

# A NOVEL CHIRP MODULATION SPREAD SPECTRUM TECHNIQUE FOR MULTIPLE ACCESS

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## ABSTRACT

This paper proposes a new approach in chirp modulation spread spectrum (CMSS). A good selection of the parameters of the linear chirps, which are used in chirp modulation, combined with antipodal signaling significantly reduces the multiple access interference (MAI) even for small time-bandwidth products. This makes such systems attractive for efficient spread spectrum multiple access. Equal energy and equal bandwidth characterize the set of chirps being used for multiple access. Analytical expressions for the time-bandwidth efficiency and the bit error rate (BER) in additive white Gaussian noise (AWGN) channels are derived. The theoretical results are then confirmed through simulations. The simulation results show that the proposed CMSS technique outperforms existing CMSS techniques in terms of bit error rate and time-bandwidth efficiency. Moreover, its performance approaches those attained by direct sequence spread spectrum (DSSS) systems.

## 1. INTRODUCTION

Over the past decade, spread spectrum (SS) techniques and spread spectrum systems have received quite some attention [1,2,3,4,5,6,7,8,9]. They are expected to be widely used in different areas of communications since their properties meet many needs for future communication systems. Among these advantages are interference rejection, multipath suppression, code division multiple access, and high resolution ranging. To some extent, spread spectrum systems also offer inherent detection protection due to their noise-like spectra.

An adequate definition of spread spectrum transmission is given in [1]: "Spread spectrum is a means of transmission in which the signal occupies a bandwidth in

excess of the minimum necessary to send the information; the band spread is accomplished by means of a code which is independent of the data, and a synchronized reception with the code at the receiver is used for despreading and subsequent data recovery." The four basic spread spectrum techniques known to satisfy this definition are direct sequence, chirp modulation, frequency hopping, and time hopping [2].

Chirp modulation or linear frequency modulation was introduced by Winkler [3] in 1962. She suggested using one pair of linear chirps that have opposite chirp rates for binary signaling. In 1971, Gott and Newsome [4] described an experimental communication system employing chirp modulation in the HF band. They measured its bit error rate (BER) in additive white Gaussian noise and carrier interference. In [5], the utility of binary chirp modulation is compared to phase-shift keying (PSK) and frequency-shift keying (FSK) in coherent, partially coherent, Rayleigh, and Rician transmission channels. Theoretically, chirp modulation is found to be superior in the partially coherent and frequency-selective fading cases over certain ranges of channel conditions. In 1974, Cook [6] assigned pairs of linear chirps with different chirp rates to several users, thus allowing multiple access within a common frequency band. However, for a given time-bandwidth product, multiple access interference (MAI) limits the number of users simultaneously accessing the shared communications resource. El-Khamy et al. [7] extended Cook's proposal in 1996. The chirp signals used for binary multi-user communication were selected such that they all have the same power as well as the same bandwidth. Closed form expressions and approximate analytical expressions for the MAI and BER of this novel chirp modulation spread spectrum (CMSS) technique were also derived in [7]. In [8], the authors of [7] went further and proposed to combine their CMSS technique with frequency hopping.

This hybrid spread spectrum technique improves communication system performance especially in multipath fading dispersive channels.

However, the performance of CMSS systems can still be further improved in the case of multiple access. In this paper, we present a novel chirp modulation spread spectrum technique that utilizes antipodal signaling in conjunction with a good selection of the chirp parameters. We then derive and compute its bit error rate and time-bandwidth efficiency.

## 2. THEORETICAL FORMULATION

For the discussion of the proposed CMSS technique, we assume a communication system as shown in Fig. 1. Note that only one transmitter-receiver pair or equivalently user  $i$ ,  $i=1,2,\dots,M$  is presented in detail;  $M$  denotes the number of users.

On the transmitting side, the binary data sequence  $b_i(t) \in \{-1; +1 | 0 \leq t \leq T_s\}$  modulates a linear chirp signal  $c_i(t)$  of energy  $E_s$  centered at some given carrier frequency  $f_c$ , i.e.,

$$s_i(t) = b_i(t)c_i(t). \quad (1)$$

Each user  $i$  is assigned a distinct chirp signal  $c_i(t)$  from the set of spreading signals

$$c_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t + \pi\alpha_i^a t^2 + \pi\theta_i^a), & 0 \leq t < \frac{T_s}{2} \\ \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + 2\pi\alpha_i^a \frac{T_s}{2} \cdot \left(t - \frac{T_s}{2}\right) + \pi\alpha_i^b \left(t - \frac{T_s}{2}\right)^2 + \pi\theta_i^b\right], & \frac{T_s}{2} \leq t \leq T_s \end{cases} \quad (2)$$

where  $T_s$  denotes the symbol duration,  $\alpha_i^a, \alpha_i^b \in \mathfrak{R}^+$  the individual chirp rates, and  $\theta_i^a, \theta_i^b \in \mathfrak{R}$  the initial phases, respectively. All user chirp signals have equal bandwidth

$$W_c = (\alpha_i^a + \alpha_i^b) \frac{T_s}{2}. \quad (3)$$

Assuming  $\alpha_M^a = \max_{i=1,2,\dots,M} \alpha_i^a$  and  $\alpha_M^b = 0$ , this can be satisfied by

$$\alpha_i^b = \alpha_M^a - \alpha_i^a. \quad (4)$$

The time-bandwidth product normalized to the number of users or simply the normalized time-bandwidth product

$$\eta = \frac{T_s W_c}{M} \quad (5)$$

may serve as a useful measure to compare and evaluate system efficiency.

The additive white Gaussian noise (AWGN) channel under consideration adds a realization of a zero mean white Gaussian noise process with variance  $N_0/2$  to the sum of transmitted signals, i.e.,

$$r(t) = \sum_{i=1}^M s_i(t) + n(t). \quad (6)$$

For simplicity, attenuation and time delay commonly caused by transmission channels are neglected here.

On the receiving side, a common array of receivers  $i$  is assumed to have knowledge of coherent, locally generated copies of the spreading signals  $c_i(t)$  normalized to unit energy. Additionally, all spreading signals are required to be coherent at reception, i.e., their symbol intervals coincide. Generally speaking, this is the case for an externally synchronized communications network with a common multi-user detector. Demodulation and detection can then be realized by an array of correlation receivers. In particular, each receiver multiplies the received signal with its coherent, locally generated replica of the spreading signal of unit energy and integrates this product over one symbol interval to obtain the decision variable

$$u_i = \frac{1}{\sqrt{E_s}} \int_0^{T_s} c_i(t)r(t)dt. \quad (7)$$

Finally, a threshold detector estimates the data symbol sent simply by detecting the sign of the decision variable  $u_i$ .

To obtain an expression of the BER for the proposed CMSS systems, we reformulate the decision variable (7) using (1) and (6)

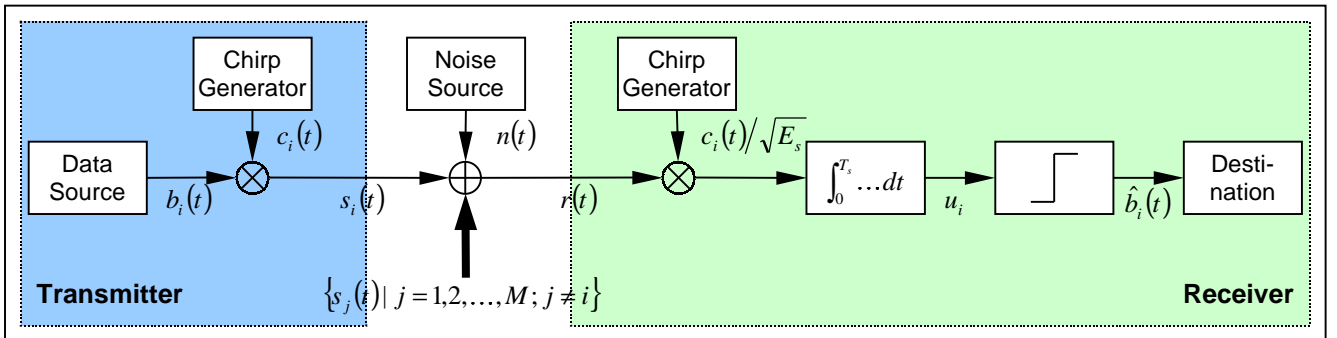


Fig. 1. Block diagram of new chirp modulation spread spectrum system.

$$u_i = \sqrt{E_s} \left( b_i(t) + \sum_{j=1, j \neq i}^M b_j(t) \rho_{ij} \right) + \frac{1}{\sqrt{E_s}} \int_0^{T_s} n(t) c_i(t) dt \quad (8)$$

where  $\rho_{ij}$  is defined as the cross-correlation between two spreading signals  $c_i(t)$  and  $c_j(t)$

$$\rho_{ij} = \frac{1}{E_s} \int_0^{T_s} c_i(t) c_j(t) dt. \quad (9)$$

Since  $n(t)$  is a realization of a white Gaussian noise process, the decision variable  $u_i$  is in fact a random variable of normal distribution [9]; its mean is given by

$$\mu_{u_i} = E[U_i] = \sqrt{E_s} \left( b_i(t) + \sum_{j=1, j \neq i}^M b_j(t) \rho_{ij} \right) \quad (10)$$

and its variance equals  $\sigma_{u_i}^2 = E[(U_i - E[U_i])^2] = N_0/2$ . A bit error occurs if  $u_i > 0$  when  $b_i(t) = -1$  or if  $u_i < 0$  when  $b_i(t) = 1$ . Due to the symmetry of the problem, it is sufficient to consider the first case  $b_i(t) = -1$  without loss of generality. Thus, the probability of a bit error at the  $i$ th receiver is found to be

$$\begin{aligned} P_{e,i} &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_{u_i}^2}} \exp\left(-\frac{(x - \mu_{u_i})^2}{2\sigma_{u_i}^2}\right) dx \\ &= Q\left(\sqrt{\frac{2E_s}{N_0}} \left[1 - \sum_{j=1, j \neq i}^M b_j(t) \rho_{ij}\right]\right) \end{aligned} \quad (11)$$

where the function  $Q(x)$  is defined as a scaled complementary error function, i.e.,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt = \frac{1}{\sqrt{2}} \operatorname{erfc}(x). \quad (12)$$

Averaging (11) over the set of possible data bit variations  $\mathbf{B}_i = \{b_j(t) \in \{-1, +1\} | j = 1, 2, \dots, M; j \neq i\}$  and subsequently over all receivers  $i = 1, 2, \dots, M$  gives a general expression for the probability of bit error. Hence, the BER of systems based on the novel CMSS technique in AWGN channels is given by

$$BER = \frac{1}{M} \sum_{i=1}^M \frac{1}{2^{M-1}} \sum_{\mathbf{B}_i} Q\left(\sqrt{\frac{2E_s}{N_0}} \left[1 - \sum_{j=1, j \neq i}^M b_j(t) \rho_{ij}\right]\right) \quad (13)$$

At this point, the selection of the chirp parameters  $\alpha_i^a$ ,  $\alpha_i^b$ ,  $\theta_i^a$ , and  $\theta_i^b$  needs to be addressed. The problem at hand is to minimize the BER (13) over the set of cross-correlation coefficients  $\rho_{ij}$ ,  $i \neq j$ . Observing that the error probability function  $Q(x)$  is a strictly monotonically decreasing function bounded by  $Q(x) \rightarrow 1$  for  $x \rightarrow -\infty$  and  $Q(x) \rightarrow 0$  for  $x \rightarrow \infty$ , this is equivalent to maximizing its argument  $x$ . Besides increasing the signal-to-noise ratio

(SNR)  $\frac{E_s}{N_0}$ , this can be accomplished by maximizing the random variable

$$Y = 1 - \sum_{j=1, j \neq i}^M b_j(t) \rho_{ij}. \quad (14)$$

The data sequences  $b_j(t)$  are assumed to be uncorrelated, wide-sense stationary discrete random processes of zero mean and unity variance. Therefore,  $Y$  is of mean  $E[Y] = 1$  and variance  $\operatorname{VAR}[Y] = \frac{1}{M-1} \sum_{j=1, j \neq i}^M \rho_{ij}^2$ . As large absolute values of  $\rho_{ij}$  dominate the variance, minimizing the maximum absolute cross-correlation coefficient

$$\rho_{ij, \max} = \max_{i, j=1, 2, \dots, M; i \neq j} |\rho_{ij}| \quad (15)$$

results in minimal bit error rates.

Substituting (2) and (4) into (9) and neglecting integration over higher frequency terms allows us to approximate the cross-correlation coefficient by

$$\begin{aligned} \rho_{ij} \approx \frac{1}{2} \int_0^{\frac{1}{2}} \left[ \cos[\pi(\alpha_i^a - \alpha_j^a) T_s^2 \tau^2 + \pi(\theta_i^a - \theta_j^a)] + \right. \\ \left. \cos[\pi(\alpha_i^a - \alpha_j^a) T_s^2 \tau(1-\tau) + \pi(\theta_i^b - \theta_j^b)] \right] d\tau. \end{aligned} \quad (16)$$

Then  $\rho_{ij}$  only depends on the difference in chirp rates and in initial phases. To achieve good spectral properties of the transmitted signals, we force the spreading signals  $c_i(t)$  to be continuous at  $t = T_s/2$  by choosing

$$\theta_i^b = \alpha_i^a \frac{T_s^2}{4} + \theta_i^a. \quad (17)$$

Additionally, for reasons that will become clear at a later point, we make  $\rho_{ij}$  an even symmetric function in  $\alpha_i^a - \alpha_j^a$  by letting

$$\theta_i^a = \alpha_i^a T_s^2 \bar{\theta} \quad (18)$$

where  $\bar{\theta} \in \mathfrak{R}$  denotes some constant independent of  $i$ . For the remainder, it is referred to as the phase parameter. If the difference in chirp rates of two spreading signals  $c_i(t)$  and  $c_j(t)$  is normalized according to

$$\Delta \bar{\alpha}_{ij} = (\alpha_i^a - \alpha_j^a) T_s^2 \quad (19)$$

the cross-correlation coefficient (16) can be rewritten as

$$\begin{aligned} \rho_{ij} \approx \frac{1}{2} \int_0^{\frac{1}{2}} \left[ \cos(\pi \Delta \bar{\alpha}_{ij} (\tau^2 + \bar{\theta})) + \right. \\ \left. \cos\left(\pi \Delta \bar{\alpha}_{ij} \left(\tau - \tau^2 + \frac{\bar{\theta}}{4}\right)\right) \right] d\tau. \end{aligned} \quad (20)$$

Obviously, the dependency of  $\rho_{ij}$  has been reduced to  $\Delta \bar{\alpha}_{ij}$  and  $\bar{\theta}$ .

It can be shown [10] that the probability of error for any user  $i$  attains a minimum if the cross-correlation coefficients are chosen pair-wise symmetric, i.e.,

$$\rho_{ij} = \rho_{kl}, \quad i, j, k, l = 1, 2, \dots, M : |i - j| = |k - l|. \quad (21)$$

This requirement can be met by choosing a constant chirp rate difference between two adjacent spreading signals,

$$\Delta\bar{\alpha}_{i(i+1)} = \Delta\bar{\alpha}, \quad \forall i = 1, 2, \dots, M - 1 \quad (22)$$

where  $\Delta\bar{\alpha} \in \mathfrak{R}^+$  denotes a constant in  $i$ . It is an important parameter for CMSS systems and we will refer to it as the chirp rate parameter. For a given time-bandwidth product, the chirp rate parameter equals twice the ratio of time-bandwidth-product and number of users. Then, the remaining unknown—the phase parameter  $\bar{\theta}$ —can easily be determined by one-dimensional optimization of the BER given in (13). Note that CMSS systems are completely specified by the parameters  $E_s$ ,  $T_s$ ,  $f_c$ ,  $\Delta\bar{\alpha}$ ,  $\bar{\theta}$ , and  $M$ .

### 3. SIMULATION RESULTS

For several communication systems based on the proposed CMSS technique, the bit error rates in an AWGN channel are computed and compared against the theoretical bit error rates as given by (13). The BER simulations were carried out with a total number of 100,000 bits per data point. Furthermore, we put the BER performance of a new CMSS system side by side with an existing CMSS system and a direct sequence spread spectrum (DSSS) system under the same time-bandwidth limitations.

The initial simulation example considers the three CMSS systems defined by the parameters given in Table 1. Note that for the proposed CMSS technique  $\eta$  equals  $\Delta\bar{\alpha}/2$  as a result of (3), (4), (5), (19), and (22). The system parameter  $\rho_{sum,max}$  denotes the maximum absolute sum of cross-correlation coefficients as computed by

$$\rho_{sum,max} = \max_{i=1,2,\dots,M} \sum_{j=1, j \neq i}^M |\rho_{ij}|. \quad (23)$$

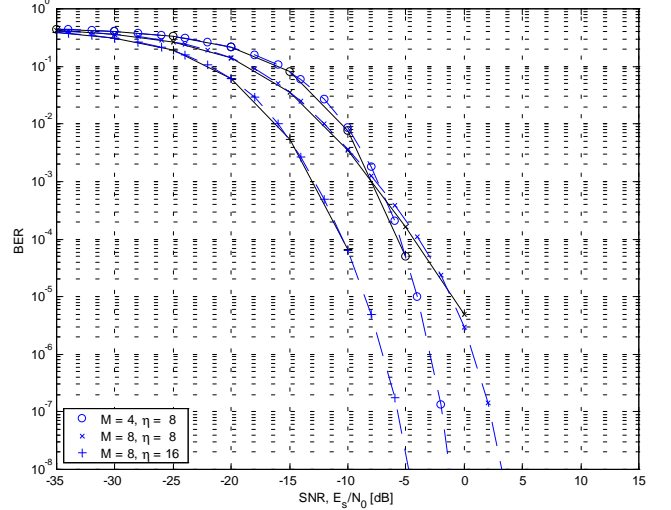
It may be regarded as a measure of multiple access interference in the CMSS system. The higher  $\rho_{sum,max}$  is, the more severe the interference among the users sharing the communications resource will be. For  $\rho_{sum,max} \geq 1$ , bit errors may even occur without channel noise; hence the CMSS system would then inherently be limited by MAI.

$M$	$\eta$	$\Delta\bar{\alpha}$	$\bar{\theta}$	$\rho_{ij,max}$	$\rho_{sum,max}$
4	8	16	1.82	0.14	0.28
8	8	16	1.97	0.16	0.74
8	16	32	1.86	0.11	0.52

**Table 1.** Parameters of CMSS systems used in Fig. 2.

Fig. 2 shows the BERs of the CMSS systems given in Table 1 as a function of the SNR  $\frac{E_s}{N_0}$ . Notice the generally

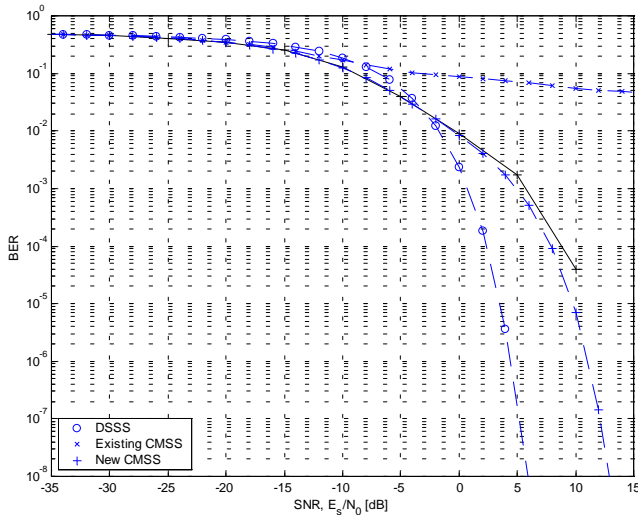
good agreement between the simulated (solid line) and theoretical (dashed line) BERs. Comparing the two CMSS systems of same  $\eta$ , the system accommodating  $M = 8$  users exhibits slightly lower BERs for high noise channels. But for SNRs above  $-7dB$ , the system with  $M = 4$  clearly shows less bit error probabilities due to the smaller cross-correlation parameters  $\rho_{ij,max}$  and  $\rho_{sum,max}$ . In the case of the two CMSS systems with the same number of users, increasing the normalized time-bandwidth product  $\eta$  by a factor of two results in considerably lower cross-correlation parameters  $\rho_{ij,max}$  and  $\rho_{sum,max}$ . Therefore, the BERs achievable are considerably smaller across the plotted range of SNRs.



**Fig. 2.** Simulated and theoretical BERs of three CMSS systems with different number of users  $M$  and/or different normalized time-bandwidth products  $\eta$ .

Fig. 3 compares the BER performance of a DSSS, and an existing and a new CMSS system. All three systems are restricted to  $M = 4$  users and the same normalized time-bandwidth product of  $\eta = 2$ . For spectrum spreading, the DSSS system employs Walsh-Hadamard codes that are known to be mutually orthogonal. For the existing CMSS system, we chose a system based on the multi-user binary chirp modulation technique as proposed by El-Khany et al. [7]. The parameters of our new CMSS system are  $\Delta\bar{\alpha} = 4$ ,  $\bar{\theta} = 1.94$ ,  $\rho_{ij,max} = 0.29$ , and  $\rho_{sum,max} = 0.72$ . For SNRs less than  $-3dB$ , the new CMSS system slightly excels over the two other spread spectrum systems. The DSSS system evidently shows the lowest BERs for higher SNRs, although the new CMSS system's performance is only

somewhat inferior. When compared to the existing CMSS system, which appears to be severely limited by MAI, the proposed technique is clearly superior. On one hand, the large cross-correlation coefficients between the spreading signals limit the BERs of the existing CMSS system. On the other hand, the proposed CMSS system minimizes the cross-correlation coefficients between its user chirp signals. For this reason, it attains BERs close to those of the DSSS system employing orthogonal spreading codes. More precisely, for a practical SNR of  $5\text{dB}$ , the new CMSS system achieves a BER of  $10^{-3}$  and thus it outperforms the existing CMSS approach by nearly two orders of magnitude. At this BER, the required SNRs of the DSSS system and the proposed CMSS system differ only by about  $4\text{dB}$ . At least for this simulation example, we can conclude that the new CMSS system approaches the DSSS system using orthogonal codes in terms of BER performance in AWGN channels.



**Fig. 3.** BER comparison of a DSSS, existing CMSS and new CMSS system for the same normalized time-bandwidth product  $\eta = 2$  and number of users  $M = 4$ .

#### 4. CONCLUSIONS

In this paper, a novel chirp modulation spread spectrum technique for efficient and flexible multiple access was proposed. Each user employs a distinct linear chirp signal for spreading and despreading of its data bit sequence. All spreading chirp signals equally occupy the same bandwidth and therefore offer inherent protection against frequency-selective fading. Application of antipodal signaling in combination with optimum phase synchronization significantly reduces multiple access interference and bit error rates accordingly. Furthermore, the novel CMSS technique allows for a flexible system design that meets

actual time-bandwidth requirements. The BER results show good agreement between theory and simulation. In comparison to other spread spectrum systems, the new CMSS system clearly outperforms the existing chirp modulation technique and approaches BER performance attainable by DSSS systems.

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