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A NOTE ON THE PRESSURE FIELD WITHIN AN OUTWARD MOVING FREE ANNULUS

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ABSTRACT

The outward radial expansion of a free liquid annulus is a common problem of both ealier and current ICF blanket design. Whether the annulus fractures or not depends on the internal pressure and surface stability. In this paper a model based on incompressible cylindrically symmetric flow is used to get a theoretical solution similar to that of the Rayleigh's solution for bubble dynamics. The pressure inside the annulus is found positive all time but the peak is lowering during the expansion. Besides, both surfaces are Taylor stable during such motion. Thus, it is concluded that an annulus in outward radial motion will not cavitate or breakup.

THE BACKGROUND

In an Inertia Confinement Fusion (ICF) reactor, the chamber wall may be protected from the neutron radiation by a neutron absorbing, falling liquid blanket, which surrounds the fusion site. The blanket has a geometry consisting of either a continuous annulus geometry[1] or an array of discrete jets[2], both of which encircle a central cavity. Shortly after a fusion event, all of the high-energy neutrons will leave the fusion site and be mostly absorbed by the falling liquid. Moreover, the attenuation of neutron energy by the liquid will occur so quickly that the liquid will not have sufficient time to expand. This process is known as isochoric heating -- i.e. heating at constant volume. The neutron absorbing liquid thus finds itself suddenly possessing much higher internal pressure than its surroundings (due to its increased internal energy), and will fragment due to the propagation of a rarefaction wave into the liquid. Interestingly, for the case of the discrete jet array, the fragmentation of the liquid results not in the formation of small liquid chunks. On the contrary, because of geometry and neutron energy density gradient, fragmentation results first in the break-off of individual annuli from each jet, which then collide with each other and consolidate into a roughly continuous annulus. In either geometry, fragmentation is expected to cause the outward radial motion of a single, free liquid annulus. In previous studies[1], it was assumed that this motion causing fracture of the liquid. In this paper, fractures during its outward motion is investigated analytically by calculating the radial pressure profile in the free annulus. In this model, we treat the fluid as an incompressible flow. We can obtain a theoretical solution following analysis similar to that of Rayleigh for bubble dynamics[3].

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THE INCOMPRESSIBLE GOVERNING EQUATIONS

We view the annulus as a free continuous body which has cylindrical inner and outer surfaces exposed to the ambient pressure as illustrated in Figure 1. The following assumptions can be reasonably made.

Figure 1. The Description of Model

<u>Assumptions</u>

1. The fluid is incompressible
2. The fluid is inviscid
3. The motion is one dimensional (radial)

Following these assumptions, we can write out the governing equations.

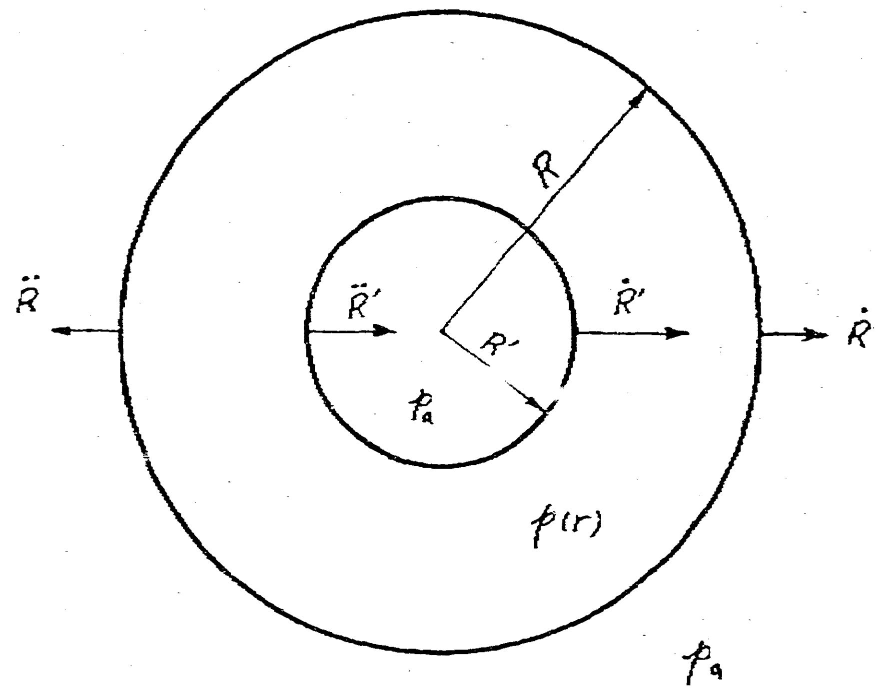

Fig. 1. The Description of Model

Continuity Equation

$$div\vec{V} = 0 \quad (1)$$

for radial flow

$$\dot{R} \cdot R = \dot{R}' \cdot R' = V_r(r) \cdot r \quad (1a)$$

in which R and R' are the instantaneous outer and inner radii of the annulus and $\dot{R}$ and $\dot{R}'$ are the respective surface velocities.

The integration (over time) of Equation 1a gives a global continuity equation,

$$R^2 - R'^2 = R_0^2 - R_0'^2 = C_1 \quad . \quad (1b)$$

Momentum Equation

The 1-D cylindrical momentum equation can be derived from Navier Stokes Equation[3], simplified for pure radial motion

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} \quad . \quad (2)$$

Boundary Conditions

The boundaries of the free annulus are exposed to the ambient pressure. In the HYLIFE II case the pressure is close to zero. That is,

$$P(R') = 0 \quad (3a)$$

$$P(R) = 0 \quad . \quad (3b)$$

Strictly, we have neglected surface tension and the normal component of viscous stress in the dynamic state.

Initial Condition

At the beginning, the annulus has its initial radii and a velocity distribution which satisfy the continuity equation. That is,

$$R = R_0 \qquad R' = R'_0$$

$$V_0(r) \cdot r = \dot{R}_0' \cdot R_0' = \dot{R}_0 \cdot R_0 \quad . \quad (4)$$

THE PRESSURE DISTRIBUTION AND OTHER RESULTS

The Pressure Distribution

Similar to Rayleigh's model of bubble dynamics in an incompressible liquid[4], we can integrate Equation 2 over space at an instant in time to obtain the relation between the pressure and the surface conditions[3]. By substituting the continuity equation into Equation 2 and integrating over r to R we can get for the instantaneous pressure distribution

$$P(r) - P(R) = \rho\left[(R\ddot{R} + \dot{R}^2)\ln\frac{R}{r} + \frac{(R\dot{R})^2}{2}\left(\frac{1}{R^2} - \frac{1}{r^2}\right)\right] \quad (5)$$

With the boundary condition, Equation 3a, we have from Equation 5

$$0 = P(R') - P(R)$$

$$= \rho\left[(R\ddot{R} + \dot{R}^2)\ln\frac{R}{R'} + \frac{(R\dot{R})^2}{2}\left(\frac{1}{R^2} - \frac{1}{R'^2}\right)\right] \quad . \quad (5a)$$

This holds for all time. (R, R', $\dot{R}$ and $\ddot{R}$ are all functions of time.)

The Total Kinetic Energy

During the outward motion, the velocity profile is known from the continuity equation. Therefore we can calculate the total kinetic energy of the liquid annulus at any instant by integrating over the whole body, i.e.

$$K.E. \;=\int_{R'}^{R} 2\pi\rho r\frac{V_r^2}{2}dr = 2\pi\rho(\dot{R}R)^2\ln\frac{R}{R'} . \quad (6)$$

Total Momentum

The total momentum can be calculated also by an integration of over the whole annulus

$$Mom. \;=\int_{R'}^{R}\rho 2\pi r V_r \cdot dr = 2\pi\rho(R\dot{R})(R - R') . \quad (7)$$

Note that the term $rV_r = R\dot{R}$ is constant in the above integration.

THE SOLUTION SCHEME

With the above equations we can set up a numerical scheme to solve the history of fluid motion and pressure distribution within the annulus.

1. Starting with the initial condition we can solve for the outer surface acceleration rate from Equation 5a,

$$\ddot{R} = -\frac{\frac{(R\dot{R})^2}{2}\left(\frac{1}{R^2} - \frac{1}{R'^2}\right)/\ln\frac{R}{R'} + \dot{R}^2}{R}$$

2. The velocity at the next time step can be obtained by using

$$\dot{R}(\Delta t) = \dot{R}(0) + \ddot{R} \cdot \Delta t$$

3. After we have the surface velocity the radius again can be calculated by

$$R(\Delta t) = R(0) + \dot{R}(0) \cdot \Delta t$$

4. The global continuity Equation 1b now can be used to determine the radius R'

5. The velocity $\dot{R}'$ can be calculated from the continuity Equation 1a

6. Using Equation 5 we can calculate the pressure distribution within the annulus

7. Increase another time step Δt, repeat step 1) to step 6). Continue until the end of the time of interest. We then have the history of radii, velocities, acceleration rates, and most importantly, the pressure profile at each instant.

8. Additionally, we can also calculate the total momentum and kinetic energy of the annulus at anytime.

Table 1. The Initial Value Used in the Calculation**

$R_0' = 0.02\ m$
$R_0 = 0.1011\ m$
$V_r(R_0') = 178.5\ m/s$
$\rho = 485\ Kg/m^3$

** The Values can be arbitrary, but the listed data are chosen for lithium[3] as in the HYLIFE study.

DISCUSSION

For illustration, initial values are chosen as given in Table 1. Following the procedure described in the last section the histories of outer and inner radii are found as shown in Figure 2. As the annulus moves outward, its thickness continually diminishes, as expected from continuity considerations. Figure 3 shows that the inner surface velocity decays with time while the outer surface is accelerated. The two velocities tend to converge as time becomes large. Surface acceleration histories are shown in Figure 4.

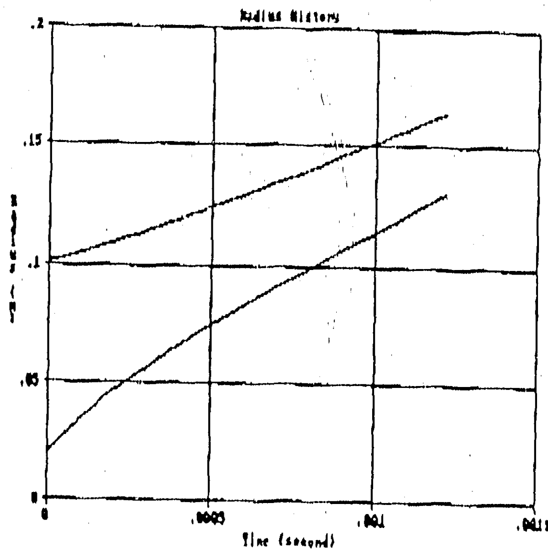

Figure 2. The Change of Radii

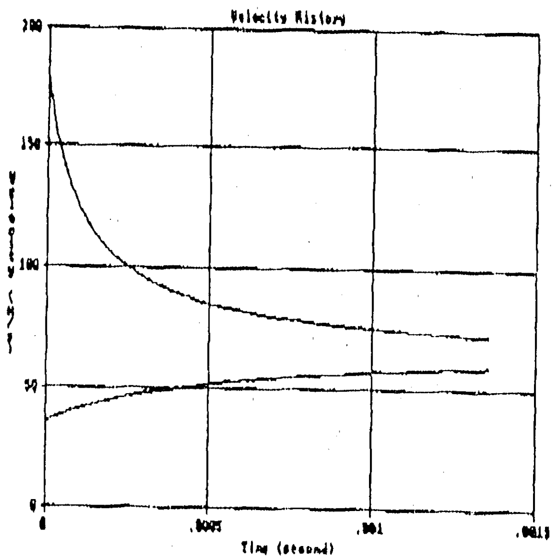

Figure 3. The History of Velocities

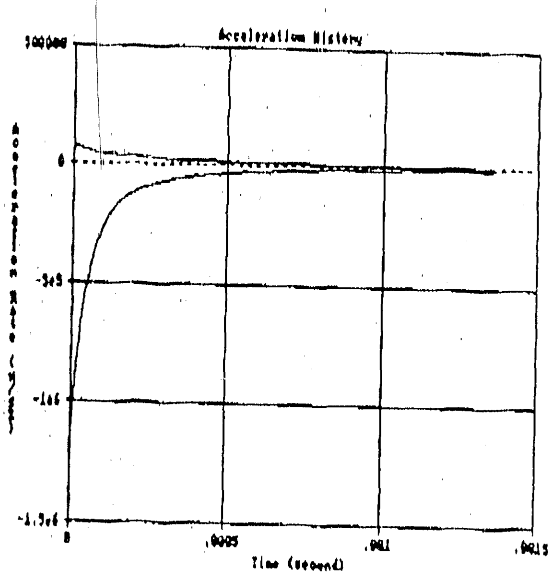

Figure 4. The History of Acceleration Rate

Figure 5 illustrates the shape of the pressure profile within the liquid. The pressure is everywhere positive except at the surfaces where it is zero. As the annulus

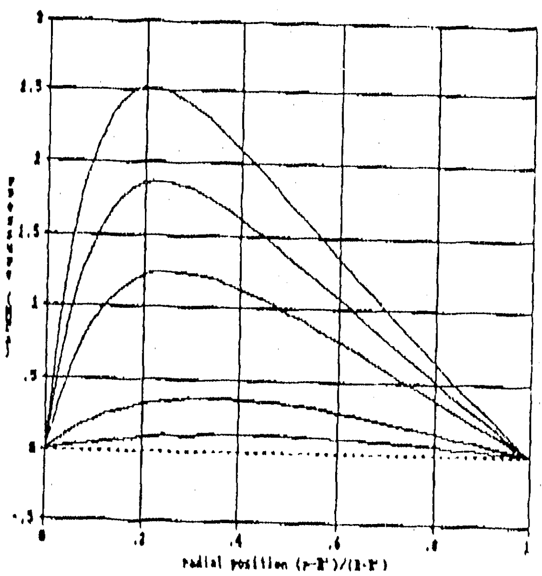

Figure 5. The Pressure Profile

moves outward and thins we see that the maximum pressure within the annulus becomes smaller and smaller. The pressure remains positive for all time, so cavitation is not expected.

We have investigated the stability of the accelerating surfaces. According to Taylor's plane interface theory[6] both cylindrical surfaces are stable. Plesset[7] derived the conditions for stability of a spherical surface. We have carried out a similar derivation for cylindrical surfaces[8] which again shows that both surfaces are stable in the present problem.

The calculations show that kinetic energy is conserved in this motion, however, the total radial momentum is found to increase with time. This surprising result is consistent with the decreasing internal pressure. A rigorous derivation of the momentum change due to th~ pressure profile is presented in the Appendix.

CONCLUSIONS

The pressure field within an outwardly moving annulus of incompressible liquid has been found analytically. Contrary the previous intuitive assumption that liquid in this motion would be subject to tension and cavitation, the prese... analysis shows that the pressure within the liquid is always positive. Furthermore stability analysis indicates that both surfaces are stable to small perturbations.

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APPENDIX

The momentum equation tells us that if there is a pressure gradient the fluid will be accelerated. It would be very clear if we rewrite Equation 2 in Lagrangian form as

$$-\frac{\partial P}{\partial r} = \rho \frac{DV_r}{Dt} = \rho\left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r}\right) \qquad (I)$$

where the $\rho \frac{DV_r}{Dt}$ is the rate of the momentum change of an unit volume fluid particle at r. The total rate of change can be obtained by integration over the whole body. That is

$$\frac{D(MV)}{Dt} = -\int_{R'}^{R} 2\pi r \frac{\partial P}{\partial r} dr \qquad (II)$$

Again by using the continuity equation, we have

$$-\int_{R'}^{R} r \frac{\partial P}{\partial r} dr = \rho \int_{R'}^{R}\left[\frac{\dot{R}^2 + R\ddot{R}}{r} - \frac{(R\dot{R})^2}{r^3}\right] r\, dr$$

$$= \rho\left[(\dot{R}^2 + R\ddot{R})(R - R') + (R\dot{R})^2\left(\frac{1}{R} - \frac{1}{R'}\right)\right] \qquad (III)$$

The total momentum change of the annulus is just the time integration of the rate

$$(MV) = (MV)_{t=0}$$

$$+ \int_0^t 2\pi\rho\left[(\dot{R} + R\ddot{R})(R - R') + (R\dot{R})^2\left(\frac{1}{R} - \frac{1}{R'}\right)\right] dt \qquad (IV)$$

And this is achievable in the simple numerical calculation along with the other calculations. What we find out is the total momentum of the annulus calculated from this equation (IV) exactly overlaps on the curve calculated from Equation 7.

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