\
 &\quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\
 &\quad + \dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\
 &\quad + s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\
 &\leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\
 &\quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\
 &\quad + 2s^3[S_b(x_{n+2}, x_{n+2}, x_{n+3}) + S_b(y_{n+2}, y_{n+2}, y_{n+3})] \\
 &\quad + \dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\
 &\quad + 2s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\
 &\leq 2(sh^n + s^2h^{n+1} + s^3h^{n+2} + \dots + s^{m-n}h^{m-1})S_{b_0} \\
 &< 2sh^n[1 + sh + (sh)^2 + \dots]S_{b_0} \\
 &= \frac{2sh^n}{1 - sh}S_{b_0} \rightarrow 0 \text{ as } n \rightarrow \infty
 \end{aligned}$$

which shows that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X . As X is complete S_b -metric space, so there exists $x, y \in X$ such that $x_n \rightarrow x$ and $y_n \rightarrow y$ as $n \rightarrow \infty$.

Now we will prove that $x = f(x, y)$ and $y = f(y, x)$. On the contrary suppose that $x \neq f(x, y)$ or $y \neq f(y, x)$. Then $S_b(x, x, f(x, y)) = l_1 > 0$ or $S_b(y, y, f(y, x)) = l_2 > 0$.

Using inequality (1)

$$\begin{aligned}
 l_1 &= S_b(x, x, f(x, y)) \\
 &\leq s[2S_b(x, x, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, f(x, y))] \\
 &= s[2S_b(x, x, x_{n+1}) + S_b(f(x_n, y_n), f(x_n, y_n), f(x, y))] \\
 &\leq 2sS_b(x, x, x_{n+1}) + s \left[a_1 \frac{S_b(x_n, x_n, x) + S_b(y_n, y_n, y)}{2} + a_2 \frac{S_b(f(x_n, y_n), f(x_n, y_n), f(x, y))S_b(x_n, x_n, x)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} \right. \\
 &\quad + a_3 \frac{S_b(f(x_n, y_n), f(x_n, y_n), f(x, y))S_b(y_n, y_n, y)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} + a_4 \frac{S_b(x_n, x_n, f(x_n, y_n))S_b(x_n, x_n, x)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} \\
 &\quad \left. + a_5 \frac{S_b(x_n, x_n, f(x_n, y_n))S_b(y_n, y_n, y)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} + a_6 \frac{S_b(x, x, f(x, y))S_b(x_n, x_n, x)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} + a_7 \frac{S_b(x, x, f(x, y))S_b(y_n, y_n, y)}{1 + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} \right].
 \end{aligned}$$

Since $\{x_n\}$ and $\{y_n\}$ are convergent to x and y , therefore by taking limit as $n \rightarrow \infty$ we get $l_1 \leq 0$, which is a contradiction, so $S_b(x, x, f(x, y)) = 0$ which gives $x = f(x, y)$.

Similarly, we can prove that $y = f(y, x)$. Also, we can prove that $x = g(x, y)$ and $y = g(y, x)$. Hence (x, y) is a common coupled fixed point of f and g .

In order to prove the uniqueness of the coupled fixed point, if possible let (p, q) be the second common coupled fixed point of f and g .

Then by using inequality (1), we have

$$\begin{aligned}
 S_b(x, x, p) &= S_b(f(x, y), f(x, y), g(p, q)) \\
 &\leq \frac{a_1}{2}(S_b(x, x, p) + S_b(y, y, q)) \\
 &\quad + a_2 \frac{S_b(f(x, y), f(x, y), g(p, q))S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} + a_3 \frac{S_b(f(x, y), f(x, y), g(p, q))S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &\quad + a_4 \frac{S_b(x, x, f(x, y))S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} + a_5 \frac{S_b(x, x, f(x, y))S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &\quad + a_6 \frac{S_b(p, p, g(p, q))S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} + a_7 \frac{S_b(p, p, g(p, q))S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &= \frac{a_1}{2}(S_b(x, x, p) + S_b(y, y, q)) + a_2 \frac{S_b(x, x, p)S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &\quad + a_3 \frac{S_b(x, x, p)S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)} + a_4 \frac{S_b(x, x, x)S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &\quad + a_5 \frac{S_b(x, x, x)S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)} + a_6 \frac{S_b(p, p, p)S_b(x, x, p)}{1 + S_b(x, x, p) + S_b(y, y, q)} \\
 &\quad + a_7 \frac{S_b(p, p, p)S_b(y, y, q)}{1 + S_b(x, x, p) + S_b(y, y, q)}
 \end{aligned}$$

Accordingly,

$$S_b(x, x, p) \leq \frac{a_1}{2}(S_b(x, x, p) + S_b(y, y, q)) + a_2 S_b(x, x, p) + a_3 S_b(x, x, p),$$

which implies

$$S_b(x, x, p) \leq \frac{a_1}{2 - a_1 - 2a_2 - 2a_3} S_b(y, y, q). \tag{4}$$

Similarly,

$$S_b(y, y, q) \leq \frac{a_1}{2 - a_1 - 2a_2 - 2a_3} S_b(x, x, p). \tag{5}$$

Adding (4) and (5) we have

$$S_b(x, x, p) + S_b(y, y, q) \leq \frac{a_1}{2 - a_1 - 2a_2 - 2a_3} [S_b(x, x, p) + S_b(y, y, q)],$$

and we get

$$\frac{2(1 - a_1 - a_2 - a_3)}{2 - a_1 - 2a_2 - a_3} [S_b(x, x, p) + S_b(y, y, q)] \leq 0.$$

Since $a_1 + a_2 + a_3 < 1$, $\frac{2(1 - a_1 - a_2 - a_3)}{2 - a_1 - 2a_2 - 2a_3} > 0$ we have that $S_b(x, x, p) + S_b(y, y, q) = 0$, which implies that $x = p$ and $y = q$ i.e., $(x, y) = (p, q)$.

Thus f and g have unique coupled common fixed point. This completes the proof. \square

Corollary 2.2. Let (X, S_b) be a complete symmetric S_b -metric space with parameter $s \geq 1$ and let the mapping $f : X^2 \rightarrow X$ satisfying

$$\begin{aligned} & S_b(f(x, y), f(x, y), f(u, v)) \\ & \leq a_1 \frac{S_b(x, x, u) + S_b(y, y, v)}{2} \\ & + a_2 \frac{S_b(f(x, y), f(x, y), f(u, v))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} + a_3 \frac{S_b(f(x, y), f(x, y), f(u, v))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_4 \frac{S_b(x, x, f(x, y))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} + a_5 \frac{S_b(x, x, f(x, y))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_6 \frac{S_b(u, u, f(u, v))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} + a_7 \frac{S_b(u, u, f(u, v))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \end{aligned}$$

for all $x, y, u, v \in X$ and $a_1, a_2, \dots, a_7 \geq 0$ with $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$ and $s < \frac{1-a_2-a_3-a_6-a_7}{a_1+a_4+a_5}$. Then f has a unique coupled fixed point in X .

Our second common coupled fixed point result is the following:

Theorem 2.3. Let (X, S_b) be a complete symmetric S_b -metric space with parameter $s \geq 1$ and let the mappings $f, g : X^2 \rightarrow X$ satisfy

$$\begin{aligned} S_b(f(x, y), f(x, y), g(u, v)) & \leq \beta_1 \frac{S_b(x, x, u) + S_b(y, y, v)}{2} \\ & + \beta_2 \frac{S_b(x, x, f(x, y))S_b(u, u, g(u, v))}{1 + s[S_b(x, x, g(x, y)) + S_b(u, u, f(u, v)) + S_b(x, x, u) + S_b(y, y, v)]} \end{aligned} \tag{6}$$

for all $x, y, u, v \in X$ and β_1, β_2 are non-negative real numbers with $\beta_1 + \beta_2 < 1$ and $s < \frac{1-\beta_2}{\beta_1}$. Then f and g have unique common coupled fixed point.

Proof. Let x_0, y_0 be arbitrary points. Define

$$\begin{aligned} x_{2k+1} &= f(x_{2k}, x_{2k}) \quad , \quad y_{2k+1} = f(y_{2k}, x_{2k}) \\ x_{2k+2} &= g(x_{2k+1}, y_{2k+1}) \quad , \quad y_{2k+2} = g(y_{2k+1}, x_{2k+1}) \end{aligned}$$

for $k = 0, 1, 2, \dots$. Then

$$\begin{aligned} & S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \\ & = S_b(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1})) \\ & = \beta_1 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}{2} \\ & + \beta_2 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1})S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})}{1 + s[S_b(x_{2k}, x_{2k}, x_{2k+2}) + S_b(x_{2k+1}, x_{2k+1}, x_{2k+1}) + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]} \\ & \leq \frac{\beta_1}{2} (S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})) + \beta_2 S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \end{aligned}$$

Accordingly we have,

$$S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \leq \frac{\beta_1}{2(1-\beta_2)} [S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]. \tag{7}$$

Similarly we can show that

$$S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \leq \frac{\beta_1}{2(1-\beta_2)} [S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] \tag{8}$$

Adding (7) and (8) we have

$$\begin{aligned} S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) & \\ \leq \frac{\beta_1}{1 - \beta_2} [S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] & \\ = k[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] & \end{aligned}$$

where $k = \frac{\beta_1}{1 - \beta_2}$.

Analogously we can show that

$$\begin{aligned} S_b(x_{2k+2}, x_{2k+2}, x_{2k+3}) + S_b(y_{2k+2}, y_{2k+2}, y_{2k+3}) & \\ \leq k[S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2})] & \\ \leq k^2[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] & \end{aligned}$$

Now, if $S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1}) = S_{b_n}$ then

$$S_{b_n} \leq kS_{b_{n-1}} \leq k^2S_{b_{n-2}} \leq \dots \leq k^n S_{b_0}$$

So, for $m > n$ we have

$$\begin{aligned} S_b(x_n, x_n, x_m) + S_b(y_n, y_n, y_m) & \leq s[2S_b(x_n, x_n, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, x_m) \\ & \quad + 2S_b(y_n, y_n, y_{n+1}) + S_b(y_{n+1}, y_{n+1}, y_m)] \\ & = s[2S_b(x_n, x_n, x_{n+1}) + 2S_b(y_n, y_n, y_{n+1})] \\ & \quad + s[S_b(x_{n+1}, x_{n+1}, x_m) + S_b(y_{n+1}, y_{n+1}, y_m)] \\ & \leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ & \quad + s^2[2S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(x_{n+2}, x_{n+2}, x_m) \\ & \quad + 2S_b(y_{n+1}, y_{n+1}, y_{n+2}) + S_b(y_{n+2}, y_{n+2}, y_m)] \\ & = 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ & \quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\ & \quad + s[S_b(x_{n+2}, x_{n+2}, x_m) + S_b(y_{n+2}, y_{n+2}, y_m)] \\ & \leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ & \quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\ & \quad + \dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\ & \quad + s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\ & \leq 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ & \quad + 2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2})] \\ & \quad + \dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\ & \quad + 2s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\ & \leq 2(sk^n + s^2k^{n+1} + \dots + s^{m-n}k^{m-1})S_{b_0} \\ & < 2sk^n[1 + sk + (sk)^2 + \dots]S_{b_0} \\ & = \frac{2sk^n}{1 - sk} S_{b_0} \rightarrow 0 \text{ as } n \rightarrow \infty \end{aligned}$$

Therefore $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X . Since X is complete S_b -metric space, there exist $x, y \in X$ such that $x_n \rightarrow x$ and $y_n \rightarrow y$ as $n \rightarrow \infty$.

Now, we will show that $x = f(x, y)$ and $y = f(y, x)$. Suppose on contrary that $x \neq f(x, y)$ or $y \neq f(y, x)$, so that $S_b(x, x, f(x, y)) = l_1 > 0$ or $S_b(y, y, f(y, x)) = l_2 > 0$. Consider the following and using inequality (6), we get

$$\begin{aligned}
 l_1 &= S_b(x, x, f(x, y)) \\
 &\leq s[2S_b(x, x, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, f(x, y))] \\
 &= sS_b(x, x, x_{n+1}) + sS_b(f(x_n, y_n), f(x_n, y_n), f(x, y)) \\
 &\leq sS_b(x, x, x_{n+1}) + s\left[\beta_1 \frac{S_b(x_n, x_n, x) + S_b(y_n, y_n, y)}{2}\right. \\
 &\quad \left. + \beta_2 \frac{S_b(x_n, x_n, f(x_n, y_n))S_b(x, x, f(x, y))}{1 + s[S_b(x_n, x_n, f(x, y)) + S_b(x, x, f(x_n, y_n)) + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)]}\right] \\
 &= sS_b(x, x, x_{n+1}) + \frac{s\beta_1}{2}[S_b(x_n, x_n, x) + S_b(y_n, y_n, y)] \\
 &\quad + \beta_2 \frac{S_b(x_n, x_n, x_{n+1})S_b(x, x, f(x, y))}{1 + s[S_b(x_n, x_n, f(x, y)) + S_b(x, x, x_{n+1}) + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)]}
 \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ we get

$$S_b(x, x, f(x, y)) \leq 0$$

Therefore

$$S_b(x, x, f(x, y)) = 0$$

which implies that $x = f(x, y)$. Similarly, we can prove that $y = f(y, x)$. Also, we can prove that $x = g(x, y)$ and $y = g(y, x)$. Hence, (x, y) is a common coupled fixed point of f and g .

In order to prove the uniqueness of the common coupled fixed point of f and g , if possible let (p, q) be the second common coupled fixed point of f and g .

Then by using inequality (6), we have

$$\begin{aligned}
 S_b(x, x, p) &= S_b(f(x, y), f(x, y), g(p, q)) \\
 &\leq \frac{\beta_1}{2}(S_b(x, x, p) + S_b(y, y, q)) \\
 &\quad + \beta_2 \frac{S_b(x, x, f(x, y))S_b(p, p, g(p, q))}{1 + s[S_b(x, x, g(p, q)) + S_b(p, p, f(x, y)) + S_b(x, x, p) + S_b(y, y, q)]}
 \end{aligned}$$

Therefore,

$$S_b(x, x, p) \leq \frac{\beta_1}{2}(S_b(x, x, p) + S_b(y, y, q)),$$

and

$$(1 - \frac{\beta_1}{2})S_b(x, x, p) \leq \frac{\beta_1}{2}S_b(y, y, q).$$

So,

$$S_b(x, x, p) \leq \frac{\beta_1}{2 - \beta_1}S_b(y, y, q) \tag{9}$$

Similarly,

$$S_b(y, y, q) \leq \frac{\beta_1}{2 - \beta_1}S_b(x, x, p) \tag{10}$$

Adding (9) and (10) we have

$$S_b(x, x, p) + S_b(y, y, q) \leq \frac{\beta_1}{2 - \beta_1}[S_b(x, x, p) + S_b(y, y, q)]$$

i.e.,

$$\frac{2(1 - \beta_1)}{2 - \beta_1} [S_b(x, x, p) + S_b(y, y, q)] \leq 0.$$

But $\frac{2(1-\beta_1)}{2-\beta_1} > 0$. Therefore $S_b(x, x, p) + S_b(y, y, q) = 0$. Which implies that $x = p$ and $y = q$, i.e., $(x, y) = (p, q)$. Thus f and g have a unique common coupled fixed point. \square

This completes the proof.

Corollary 2.4. Let (X, S_b) be a complete symmetric S_b -metric space with parameter $s \geq 1$ and let the mapping $f : X^2 \rightarrow X$ satisfying

$$S_b(f(x, y), f(x, y), f(u, v)) \leq \beta_1 \frac{S_b(x, x, u) + S_b(y, y, v)}{2} + \beta_2 \frac{S_b(x, x, f(x, y))(u, u, f(u, v))}{1 + s[S_b(x, x, f(u, v)) + S_b(u, u, f(x, y)) + S_b(x, x, u) + S_b(y, y, v)]}$$

for all $x, y, u, v \in X$ and β_1, β_2 are non-negative real numbers with $\beta_1 + \beta_2 < 1$ and $s < \frac{1-\beta_2}{\beta_1}$. Then f has a unique coupled fixed point.

Example 2.5. Let $X = \mathbb{R}$ be the set of real numbers,

$$S_b(x, y, z) = |x - z| + |y - z| \text{ for all } x, y, z \in \mathbb{R}$$

where $s = 2 < 3$. Then (X, S_b) is an S_b -metric space. Let $f(x, y) = g(x, y) = \frac{2x-y+11}{12}$.

For the pair (f, g) as well as for all $x, y, u, v \in \mathbb{R}$ we have

$$\begin{aligned} S_b(f(x, y), f(x, y), g(u, v)) &= 2 \left| \frac{2x - y + 11}{12} - \frac{2u - v + 11}{12} \right| \\ &= \frac{|x - u|}{3} + \frac{|y - v|}{6} \\ &\leq \frac{1}{3} (|x - u| + |y - v|) \\ &= \frac{1}{3} \left(\frac{S(x, x, u) + S(y, y, v)}{2} \right) \end{aligned}$$

In equation (1) is satisfied with $a_1 = \frac{1}{3}$ and $a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0$. Note that corollary 2.2 is also satisfied. Therefore the pair (f, g) has a unique coupled fixed point $(x, y) = (1, 1)$. \square

Competing interests

The authors declare that they have no competing interests.

Author’s contributions

All authors contributed equally in preparation of the final manuscript. All the authors read and approved the final manuscript.

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