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A Novel Approach for Minimizing Processing Times of Three-Stage Flow Shop Scheduling Problems under Fuzziness

Alhanouf Alburaikan [1], Harish Garg [2,3,4,*] and Hamiden Abd El-Wahed Khalifa [1,5]

1 Department of Mathematics, College of Science and Arts, Qassim University, Al-Badaya 51951, Saudi Arabia
2 School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala 147004, Punjab, India
3 Department of Mathematics, Graphic Era Deemed to be University, Dehradun 248002, Uttarakhand, India
4 Applied Science Research Center, Applied Science Private University, Amman 11931, Jordan
5 Department of Operations and Management Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt
* Correspondence: harishg58iitr@gmail.com; Tel.: +91-86990-31147

Abstract: The purpose of this research is to investigate a novel three-stage flow shop scheduling problem with an ambiguous processing time. The uncertain information is characterized by Pentagonal fuzzy numbers. To solve the problem, in this paper, two different strategies are proposed; one relies on the idea of a ranking function, and the other on the close interval approximation of the pentagonal fuzzy number. For persons that need to be more specific in their requirements, the close interval approximation of the Pentagonal fuzzy number is judged to be the best appropriate approximation interval. Regarding the rental cost specification, these methods are used to reduce the rental cost for the concerned devices. In addition, a comparison of our suggested approach's computed total processing time, total machine rental cost, and machine idle time to the existing approach is introduced. A numerical example is shown to clarify the benefits of the two strategies and to help the readers understand it better.

Keywords: flow shop scheduling problem; processing time; rental policy; utilization time; pentagonal fuzzy number; ranking function; close-interval approximation

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1. Introduction

A scheduling problem is one that involves locating an ideal, or nearly ideal, timetable while taking into account various constraints. Different techniques to solving the scheduling problem, which aims to establish the order of processing work on a given set of machines, were explored. It is important and relevant to what the industry has marketed. Scheduling calls for a range of actions in order to complete a specific task within the allotted time and budget. The flow shop is the most traditional settings for manufacturing and study in the scheduling literature. One of the early results in flow shop scheduling theory was presented by Johnson [1], who presented an algorithm for scheduling jobs in a two- or three-machine flow shop to minimize the time at which all jobs are completed. The process of planning, organizing jobs and their flow through the production method plays a crucial role in every contemporary manufacturing system. The n-job on m-machine scheduling problem is known as the flow shop scheduling problem (FSSP). Each machine can only handle one job at a time, and each job can only be completed on one machine at a time. All jobs enter through all machines in the same order. Numerous scholars have conducted research in this area [2–7]. Ueno et al. [8] investigated FSSP with multi-stages in relation to the steel works. Yuan et al. [9] developed a mathematical theory and technique to address the specific blocking constraint in the two-stage flow shop group scheduling problem. Vahedi-Nouri et al. [10] proposed a fresh flow shop method to reduce the mean flow rate. Ren et al. [11] conducted research on the flow shop process to reduce publication time.

Two flow shop machines were represented by a model by Laribi et al. in [12]. Yazdani and Naderi [13] used mixed integer linear programming to formulate a scheduling problem. Qu et al. [14] used a hormone modulation approach for no wait flow shop scheduling. The fuzzy number is said to indicate a range of potential values rather than just one particular value [15]. The membership function, which has weights ranging from zero to one for each potential value, is used in membership functions. Numerous types of fuzzy numbers have been studied in the previous literature, including the triangular fuzzy number [16], the trapezoidal fuzzy number [17], the pentagonal fuzzy number (PFN) [18,19], etc. based on data from several observations and flaws in measuring processes, equipment faultiness, etc. Assume we are taking temperature and humidity readings. The temperature is approximately 30 °C when the humidity in the air is typical, meaning that neither a temperature less than 30 °C nor a temperature greater than 30 °C is ideal. As a result, temperature variation will also be influenced by the amount of humidity. It is a widespread phenomenon. The PFN results from this idea of variation. A PFN is often a five-tuple subset of a real number R with five parameters. Pathinathan and Ponnivalavan introduced PFNs for the first time in [20]. The fundamental ideas of PFN were created by Rajkumar and Pathinathan in [21]. Prameela and Kumar [22] looked at a PFN application to ascertain a queuing model's performance metrics. Although the processing time knowledge is not deterministic, we frequently utilize fuzzy numbers to describe it. Numerous researchers have researched fuzzy FSSP.

According to MacCahone and Lee [23], the job sequencing affecting the fuzzy processing time (FPT) was handled. An algorithm for calculating the minimization of the payment cost of machines with triangular FPT was processed by Sathish and Ganesan [24]. For a machine scheduling problem, Khalifa [25] considered the different due dates with a fuzzy sense. The FSSP was solved by Khalifa et al. [26] assuming that the due dates were ambiguous. Alharbi and Khalifa created an FSSP in [27] by employing processing time as PFNs. The bi-objective joint optimization of both preventative and corrective maintenance costs in assembly permutation flow shop scheduling was given by Zhang et al. [28]. Using effective scheduling, Jabbari et al. [29] established a mathematical approach to reduce the time needed to complete all goods (makespan). Flexible job shop scheduling with type-2 fuzzy processing time was presented by Li et al. [30]. Zhou et al. [31] looked at the structure of n-job flow shop scheduling with fuzzy piecewise quadratic processing times and three machines. Some of the advanced methods to examine the FSP are listed in Ren et al. [32]; Wang et al. [33,34]; Jemmai and Hidri [35]; Wang Gai- Ge et al. [36], and Kou-lamas and Kyparisis [37].

In this study, a novel three-stage flow shop scheduling problem with an uncertain processing time was investigated. Pentagonal fuzzy numbers were used to describe the uncertain information. In this study, two different approaches to solving the problem were put forth; one was based on the concept of a ranking function, while the other was based on the near interval approximation of the pentagonal fuzzy number. The study's primary contributions and novelties were as follows:

(1) Introducing suitable terminologies and measures that consider the properties of a possible optimal scheduling;
(2) Defining two methods for determining the best schedule, one based on the ordering of pentagonal fuzzy numbers and the other on PFN interval confidence;
(3) Interacting the analyst with the DM to arrive to the optimal sequence.

The main objectives of the proposed study were:

(1) To minimize the fuzzy professing time of the machines subject to the rental policy;
(2) To study the inclusive study of pentagonal fuzzy numbers in the scheduling problem;
(3) To specific the concept of the optimal scheduling for the scenario;
(4) To validate the proposed study with the support of illustrative example.

The remainder of this study is structured as follows: Section 2 has discussed several fundamental ideas. Some of the necessary presumptions and concepts are explored in

Section 3 in relation to the topic being studied. In Section 5, a solution method is presented. A numerical example is provided as an illustration in Section 6. A brief comparison with a few current approaches is offered in Section 7. Conclusions and upcoming research are reported in Section 8.

2. Preliminaries

Some fundamental definitions of PFNs, their arithmetic operations, ranking function, and interval confidence are recalled in this section.

Definition 1. *(Zadeh [15]). A fuzzy set $\widetilde{A}$ defined on the set of $\mathbb{R}$ is said to be fuzzy membership function $\mu_{\tilde{A}}(x) : \mathbb{R} \to [0,1]$, have the following properties:*

(1) $\boldsymbol{\mu_{\tilde{A}}(x)}$ *is an upper semi—continuous membership function;*

(2) $\tilde{\boldsymbol{A}}$ *is convex fuzzy set, i.e.,* $\boldsymbol{\mu_{\tilde{A}}(w\,x + (1-w)\,y) \geq min\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\right\}}$ *for all* $\boldsymbol{x, y} \in \mathbb{R}$; $\boldsymbol{0 \leq w \leq 1}$; $\tilde{\boldsymbol{A}}$ *is normal, i.e.,* $\exists\, \boldsymbol{x_0} \in \mathbb{R}$ *for which* $\boldsymbol{\mu_{\tilde{A}}(x_0) = 1}$;

(3) $Supp\ (\widetilde{A}) = \left\{x \in \mathbb{R} : \mu_{\tilde{A}}(x) > 0\right\}$ *is the support of* $\widetilde{A}$*, and the closurecl*$(Supp(\widetilde{A}))$ *is compact set.*

Definition 2. *(Panda and Pal [18,19]). A fuzzy number $\widetilde{A}_P$ is a pentagonal fuzzy number (PFN), represented by $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are reals. The middle point a_3 possesses the degree of membership 1 and w, v are the respective membership values of points a_2, a_4. The membership function $\mu_{\tilde{A}_P}(x)$ for PFN is written as follows (Figure 1):*

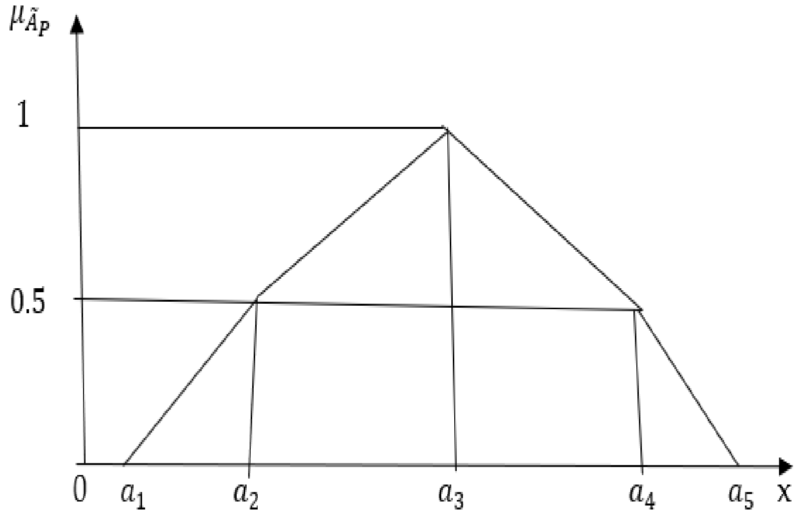

Figure 1. Graphical Representation of a symmetric PFN [18].

$$\mu_{\tilde{A}_P}(x, w, v) = \begin{cases} 0, & x < a_1 \\ w\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1-(1-w)\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 1, & x = a_3 \\ 1-(1-v)\frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ v\frac{x-a_5}{a_4-a_5}, & a_4 \leq x \leq a_5 \\ 0, & x \geq a_5 \end{cases} \tag{1}$$

Remark 1. *The other fuzzy numbers, specifically the triangular and trapezoidal fuzzy numbers, are regarded as a special issue for a given PFN and are listed as follows:*
Case I *(Panda and Pal [18]).*

For $w = v = 0$, the PFN converts to triangular fuzzy number i.e., $(a_1,\ a_2, a_3, a_4, a_5) \cong (a_2, a_3, a_4)$ and the membership function becomes:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_2 \\ 1 - \frac{a_2 - x}{a_2 - a_3}, & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ 1 - \frac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & x \ge a_4 \end{cases} \tag{2}$$

Case II *(Panda and Pal [18]).*

For $w = v = 1$, the PFN converts to trapezoidal fuzzy number i.e., $(a_1,\ a_2, a_3, a_4, a_5) \cong (a_1,\ a_2, a_4, a_5)$ and the membership function becomes:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_4 \\ \frac{a_4 - x}{a_5 - a_4}, & a_4 \le x \le a_5 \\ 0, & x \ge a_5 \end{cases} \tag{3}$$

Definition 3. *(Panda and Pal [18]). Let $\boldsymbol{A} = (\boldsymbol{a_1},\ \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4}, \boldsymbol{a_5})$ and $\boldsymbol{B} = (\boldsymbol{b_1},\ \boldsymbol{b_2}, \boldsymbol{b_3}, \boldsymbol{b_4}, \boldsymbol{b_5})$ be two PFNs. All PFN is connected with two weights $\boldsymbol{w}, \boldsymbol{v}$. To avoid misunderstanding, we will use the notations $\boldsymbol{w_1}$, $\boldsymbol{v_1}$ to represent the weights of $\widetilde{\boldsymbol{A}}_{\boldsymbol{P}}$, and $\boldsymbol{w_2}$, $\boldsymbol{v_2}$ to represent the weights of $\widetilde{\boldsymbol{B}}_{\boldsymbol{P}}$. The arithmetic operations of PFN can be defined as follows:*

Addition: $\boldsymbol{A} \oplus \boldsymbol{B} = (\boldsymbol{a_1} + \boldsymbol{b_1},\ \boldsymbol{a_2} + \boldsymbol{b_2}, \boldsymbol{a_3} + \boldsymbol{b_3}, \boldsymbol{a_4} + \boldsymbol{b_4}, \boldsymbol{a_5} + \boldsymbol{b_5})$ with $\boldsymbol{w_3} \ge \boldsymbol{max}(\ \boldsymbol{w_1},\ \boldsymbol{w_2})$ and $\boldsymbol{v_3} \ge \boldsymbol{max}(\ \boldsymbol{v_1},\ \boldsymbol{v_2})$

Subtraction: $\boldsymbol{A} \ominus \boldsymbol{B} = (\boldsymbol{a_1} - \boldsymbol{b_5},\ \boldsymbol{a_2} - \boldsymbol{b_4}, \boldsymbol{a_3} - \boldsymbol{b_3}, \boldsymbol{a_4} - \boldsymbol{b_2}, \boldsymbol{a_5} - \boldsymbol{b_1})$ with $\boldsymbol{w_3} \ge \boldsymbol{max}(\ \boldsymbol{w_1},\ \boldsymbol{w_2})$ and $\boldsymbol{v_3} \ge \boldsymbol{max}(\ \boldsymbol{v_1},\ \boldsymbol{v_2})$

Scalar Multiplication: let $\boldsymbol{h}$ be a real number.

If $\boldsymbol{h} \ge \boldsymbol{0}$, $\boldsymbol{hA} = (\boldsymbol{ha_1}, \boldsymbol{ha_2}, \boldsymbol{ha_3}, \boldsymbol{ha_4}, \boldsymbol{ha_5})$, if $\boldsymbol{h} \le \boldsymbol{0}$, $\boldsymbol{h} \odot \boldsymbol{A} = (\boldsymbol{ha_5}, \boldsymbol{ha_4}, \boldsymbol{ha_3}, \boldsymbol{ha_2}, \boldsymbol{ha_1})$.

Multiplication: $\boldsymbol{A} \otimes \boldsymbol{B} = (\boldsymbol{a_1 b_1},\ \boldsymbol{a_2 b_2}, \boldsymbol{a_3 b_3}, \boldsymbol{a_4 b_4}, \boldsymbol{a_5 b_5})$ with $\boldsymbol{w_3} \ge \boldsymbol{max}(\ \boldsymbol{w_1},\ \boldsymbol{w_2})$ and $\boldsymbol{v_3} \ge \boldsymbol{max}(\ \boldsymbol{v_1},\ \boldsymbol{v_2})$.

Inverse: $\boldsymbol{A}^{-1} = \left(\frac{\boldsymbol{1}}{\boldsymbol{a_5}}, \frac{\boldsymbol{1}}{\boldsymbol{a_4}}, \frac{\boldsymbol{1}}{\boldsymbol{a_3}}, \frac{\boldsymbol{1}}{\boldsymbol{a_2}}, \frac{\boldsymbol{1}}{\boldsymbol{a_1}}\right)$, $\boldsymbol{a_i} \neq \boldsymbol{0}$, $\boldsymbol{1} \le \boldsymbol{i} \le \boldsymbol{5}$.

Division: $\frac{\boldsymbol{A}}{\boldsymbol{B}} = \boldsymbol{AB}^{-\boldsymbol{1}} = \left(\frac{\boldsymbol{a_1}}{\boldsymbol{b_5}}, \frac{\boldsymbol{a_2}}{\boldsymbol{b_4}}, \frac{\boldsymbol{a_3}}{\boldsymbol{b_3}}, \frac{\boldsymbol{a_4}}{\boldsymbol{b_2}}, \frac{\boldsymbol{a_5}}{\boldsymbol{b_1}}\right)$ with $\boldsymbol{w_3} \ge \boldsymbol{max}(\ \boldsymbol{w_1},\ \boldsymbol{w_2})$ and $\boldsymbol{v_3} \ge \boldsymbol{max}(\ \boldsymbol{v_1},\ \boldsymbol{v_2})$.

Exponent: $A^n = ({a_1}^n, {a_2}^n, {a_3}^n, {a_4}^n, {a_5}^n)$ with n being a real number.

3. Notations and Assumptions

In this part, we present the notations, presumptions, and rental policy required to construct the problem for a three-stage fuzzy FSSP.

3.1. Notations

$\boldsymbol{\mathcal{S}_k}$: Job sequencing order, $\boldsymbol{k} = \boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3}, \ldots, \boldsymbol{m}$.

$\widetilde{\boldsymbol{\mathcal{A}}}_{\boldsymbol{ij}}$: PFPT of the $\boldsymbol{i^{th}}$ job on machine $\boldsymbol{\mathcal{M}_j}$, $\boldsymbol{i} = \boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3}, \ldots, \boldsymbol{n}$; $\boldsymbol{j} = \boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3}$.

$\widetilde{\boldsymbol{\mathcal{T}}}_{\boldsymbol{ij}}$: Completion time of the $\boldsymbol{i^{th}}$ job on machine $\boldsymbol{\mathcal{M}_j}$ of sequence $\boldsymbol{\mathcal{S}_k}$

$\widetilde{\boldsymbol{\mathcal{T}}}_{ij} = \mathbf{max}\left(\widetilde{\boldsymbol{\mathcal{T}}}_{i-1,j} \oplus \widetilde{\boldsymbol{\mathcal{T}}}_{i,j-1}\right) \oplus \widetilde{\boldsymbol{\mathcal{A}}}_{ij};\ \forall j > 2.$

$\widetilde{\boldsymbol{\mathfrak{I}}}_{ij}$: Idle time of machine $\boldsymbol{\mathcal{M}}_j$ for the $\boldsymbol{i^{th}}$ job in sequence $\boldsymbol{\mathcal{S}}_k$

$\widetilde{\boldsymbol{\mathcal{U}}}_j$: Utilization for machine $\boldsymbol{\mathcal{M}}_j$

$\widetilde{\boldsymbol{\mathcal{R}}}$: Total rental cost of machine of all machines of sequence $\boldsymbol{\mathcal{S}}_k$

$\widetilde{\boldsymbol{\mathcal{C}}}_j$: Rental cost of machine $\boldsymbol{\mathcal{M}}_j$

$\boldsymbol{\mathcal{CT}}$: Total time required for completing all jobs of sequence $\boldsymbol{\mathcal{S}}_i$

3.2. Assumptions

i. Preemption was prohibited in all jobs. The machine only handled one work at a time.
ii. All jobs were open at the beginning of schedule time. The duration of the production was independent of the schedule.
iii. Each machine's initial setup time was disregarded. Any machine could be unoccupied.
iv. The deterministic phase was used to process each job. A task must be completed after it had been started.
v. Before the second machine could handle the second work, the first job must have been completed in the first machine.
vi. PFNs were used to represent the due dates.

For every task, $\boldsymbol{m}$ operations were required.

4. Statement of the Problem

We first defined the meaning of the rental policy before introducing the mathematical model of the issue: the first machine was rented when processing the first work, the second machine was rented when processing the first job on the second machine, and the third machine was rented when processing the second job on the second machine. Now, some job $\boldsymbol{i}$ $\left(\boldsymbol{i} = \overline{\mathbf{1},\ \boldsymbol{n}}\right)$ could be processed on three machines $\boldsymbol{\mathcal{M}}_j(\boldsymbol{j} = \mathbf{1},\ \mathbf{2}, \mathbf{3})$ constrained by the specific rental policy $\boldsymbol{\mathcal{P}}$. Let $\widetilde{\boldsymbol{\mathcal{A}}}_{ij}$ be the pentagonal fuzzy processing time (PFPT) of the $\boldsymbol{i^{th}}$ job on the $\boldsymbol{j^{th}}$ machines. Our aim was to find the sequence $\{\mathcal{S}_k\}$ of the jobs which minimized the rental cost of the machines. We present the matrix-form description of the model in Table 1 as follow.

Table 1. Mathematical model representation in matrix form.

T	i	1	2	...	n
Machine $\mathcal{M}_1$	$\widetilde{\mathcal{A}}_{i1}$	$\widetilde{\mathcal{A}}_{11}$	$\widetilde{\mathcal{A}}_{12}$	...	$\widetilde{\mathcal{A}}_{1n}$
Machine $\mathcal{M}_2$	$\widetilde{\mathcal{A}}_{i2}$	$\widetilde{\mathcal{A}}_{21}$	$\widetilde{\mathcal{A}}_{22}$	...	$\widetilde{\mathcal{A}}_{2n}$
Machine $\mathcal{M}_3$	$\widetilde{\mathcal{A}}_{i3}$	$\widetilde{\mathcal{A}}_{31}$	$\widetilde{\mathcal{A}}_{32}$	...	$\widetilde{\mathcal{A}}_{3n}$

The mathematical model of this problem is given as follows:

$$\text{Minimize } \widetilde{\mathcal{R}} = \sum_{i=1}^{n} \widetilde{\mathcal{A}}_{i1}\widetilde{\mathcal{C}}_1 + \widetilde{\mathcal{U}}_2(\mathcal{S}_k)\widetilde{\mathcal{C}}_2 + \widetilde{\mathcal{U}}_3(\mathcal{S}_k)\widetilde{\mathcal{C}}_3 \quad (4)$$

Subject to: the specified rental policy $\mathcal{P}$.

5. Solution Procedure

This section introduces an approach for minimizing utilization time of machines, which leads to the minimization of rental cost for the machines following the PFPT. Let $\widetilde{\mathcal{A}}_{i1},\ \widetilde{\mathcal{A}}_{i2},\ \widetilde{\mathcal{A}}_{i3}, \ldots,\ \widetilde{\mathcal{A}}_{im}$ be the PFPT of machines $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \ldots, \mathcal{M}_m$, respectively.

Step 1: Estimate the associated ordinary number for the pentagonal fuzzy using ranking function.

Step 2: Check the condition either

$$\min_i \widetilde{\mathcal{A}}_{i1} \geq \max_i \widetilde{\mathcal{A}}_{ij},\ j =, 2,\ 3, \ldots,\ m-1 \text{ or} \quad (5)$$

$$\min_i \widetilde{\mathcal{A}}_{im} \geq \max_i \widetilde{\mathcal{A}}_{ij},\ j = 2,\ 3, \ldots,\ m-1 \tag{6}$$

Step 3: Organize two machines $\boldsymbol{X}$ and $\boldsymbol{Y}$ such that

$$\widetilde{X}_i = \sum_{j=1}^{m-1} \widetilde{\mathcal{A}}_{ij},\ i = 1,\ 2,\ 3, \ldots,\ n. \tag{7}$$

$$\widetilde{Y}_i = \sum_{j=2}^{m} \widetilde{\mathcal{A}}_{ij},\ i = 1,\ 2,\ 3, \ldots,\ n, \tag{8}$$

where, $\widetilde{X}_i$, $\widetilde{Y}_i$are the PFPTs for job i on machines X and Y; respectively.

Step 4: Find the sequence $\{\mathcal{S}_k\}$ of jobs on machines X and Y using a suitable ranking method.

A flow chart of the proposed method is shown in the Figure 2.

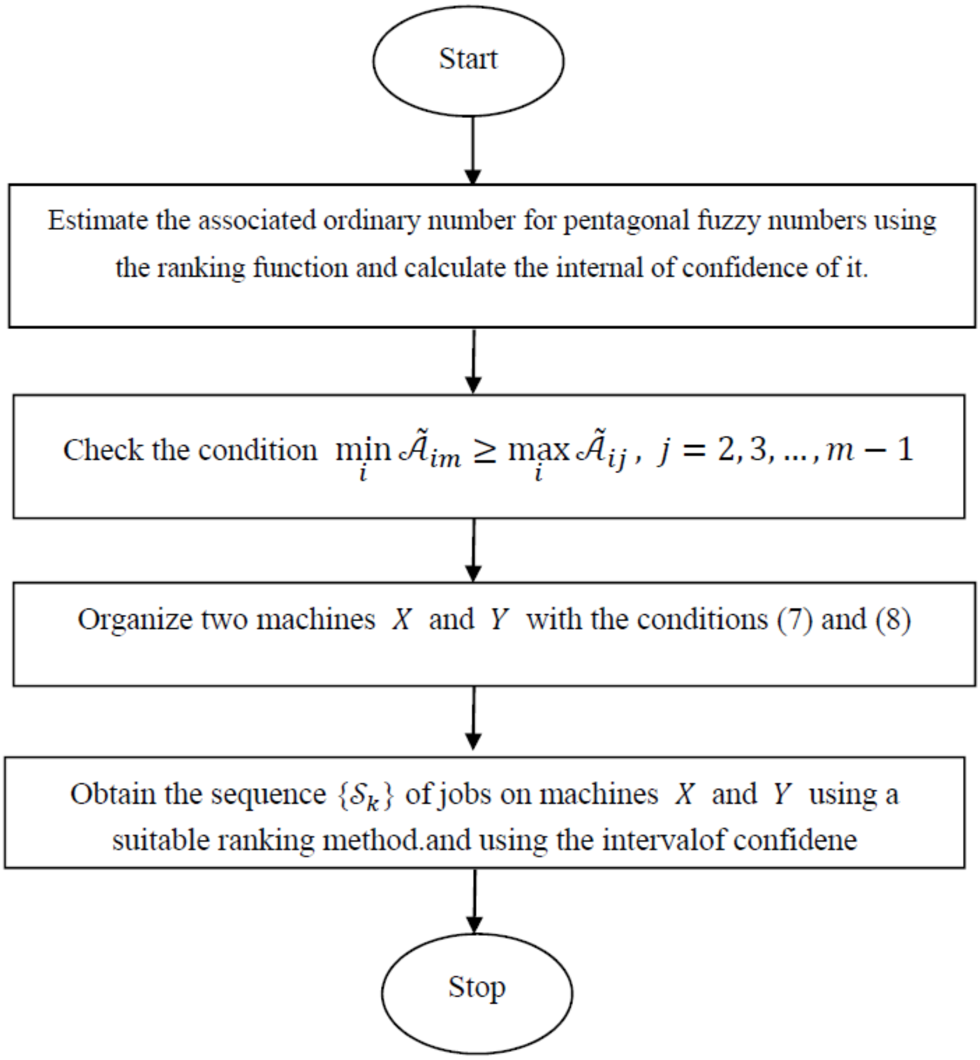

Figure 2. Flow chart of the proposed method.

6. Numerical Example

In Table 2, Consider five jobs and three-machines FSSP with PFPT. The rental cost of machines $\mathcal{M}_1, \mathcal{M}_2$, and $\mathcal{M}_3$ per unit time was four units, two units, and three units respectively, subject to the rental policy $\mathcal{P}$. The main goal was to find an optimal schedule.

Table 2. PFPT data for representation.

Job	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
1	(7, 7.5, 8, 8.5, 9)	(6, 6.5, 7, 7.5, 8)	(3, 3.5, 4, 4.5, 5)
2	(12, 12.5,13, 13.5, 14)	(5, 5.5, 6, 6.5, 7)	(4, 4.5, 5, 5.5, 6)
3	(8, 9, 10, 11, 12)	(4, 4.5, 5, 5.5, 6)	(6, 6.5, 7, 7.5, 8)
4	(10, 10.5, 11, 11.5, 12)	(5, 5.5, 6, 6.5, 7)	(11, 11.5, 12, 12.5, 13)
5	(9, 9.5, 10, 10.5, 11)	(5, 5.5, 6, 6.5, 7)	(8, 8.5, 9, 9.5, 10)

1st approach: Ranking method solution

For the first step of the proposed approach

$$\min_i \widetilde{\mathcal{A}}_{i1} = (7,\ 7.5,\ 8,\ 8.5,\ 9), \tag{9}$$

$$\max_i \widetilde{\mathcal{A}}_{i2} = (6,\ 6.5,\ 7,\ 7.5,\ 8) \tag{10}$$

$$\min_i \widetilde{\mathcal{A}}_{i3} = (3,\ 3.5,\ 4,\ 4.5,\ 5). \tag{11}$$

It was observed that $\min_i \widetilde{\mathcal{A}}_{i1} > \max_i \widetilde{\mathcal{A}}_{i2}$, then the problem could be converted to two machines. Assume X and Y were two fictions machines such that

$$\widetilde{X}_i = \sum_{j=1}^{2} \widetilde{\mathcal{A}}_{ij}, \tag{12}$$

$$\widetilde{Y}_i = \sum_{j=2}^{3} \widetilde{\mathcal{A}}_{ij} \tag{13}$$

The PFPTs for job i on machines X and Y are listed in Table 3.

Table 3. PFPTs of Machines X and Y.

Job	X	Y
1	(13,14,15,16,17)	(9,10,11,12,13)
2	(17,18,19,20,21)	(9,10,11,12,13)
3	(12,13.5, 15,16.5,18)	(10,11,12,13,14)
4	(15,16,17,18,19)	(16,17,18,19,20)
5	(14,15,16,17,18)	(13,14,15,16,17)

Using sub-interval average method [28], the optimal order of sequencing was as follows: $4 \to 5 \to 2 \to 3 \to 1$.

From Table 4, the following results were noticed:

Table 4. Machines Time in and Time out.

Job	Machine 1		Machine 2		Machine 3	
	Time in	Time out	Time in	Time out	Time in	Time out
4	(0, 0, 0, 0, 0)	(10, 10.5, 11, 11.5, 12)	(10, 10.5, 11, 11.5, 12)	(10, 10.5, 11, 11.5, 12)	(15, 16, 17, 18, 19)	(26, 27.5, 29, 30.5, 32)
5	(10, 10.5, 11, 11.5, 12)	(19, 20, 21, 22, 23)	(19, 20, 21, 22, 23)	(19, 20, 21, 22, 23)	(26, 27.5, 29, 30.5, 32)	(34, 36, 38, 40, 42)
2	(19, 20, 21, 22, 23)	(31, 32.5, 34, 35.5, 37)	(31, 32.5, 34, 35.5, 37)	(31, 32.5, 34, 35.5, 37)	(36, 38, 40, 42, 44)	(40, 42.5, 45, 47.5, 50)
3	(31, 32.5, 34, 35.5, 37)	(39, 41.5, 44, 46.5, 49)	(39, 41.5, 44, 46.5, 49)	(39, 41.5, 44, 46.5, 49)	(43, 46, 49, 52, 55)	(49, 52.5, 56, 59.5, 63)
1	(39, 41.5, 44, 46.5, 49)	(46, 49, 52, 55, 58)	(46, 49, 52, 55, 58)	(46, 49, 52, 55, 58)	(52, 55.5, 59, 62.5, 66)	(55, 59, 63, 67, 71)

Total time required for completing all jobs of the obtained sequence,

$$\mathcal{CT}(\mathcal{S}_i) = (55,\ 59,\ 63,\ 67,\ 71) \tag{14}$$

Idle time of $\mathcal{M}_1$ is,

$$\widetilde{\mathcal{I}}_1 = (55,59,63,67,71) - (46,49,52,55,58) = (-3,\ 4,11,18,25) \text{ hrs.}$$

Idle time of $\mathcal{M}_2$ is,

$$\widetilde{\mathcal{I}}_2 = (-13,\ 2.5,18,33.5,49) \text{ hrs.}$$

Idle time of $\mathcal{M}_3$ is,

$$\widetilde{\mathcal{I}}_3 = (-24,-7.5,9,25.5,42) \text{ hrs.}$$

Utilization time of $\mathcal{M}_1$,

$$\widetilde{\mathcal{U}}_1 = (46,49,52,55,58) \text{ hrs.}$$

Utilization time of $\mathcal{M}_2$,

$$\widetilde{\mathcal{U}}_2 = (3,22,41,60,79) \text{ hrs.}$$

Utilization time of $\mathcal{M}_3$,

$$\widetilde{\mathcal{U}}_3 = (13,33.5,54,74.5,95) \text{ hrs.}$$

Rental cost of machine $\mathcal{M}_1$,

$$\widetilde{\mathcal{C}}_1 = 4 * (46,49,52,55,58) = (184,196,208,220,232) \text{ units.}$$

Rental cost of machine $\mathcal{M}_2$,

$$\widetilde{\mathcal{C}}_2 = 2 * (3,22,43,60,79) = (6,44,82,120,158) \text{ units.}$$

Rental cost of machine $\mathcal{M}_3$,

$$\widetilde{\mathcal{C}}_3 = 3 * (13,33.5,54,74.5,95) = (39,100.5,162,223.5,285) \text{ units.}$$

Thus, the total rental cost,

$$\widetilde{\mathcal{R}}(\mathcal{S}_k) = \sum_{j=1}^{3} \widetilde{\mathcal{C}}_j = (229,\ 340.5,\ 452,\ 563.5,\ 675) \text{ units.} \tag{15}$$

2nd Approach named as Interval Confidence based method:

For people, it needs to be more precise in their requirements. Let us propose the following alternative approach:

1. Make the optimization with the associated ordinary;
2. Compute the true optimum for the intervals of confidence at the level $\alpha = 0$;
3. Compare the results with those obtained using the pentagonal fuzzy numbers. If the divergence is very small within acceptable limits, keep the results obtained by using the PFNs.

The previous example was resolved by this approach and the obtained results were shown in the following Tables 5–7.

For the first step of the approach,

$$\min_i \widetilde{\mathcal{A}}_{i1} = [7,\ 9],$$
$$\max_i \widetilde{\mathcal{A}}_{i2} = [6,\ 8],$$
$$\min_i \widetilde{\mathcal{A}}_{i3} = [3,\ 5].$$

Table 5. Processing Times at $\alpha = 0$.

Jobs	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
1	[7, 9]	[6, 8]	[3, 5]
2	[12, 14]	[5, 7]	[4, 6]
3	[8, 12]	[4, 6]	[6, 8]
4	[10, 12]	[5, 7]	[11, 13]
5	[9, 11]	[5, 7]	[8, 10]

Table 6. Processing Times of Machines X and Y at $\alpha = 0$.

Job	X	Y
1	[13, 17]	[9, 13]
2	[17, 21]	[9, 13]
3	[12, 18]	[10, 14]
4	[15, 19]	[16, 20]
5	[14, 18]	[13, 17]

Table 7. Machines' Time in as well as Time out at $\alpha = 0$.

Job	Machine-1		Machine-2		Machine-3	
	Time in	Time out	Time in	Time out	Time in	Time out
4	[0, 0]	[10, 12]	[10, 12]	[15, 19]	[15, 19]	(26,32)
5	[10, 12]	[19, 23]	[19, 23]	[24, 30]	[26, 32]	(34,42)
2	[19, 23]	[31, 37]	[31, 37]	[36, 44]	[36, 44]	[40, 50]
3	[31, 37]	[39, 49]	[39, 49]	[43, 55]	[43, 55]	[49, 63]
1	[39, 49]	[46, 58]	[46, 58]	[52, 66]	[52, 66]	[55, 71]

It was observed that $\min_i \widetilde{\mathcal{A}}_{i1} > \max_i \widetilde{\mathcal{A}}_{i2}$, then the problem could be converted to two machines. Assume X and Y were two fictions machines such that

$$\widetilde{X}_i = \sum_{j=1}^{2} \widetilde{\mathcal{A}}_{ij},\ \widetilde{Y}_i = \sum_{j=2}^{3} \widetilde{\mathcal{A}}_{ij} \tag{16}$$

Using sub-interval average method [28], the optimal order of sequencing was, $4 \to 5 \to 2 \to 3 \to 1$.

Total time required for completing all jobs of the obtained sequence,

$$\mathcal{CT}(\mathcal{S}_i) = [55,\ 71],$$

Idle time of $\mathcal{M}_1$ is,

$$\widetilde{\mathcal{I}}_1 = [55,\ 71] - [46,\ 58] = [-3,\ 25] \text{ hrs.}$$

Idle time of $\mathcal{M}_2$ is,

$$\widetilde{\mathcal{I}}_2 = [0,\ 8] + [1,\ 13] + [-5,\ 13] + [-9,\ 15] = [-13,\ 49] \text{ hrs.}$$

Idle time of $\mathcal{M}_3$ is,

$$\widetilde{\mathcal{I}}_3 = [-6,\ 10] + [-7,\ 15] + [-11,\ 17] = [-24,\ 42] \text{ hrs}$$

Utilization time of $\mathcal{M}_1$,

$$\widetilde{\mathcal{U}}_1 = [46,\ 58] \text{ hrs.}$$

Utilization time of $\mathcal{M}_2$,

$$\widetilde{\mathcal{U}}_2 = [52,\ 66] - [-13,\ 49] = [3,\ 79] \text{ hrs}$$

Utilization time of $\mathcal{M}_3$,

$$\widetilde{\mathcal{U}}_3 = [55,\ 71] - 24,\ 42 = [13,\ 95] \text{ hrs.}$$

Rental cost of machine $\mathcal{M}_1$,

$$\widetilde{\mathcal{C}}_1 = 4 * [46,\ 58] = [184,\ 232] \text{ units.}$$

Rental cost of machine $\mathcal{M}_2$,

$$\widetilde{\mathcal{C}}_2 = 2 * [3,\ 79] = [6,\ 258] \text{ units.}$$

Rental cost of machine $\mathcal{M}_3$,

$$\widetilde{\mathcal{C}}_3 = 3 * [13,\ 95] = [39,\ 285] \text{ units.}$$

Thus, the total rental cost,

$$\widetilde{\mathcal{R}}(\mathcal{S}_k) = \sum_{j=1}^{3} \widetilde{\mathcal{C}}_j = [\,229,\ 675] \text{ units.} \tag{17}$$

7. Comparative Study

This Section includes a comparison of the results we obtained with those found in [24]. The PFN was a generalized version of the triangular fuzzy number, as is known from Section 2. A triangular fuzzy number was transformed into the PFN, and vice versa. Tables 7–9 show comparisons between our calculated total processing time, total machine rental cost, and total machine idle time with that reported in [24].

Table 8. Processing Time.

Type of Fuzzy Number	Our Proposed Algorithm	Algorithm by Sathish and Ganesan [24]
Pentagonal fuzzy number	(55, 59, 63, 67, 71)	(61, 62, 63, 64, 65)
Triangular fuzzy number	(59, 63, 67)	(61, 63, 65)
Fuzziness index triangular fuzzy number	(63, 4, 4)	(63, 2, 2)

Table 9. Total Renal cost.

Type of Fuzzy Number	Proposed Algorithm	Algorithm by Sathish and Ganesan [24]
Pentagonal fuzzy number	(229, 340.5, 452, 563.5, 675)	(444, 448, 452, 456, 460)
Triangular fuzzy number	(340.5, 452, 563.5)	(444, 452, 460)
Fuzziness index triangular fuzzy number	(452, 111.5, 111.5)	(452, 8, 8)

From the above Tables 8–10, it was observed that:

(a) The intermediate value of the pentagonal and the triangular fuzzy numbers was equal i.e., when the membership $\mu_A(x) = 1,\ x = 452$ for the total rental cost and $x = 63$ for the total processing time;

(b) The left and right fuzziness index values, defined in [24] were smaller than the values we obtained for the idle times of machines, total processing time and the rental cost;

(c) From Table 9, it was observed that the proposed rental cost was nonlinear in nature, while the Sathish and Ganesan [24] results showed it was linear in nature. Thus, the presented approach result showed a better representation than the previously existing ones.

Table 10. Idle time of machines.

Type of Fuzzy Number	$\mathcal{J}_j$	Proposed Algorithm	Algorithm by Sathish and Ganesan [24]
	$\mathcal{J}_1$	$(-3,\ 4,\ 11,\ 18,\ 25)$	(9, 10, 11, 12, 13)
Pentagonal fuzzy number	$\mathcal{J}_2$	$(-13,\ 2.5,\ 18,\ 33.5,\ 49)$,	(16, 17, 18, 19, 20)
	$\mathcal{J}_3$	$(-24, -7.5,\ 9,\ 25.5,\ 42)$	(7, 8, 9, 10, 11)
	$\mathcal{J}_1$	$(\ 4,\ 11,\ 18)$	(9, 11, 13)
Triangular fuzzy number	$\mathcal{J}_2$	$(\ 2.5,\ 18,\ 33.5)$,	(16, 18, 20)
	$\mathcal{J}_3$	$(-7.5,\ 9,\ 25.5)$	(7, 9, 11)
	$\mathcal{J}_1$	(11, 7, 7)	(11, 2, 2)
Fuzziness index triangular fuzzy number	$\mathcal{J}_2$	(18, 15.5, 15.5)	(18, 2, 2)
	$\mathcal{J}_3$	(9, 16.5, 16.5)	(9, 2, 2)

Advantages/Limitations of the Proposed Algorithm

The ranking function, interval confidence of the PFN, and DM's vision formed a novel combination that is the main benefit of the suggested solution approach. This combination makes use of the advantages of using uncertainty to efficiently scan the search space, the advantages of the PFN, which uses the DM's vision to rate potential solutions, and the advantages of incorporating the DM's vision. There may be some restrictions when using the proposed algorithm to solve real-world issues, such as:

(1) The methodology does not involve a unified method because the DM's vision, kind of fuzzy number, and $\alpha-$level set vary from one another, making it impossible to assign a united way for allocating the intriguing scenarios for the DM.
(2) Many factors must be considered such as: (i) the possibility of formulating the problem, (ii) the possibility of formulating the problem and choosing the $\alpha-$ level set, and (iii) the capability of solving the problem's selected scenarios and finding their exact optimal scheduling.

8. Conclusions and Future Work

Two methods for using PFPT to solve the novel structured three-stage FSSP have been described in this paper. The major goal of this study was to ascertain how to reduce the overall cost of machine rentals. The membership $\mu_A(x) = 1,\ x = 452$ for the total rental cost, and $x = 63$ for the total processing time made up the quantitative results. The operation sequencing produced by the subintervals ranking approach differed from that acquired by Johanson [22]. Although the jobs were done in a different order, the findings were comparable to those obtained by Sathish and Ganesan [24]. The values of the left and right fuzziness indices were higher than those found in [24] for machine idle times, overall processing time, and rental costs. This indicated that the algorithm was more trustworthy where it expanded the decision-selecting region, enabling the decision-maker to select the appropriate values in accordance with their objectives. Additionally, the suggested methods incorporate the DM's insight into the process of determining the ideal scheduling, and a ranking function was utilized to rank the various alternatives so that the appropriate optimal scheduling may be quickly found. An illustration was given to demonstrate the effectiveness of the suggested method, and findings from the genetic algorithm (GA), one of the most well-known evolutionary algorithms, were compared to the simulation results in order to confirm their accuracy and dependability.

Future work may include the expansion of this research to other fuzzy-like structures, such as Neutrosophic sets, interval-valued fuzzy sets, spherical fuzzy sets, Pythagorean fuzzy sets, etc. One can also take into account brand-new fuzzy systems with applications in decision-making, such as interval type-2, interval type-3, etc. Additionally, in future, a neural-network and convolution neural network approaches will be implemented to improve the solutions and the total cost [38,39].

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References

1. Johnson, S.M. Optimal two and three stages production schedule with Set up times included. *Nav. Res. Logist. Q.* **1954**, *1*, 61–68. [CrossRef]
2. Mir, M.S.S.; Rezaeian, J.; Mohamadian, H. Scheduling parallel machine problem under general effects of deterioration and learning with past-sequence-dependent setup time: Heuristic and meta-heuristic approaches. *Soft Comput.* **2000**, *24*, 1335–1355.
3. Luo, W.; Xu, Y.; Tong, W.; Lin, G. Single-machine scheduling with job-dependent machine deterioration. *J. Sched.* **2019**, *22*, 691–707. [CrossRef]
4. Chen, R.; Yuan, J. Unary NP-hardness of single-machine scheduling to minimize the total tardiness with deadlines. *J. Sched.* **2019**, *22*, 595–601. [CrossRef]
5. Luo, W.; Liu, F. On single-machine scheduling with workload-dependent maintenance duration. *Omega* **2017**, *68*, 119–122. [CrossRef]
6. Atakan, S.; Kerem, B.; Noyan, N. Minimizing value-at-risk in single-machine scheduling. *Anal. Oper. Res.* **2017**, *248*, 25–73. [CrossRef]
7. Zhang, L.; Deng, Q.; Gong, G.; Han, W. A new unrelated parallel machine scheduling problem with tool changes to minimize the total energy consumption. *Int. J. Prod. Res.* **2019**, *58*, 6826–6845. [CrossRef]
8. Ueno, N.; Sotojima, S.; Takeda, J. Multi-Stage Flow-Shop in Steel Works. In Proceedings of the 24th Annual Simulation Symposium, New Orleans, LO, USA, 1 April 1991; Volume 21, pp. 332–337.
9. Yuan, S.; Li, T.; Wang, B.; Yu, N. Model and algorithm for two-stage flow shop group scheduling problem with special blocking constraint. *Control. Decis.* **2020**, *35*, 1773–1779.
10. Vahedi-Nouri, B.; Fattahi, P.; Tavakkoli-Moghaddam, R.; Ramezanian, R. A algorithm for flow shop scheduling problem with consideration of position-based learning effect and multiple availability constraints. *Int. J. Adv. Manuf. Technol.* **2014**, *73*, 601–611. [CrossRef]
11. Ren, T.; Guo, M.; Lin, L.; Miao, Y. A local search algorithm for the flow-shop scheduling problem with release dates. *Discret. Dyn. Nat. Soc.* **2015**, *2015*, 320140. [CrossRef]
12. Laribi, I.; Yalaoui, F.; Belkaid, F.; Sari, Z. Heuristics for solving flow shop scheduling problem under resources constraints. *IFAC-Pap. Online* **2016**, *49*, 1478–1483. [CrossRef]
13. Yazdani, M.; Naderi, B. Modeling and scheduling no-idle hybrid flow shop problem. *J. Optim. Ind. Eng.* **2017**, *10*, 59–66.
14. Qu, C.; Fu, Y.; Yi, Z.; Tan, J. Solutions to no wait flow-shop scheduling problem using the flower pollination algorithm based on the hormone modulation mechanism. *Complexity* **2018**, *2018*, 1973604. [CrossRef]
15. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
16. Shymala, A.K.; PAL, M. Triangular fuzzy matrices. *Iran. J. Fuzzy Syst.* **2007**, *4*, 75–87.
17. Shaw, A.K.; Roy, T.K. Generalized trapezoidal fuzzy number with its arithmetic operations and its application in fuzzy system reliability analysis. *Int. J. Pure Appl. Sci. Technol.* **2011**, *5*, 60–76.
18. Panda, A.; Pal, M. A study on pentagonal fuzzy number and its corresponding matrices. *Pac. Sci. Rev. B Humanit. Soc. Sci.* **2015**, *1*, 131–139. [CrossRef]
19. Chakraborty, A.; Mondal, S.P.; Alam, S.; Ahmadian, A.; Senu, N.; De, D.; Salahshour, S. The pentagonal fuzzy number: Its different representations, properties, ranking, defuzzification and application in game problems. *Symmetry* **2019**, *11*, 248. [CrossRef]
20. Pathinathan, T.; Ponnivalavan, K. Pentagonal fuzzy number. *Int. J. Comput. Algorithm* **2014**, *3*, 1003–1005.
21. Kumar, R.; Pathinathan, T. Sieving out the poor using fuzzy decision-making tools. *Indian J. Sci. Technol.* **2015**, *8*, 1–7. [CrossRef]
22. Prameela, K.U.; Kumar, P. Execution proportions of multi-server queuing model with pentagonal fuzzy number: DSW algorithm approach. *Int. J. Innov. Technol. Explor. Eng.* **2019**, *8*, 1047–1051.
23. McCahon, C.S.; Lee, E.S. Job sequencing with fuzzy processing times. *Comput. Math. Appl.* **1990**, *19*, 31–41. [CrossRef]
24. Sathish, S.; Ganesan, K. Flow shop scheduling problem to minimize the rental cost under fuzzy environment. *J. Nat. Sci. Res.* **2012**, *2*, 62–68.
25. Khalifa, H.A. On single machine scheduling problem with distinct due dates under fuzzy environment. *Int. J. Supply Oper. Manag.* **2020**, *7*, 272–278.

26. Khalifa, H.A.; Alodhaibi, S.S.; Kumar, P. Solving constrained flow-shop scheduling problem through Multistage Fuzzy Binding Approach with Fuzzy Due Dates. *Adv. Fuzzy Syst.* **2021**, *2021*, 6697060. [CrossRef]
27. Alharbi, M.; Khalifa, H.A. On a flow-shop scheduling problem with fuzzy pentagonal processing time. *Hindawi J. Math.* **2021**, *2021*, 6695174. [CrossRef]
28. Zhang, Z.; Tang, Q.; Chica, M. Maintenance costs and makespan minimization for assembly permutation flow shop scheduling by considering preventive and corrective maintenance. *J. Manuf. Syst.* **2021**, *59*, 549–564. [CrossRef]
29. Jabbari, M.; Tavana, M.; Fattahi, P.; Daneshamooz, F. A parameter tuned hybrid algorithm for solving flow shop scheduling problems with parallel assembly stages. *Sustain. Oper. Comput.* **2022**, *3*, 22–32. [CrossRef]
30. Li, R.; Gong, W.; Lu, C.; Wang, A. Learning-based Memetic Algorithm for Energy-Efficient Flexible Job Shop Scheduling with Type-2 Fuzzy Processing Time. *IEEE Trans. Evol. Comput.* **2022**. [CrossRef]
31. Zhou, T.; Khalifa, H.A.; Najafi, S.E.; Edalatpanah, S.A. Minimizing the machine processing time in a flow shop scheduling problem under piecewise quadratic fuzzy numbers. *Discret. Dyn. Nat. Soc.* **2022**, *2022*, 3495228. [CrossRef]
32. Ren, J.; Ye, C.; Yang, F. Solving flow-shop scheduling problem with a reinforcement-learning algorithm that generalizes the value function with neural network. *Alex. Eng. J.* **2021**, *60*, 2787–2800. [CrossRef]
33. Wang, C.-N.; Hsu, H.-P.; Fu, H.-P.; Phan, N.K.P.; Nguyen, V.T. Scheduling flexible flow shop in labeling companies to minimize the makespan. *Comput. Syst. Sci. Eng.* **2022**, *40*, 17–36. [CrossRef]
34. Wang, C.-N.; Porter, G.A.; Huang, C.C.; Nguyen, V.T.; Husain, S. TFlow-shop scheduling with transportation capacity and time consideration. *Comput. Mater. Contin.* **2022**, *70*, 3031–3048.
35. Jemmali, M.; Hidri, L. Hybrid Flow Shop with Setup Times Scheduling Problem. *Comput. Syst. Sci. Eng.* **2023**, *44*, 563–577. [CrossRef]
36. Wang, G.-G.; Gao, D.; Pedrycz, W. Solving Multi-Objective Fuzzy Job-shop Scheduling Problem by a Hybrid Adaptive Differential Evolution Algorithm. *IEEE Trans. Ind. Inform.* **2022**, *18*, 8519–8528. [CrossRef]
37. Koulamas, C.; Kyparisis, G.J. Flow shop scheduling with two distinct job due dates. *Comput. Ind. Eng.* **2022**, *163*, 107835. [CrossRef]
38. Alajanbi, M.; Malerba, D.; Liu, H. Distributed Reduced Convolution Neural Networks. *Mesop. J. Big Data* **2021**, *2021*, 26–29. [CrossRef]
39. Ali, A.H.; Mohammed, M.A.; Ahmed, M.A. Character Recognition by Implementing FPGA-Based Artificial Neural Network. *Mesop. J. Comput. Sci.* **2021**, *2021*, 14–19. [CrossRef]