A Novel Application to Image Restoration Based on Regularized SL0 Algorithm in Frequency Domain

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ABSTRACT

In these recent years, Compressive Sensing (CS) is becoming an attractive topic in the field of Information Theory. It is widely used in several area including networking, image processing and digital camera. In particular, image reconstruction based on small number of measured components is known as the most useful application. In this paper, SL0 algorithm is specially used for the reconstruction process. It significantly decrease the processing time by utilizing a matrix in which the number of row is much smaller than number of column. Therefore, SL0 is known as one of the fastest and most accurate algorithm in CS. However due to ill-posed condition, if the prior information of the original image is undetermined, the reconstruction procedure of SL0 is much affected by the noise. Unfortunately, the investigation for solving this SL0 ill-posed condition is very limited therefore SL0 is not widely applied in many application. Consequently, this paper proposes a novel regularization technique for SL0 algorithm in the frequency domain. In order to reduce and constraint the space of reconstructed image, the frequency domain Tikhonov regularization technique is employed. It is shown that the quality of the reconstructed image is much better compared to the traditional algorithm under the noisy environment. The experimental result is exclusively simulated for 3 images: Lena, Sussie and Cameraman under both Gaussian and Non-Gaussian noise models (such as AWGN, Poisson noise, Salt & Pepper noise and Speckle noise) at different noise powers.

Keywords: Compressive Sensing (CS), Digital Image Reconstruction, SL0 Algorithm, Frequency Domain Tikhonov Regularization.

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1. INTRODUCTION

Shannon and Nyquist sampling theorem has been applying for a long time as a classical method for a digital representation of analog signals. It is stated that, the analog signal can be perfectly reconstructed if the sampling rate exceeds 2B samples per second hence the whole bandwidth has to be explored. However in most of the case, the reconstructed data can be compressed with a much smaller size as the information is only stand at some particular part. The Shannon and Nyquist sampling theorem is not efficiency in these cases. There are several ways to improve this situation and domain transform is exclusively applied for sparse representation of the signal and the source coding is implemented afterward to reduce the size of processed signal. JPEG format [1] is the most famous application of source coding in which image is sparsely represented and compressed in DCT domain. With these great impact, source coding is used widely in the field of video and image process. Unfortunately, it is still not provided the optimal solution as the raw data has to be stored before compression. It is a waste when huge amount of data is kept for compression and most of them is discarded when the process is finished. The situation is even worst when the data is stored in terabytes or it is expensive and physical limiting to keep the measurements.

Subsequently, Compressive Sensing (CS) [7] is introduced to deals with efficient recovery of sparse vector from the linear measurements. The size of sampled data is then reduced dramatically as only useful information is captured for the reconstruction process. In particular, images are linearly fixed with a set of independent waveforms in which it is compressible. In many situations, it can be extracted with respect to the combination of several waveform rather than a basis in order to obtain much sparser representations. An example of those transformed domains [11, 12] are: Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Wavelets Transform (WT) and Curvelet Transform. The procedure of CS algorithms are generally divided into 2 steps: small amount of sensors are used first to efficiently capture the sparse information, subsequently optimization algorithms are then applied to reconstruct the full length signal from small amount of collected data (the sparse information).

The SL0 algorithm is known as one of the most accurate algorithms in CS. However it is usually ill-posed problem [3-6] and it is highly ill-posed problem under noisy environments. Simultaneously, a regularization technique is known as one of the most effective techniques for solving this ill-posed problem. As a result, this paper proposes the novel frequency domain Tikhonov regularization for SL0 algorithm. Moreover, the proposed reconstruction algorithm is exclusively presented for image reconstruction from small amount of measurement under frequency domain. The experimental results obviously confirm the performance of the proposed algorithm for all the simulation cases.

The organization of the paper is as follow: section 2 provides an outlines about the SL0 algorithms and the issue of ill-posed problem. Section 3 give a general ideal of using CS for the image reconstruction. Section 4 proposes a novel frequency domain Tikhonov regularization for improving the quality of the reconstructed image. Later, section 5 presents the analysis of experimental result. Finally, section 6 discusses the conclusion.

2. THE INTRODUCTION OF COMPRESSIVE SENSING

A. Introduction of Classical Image Restoration based on SL0 Norm Estimation [4-5]

There are several groups of CS, their performance is mainly evaluated based on the accuracy and time processing. The reconstruction process of CS algorithms are executed based on norm estimation. Smoothed l^0 norm (SL0) algorithm is exclusively presented in this section as the very first method that employ l^0norm.

Assume that there are totally N components in signal x and has a sparse representation with respect to a basis $\Psi(\alpha = \Psi x)$. Suppose Φ is an $M \times N$ measurement matrix ($M < N$), so the observation signal y is measured by:

$$y = \Phi\alpha \ or \ y = \Phi\Psi x = Ax \tag{1}$$

If there is no additional noise in the system then the signal can be reconstructed by minimizing l^0 norm optimization.

$$\hat{\alpha} = \min_{\alpha} \|\alpha\|_0 \ subject \ to \ y = A\alpha \tag{2}$$

where $\|\alpha\|_0$ counts the number of non-zero components of α.

l^0 norm of a vector is known as a discontinuous function of that vector so it is easily trapped in local minimization and very sensitive with the noise. SL0 replaces this discontinuous function by a suitable continuous one, and minimizes it by means of minimization algorithm for continuous functions (e.g. steepest descent method). There are several families of functions, which can be used for the minimization process but they should have the following property.

$$f_\sigma(\alpha) \approx \ 1 \ if \ |\alpha| \leq \sigma \ and \ f_\sigma(\alpha) \approx \ 0 \ if \ |\alpha| > \sigma \tag{3}$$

The Gaussian family of function is typically used in the original algorithm as follow:

$$f_\sigma(\alpha) = \exp(-\alpha^2/2\sigma^2) \tag{4}$$

According to (4), when the value of $f_\sigma(\alpha)$ is close to zero then the signal α is detected as sparse signal. Thereby, the behaviour of the approximation function is affected by σ for difference value of the signal α. The definition of the estimator of the algorithm can be rewritten as the following maximization problem:

$$F_\sigma(\alpha) = \sum_{i=1}^{M} f_\sigma(\alpha_i) \tag{5}$$

$$\|\alpha\|_0 = M - \sum_{i=1}^{m} f_\sigma(\alpha_i) = M - F_\sigma(\alpha) \tag{6}$$

By the steepest descent method, the solution of problem is:

$$\hat{\alpha}_{i+1} = \hat{\alpha}_i - \Phi^{-1}(\Phi(\alpha_i - \mu\delta) - y) \tag{7}$$

where $\mu > 0$ and is small enough number, A is an $M \times N$ mixing matrix and

$$\delta = \left[\alpha_1 \exp(-\alpha_1^2/2\sigma^2), \ldots, \alpha_n \exp(-\alpha_n^2/2\sigma^2)\right] \tag{8}$$

Initialization:

1) Let α_0 be the minimum l^2 norm solution of $y = A\alpha$ obtained by pseudo-inverse of A.
2) Choose a suitable value for σ and decreasing sequence $[\sigma_1, \sigma_2, \ldots, \sigma_J]$.

Main loop

for $j = 1,2,\ldots,J$

Set $\sigma = \sigma_1, \alpha = \hat{\alpha}_j$

2.1 $\delta = [\alpha_1 exp(-\alpha_1^2/2\sigma^2), \ldots, \alpha_n exp(-\alpha_n^2/2\sigma^2)]$

2.2 $\alpha \leftarrow \alpha - \mu\,\delta$

2.3 $\alpha \leftarrow \alpha - A^T(AA^T)^{-1}(A\alpha - y)$

end loop

Final Answer: $\hat{\alpha} = \hat{\alpha}_J$

Fig.1: *Procedure for SL0 algorithm.*

The procedure of SL0 algorithm is basically divided into 2 steps. The sparse components of the signal or image is determined first. Furthermore, the classical ML estimation is applied for recovering of original signal or image from components in first step. Therefore, the second step of SL0 algorithm is also stated as recovery process. Hence, SL0 algorithm is attained base on the Maximum Likelihood (ML) estimation algorithms.

B. Inverse Problems.

1) Definition of an Inverse Problem.

If the formulation of 2 problems is related with each other, they are defined as inverse problem. [15-19]. In particular, if the first problem is the direct problem then the inverse problem is its complementary part. Therefore the response of the inverse problem is calculated from the result of the direct problem.

2) Well-Posed and Ill-Posed Problem.

According to Hadamard [20], the well-posed problem is defined as follow:

a. Existence of solution: The solution of problem or model must exist. In term of mathematical, the solution is available for most of the case. However if the solution is too difference with the original, it is meaningless.
b. Uniqueness of solution: The solution of the problem must be unique. For example if the input data is taken from difference path and in difference time, the solution of the problem may not be the same. In discrete linear inverse problems, non-unique solution is mainly caused by rank insufficient.
c. Dependence of solution: The solution of the problem is found from input data. Moreover, the solution of inverse problem is also very sensitive with the nature of the data, it may be diversed when the data is changed. As a result it may not present the properties of solution.

The problem is ill-posed if it does not satisfy all of these 3 requirements. In fact the information is often loosed or disturbed in the observation process, so the content of solution is often lower than the original information. In general, the direct reconstruction problem is not affected much with the loss of information. However in case of inverse problem, the reconstruction signal is determined from the observed information; this information loss has serious issue. Due to the loss of information, it is not possible to recover exactly the original information therefore the inverse problem fails to have unique solution.

C. Regularized Technique and the Solutions to Ill-Posed Problems

Regularization [5] employs additional information to compensate for the information loss. This additional information typically cannot be obtained from the observed data and must be known in advance. Basically, the prior information cover some desired characteristics of the solution like total energy, smoothness, positivity and so on. When the priori information is supplemented, the space of solutions is constrained or scaled down to the expectation which is compatible with the observed data. A general theory of regularization algorithms to ill-posed problems was first introduced by A. N. Tikhonov.

In the Tikhonov approach, a family of approximate solutions to the inverse problem is constructed. In general, it is primarily controlled by a nonnegative real-valued regularization parameter. If there is no noise in system, the solution is constructed exactly the same to the original problem as the regularization parameter goes to zeros. While the noise is existed, an optimal approximate solution is obtained when the regularization parameter is positive value. Later, various stochastic and regularization theories have been purposed to improve the performance. In these proposed regularization model, the addition of a priori information is resulted in a new well-posed problem which is closely related to the original ill-posed one. This solution of resulted well-posed problem has to satisfy the Hadamard requirements. Moreover it is formulated so that its unique solution is meaningful with respect to the original ill-posed one. It is therefore very important to ensure that the priori constraints used accurately reflect the required characteristics of the solution.

According to regularization theories, the error between the reconstructed signal and the original signal is estimated by a cost function. In general, cost function is initialled based on two conditions. The first term is a corresponding error between the observed signal and the proposed solution. The second term is the constraint criterion which penalizes for lack of "smoothness". According to the inverse problem, the solution is estimated which minimizes the cost function. The Tikhonov regularization supports the same idea in which the problem is posed as constraining the admissible solution set. The regularization parameters must be chosen first based on the properties of signal system, the reconstruction signal is then very sensitive to these parameters and to the additional noise. Therefore, the solution of inverse problem or reconstruction image in this case is improved by limiting the solution space. The affections of the outlier to the cost function are then reduced.

Up to now, there are several researches about regularization algorithms. Each of them has advantage and disadvantage for specific case. However, one of the most common algorithms is to constrain the spatial energy of first or second spatial derivative of the solution. In this paper we propose the problems to incorporate constraints on second spatial derivative (Laplacian) of the solution. The general framework for the inverse problem with regularization is presented in Figure 2.

D. SL0 as an Ill-Posed Inverse Problem

1) SL0 is an Inverse Problem

In the field of image processing, SL0 is applied for recovering original image from the incomplete observation. The whole framework is actually an inverse problem because the reconstruction image is estimated by the observed image [21]. If the characteristics of the imaging process and system are granted, the simulation is the forward problem, while the reconstruction is the inverse problem.

2) SL0 is an Ill-Posed Problem

It is stated above that ill-posed problem implies the failure of any of the Hadamard conditions. SL0 algorithm may fail to satisfy Hadamard conditions because only small amount of the observed signals is used for the recovery process. Little change in observed signal may cause a big change in the solution.

2.1 Nonexistence of the Solution: If the noise is existed in the observation process, the characteristic of the image system may be changed [15]. There are many cases in which the reconstructed image cannot represent the properties of original image. Therefore the system is non-invertible and the expected result cannot be obtained.

2.2 Uniqueness of Solution: lack of information in the reconstructed process usually makes the solution of SL0 to be non uniquesss [20].

2.3 Dependence of solution: The inverse problem can be extremely sensitive to the outliers for some specific imaging system [15, 16]. Little amount of the noise in the system can lead to an arbitrarily large unauthentic signal in the restoration process.

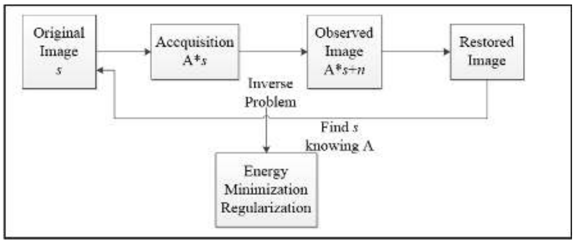


Fig.2: *General Model for Inverse Problem with Regularization.*

3. THE IMAGE RESTORATION IN CS

A. Classical Image Restoration Algorithm in Spatial Domain

This section represents the classical image restoration in spatial domain and the necessary for the regularization of inverse problem. In general, the purpose of image restoration is to produce the best estimate of the source image, given the recorded image and some a priori knowledge. However, in the inverse process the observed image is always degraded. Therefore, the restoration of degraded images is a crucial problem because it allows the recovery of lost information. In spatial domain, there are 2 common types of degradation: blurring is due to lens aberrations, atmospheric turbulence and motion; whereas point degradations are caused by photo electronic, photochemical and electronic random noises. In the presence of additional noise, spatial degradation is ill-conditioned and is very difficult to invert the solution. Regularization is one of the most effective approaches in ill-posed recovery problem. It is stated that, recorded data and priori knowledge are use in a complementary way to obtain the best possible solution. The regularization parameter trades off fidelity to the available data to smoothness of the solution while the smoothness properties of the image are captured by the regularization operator.

B. Proposed Image Restoration Algorithm in Frequency Domain

In this section, the novel algorithm for image restoration in frequency domain is proposed. It is then proved that the new algorithm has many benefits over the image restoration in spatial domain. As stated above, SL0 is used to restore the image from small amount of observed information. It overcomes the classical image restoration in term of time and required information. However, in spatial domain, the information of an image is spread all over or it is not in a sparse form. Hence, the image is first transformed into frequency domain (by FFT in this case) where the sparse representation of image is obtained. According to image property: the Fourier Transform of image contains component of all frequencies, but most of the energy is concentrated in few transform coefficients and the other is almost zero. These frequency components are then employed by SL0 to recover the original image. In general, most of the image's structure can be restored by SL0. However, the detail and the smoothness of the image are stayed on high frequency components. The exact restoration of these components cannot be implemented by SL0. The situation is even worst in case of noise where SL0 algorithm may mistakenly detect noise as restoration signal. As a result, the novel regularization for SL0 algorithm in frequency domain is proposed to give a better performance for image restoration algorithm in CS.

4. PROPOSED PROBLEM DEFINITION AND FORMULATION

Tikhonov's Regularization Algorithm.

The difficulties of the ill-posed problem can be solved by regularization algorithms. In general, the prior information is added up to the process, so that the solution is put under some constraints. When the process is affected by noise, Eq. (1) is rewritten as follow:

$$y(i) = \Psi\alpha(i) + \varepsilon(i) \tag{9}$$

where $y(i)$ is a measured image or signal at time instant $(i), \alpha(i)$ is the unknown original signal or image at the same time, Φ is the measurement matrix and $\varepsilon(i)$ is the additional noise. The measurement matrix Φ is often ill-posed for the Cauchy problem of the Laplace equation, so regularization algorithm which regularizes the measurement matrix is essential. Instead of solving Eq. (9) directly, Tikhonov proposed a method to transform ill-posed problem into a well-posed one by solving the following problem:

$$minimize\|\alpha\|^2 subject\ to\ \|\Phi_i\alpha - y_i\|^2 \leq \varepsilon \quad (10)$$

where ε is the prescribed error tolerance. The proposed problem in Eq. (10) can be also resolved as:

$$\hat{\alpha} = \arg\min_{\hat{\alpha}}(\|y - \Phi\hat{\alpha}\|^2 + \Upsilon\|R\hat{\alpha}\|^2) \quad (11)$$

where Υ is the regularization parameter (Lagrange parameter).

According to [33], the regularization parameter Υ has strong effect to the inverse problem so the selection of it is very important. There are several ways to search for it but the selection is based on a priori properties of the system. In general, various properties are put under consideration: for example the additional noise in the system or the basis structure of the model. Normally, small values of ? direct the solution to the least square results of $\hat{\alpha} = (\Phi^T\Phi)^{-1}\Phi y$. This is also called as under-regularization which may cause ill-posed condition with the presence of outliers. Vice versa, large value of regularization parameter can lead the solution toward zero (over regularization). Therefore, an optimal choice of regularization parameter is required to balance accuracy and smoothness in the solution.

B. Proposed Regularization for SL0 Algorithm.

1) General Regularization for SL0 in Frequency Domain

As stated above, SL0 is an ill-posed problem for the under-determined cases therefore infinite number of solutions can be obtained. The solution for squared and over-undetermined cases is known as unstable and additional noise in the measurements produce large disturbance in the reconstructed image. The regularization algorithms compensate the disturbed information with some general prior information about the desire solution, it remove artifacts from the final answer and improve the rate of convergence. The recovery process of SL0 algorithm with regularization function is rewritten as the following minimization problems:

$$\alpha = \mathrm{Arg}\min\left\{\sum_{i=1}^{M}(\Phi_i\alpha - y_i) + \lambda\Upsilon(\alpha)\right\} \quad (12)$$

where $\Upsilon(\alpha)$ is the regularization function and λ is scalar defining the regularization parameter.

2) Laplacian Regularization for SL0 in Frequency Domain

In general, Tikhonov regularization $\Upsilon(\alpha)$ is a matrix recognition of the Laplacian kernel (the simple and small kernel). There are different sets of integer and kernel and this paper apply the simplest form of kernel in spatial domain:

$$\Gamma_{SD} = [0, 1, 0; 1, -4, 1 : 0, 1, 0] \quad (13)$$

Fourier transformation of filter is also executed as the image reconstruction process is taken under frequency domain. The most classical and simplest regularization norm function $\rho_{REG}(\cdot)$, is $\rho_{REG}(\cdot) = (\cdot)^2$ thus the solution of the SL0 is defined as:

$$\alpha = \mathrm{Arg}\min\left[\sum_{i=1}^{M}(\Phi_i\alpha - y_i) + \lambda\cdot(\Gamma_{FD}\alpha)^2\right] \quad (14)$$

Basically, the frequency domain filtering selects a transfer function that can modifies regularization function "(14)" in specific manner. Fig. 3 and Fig. 4 expresses the Laplacian regularization function in spatial domain and frequency domain respectively. Fig. 5 illustrates the image presentation in frequency domain. It is shown that the components that contain information are mainly located at low frequency.

In frequency domain, low frequencies present smooth areas while high frequencies show edges and noise. The frequency domain Laplacian regularization is actually used as filtering. It mainly allow the low frequency value while dismiss the high frequency one. Therefore when it is employed for SL0 algorithm, the reconstructed value is constrained and is more robust to the noise.

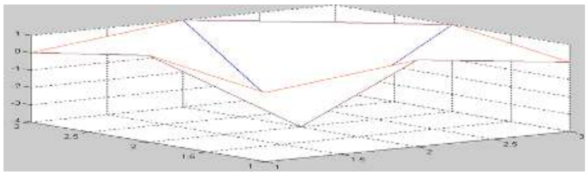


Fig.3: *Proposed Laplacian Regularized Kernal Function in Spatial Domain (Γ_{SD}).*

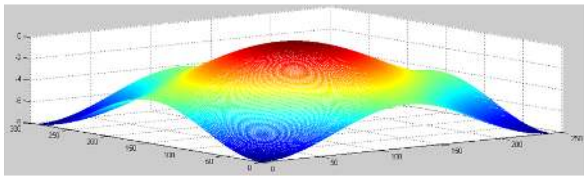


Fig.4: *Proposed Laplacian Regularized Kernal Function in Frequency Domain (Γ_{FD}).*

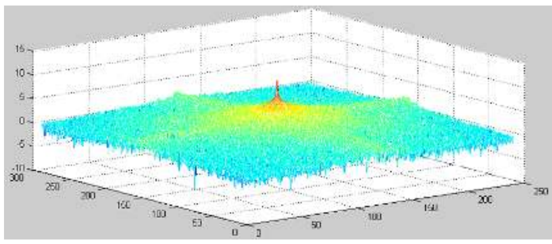


Fig.5: *Example of Reconstructed Image in Frequency Domain (α).*

Due to image characteristic, the classical regularized technique in spatial domain is Laplacian regularized kernel, which is expressed in Eq. (13). From Eq. (13) we can observed that the Laplacian kernel (Γ_{SD}) is one of the high pass filters in spatial domain (or edge amplification) therefore the frequency regularization Γ_{FD} which is used for cooperating the optimization problem can be formulated from the Laplacian regularized function.

The general framework for SL0 regularization algorithm is exclusively presented in Fig. 6.

5. NUMERICAL RESULTS AND DISCUSSION

This section presents the image reconstruction simulation which is obtained by SL0 and SL0 regularized algorithm. It is simulated in MATLAB for 3 images such as Lena, Cameraman and Sussie. Images are firstly converted to the Frequency Domain by a Fourier Transform. The additional noises are then randomly added to the system. There are 3 different types of noises model: Additive White Gaussian Noise (AWGN), Salt & Pepper Noise, and Speckle Noise. The power of each noise model is also varied from low to high.

There are only small numbers of components (Sparsity Number) are used for the reconstruction process. In the reconstruction process, image is reconstructed column by column. The Sparsity Number (K) is calculated as the percentage of total components at each column. Therefore, each column is reconstructed by both SL0 algorithms based on K. For example, assume that an image has size of 200×250 and K is 5%. K is first calculated as 5%*200=10; therefore each row is reconstructed by using only 10 mostly sparse components. As a result, the reconstructed image is a combination of the reconstruction of 250 rows. Table 1, 2 and 3 indicates the results in PSNR (dB) between the reconstructed image and original image. Due to page limitation, the illustrations of some reconstructed images (K =5%) are shown in Fig. 13, 14 and 15. At each figure, "a" represents the original image, "b" express the reconstructed image by SL0 and "c" shows the reconstructed image by our proposed SL0 regularization.

A. Experimental Estimation of Regularization Parameter

According to (14), Regularization Parameter (λ) is undetermined. In this section, the relationship between the PSNR of the reconstructed image and λ is demonstrated. The result is displayed in table in which: the first row present the values of λ and the first column represent the noise model. The value of λ is varied from 0.01 to 0.1, whereas the power of noise model is varied from low to high. There are 3 demonstration figures: Lena is shown in Fig. 7, Susie (40th frame) sequence is shown in Fig. 8 and Cameraman is shown in Fig. 9. At each table, the last row represents the average PSNR value for all different noise power. The best result for S is estimated as the one with highest averaged PSNR value. For the best chosen S, the illustration of restoration image is also presented for the lowest and highest noise power. Additionally, the graph in Fig. 10, 11 and 12 is exclusively present the average PSNR value for each type of noise. The row express the regularization parameter which is varied from 0.01 to 0.1 and the column is the average PSNR value. When the value of λ is varied, the average PSNR value tend to increase to the peak value before decreasing and saturating.

In theoretical point of view, λ can be set from 0-1. The higher the noise power is, the Regularization Parameter is set to larger and closer to 1. Vice versa the lower the noise power is, λ is set to lower and closer to 0. Following the observation from large amount of experimental, if the noise has low power, it is suggested to set λ=0.02 to obtain the best performance. Furthermore if the noise has high power, λ is set to 0.04 to obtain the best performance. According to Fig. 10, 11 and 12 the PSNR value for the reconstructed image is dropped and saturated when λ is greater than 0.05.

B. Performance Evaluation

In this section, the results from the simulation are analysed. It is shown that the result of our proposed regularization algorithms overcome original SL0 based algorithm in both conception and perception.

- According to Table 1, 2 and 3, the result in PSNR by the proposed SL0 regularization is much higher than by the original SL0 about 2-3 dB in all tested images. Moreover the illustration of the reconstructed image in Fig. 13, 14 and 15 give a much clearer vision compared to the original SL0.

- In order to virtually illustrate the proposed algorithm performance, an example of single column reconstruction from image is exclusively presented in Fig. 16. A 'log' function is executed before plotting to give a clearer view of vector representation. There are 2 columns: at each column, the original vector and the reconstructed vector by difference algorithms are presented. The column on the left hand side expresses the vectors in frequency domain. Meanwhile, the column on the right hand side conveys the vectors in spatial domain. It is shown in Fig. 16(b-1) and Fig. 16(b-2) that the original SL0 algorithm just recovers the sparse number while treats small number as zero. In fact, each small value has its own contribution for the image reconstruction process. Therefore classical SL0 algorithm does not give a clear view of reconstructed image. On the contrary, by using the proposed regularization SL0 algorithms (shown in Fig. 16(c-1) and Fig. 16(c-2)), all the component are constrained and recovered as close to the original value as possible. Thus, the reconstructed image by the proposed algorithm is much better than classical algorithm in both subjective and objective view.

It is shown that, a good reconstructed image is given by original SL0 algorithm under the noise free or noise with low power. However if the observed image is effected by the non-Gaussian noise model or Gaussian noise model with high power, the original SL0 fail to give an acceptance reconstruction image. The cause of this situation is in the recovery process where the error between the reconstruction image and the observed image is multiplied with the inverse of the matrix; this step is taken iteratively until the expected result is obtained. Nevertheless if the error is too big, the whole process may be diverted and the result is too much difference with the original one. The regularization filter is then added up to the original algorithm to constraint the error. As a result, the error is put under a boundary and the overall process is improved. The shape of reconstruction vector by regularization algorithms is much closed to the original vector compared to the non-regularized ones. The image illustration for the single column reconstructed is exclusively represented in Fig. 17 to illustrate the reconstruction process of each image.

6. CONCLUSIONS

In general, SL0 algorithm can be used to reconstruct the image based on small number of components based on Fourier Domain transformation. The quality of the reconstructed image is recognized for the noiseless environments but it is very poor under the noisy environment. In this paper we propose a novel regularization method for SL0 algorithm which is based on Tikhonov's regularization for ML estimation in frequency domain. The regularization function compromises fidelity to the available data to smoothness of the solution. From the simulation result, we can confirm that the frequency domain regularization technique evidently improve the SL0 performance. The proposed algorithm yields a better performance in both subjective and objective measurement.

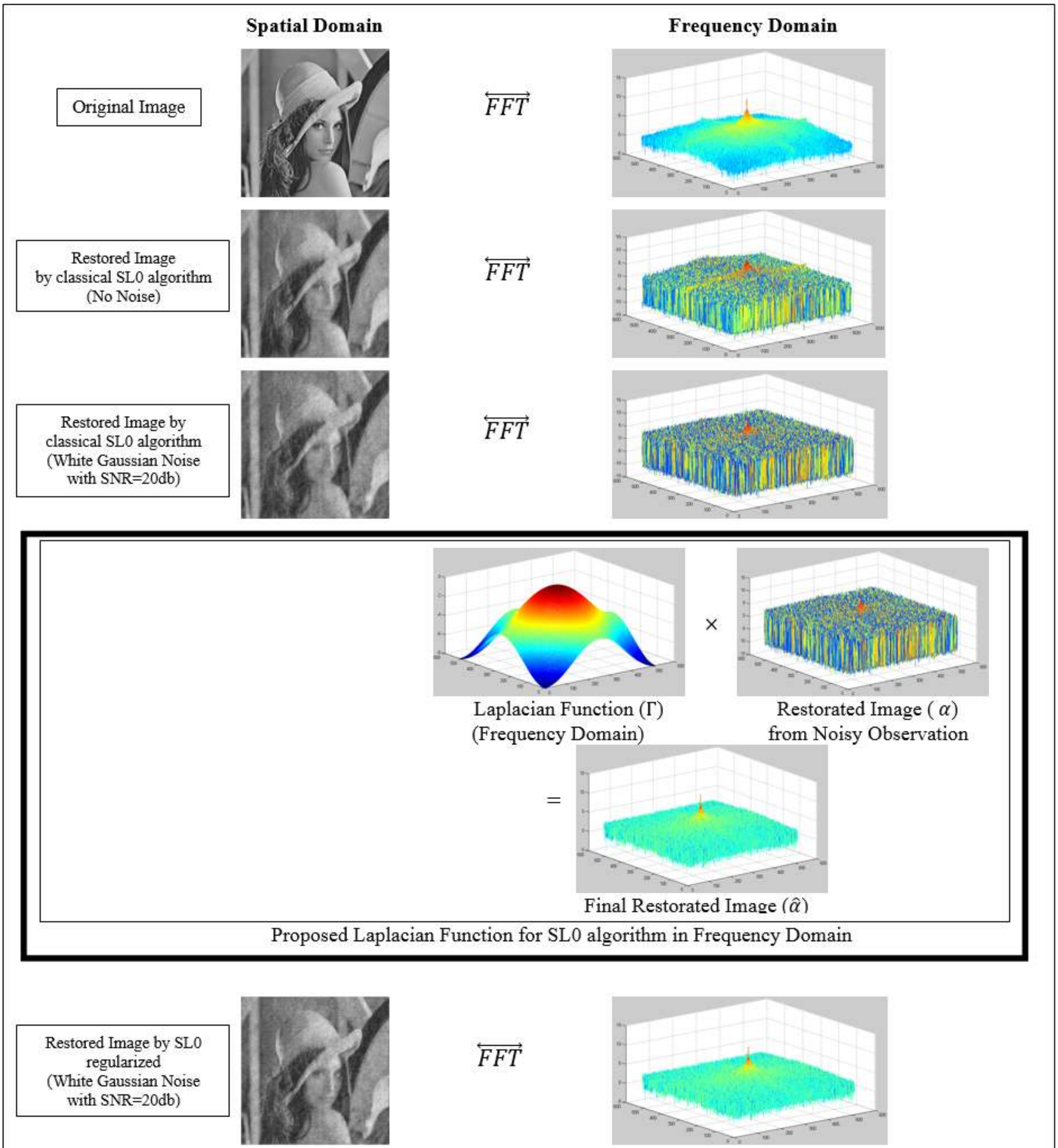
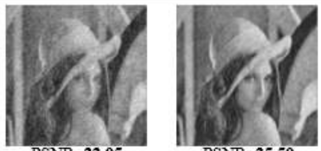


Figure 6. General Framework for SL0 Regularization Algorithm.

Sparse Number =5%

AWGN

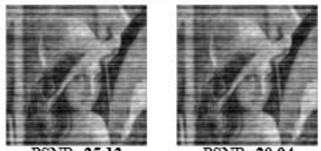
	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	21.85	22.66	**22.95**	20.55	5.73	5.70	5.68	5.68	5.68	5.69
17.5dB	22.49	23.18	**23.53**	23.32	5.86	5.69	5.68	5.68	5.69	5.69
20dB	23.33	23.94	**24.31**	24.35	5.97	5.71	5.70	5.69	5.69	5.69
22.5dB	23.95	24.44	**24.72**	24.77	6.40	5.75	5.72	5.71	5.71	5.71
25dB	24.88	25.36	**25.59**	25.68	7.23	5.76	5.71	5.71	5.70	5.71
Average	*23.30*	*23.92*	**24.22**	*23.73*	*6.24*	*5.72*	*5.70*	*5.70*	*5.70*	*5.70*



PSNR=22.95 PSNR=25.59

Salt&Pepper

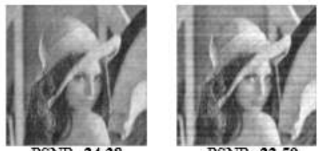
	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	24.33	24.77	24.99	**25.13**	22.98	7.57	5.90	5.72	5.68	5.68
V=0.02	24.72	25.16	25.37	**25.46**	16.77	6.57	5.82	5.70	5.68	5.68
V=0.03	23.65	24.07	24.29	**24.41**	10.64	5.86	5.70	5.70	5.71	5.70
V=0.04	22.02	22.41	22.62	**22.77**	7.67	5.80	5.71	5.70	5.70	5.70
V=0.05	20.48	20.79	20.90	**20.94**	15.52	6.49	5.79	5.70	5.68	5.67
Average	*23.68*	*24.10*	*24.32*	**24.44**	*14.51*	*6.45*	*5.78*	*5.70*	*5.69*	*5.69*



PSNR=25.13 PSNR=20.94

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	23.58	24.05	24.27	**24.38**	23.37	7.53	5.91	5.71	5.69	5.68
D=0.02	23.01	23.32	23.48	**23.57**	22.33	7.03	5.91	5.71	5.68	5.67
D=0.03	22.25	22.50	22.62	**22.69**	21.71	6.89	5.82	5.70	5.67	5.67
D=0.04	22.08	22.39	22.50	**22.51**	19.98	6.58	5.80	5.69	5.67	5.66
D=0.05	21.95	22.27	22.46	**22.59**	21.98	7.40	5.84	5.69	5.67	5.67
Average	*22.32*	*22.62*	*22.77*	**22.84**	*21.50*	*6.98*	*5.84*	*5.70*	*5.67*	*5.67*



PSNR=24.38 PSNR=22.59

Sparse Number =10%

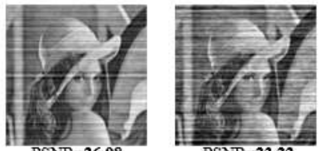
AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	21.25	22.23	**22.79**	7.62	5.69	5.70	5.70	5.70	5.70	5.71
17.5dB	22.91	23.87	**24.07**	10.52	5.70	5.70	5.70	5.70	5.70	5.70
20dB	24.78	25.70	**25.97**	14.00	5.71	5.68	5.70	5.69	5.69	5.69
22.5dB	26.14	26.96	**27.34**	19.14	5.75	5.68	5.68	5.68	5.69	5.69
25dB	27.51	28.17	**28.45**	24.44	5.96	5.73	5.73	5.71	5.71	5.71
Average	*24.52*	*25.39*	**25.72**	*15.14*	*5.76*	*5.70*	*5.70*	*5.69*	*5.70*	*5.70*

PSNR=22.79 PSNR=28.45

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	26.74	26.90	**26.98**	26.93	11.44	5.94	5.68	5.64	5.64	5.64
V=0.02	25.56	25.89	**26.10**	26.13	10.52	5.92	5.67	5.65	5.65	5.64
V=0.03	25.16	25.40	**25.53**	25.53	10.40	5.92	5.67	5.64	5.64	5.64
V=0.04	23.65	23.73	**23.66**	23.57	10.92	5.91	5.67	5.64	5.63	5.65
V=0.05	23.33	23.31	**23.22**	23.09	9.47	5.87	5.67	5.64	5.64	5.63
Average	*24.89*	*25.05*	**25.10**	*25.05*	*10.55*	*5.91*	*5.67*	*5.64*	*5.64*	*5.64*



PSNR=26.98 PSNR=23.22

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	28.37	28.73	**28.87**	28.67	7.90	5.79	5.69	5.68	5.69	5.69
D=0.02	27.50	27.94	**28.16**	28.14	8.05	5.77	5.69	5.71	5.70	5.70
D=0.03	17.93	17.68	**17.63**	17.64	6.52	5.74	5.70	5.70	5.69	5.71
D=0.04	21.56	21.84	**21.91**	21.86	6.47	5.73	5.71	5.70	5.70	5.70
D=0.05	16.42	16.33	**16.30**	16.23	6.11	5.73	5.70	5.70	5.70	5.69
Average	*22.36*	*22.50*	**22.57**	*22.51*	*7.01*	*5.75*	*5.70*	*5.70*	*5.70*	*5.70*

PSNR=28.87 PSNR=16.30

Sparse Number =15%

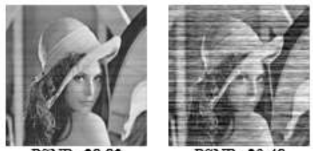
AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	26.74	26.90	**26.98**	26.93	11.44	5.94	5.68	5.64	5.64	5.64
17.5dB	25.56	25.89	**26.10**	26.13	10.52	5.92	5.67	5.65	5.65	5.64
20dB	25.16	25.40	**25.53**	25.53	10.40	5.92	5.67	5.64	5.64	5.64
22.5dB	23.65	23.73	**23.66**	23.57	10.92	5.91	5.67	5.64	5.63	5.65
25dB	23.33	23.31	**23.22**	23.09	9.47	5.87	5.67	5.64	5.64	5.63
Average	*24.89*	*25.05*	**25.10**	*25.05*	*10.55*	*5.91*	*5.67*	*5.64*	*5.64*	*5.64*

PSNR=26.98 PSNR=23.22

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	28.35	28.70	**28.83**	24.55	6.22	5.73	5.71	5.71	5.69	5.70
V=0.02	20.00	20.23	**20.41**	19.77	6.18	5.73	5.71	5.70	5.71	5.70
V=0.03	14.48	14.63	**14.86**	14.57	5.98	5.73	5.73	5.71	5.70	5.59
V=0.04	21.79	22.24	**22.50**	19.40	5.76	5.69	5.69	5.69	5.68	5.69
V=0.05	19.75	20.18	**20.48**	18.17	5.79	5.70	5.68	5.69	5.69	5.69
Average	*20.87*	*21.20*	**21.42**	*19.29*	*5.99*	*5.72*	*5.70*	*5.70*	*5.70*	*5.67*



PSNR=28.83 PSNR=20.48

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	29.43	29.73	**29.80**	29.33	7.33	5.76	5.70	5.70	5.70	5.69
D=0.02	27.32	27.66	**27.80**	27.54	8.19	5.80	5.69	5.68	5.69	5.71
D=0.03	23.86	24.08	**24.22**	24.15	7.97	5.75	5.70	5.70	5.71	5.69
D=0.04	23.28	23.12	**23.07**	22.88	6.54	5.73	5.71	5.71	5.70	5.69
D=0.05	23.14	22.98	**22.91**	22.70	7.20	5.73	5.69	5.70	5.70	5.75
Average	*25.40*	*25.51*	**25.56**	*25.32*	*7.44*	*5.75*	*5.70*	*5.70*	*5.70*	*5.71*

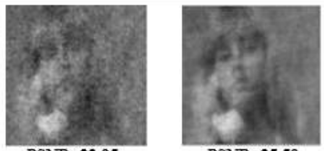
PSNR=29.80 PSNR=22.91

Figure 7. Experimental Estimation of Parameter Estimation for Lena (512 × 512)

Sparse Number =5%

AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	21.85	22.66	**22.95**	20.55	5.73	5.70	5.68	5.68	5.68	5.69
17.5dB	22.49	23.18	**23.53**	23.32	5.86	5.69	5.68	5.68	5.69	5.69
20dB	23.33	23.94	**24.31**	24.35	5.97	5.71	5.70	5.69	5.69	5.69
22.5dB	23.95	24.44	**24.72**	24.77	6.40	5.75	5.72	5.71	5.71	5.71
25dB	24.88	25.36	**25.59**	25.68	7.23	5.76	5.71	5.71	5.70	5.71
Average	23.30	23.92	**24.22**	23.73	6.24	5.72	5.70	5.70	5.70	5.70



PSNR=22.95 PSNR=25.59

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	24.33	24.77	24.99	**25.13**	22.98	7.57	5.90	5.72	5.68	5.68
V=0.02	24.72	25.16	25.37	**25.46**	16.77	6.57	5.82	5.70	5.68	5.68
V=0.03	23.65	24.07	24.29	**24.41**	10.64	5.86	5.70	5.70	5.71	5.70
V=0.04	22.02	22.41	22.62	**22.77**	7.67	5.80	5.71	5.70	5.70	5.70
V=0.05	20.48	20.79	20.90	**20.94**	15.52	6.49	5.79	5.70	5.68	5.67
Average	23.68	24.10	24.32	**24.44**	14.51	6.45	5.78	5.70	5.69	5.69

PSNR=25.13 PSNR=20.94

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	23.58	24.05	24.27	**24.38**	23.37	7.53	5.91	5.71	5.69	5.68
D=0.02	23.01	23.32	23.48	**23.57**	22.33	7.03	5.91	5.71	5.68	5.67
D=0.03	22.25	22.50	22.62	**22.69**	21.71	6.89	5.82	5.70	5.67	5.67
D=0.04	22.08	22.39	22.50	**22.51**	19.98	6.58	5.80	5.69	5.67	5.66
D=0.05	21.95	22.27	22.46	**22.59**	21.98	7.40	5.84	5.69	5.67	5.67
Average	22.32	22.62	22.77	**22.84**	21.50	6.98	5.84	5.70	5.67	5.67

PSNR=24.38 PSNR=22.59

Sparse Number=10%

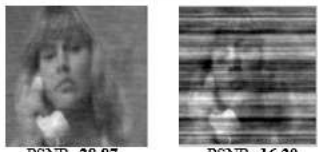
AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	21.25	22.23	**22.79**	7.62	5.69	5.70	5.70	5.70	5.70	5.71
17.5dB	22.91	23.87	**24.07**	10.52	5.70	5.70	5.70	5.70	5.70	5.70
20dB	24.78	25.70	**25.97**	14.00	5.71	5.68	5.70	5.69	5.69	5.69
22.5dB	26.14	26.96	**27.34**	19.14	5.75	5.68	5.68	5.68	5.69	5.69
25dB	27.51	28.17	**28.45**	24.44	5.96	5.73	5.73	5.71	5.71	5.71
Average	24.52	25.39	**25.72**	15.14	5.76	5.70	5.70	5.69	5.70	5.70

PSNR=22.79 PSNR=28.45

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	28.37	28.73	**28.87**	28.67	7.90	5.79	5.69	5.68	5.69	5.69
V=0.02	27.50	27.94	**28.16**	28.14	8.05	5.77	5.69	5.71	5.70	5.70
V=0.03	17.93	17.68	**17.63**	17.64	6.52	5.74	5.70	5.70	5.69	5.71
V=0.04	21.56	21.84	**21.91**	21.86	6.47	5.73	5.71	5.70	5.70	5.70
V=0.05	16.42	16.33	**16.30**	16.23	6.11	5.73	5.70	5.70	5.70	5.69
Average	22.36	22.50	**22.57**	22.51	7.01	5.75	5.70	5.70	5.70	5.70



PSNR=28.87 PSNR=16.30

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	26.74	26.90	**26.98**	26.93	11.44	5.94	5.68	5.64	5.64	5.64
D=0.02	25.56	25.89	**26.10**	26.13	10.52	5.92	5.67	5.65	5.65	5.64
D=0.03	25.16	25.40	**25.53**	25.53	10.40	5.92	5.67	5.64	5.64	5.64
D=0.04	23.65	23.73	**23.66**	23.57	10.92	5.91	5.67	5.64	5.63	5.65
D=0.05	23.33	23.31	**23.22**	23.09	9.47	5.87	5.67	5.64	5.64	5.63
Average	24.89	25.05	**25.10**	25.05	10.55	5.91	5.67	5.64	5.64	5.64

PSNR=26.98 PSNR=23.22

Sparse Number=15%

AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	18.98	20.06	**20.58**	5.91	5.69	5.70	5.69	5.69	5.69	5.68
17.5dB	21.33	22.36	**22.88**	5.98	5.69	5.69	5.70	5.71	5.70	5.69
20dB	23.51	24.53	**24.79**	6.29	5.70	5.70	5.69	5.69	5.69	5.70
22.5dB	25.56	26.54	**26.65**	7.39	5.69	5.67	5.68	5.68	5.69	5.68
25dB	27.66	28.53	**28.70**	10.42	5.71	5.68	5.68	5.68	5.68	5.68
Average	23.41	24.40	**24.72**	7.20	5.70	5.69	5.69	5.69	5.69	5.69

PSNR=20.58 PSNR=28.70

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	28.35	28.70	**28.83**	24.55	6.22	5.73	5.71	5.71	5.69	5.70
V=0.02	20.00	20.23	**20.41**	19.77	6.18	5.73	5.71	5.70	5.71	5.70
V=0.03	14.48	14.63	**14.86**	14.57	5.98	5.73	5.73	5.71	5.70	5.59
V=0.04	21.79	22.24	**22.50**	19.40	5.76	5.69	5.69	5.69	5.68	5.69
V=0.05	19.75	20.18	**20.48**	18.17	5.79	5.70	5.68	5.69	5.69	5.69
Average	20.87	21.20	**21.42**	19.29	5.99	5.72	5.70	5.70	5.70	5.67

PSNR=28.83 PSNR=20.48

Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	29.43	29.73	**29.80**	29.33	7.33	5.76	5.70	5.70	5.70	5.69
D=0.02	27.32	27.66	**27.80**	27.54	8.19	5.80	5.69	5.68	5.69	5.71
D=0.03	23.86	24.08	**24.22**	24.15	7.97	5.75	5.70	5.70	5.71	5.69
D=0.04	23.28	23.12	**23.07**	22.88	6.54	5.73	5.71	5.71	5.70	5.69
D=0.05	23.14	22.98	**22.91**	22.70	7.20	5.73	5.69	5.70	5.70	5.75
Average	25.40	25.51	**25.56**	25.32	7.44	5.75	5.70	5.70	5.70	5.71

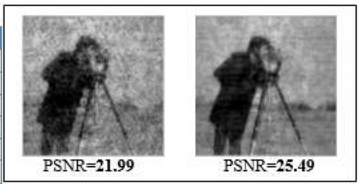
PSNR=29.80 PSNR=22.91

Figure 8. Experimental Estimation of Parameter Estimation for Sussie (160 × 160)

Sparse Number=5%

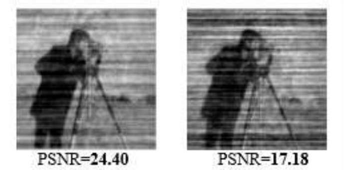
AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	20.65	21.54	**21.99**	18.29	5.47	5.45	5.45	5.44	5.45	5.45
17.5dB	22.01	22.84	**23.15**	22.32	5.53	5.44	5.45	5.44	5.45	5.45
20dB	23.07	23.78	**24.09**	23.86	5.60	5.45	5.45	5.44	5.44	5.44
22.5dB	23.74	24.40	**24.78**	24.79	6.09	5.46	5.44	5.45	5.45	5.45
25dB	24.62	25.19	**25.49**	25.51	6.67	5.48	5.45	5.45	5.45	5.45
Average	*22.82*	*23.55*	**23.90**	*22.95*	*5.87*	*5.46*	*5.45*	*5.45*	*5.45*	*5.45*



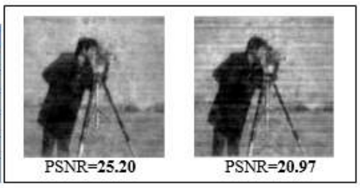
Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	23.57	24.02	24.25	24.36	**24.40**	23.66	13.13	6.52	5.65	5.49
V=0.02	22.77	23.12	23.27	23.36	**23.41**	22.66	12.48	6.55	5.55	5.46
V=0.03	19.42	19.45	19.38	19.34	**19.29**	18.78	11.29	6.11	5.45	5.44
V=0.04	17.01	17.05	17.00	16.94	**16.91**	16.71	10.22	5.79	5.45	5.44
V=0.05	16.54	16.83	17.00	17.11	**17.18**	16.67	9.94	5.73	5.45	5.43
Average	*19.86*	*20.10*	*20.18*	*20.22*	**20.24**	*19.70*	*11.41*	*6.14*	*5.51*	*5.45*



Speckle

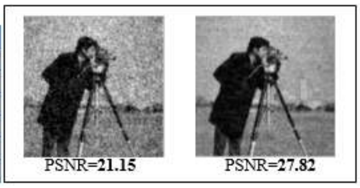
	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	24.47	24.89	25.09	25.18	**25.20**	24.28	12.22	6.27	5.61	5.48
D=0.02	23.89	24.24	24.37	24.40	**24.37**	23.24	12.47	6.58	5.65	5.47
D=0.03	22.54	22.74	22.82	22.85	**22.85**	22.20	11.87	6.24	5.60	5.46
D=0.04	22.08	22.13	22.08	22.03	**21.98**	21.54	9.80	5.98	5.54	5.46
D=0.05	20.68	20.87	20.93	20.96	**20.97**	20.33	11.12	6.24	5.60	5.45
Average	*22.73*	*22.97*	*23.06*	23.08	**23.08**	*22.32*	*11.50*	*6.26*	*5.60*	*5.46*



Sparse Number=10%

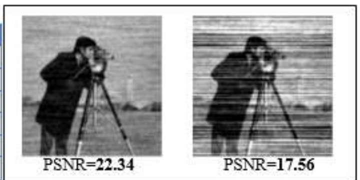
AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
15dB	19.43	20.47	**21.15**	7.16	5.45	5.45	5.45	5.45	5.44	5.45
17.5dB	21.51	22.55	**23.17**	8.19	5.45	5.45	5.45	5.45	5.44	5.45
20dB	23.42	24.46	**24.78**	12.01	5.46	5.45	5.45	5.45	5.45	5.45
22.5dB	25.15	26.13	**26.40**	17.83	5.48	5.45	5.45	5.45	5.45	5.45
25dB	26.64	27.51	**27.82**	21.80	5.50	5.45	5.45	5.44	5.44	5.44
Average	*23.23*	*24.22*	**24.67**	*13.40*	*5.47*	*5.45*	*5.45*	*5.45*	*5.45*	*5.45*



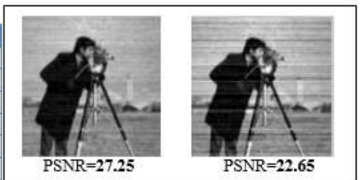
Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	21.94	22.17	22.27	22.35	**22.34**	7.50	5.61	5.47	5.45	5.45
V=0.02	21.35	21.50	21.57	21.63	**21.68**	7.88	5.60	5.47	5.43	5.48
V=0.03	20.61	20.85	20.93	20.98	**21.01**	8.35	5.66	5.47	5.44	5.44
V=0.04	17.52	17.48	17.48	17.53	**17.56**	8.43	5.67	5.46	5.45	5.44
V=0.05	16.36	16.29	16.25	16.26	**16.29**	7.81	5.62	5.46	5.43	5.44
Average	*18.96*	*19.03*	*19.06*	*19.10*	**19.14**	*8.12*	*5.64*	*5.46*	*5.44*	*5.45*



Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	27.13	27.26	**27.25**	27.20	27.08	7.75	5.60	5.46	5.45	5.45
D=0.02	24.91	25.12	**25.19**	25.22	25.19	7.87	5.59	5.47	5.45	5.45
D=0.03	23.32	23.32	**23.26**	23.20	23.12	8.36	5.66	5.47	5.44	5.46
D=0.04	22.82	22.99	**23.04**	23.05	23.05	7.24	5.57	5.46	5.45	5.44
D=0.05	22.65	22.64	**22.65**	22.65	22.63	7.42	5.57	5.46	5.45	5.45
Average	*24.17*	*24.27*	**24.28**	*24.26*	*24.21*	*7.73*	*5.60*	*5.47*	*5.45*	*5.45*



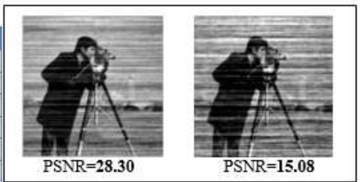
Sparse Number=15%

AWGN

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	S=0.1
15dB	16.85	17.83	**17.85**	5.50	5.45	5.45	5.45	5.45	5.45	5.45
17.5dB	19.15	20.20	**20.57**	5.61	5.45	5.45	5.45	5.45	5.44	5.44
20dB	21.45	22.52	**22.99**	5.80	5.45	5.45	5.45	5.45	5.44	5.45
22.5dB	23.68	24.75	**25.06**	5.99	5.45	5.45	5.45	5.45	5.45	5.45
25dB	25.78	26.86	**27.06**	8.22	5.46	5.44	5.45	5.45	5.45	5.45
Average	*21.38*	*22.43*	**22.71**	*6.22*	*5.45*	*5.45*	*5.45*	*5.45*	*5.45*	*5.45*

Salt&Pepper

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
V=0.01	28.27	28.34	**28.30**	28.25	27.81	5.65	5.46	5.45	5.45	5.45
V=0.02	17.64	17.81	**18.01**	18.23	18.12	5.76	5.47	5.45	5.44	5.44
V=0.03	16.84	16.92	**17.04**	16.85	5.46	5.09	5.06	5.07	5.06	5.07
V=0.04	16.65	16.78	**16.95**	17.16	16.07	5.54	5.46	5.45	5.47	5.44
V=0.05	14.50	14.79	**15.08**	15.35	14.87	5.59	5.46	5.46	5.43	5.45
Average	*19.26*	*19.26*	**19.59**	*19.59*	*19.22*	*19.22*	*5.46*	*5.46*	*5.45*	*5.45*



Speckle

	λ=0.01	λ=0.02	λ=0.03	λ=0.04	λ=0.05	λ=0.06	λ=0.07	λ=0.08	λ=0.09	λ=0.1
D=0.01	25.53	25.63	25.70	**25.76**	25.59	5.56	5.46	5.45	5.45	5.45
D=0.02	25.34	25.38	25.22	**24.80**	6.58	5.08	5.06	5.06	5.06	5.09
D=0.03	23.45	23.52	23.57	**23.62**	23.35	5.53	5.46	5.45	5.45	5.43
D=0.04	21.91	21.94	22.04	**22.15**	22.10	5.56	5.46	5.45	5.45	5.44
D=0.05	20.46	20.49	20.54	**20.60**	20.56	5.56	5.46	5.45	5.44	5.46
Average	*22.84*	*22.90*	*22.96*	**23.03**	*22.90*	*5.55*	*5.46*	*5.45*	*5.45*	*5.45*

Figure 9. Experimental Estimation of Parameter Estimation for Cameraman (256 × 256)

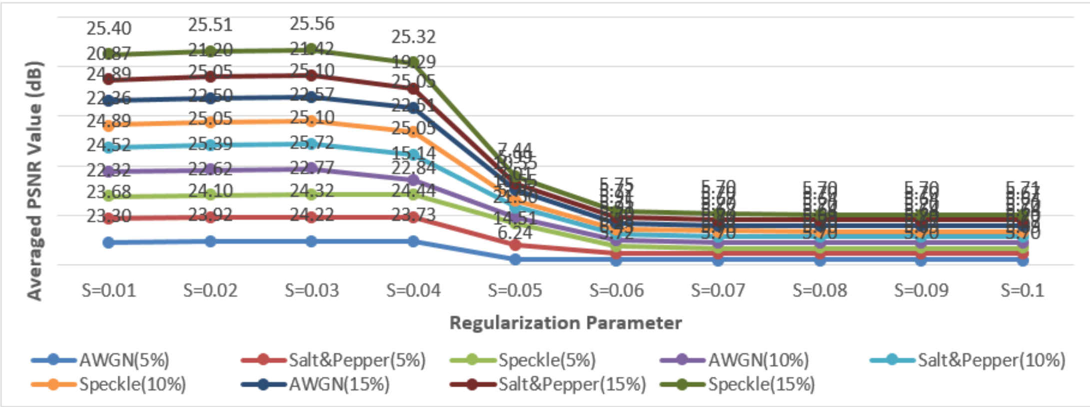


Figure 10. Regularized Parameter Estimation for Lena (512 × 512)

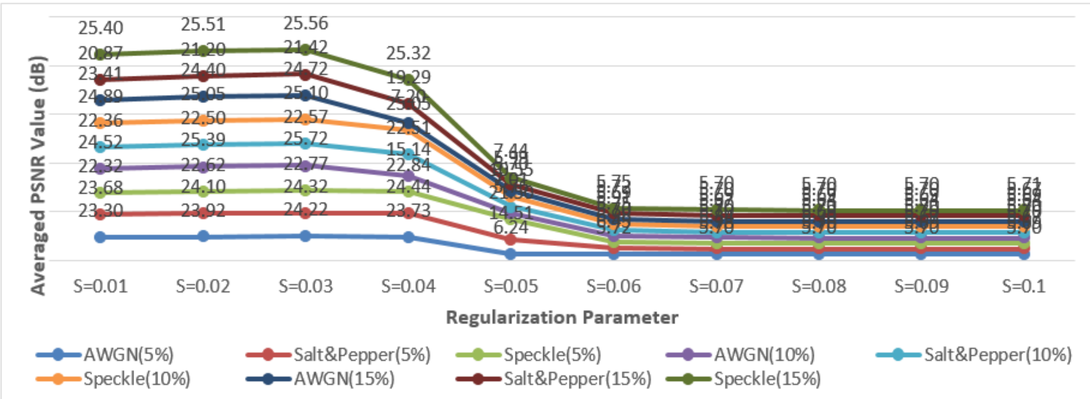


Figure 11. Regularized Parameter Estimation for Sussie (160× 160)

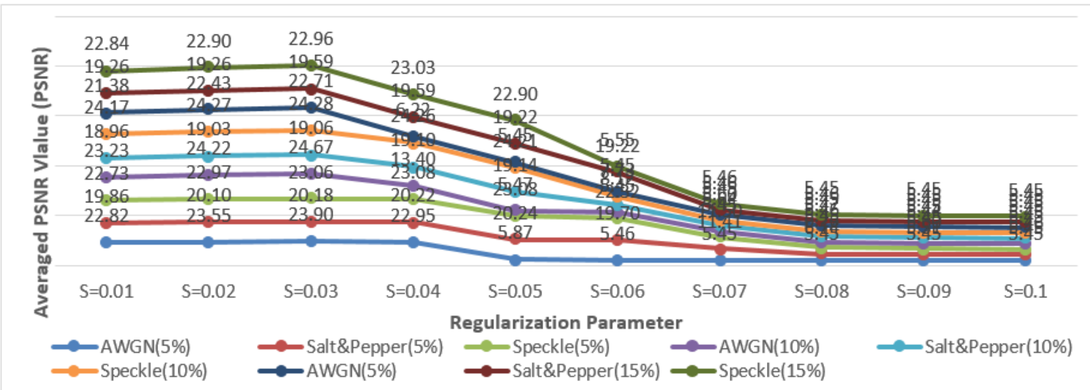


Figure 12. Regularized Parameter Estimation for Cameraman (256× 256)

Table 1: *Experimental Result of Lena.*

Noise Model		K=5%		K=10%		K=15%	
		SL0	SL0 Regularization	SL0	SL0 Regularization	SL0	SL0 Regularization
AWGN	15dB	17.75	20.79	16.13	19.52	14.11	16.46
	17.5 dB	19.38	23.50	18.28	21.17	16.26	16.92
	20 dB	20.94	24.63	20.41	23.62	18.61	19.25
	22.5 dB	22.05	25.51	22.46	25.13	20.78	21.43
	25 dB	22.78	25.76	24.28	26.80	25.18	26.81
Salt &Pepper	0.01	22.88	25.20	25.15	26.90	24.83	26.50
	0.02	21.79	24.16	22.59	24.32	15.90	17.79
	0.03	15.88	17.65	19.33	21.07	15.34	17.13
	0.04	14.80	17.10	15.84	17.29	14.14	15.70
	0.05	14.54	16.20	13.92	15.68	11.73	12.68
Speckle	0.01	23.44	25.15	26.94	28.19	26.46	27.50
	0.02	22.16	23.95	23.80	24.63	24.90	25.53
	0.03	21.58	23.18	22.19	23.17	22.40	23.12
	0.04	21.31	23.16	21.90	22.70	21.03	22.36
	0.05	20.63	22.19	20.87	21.81	20.94	21.99
Poisson		23.86	26.32	28.76	30.32	30.22	31.75

Table 2: *Experimental Result of Cameraman.*

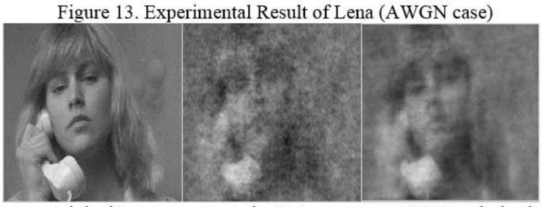
Noise Model		K=5%		K=10%		K=15%	
		SL0	SL0 Regularization	SL0	SL0 Regularization	SL0	SL0 Regularization
AWGN	15dB	15.20	18.61	14.48	17.40	12.98	14.96
	17.5 dB	16.66	19.77	16.36	18.00	14.91	17.03
	20 dB	17.57	20.22	18.07	20.87	17.29	19.47
	22.5 dB	18.01	20.59	19.85	21.02	19.07	21.22
	25 dB	18.78	21.21	21.08	23.09	21.09	23.14
Salt &Pepper	0.01	17.28	18.32	20.76	22.49	17.31	18.66
	0.02	16.49	18.05	15.91	17.72	14.05	15.83
	0.03	13.18	14.15	14.66	16.55	13.56	15.36
	0.04	12.50	13.78	14.60	16.33	13.49	14.42
	0.05	10.92	12.06	12.30	13.76	10.45	11.62
Speckle	0.01	18.64	20.56	22.10	23.14	24.19	25.23
	0.02	18.49	20.17	20.77	22.25	22.35	23.58
	0.03	18.12	19.75	20.52	21.47	21.14	22.19
	0.04	17.78	19.15	20.01	21.18	19.59	20.57
	0.05	17.30	18.64	19.51	20.82	18.70	19.58
Poisson		19.57	21.39	23.15	24.55	25.11	26.80

Table 3: *Experimental Result of Resolution Chart.*

Noise Model		K=5%		K=10%		K=15%	
		SL0	SL0 Regularization	SL0	SL0 Regularization	SL0	SL0 Regularization
AWGN	15dB	9.64	12.67	11.32	14.16	10.96	13.26
	17.5 dB	10.22	12.77	12.96	15.41	13.13	15.14
	20 dB	11.06	13.65	14.20	16.40	14.92	16.84
	22.5 dB	11.14	13.38	15.23	17.28	16.93	18.75
	25 dB	11.22	13.61	16.25	17.98	18.43	20.02
Salt &Pepper	0.01	11.18	13.29	16.65	17.88	19.00	19.83
	0.02	10.65	12.99	12.80	14.44	17.40	18.24
	0.03	10.41	12.83	11.15	12.90	12.20	13.26
	0.04	8.83	10.68	10.44	12.26	11.09	12.02
	0.05	7.71	9.16	9.83	11.29	10.88	11.90
Speckle	0.01	11.40	13.37	17.61	19.05	21.96	23.05
	0.02	11.28	13.73	17.01	18.58	21.03	22.20
	0.03	11.16	13.54	16.31	17.83	20.39	21.64
	0.04	11.45	13.43	15.90	17.79	19.62	20.58
	0.05	11.31	13.92	15.65	17.28	19.07	20.18
Poisson		11.61	13.77	16.94	18.09	21.47	22.33

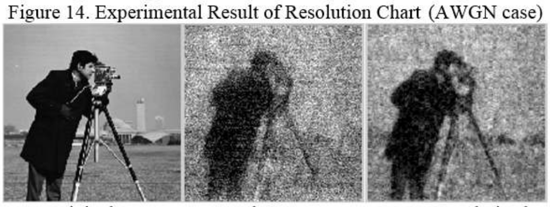
a. Original Image b. SL0 c. SL0 Regularized

Figure 13. Experimental Result of Lena (AWGN case)



a. Original Image b. SL0 c. SL0 Regularized

Figure 14. Experimental Result of Resolution Chart (AWGN case)



a. Original Image b. SL0 c. SL0 Regularized

Figure 15. Experimental Result of Cameraman (AWGN case)

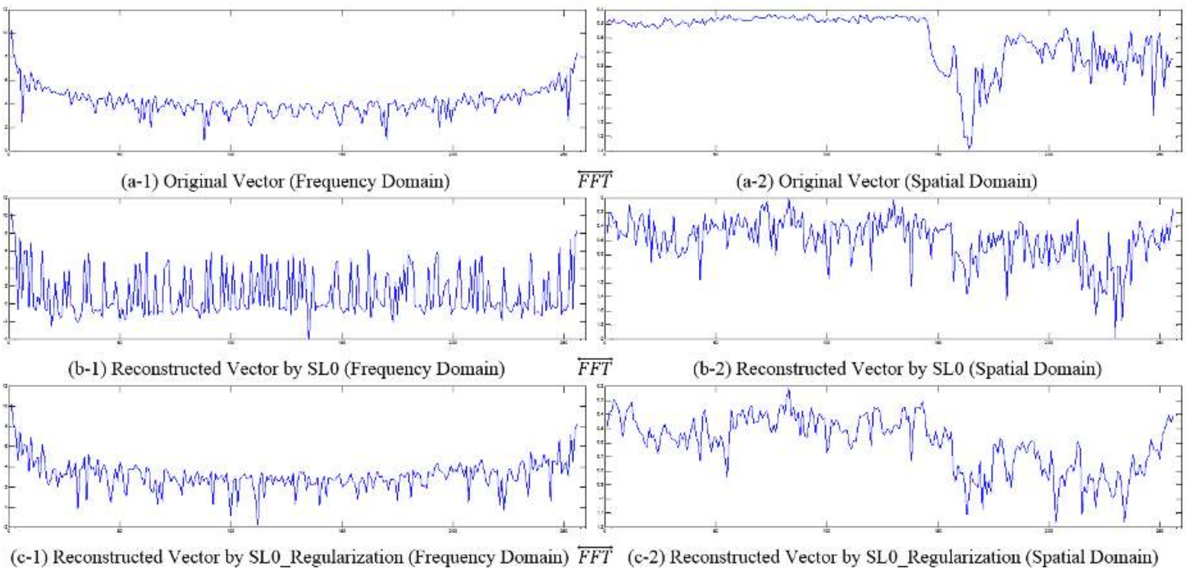


Figure 16. Vector Representation of Cameraman Figure in Frequency Domain (Left) and Spatial Domain (Right).

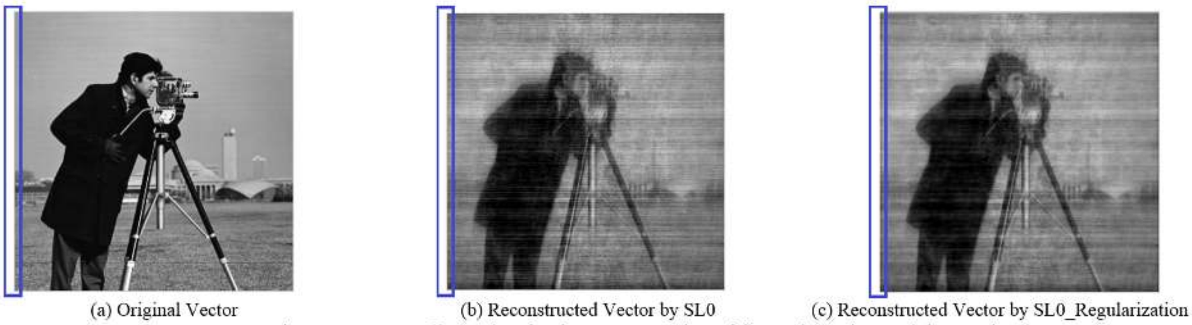


Figure 17. Image Illustration for the reconstruction of first column in Spatial Domain.

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