A Novel Detection Algorithm using the Sorted QR Decomposition based on Log-Likelihood Ratio in V-BLAST Systems

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***Abstract*— In this paper, we propose a novel detection algorithm using Log-Likelihood Ratio (LLR) to detect signals in V-BLAST systems. This algorithm utilizes the sorted QR decomposition (SQRD) of the channel matrix, and applies LLR to determine the order of detection. Simulation results show that the proposed algorithm provides a better performance than the conventional SQRD with a few additional computations. Approximately, the average BER performance of our algorithm is better than that of the conventional SQRD algorithm by 6dB for BPSK and by 2dB for QPSK respectively at 10^{-3} target BER.**

I. Introduction

Using multiple antennas at the transmitter and the receiver, we can obtain high spectral efficiency with the help of the layered space-time (LST) codes [1]. The simplified version of LST codes is known as V-BLAST (Vertical Bell Lab. Layered Space-Time). In order to detect LST codes at the receiver, several detection algorithms were proposed in [2]-[6]. Especially, ZF-OSIC (Zero-forcing Ordered Successive Interference Cancelation) was introduce in [2]. As a similar approach, the LLR based ZF-OSIC was proposed in [3], and was widely adopted. It decides the order of detection by LLR with aid of the SNR as well as the instantaneous noise. Although these two approaches provide reasonable BER performance at the receiver, the computational efforts which are required to resolve the transmitted signals are quite high. This is because OSIC based detector needs the pseudo-inversion of the channel matrix repeatedly until it finds all the transmitted symbols [6]. To reduce this complexity, [6] was devised to use the sorted QR decomposition (SQRD) by sorting the detection sequence based on SNR followed by exchanging the columns of the channel matrix. However, it showed some performance degradation in comparison with that of ZF-OSIC due to the non-optimal sorting processes.

In this paper, we propose a new detection algorithm for V-BLAST which is applying SQRD of the channel matrix by LLR to determine the order of detection. Like LLR-based ZF-OSIC of [3], the proposed scheme outperforms the conventional SQRD algorithm with just small amount of additional computations.

The remainder of this paper is organized as follows. In Section II, the system model is described. Section III reviews the traditional detection algorithms. And in Section IV, a new detection algorithm is introduced. Section V provides the simulation results and some discussions. Finally, we present conclusions in Section VI.

II. System Model

We consider the multiple antenna systems with n_T transmit antennas and $n_R(\geq n_T)$ receive antennas. The data stream is de-multiplexed into n_T sub-streams. Each sub-stream is encoded into M-PSK symbols. The $n_T \times 1$ transmit signal vector $\mathbf{x} = [x_1, ..., x_{n_T}]^T$ is sent from each transmit antenna over Rayleigh fading channel. At each time slot, the $n_R \times 1$ received vector can be represented as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is the $n_R \times n_T$ channel matrix which contains i.i.d. complex Gaussian gains with zero mean and unit variance. And each element $h_{j,i}$ of $\mathbf{H}$ describes the tap gain between transmitter i and receiver j, and it is assumed to be perfectly known at the receiver. The $n_R \times 1$ noise vector $\mathbf{n}$ is assumed to be i.i.d. complex white Gaussian with zero mean and variance $N_0/2$ per dimension.

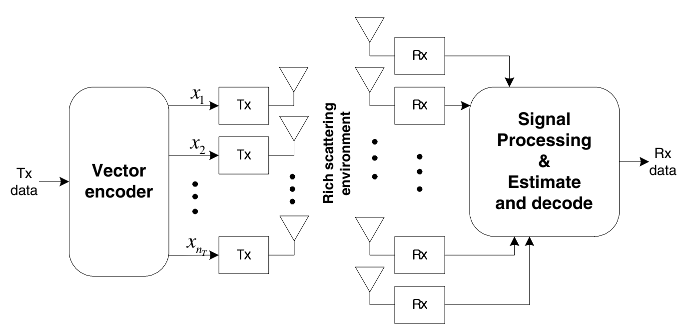

Fig. 1. System model of V-BLAST system

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III. Detection Algorithms

In this section, we review several detection algorithms. First, the ZF-OSIC algorithm of [2] is described and then ZF-OSIC algorithm based on the LLR ordering is explained [3], and finally the detection algorithm using SQRD algorithm is introduced [6].

A. ZF-OSIC Algorithm

This algorithm was proposed in [2] using OSIC based on the zero-forcing solution of channel matrix $\mathbf{H}$. This algorithm is comprised of interference nulling, interference cancelation, and ordering.

Consider a linear zero-forcing solution [7]

$$\mathbf{W} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H = [\mathbf{w}_1\mathbf{w}_2...\mathbf{w}_{n_T}]^T, \quad (2)$$

where $\mathbf{w}_i$ is the $1\times n_R$ nulling vector which satisfies as follows

$$\mathbf{w}_i \cdot \mathbf{h}_l = \begin{cases} 1, & l = i \\ 0, & l \neq i \end{cases} \quad (3)$$

where $\mathbf{h}_l$ represents the l-th column vector of channel matrix $\mathbf{H}$. Let $\mathbf{r}^{(0)} = \mathbf{r}$ and $\mathbf{H}^{(0)} = \mathbf{H}$, then we can represent interference nulling as follows

$$y_i = \mathbf{w}_i\mathbf{r} = x_i + \mathbf{w}_i\mathbf{n} \quad (4)$$

and $\hat{x}_i$ is estimated by $\hat{x}_i = \mathcal{Q}(y_i)$, where $\mathcal{Q}(\cdot)$ is the quantization function which maps to the constellation of symbols. For example, in BPSK case, $\mathcal{Q}(y_i) = +\sqrt{E_S}$ or $-\sqrt{E_S}$ if $y_i \geq 0$ or $y_i < 0$, respectively, where E_S is the transmit energy per symbol. The effect of x_i from $\mathbf{r}^{(0)}$ is canceled out by following method,

$$\mathbf{r}^{(1)} = \mathbf{r}^{(0)} - \hat{x}_i\mathbf{h}_i \quad (5)$$

and the corresponding column of channel matrix is eliminated,

$$\mathbf{H}^{(1)} = [\mathbf{h}_1...\mathbf{h}_{i-1}\mathbf{h}_{i+1}...\mathbf{h}_{n_T}] \quad (6)$$

The repeated processes from (4) to (6) are performed for the remaining signals for estimation according to the detection sequence. The error performance of ZF-OSIC algorithm is affected by the detection sequence [2]. In order to get the minimum error probability, ordering of the detection sequence will be based on the post detection SNR from (4). And the post detection SNR for x_i as follows:

$$SNR_i = \frac{E[|x_i|^2]}{E[\|\mathbf{w}_i\mathbf{n}\|^2]} = \frac{E[|x_i|^2]}{\|\mathbf{w}_i\|^2 N_0} \sim \frac{1}{\|\mathbf{w}_i\|^2} \quad (7)$$

always chooses y_i with the largest post detection SNR for the remaining signals to be detected. Since the signal with the largest SNR is behaving as the largest interference, we first detect and cancel the signal from the received signal.

But the computational efforts of this algorithm are very costly due to the pseudo-inverse process of the channel matrix. In III-C, we will review the algorithm with less complexity using SQRD.

B. Detection Algorithm based on the LLR Ordering

In [3], a new detection ordering technique based on LLR was proposed for the OSIC algorithm. Assuming that the modulation is BPSK and x_i is equi-probable, i.e. $x_i \in \{+\sqrt{E_S}, -\sqrt{E_S}\}$, then LLR, Λ_i, is given by

$$\Lambda_i = \ln\frac{P(x_i = +\sqrt{E_S}|y_i)}{P(x_i = -\sqrt{E_S}|y_i)} = \frac{4\sqrt{E_S}}{N_0} \cdot \frac{\mathrm{Re}\{y_i\}}{\|\mathbf{w}_i\|^2} \quad (8)$$

This algorithm is to search the sub-stream of $\mathbf{x}$ that has the largest $|\Lambda_i|$, where

$$\begin{aligned} |\Lambda_i| &= \frac{4\sqrt{E_S}}{N_0} \cdot \frac{|\mathrm{Re}\{y_i\}|}{\|\mathbf{w}_i\|^2} \quad (9) \\ &= \frac{4\sqrt{E_S}\,|x_i + \mathrm{Re}\{\mathbf{w}_i\mathbf{n}\}|}{N_0 \cdot \|\mathbf{w}_i\|^2} \quad (10) \end{aligned}$$

And it can be extended to M-ary case. Let $x_i \in \{s_1, ..., s_M\}$, then the simplified LLR is given by

$$\Lambda_{i,m^*} = \min_{s_m \neq \hat{x}_i} (|y_i - s_m|^2 - |y_i - \hat{x}_i|^2)/(\|\mathbf{w}_i\|^2 N_0) \quad (11)$$

If equi-energy signaling such as M-PSK, i.e. $|s_m|^2 = |x_i|^2$ for all m and i, is assumed, then LLR can be simplified as

$$\Lambda_{i,m^*} = \min_{s_m \neq \hat{x}_i} 2 \cdot \mathrm{Re}\{y_i(\hat{x}_i - s_m)\}/(\|\mathbf{w}_i\|^2 N_0) \quad (12)$$

Using the relation $\mathrm{Re}\{xy\} \leq |x| \cdot |y|$, then it is rewritten by

$$\begin{aligned} \Lambda_{i,m^*} &\leq \min_{s_m \neq \hat{x}_i} |y_i| \cdot |\hat{x}_i - s_m|/(\|\mathbf{w}_i\|^2 N_0) \quad (13) \\ &= 2 \cdot |y_i| \cdot d_{\min}/(\|\mathbf{w}_i\|^2 N_0) \quad (14) \end{aligned}$$

where $d_{\min}$ is the minimum Euclidean distance between two symbols in a constellation. [3] proposed the cancelation scheme in order of $|y_i| \,/\, \|\mathbf{w}_i\|^2$. The proposed ordering algorithm in [3] uses the instantaneous noise when selecting x_i that has the same phase as $\mathbf{w}_i\mathbf{n}$. And it provides a larger LLR magnitude than that of using only SNR. Therefore this algorithm obtains a better error performance than that of SNR based ordering. Since the OSIC process of this algorithm is similar to that of III-A, this algorithm also requires a lot of computations for the pseudo-inversion process of the channel matrix.

C. Detection Algorithm using the Sorted QR decomposition

In [4], it used QR decomposition of channel matrix $\mathbf{H}$ given by

$$\mathbf{H} = \mathbf{Q} \cdot \mathbf{R} \quad (15)$$

where $\mathbf{Q}$ is the $n_R \times n_T$ unitary matrix composed of orthogonal columns with unit norm and the $n_T \times n_T$ matrix $\mathbf{R}$ is upper triangular. Multiplying the received vector $\mathbf{r}$ with $\mathbf{Q}^H$, we can obtain a $n_T \times 1$ modified receive vector $\mathbf{y}$ as

$$\mathbf{y} = \mathbf{Q}^H \cdot \mathbf{r} = \mathbf{R} \cdot \mathbf{x} + \eta \quad (16)$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_{n_T} \end{pmatrix} = \begin{pmatrix} r_{1,1} & \cdots & r_{1,n_T} \\ & \ddots & \vdots \\ \mathbf{0} & & r_{n_T,n_T} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_{n_T} \end{pmatrix} + \eta \quad (17)$$

where $\eta = \mathbf{Q}^H \cdot \mathbf{n}$ is statistically identical to $\mathbf{n}$. Because of the upper triangular structure of $\mathbf{R}$, the i-th element of $\mathbf{y}$ is given by

$$y_i = r_{i,i} \cdot x_i + \eta_i + d_i \tag{18}$$

where $d_i = \sum_{j=i+1}^{n_T} r_{i,j} \cdot x_j$ is the interference term. Assuming that all previous decisions $\hat{x}_{i+1}, ..., \hat{x}_{n_T}$ are correct ($\hat{d}_i = d_i$), the signal canceled out in the modified received signal y_i is given by

$$z_i = y_i - \hat{d}_i = r_{i,i} \cdot x_i + \eta_i \tag{19}$$

and it is free of interference. And we can detect x_i with $\hat{x}_i = \mathcal{Q}(z_i / r_{i,i})$. Because the improper ordering of detection is causing the error propagation, the order of detection should be properly determined such as the OSIC algorithm. The ordering of detection is achieved by permuting the elements of $\mathbf{x}$ and the corresponding columns of $\mathbf{H}$ in each detection step. The ordering presented in [5] searches $\mathbf{R}$ that maximizes

$$SNR_i = \frac{E[|x_i|^2] \cdot |r_{i,i}|^2}{E[|n_i|^2]} \sim |r_{i,i}|^2 \tag{20}$$

in each step of the detection process. The algorithm which was proposed in [6] uses the modified Gram-Schmidt algorithm in each orthogonalization step and determines the detection sequence $\mathcal{S}$ to achieve small SNR_i in the upper layer. As a result, the effect of the error propagation can be reduced. Since the ordered detection sequences proposed in [6] are not optimal, it causes the small degradation compared to ZF-OSIC in error performance. The algorithm of [6] is summarized as Fig. 2.

SQRD Algorithm

(1) $\mathbf{R} = \mathbf{0},\ \mathbf{Q} = \mathbf{H},\ \mathcal{S} = (1, ..., n_T)$
(2) for $i = 1, ..., n_T$
(3) $\quad k_i = \arg\min_{l=i,...,n_T} (||\mathbf{q}_l||^2)$
(4) $\quad$ exchange col. i and k_i in $\mathbf{Q}, \mathbf{R}, \mathcal{S}$
(5) $\quad r_{i,i} = ||\mathbf{q}_l||$
(6) $\quad \mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
(7) $\quad$ for $l = i+1, ..., n_T$
(8) $\quad\quad r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
(9) $\quad\quad \mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
(10) $\quad$ end
(11) end
, where $\mathbf{r}$ is the received signal vector

Signal detection

(12) $\hat{\mathbf{x}} = \mathbf{x}(\mathcal{S}),\ y = \mathbf{Q}^H \cdot \hat{\mathbf{x}}$
(13) for $i = n_T, ..., 1$
(14) $\quad \hat{d}_i = \sum_{j=i+1}^{n_T} r_{i,j} \cdot \hat{x}_j$
(15) $\quad z_i = y_i - \hat{d}_i$
(16) $\quad x_i = \mathcal{Q}[z_i / r_{i,i}]$
(17) end

Fig. 2. SNR-based SQRD algorithm and signal detection

IV. Proposed Algorithm

Now we propose a novel detection algorithm which uses SQRD algorithm based on the LLR ordering. Assuming that all previous decisions are correct, we can get (19). And it can be rewritten by

$$b_i = \frac{z_i}{r_{i,i}} = x_i + \frac{\eta_i}{r_{i,i}} \tag{21}$$

where $\eta_i / r_{i,i}$ has zero mean and variance N_0. We use the LLR ordering algorithm of III-B to detect the signal causing the highest interference for the remaining signals. Using (11) and (21), the simplified LLR is given by

$$\begin{aligned} \Lambda_{i,m^*} &= \min_{s_m \neq \hat{x}_i} (|b_i - s_m|^2 - |b_i - \hat{x}_i|^2) / (\|\mathbf{q}_i\|^2 N_0 / |r_{i,i}|^2) \\ &= \min_{s_m \neq \hat{x}_i} (|b_i - s_m|^2 - |b_i - \hat{x}_i|^2) / (N_0) \end{aligned} \tag{22}$$

where $\mathbf{q}_i$ is not the i-th column of the complete unitary matrix $\mathbf{Q}$. It means that $\mathbf{q}_i$ is updated at each orthogonalization step [6]. If the modulated symbols are equi-energy signals as M-PSK, (22) can be rewritten by

$$\Lambda_{i,m^*} = \min_{s_m \neq \hat{x}_i} 2 \cdot \mathrm{Re}\{b_i (\hat{x}_i - s_m)\} / (N_0) \tag{23}$$

$$\leq \min_{s_m \neq \hat{x}_i} |b_i| \cdot |\hat{x}_i - s_m| / (N_0) \tag{24}$$

$$= 2 \cdot |b_i| \cdot d_{\min} / (N_0) \tag{25}$$

From (21) to (22), this algorithm is performed so that $|\Lambda_{i,m^*}|$ of the upper layer is smaller than that of the lower layer in the structure of $\mathbf{R}$ matrix. It results in reducing the effect of the error propagation as the conventional SQRD algorithm.

Fig. 3 shows the LLR-based SQRD algorithm using LLR instead of SNR to sort the order of detection. The proposed algorithm utilizes the SNR together with the instantaneous noise in the sorting process. Hence we can acquire gains compared to the comventional algorithms in the error performance with just small additional computations for $|\mathbf{q}_l^H \cdot \mathbf{r}|$ in the third step shown in Fig. 3.

LLR-based SQRD Algorithm

(1) $\mathbf{R} = \mathbf{0},\ \mathbf{Q} = \mathbf{H},\ \mathcal{S} = (1, ..., n_T)$
(2) for $i = 1, ..., n_T$
(3) $\quad k_i = \arg\min_{l=i,...,n_T} (|\mathbf{q}_l^H \cdot \mathbf{r}| / ||\mathbf{q}_l||)$
(4) $\quad$ exchange col. i and k_i in $\mathbf{Q}, \mathbf{R}, \mathcal{S}$
(5) $\quad r_{i,i} = ||\mathbf{q}_l||$
(6) $\quad \mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
(7) $\quad$ for $l = i+1, ..., n_T$
(8) $\quad\quad r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
(9) $\quad\quad \mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
(10) $\quad$ end
(11) end
, where $\mathbf{r}$ is the received signal vector

Fig. 3. LLR-based SQRD algorithm

The signal detection process is identical to that of Fig. 2. which considers the exchanged sequences of the columns of the channel matrix $\mathbf{H}$ and transmitted signal vector $\mathbf{x}$.

V. Simulation Results

In this section, we compare BER performance and processing time of the proposed algorithm with those of the conventional algorithms such as SNR-based ZF-OSIC, LLR-based ZF-OSIC and the SQRD based detection algorithm. Our experimental setup corresponds to the spatial multiplexing MIMO systems with $n_T = 4$ transmit and $n_R = 4$ receive antennas. The channels are assumed as complex Gaussian gains with zero mean and unit variance. We also consider two cases of modulation, BPSK and QPSK. The relative processing time is measured by MATLAB 7.0 command 'etime'. The simulation results are shown as TABLE I, Fig. 4 and Fig. 5.

As we can see in TABLE I, OSIC-based algorithms and LLR-based algorithms perform more computations than SQRD-based algorithms and SNR-based algorithms, respectively. Although the proposed algorithm does not have the smallest amount of computations, from Fig. 4 and Fig. 5, the proposed algorithm significantly outperforms the SNR-based SQRD algorithm. Specifically, the proposed algorithm obtains about 6dB E_b/N_0 gains in case of BPSK and the E_b/N_0 gains by approximately 2dB in case of QPSK at 10^{-3} target BER compared to the SNR-based SQRD algorithm. Hence, we conclude that the proposed algorithm is the best choice of detection algorithms explained in the paper.

Furthermore, the higher order modulation schemes in Fig. 5 obtain less gains than the lower order modulation ones in Fig. 4. This may be caused by the difference of updating periods of the sorting process between BPSK (1symbol=1bit) and QPSK (1symbol=2bits) [3].

TABLE I
Relative processing time of detection algorithms

Algorithms	*Relative Processing Time*
SNR-based ZF-OSIC	1
LLR-based ZF-OSIC	1.083
Proposed algorithm	0.658
SNR-based SQRD	0.541

VI. Conclusions

In this paper, we propose a novel detection algorithm that utilizes the SQRD algorithm using the LLR ordering in the sorting processes. The achievements of this paper were the reduction of the computational effort by using QR decomposition of the channel matrix instead of using the pseudo-inversion processes and the better error performance by using LLR in the sorting processes. Therefore, in many cases of interest, the proposed algorithm can be one of the candidates of good detection algorithms for V-BLAST systems.

VII. Acknowledgement

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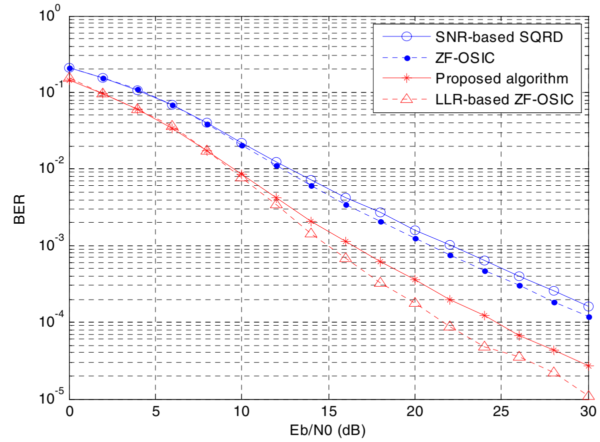

Fig. 4. Performance of the conventional algorithms and the proposed algorithm ; $n_T = n_R = 4$, BPSK

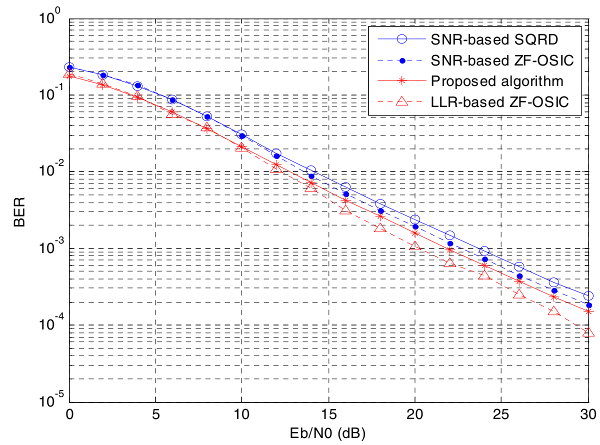

Fig. 5. Performance of the conventional algorithms and the proposed algorithm ; $n_T = n_R = 4$, QPSK

References

[1] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment using multiple antennas," *Bell Labs Technical Journal*, vol.1, No.2, pp.41-59, Autumn 1996
[2] P. W. Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *Proc. ISSE '98*, Pisa, Italy, Sep. 1998
[3] S. W. Kim, "Log-Likelihood Ratio based Detection Ordering for the V-BLAST," *GLOBECOM '03*, vol.1, pp.292-296, Dec.2003
[4] D. Shiu and J. M. Kahn, "Layered Space-Time Codes for Wireless Communications using Multiple Transmit Antennas," in *Proc. ICC '99*, Vancouver, B.C., Jun. 1999
[5] G. J. Foschini, G. D. Golden, A. Valenzuela and P. W. Wolniansky, "Simplified Processing for High Spectral Efficiency Wireless Communications Employing Multi-element Arrays," *IEEE Journal on Selected Areas in Commnunications*, vol.17, no.11, pp.1841-1852, Nov. 1999
[6] D. Wübben, J. Rinas, R. Böhnke, V. Kühn and K. D. Kammeyer, "Efficient Algorithm for Detection Layered Space-Time Code," in *Proc. ITG Conference on Source and Channel Coding*, Berlin, Germany, pp.399-405, Jan. 2002
[7] G. Strang, *Linear Algebra and its Applications*, 3rd ed., Harcout Brace Jovanovich Collage Publishers, Orlando, Florida, 1988