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A Novel Fuzzy System With Adaptive Neurons for Earthquake Modeling

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ABSTRACT Data driven fuzzy neural networks have some disadvantages, such as high dimensions and complex learning process. Also, the obtained models are difficult to interpret. In this paper, we propose a novel simple fuzzy system, which uses fuzzy adaptive neurons. This novel model takes the advantages of the interpretability of the fuzzy system and good approximation ability of the neural networks. We propose a simple learning algorithm for the novel fuzzy system. The stability analysis is given. We successfully apply this fuzzy model for the earthquake modeling. Comparisons with the popular fuzzy neural model are proposed.

INDEX TERMS Fuzzy system, neural networks, earthquake modeling, stability.

I. INTRODUCTION

The model of a system is the representation of the structure, properties, of the system. The choice in which model is developed depends on what is expected to represent it. Obtaining models can be done in different ways, such as through physical laws, mathematical modeling. It is the most common form, but this type of technique needs knowing exactly the environment in which the system operates, as well as making the biggest amount of theoretical considerations as possible. Another way to obtain models is to include measurements of the aspects of interest, together with some equations that describe the system behavior, achieving high robustness and adaptability. Neural networks (NNs) and fuzzy systems are very common to use as gray box models [1]–[6]. The use of neural networks and fuzzy systems can generate models with the characteristics, either for system modeling or adaptive control.

Fuzzy systems use fuzzy rules of the IF-THEN type to model systems. There are two main types of fuzzy systems: Mamdani fuzzy systems and Takagi-Sugeno (TS) fuzzy systems. Several comparisons between them are made [7]–[10].

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Fuzzy systems represent expert's knowledge but can be constructed in such a way that they emulate an expert, through learning processes, like neural networks (NNs) [11], [12]. It is the famous ANFIS (adaptive network based fuzzy inference system), which is based on a TS fuzzy system and transform fuzzy systems into something similar to NNs. If the consequences are taken as nonlinear functions, it is possible to obtain better results in the general performance. The inclusion of NNs of different types in ANFIS systems was introduced and discussed in many works, such as [12]–[14]. More recent works on this topic use RBFNN (radial basis function neural networks) [15], DNN (delayed neural networks) [16] and RNNs (recurrent neural networks) [3], to estimate the consequences in fuzzy systems. For example, the wavelet network (WN) is used in [17]. In [18]–[20], different types of fuzzy systems are applied, which are structured with FNNs (fuzzy neural networks) and conventional representations.

Fuzzy systems require proper membership functions for systems identification [21]. Usually, fuzzy data-based models include fuzzy inference rules and neural network learning methods. The fuzzy neural networks required to successfully solve the precision problems in fuzzy identification of the system, which need good learning algorithms and mathematical models [8], [22]–[28]. However, for many engineering applications [5], [9], [29], [30], they could be very complex.

Membership functions with different shapes, such as triangular, trapezoidal, sigmoidal, bell, Gaussian, among others, have been developed. The Gaussian membership function is the most popular [21]. The sigmoid activation function is configured in series [31], [32].

In order to create a system that reacts with better approximation, the fuzzy adaptive neuron (FAN) method is employed inside the structure of the fuzzy system, instead of fuzzy neural networks. We have the following contributions in this paper

1) We propose the FAN based fuzzy model, which is simpler than the TS structure. This model is established by the fuzzy system and benefited by the neural network.

2) A learning process for this fuzzy network is proposed, it performs simple learning and it is feasible. The stability of the proposed model considering the training algorithm is proved.

To show the advantages of the novel fuzzy system, we use three different activation functions: Gaussian, parabola and sigmoid. The proposed method is compared with fuzzy RBFNN by using the seismic accelerograms of six Mexican seismological stations for the earthquake modeling.

The paper is organized as follows. After the introduction, we show how to use adaptive neurons in fuzzy system in Section II. In Section III, we give the training method of the proposed fuzzy system. The stability analysis of the modeling process is given in Section IV. In Section V, we apply the proposed fuzzy system to the earthquake modeling. Finally, we conclude this paper.

II. FUZZY SYSTEM WITH ADAPTIVE NEURONS FOR NONLINEAR SYSTEM MODELING

The unknown nonlinear system in discrete time can be represented as:

$$y(k) = \Phi\left[y(k-1), y(k-2), \ldots, u(k-1), u(k-2), \ldots\right] \quad (1)$$

where $\Phi(\cdot)$ is an unknown nonlinear difference equation, the plant dynamics, $u(k)$ and $y(k)$ are the input and output of the system. This is the NARMA (nonlinear autoregressive-moving average) model. In multivariable NARMA form [13],

$$Y(k) = \Phi\left[\mathrm{H}(k)\right] \quad (2)$$

where

$$\begin{aligned} H(k) &= \left[Y(k-1), \ldots, U(k-d_t), \ldots\right]^T. \\ U(k) &= \left[u(k), u(k+1), \ldots, u(k+n-2), \ldots\right]^T \\ Y(k) &= \left[y(k), y(k+1), \ldots, y(k+n-1), \ldots\right]^T \end{aligned}$$

To model the system (1) and avoid some problems, such as slow convergence and difficult to design hyper-parameters. In this paper, we use the FANs for the fuzzy system.

We consider the following two types of fuzzy systems:

A) If we normalize the input and output of the unknown system (1) into [0,1], then we use the following fuzzy IF-THEN rules for the unipolar system,

$$R^i: \text{ IF } h_1 > 1 \text{ or } h_2 > 1 \text{ or } \cdots h_n > 1$$

$$\text{THEN } \hat{h}_1 = h_1 \cdot r_e, \hat{h}_2 = h_2 \cdot r_e, \cdots, \hat{h}_n = h_n \cdot r_e \quad (3)$$

where $r_e \in \mathbb{R}$. We use the synaptic operation, somatic Gupta-type aggregation, and the fuzzy integrator operation.

Because the fuzzy unipolar system is in the interval [0,1], the following synaptic operation, aggregation operation, and nonlinear operation are also the somatic operations,

$$\tilde{V}_{minj}(\mathrm{k}) = \min\left(z_{inj}(k), w_{inj}(k)\right) \quad (4)$$

$$\tilde{V}_{max}(\mathrm{k}) = MAX_{j=1}^{N}\tilde{V}_{minj}(\mathrm{k}) \quad (5)$$

$$\tilde{V}_{out}(\mathrm{k}) = \max\left(\tilde{V}_{max}(\mathrm{k}), V_{threshold}(k)\right) \quad (6)$$

$$\tilde{y}_{SAF}(k) = \frac{1}{1+e^{\left(-min\left(\gamma,\tilde{V}_{out}(k)\right)\cdot a+b\right)}} \quad (7)$$

$$e(k) = \tilde{y}_{SAFref}(k) - \tilde{y}_{SAF}(k) \quad (8)$$

where:

a, b, c	real numbers.
k	time variable
$z_{inj}(k)$	dendrite inputs.
$w_{inj}(k)$	synaptic weights.
$V_{threshold}(k)$	threshold.
γ (k)	learning factor, $0 < \gamma \leq 1$.
$\tilde{y}_{SAF}(k)$	sigmoid activation function (SAF).
$e(k)$	modeling error.

B) If we normalize the input and output of the unknown system (1) in the interval [-1,1], then the following synaptic operation, aggregation operation, and nonlinear operation with threshold are also the somatic operations,

$$\tilde{V}_{min\,j}(\mathrm{k}) = \min\left(z_{inj}(k), w_{inj}(k)\right) \quad (9)$$

$$\tilde{V}_{max}(\mathrm{k}) = MAX_{j=1}^{N}\tilde{V}_{min\,j}(\mathrm{k}) \quad (10)$$

$$\tilde{V}_{out}(\mathrm{k}) = \max\left(\tilde{V}_{max}(\mathrm{k}), V_{threshold}(k)\right) \quad (11)$$

$$\tilde{y}_{SAF}(k) = \frac{2}{1+e^{\left(-min\left(\gamma,\tilde{V}_{out}(k)\right)\cdot c\right)}} - 1 \quad (12)$$

$$e(k) = \tilde{y}_{SAFref}(k) - \tilde{y}_{SAF}(k) \quad (13)$$

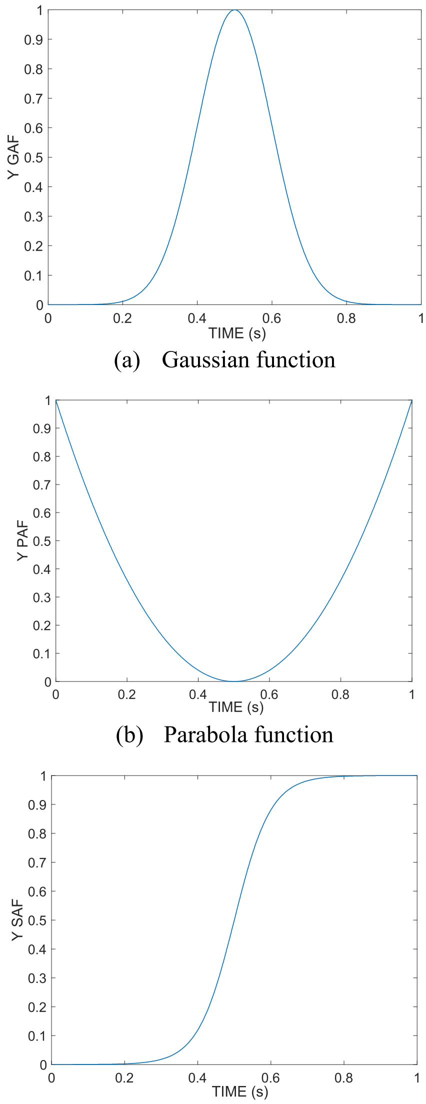
Based on the above operations, the unknown nonlinear system can be expressed by the following fuzzy system

$$\hat{Y}(k) = W_1(k)\cdot\Phi\left[H(k), W_2(k)\right]\cdot\gamma\,(\mathrm{k}) \quad (14)$$

where γ (k) is a scalar,

$$W_1(k) = \begin{bmatrix} w_{11} & \cdots & w_{1l} \\ \vdots & \ddots & \vdots \\ w_{m1} & \cdots & w_{ml} \end{bmatrix} \in \mathcal{R}^{m\times l}$$

$\hat{Y}(k) \in \mathcal{R}^{m\times 1}, \Phi \in \mathcal{R}^{l\times 1}, W_1 \in \mathcal{R}^{m\times 1}, W_2(k) \in \mathcal{R}^{l\times 1}, H(k) \in \mathcal{R}^{l\times l}$, $\Phi(\cdot)$ is the nonlinear function corresponding to the membership functions of the fuzzy system. In this paper, we will use three types of functions for $\Phi(\cdot)$, Gaussian function, parabola function and sigmoid function. We fix $W_2(k)$, and only train $W_1(k)$. $W_2(k)$ are selected randomly in (0,1)



(a) Gaussian function

(b) Parabola function

(c) Sigmoid function

FIGURE 1. Three types of membership functions.

The three types of membership functions, Gaussian function, parabola function and sigmoid function, are shown in Fig. 1.

The parabola membership function is,

$$\tilde{y}_{PAF}(k) = min\left(\gamma, \frac{\left(\tilde{V}_{out}(k) - \delta(k)\right)^2}{\sigma(k)}\right) \quad (15)$$

where,

$\delta(k)$ delay, $0 \leq \delta(k) \leq 1$.
$\sigma(k)$ real number, $0 < \sigma(k)$.

For bipolar fuzzy systems $[-1,1]$, the membership function is expressed in (16),

$$\tilde{y}_{PAF}(k) = min\left(\gamma, \frac{\left(\tilde{V}_{out}(k) - \delta(k)\right)^2}{\sigma(k)}\right) \cdot 2 - 1 \quad (16)$$

The novel Gaussian function is,

$$\tilde{y}_{GAF}(k) = e^{-\frac{min\left(\gamma, \left(\tilde{V}_{out}(k)-\delta(k)\right)^2\right)}{2\cdot\sigma(k)^2}} \quad (17)$$

where,

$\delta(k)$ delay, $0 \leq \delta(k) \leq 1$.
$\sigma(k)$ compressive factor, $0 < \sigma(k)$.

For bipolar fuzzy systems $[-1,1]$, the membership function is,

$$\tilde{y}_{GAF}(k) = 2 \cdot e^{-\frac{min\left(\gamma, \left(\tilde{V}_{out}(k)-\delta(k)\right)^2\right)}{2\cdot\sigma(k)^2}} - 1 \quad (18)$$

These three membership functions have similar thresholds and the shapes. We will use the same learning algorithm to train all the weights of the fuzzy system, FAN-RBFNN.

The fuzzy Gupta integrator is $V_{max}(k)$, the somatic membership function is $\varphi(\cdot)$ with the threshold $V_{threshold}(k)$. The output system is $\tilde{y}(k)$.

$$\begin{aligned} V_{max}(k) &= MAX_{i=1}^{l}\left(\min\left(w_{1i}(k), h_{1i}(k)\right)\right) \\ \tilde{y}(k) &= \varphi\left(V_{max}(k), V_{threshold}(k), \gamma(k)\right) \end{aligned} \quad (19)$$

where $\gamma(k)$ is the learning factor.

The scheme of the proposed fuzzy system is shown in Fig.2.

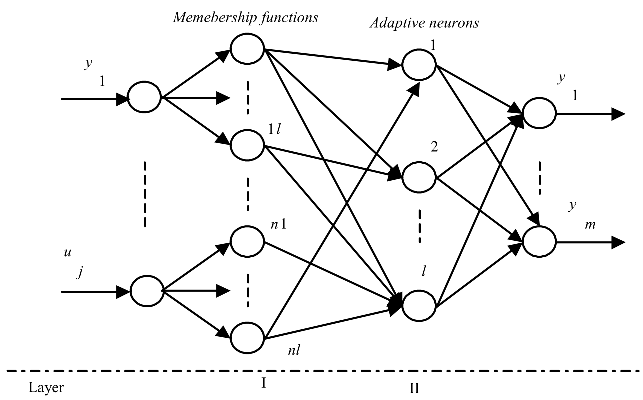


FIGURE 2. The block diagram of the proposed fuzzy system.

III. FUZZY SYSTEM TRAINING

The novel fuzzy model (14) is a multi-input, single-output system, its inputs are $H(k) = [h_{11}(k), \ldots, h_{1l}(k)]$, the weights are $W_1(k)$ and $W_2(k)$.

This fuzzy mode allows us to approximate the output $y(k)$ with $\hat{y}(k)$. The input the to the plant and the model is the same, $H(k)$.

The main idea of fuzzy neural modeling is to find the values of $W_1(k)$ and $W_2(k)$, such that the output $\hat{Y}(k)$ of the proposed model (14), can follow $Y(k)$ output of the nonlinear plant. The identification error between (2) and (14), $e(k) \in \mathcal{R}^{m\times 1}$, is defined as,

$$e(k) = Y(k) - \hat{Y}(k) \quad (20)$$

The modeling error $e(k)$ is used to train the FANs online so that (14), can approximate $Y(k)$.

We only consider to train $W_1(k)$. We define $W_{FAN}(k)$ as unit weights. For our novel fuzzy model (14), we use the following stable training algorithm. In next section, we will prove the stability of this training method. In matrix form,

$$\begin{aligned} W(k+1) &= W(k) + \Gamma(k) \cdot E(k) \cdot H(k) \quad (21) \\ \Gamma(k+1) &= \Gamma(k) + \Gamma(k) \cdot E(k) \cdot H(k) \\ & 0 < \Gamma(k) \le 1, 0 \le W(k) \le 1 \\ \gamma(k+1) &= \gamma(k) + \gamma(k) \cdot h_{in}(k) \cdot e(k) \quad (22) \end{aligned}$$

where $w(k+1) = w(k) + \Delta w(k)$, $\gamma(k+1) = \gamma(\mathrm{k}) + \Delta\gamma(k)$.

Because we use the adaptive neurons as in (4)-(7) and (9)-(12), the training algorithm (21) and (22) is simpler than the gradient based ANFIS [13], [14].

IV. STABILITY ANALYSIS

The fuzzy modeling can be represented by

$$\begin{aligned} &\text{Plant} : y = W^* \cdot \Phi[H(k)] + d(t) \quad (23) \\ &\text{Fuzzy model} : \hat{y} = W(k) \cdot \Phi[H(k)] \quad (24) \\ &\text{Training error} : y - \hat{y} = \left(W^* - W(k)\right) \cdot \Phi[H(k)] \quad (25) \end{aligned}$$

where $\Phi[\cdot]$ is a function of $H(k)$. In matrix form,

$$Y(k) = W^*\Phi[H(k)] + W_d^*\Phi[H_d(k)] + d(t) \quad (26)$$

where W^* are the unknown weights to minimize unmodeled dynamic $W_d^*\Phi[H_d(k)]$, d(t) is the unmodeled dynamic.

So the identification error can be reformed by (20) and (21),

$$\begin{aligned} e(k) &= W^* \cdot \Phi[H(k)] + W_d^* \cdot \Phi[H_d(k)] \\ &\quad - W(k) \cdot \Phi[H(k)] \\ e(k) &= \tilde{W}(k) \cdot \Phi[H(k)] + W_d^* \cdot \Phi[H_d(k)] \\ e(k) &= \tilde{W}(k) \cdot \Phi[H(k)] + \mu_d(k) \quad (27) \end{aligned}$$

where,

$$\tilde{W}(k) = W^* - W(k), \quad \text{and } \mu_d(k) = W_d^* \cdot \Phi[H_d(k)].$$

We are interested in open-loop systems identification, we assume plant (1), is bounded-input-bounded-output (BIBO) stable, i.e. $y(k)$ and $u(k)$ in (1) are bounded. The membership function $\Phi(\cdot)$ is bounded. The following theorem provides the stability analysis for nonlinear system modeling with the novel fuzzy system.

Theorem :. If the unknown nonlinear system (2) is modeled by the fuzzy system (14), the membership functions are updated by (21) and (22), then the modeling error $e(k)$ is uniformly ultimately bounded (UUB). And the normalized identification error,

$$E_N(k) = \frac{W_N(k+1) - W_N(k)}{\Gamma_N(k) \cdot H(k)} \quad (28)$$

satisfies the following average performance:

$$\lim_{T\to\infty} \sup \frac{1}{T} \sum_{k=1}^{T} \|E_N(k)\|^2 \le max_k \left[\left\| W_d^*\Phi[H_d(k)] \right\|^2 \right]$$

$$\lim_{T\to\infty} \sup \frac{1}{T} \sum\nolimits_{k=1}^{T} \|E_N(k)\|^2 \le \bar{\mu}_d \quad (29)$$

Proof: For unipolar systems with values in [0,1], the conditions for $W(k+1)$ and $\Gamma(k+1)$ are

IF $W(k+1) > 1$ THEN $W_N(k+1) = 1$.
IF $W(k+1) < 0$ THEN $W_N(k+1) = 0$.
ELSE $W_N(k+1) = W_N(k) + \Delta W_N(k)$.
IF $\Gamma(k+1) > 1$ THEN $\Gamma_N(k+1) = 1$.
IF $\Gamma(k+1) < 0$ THEN $\Gamma_N(k+1) = n$
ELSE $\Gamma_N(k+1) = \Gamma_N(k) + \Delta\Gamma_N(k)$.

where, $0 < n \le 1$

Therefore,

$$\begin{aligned} \Gamma_N(k+1) &= \Gamma_N(k) + \Gamma_N(k) \cdot E(k) \cdot H(k) \quad (30) \\ W_N(k+1) &= W_N(k) + \Gamma_N(k) \cdot E(k) \cdot H(k) \quad (31) \end{aligned}$$

We selected a positive defined scalar L_k as,

$$L_k = \left\| \tilde{W}(k) \right\|^2 \quad (32)$$

where $\|\cdot\|$ denotes the Euclidean norm.

By the updating law (29), we have,

$$\tilde{W}(k+1) = \tilde{W}(k) + \Gamma(k) \cdot E(k) \cdot H(k)^T \quad (33)$$

Using the inequalities,

$$\|q + r\| \le \|q\| + \|r\|, \quad \|q \cdot r\| = \|q\| \cdot \|r\|$$

For any "q" and "r". By using (33) and $0 < \Gamma_N(k) \le \Gamma(k) \le 1$, we have,

$$\begin{aligned} \Delta L_k &= L_{k+1} - L_k \\ &= \left\| \tilde{W}(k) + \Gamma(k) \cdot E(k) \cdot H(k)^T \right\|^2 - \left\| \tilde{W}(k) \right\|^2 \\ &= 2 \left\| \Gamma(k) \cdot E(k) \cdot H(k)^T \cdot \tilde{W}(k) \right\| \\ &\quad + \left\| \Gamma(k) \cdot E(k) \cdot H(k)^T \right\|^2 \\ &= \|\Gamma(k)\|^2 \cdot \|\mathrm{E}(k)\|^2 \cdot \left\| H(k)^T \right\|^2 \\ &\quad + 2 \left\| \Gamma(k) \cdot E(k) \cdot H(k)^T \cdot \frac{E(k) + W_d^*\Phi[H_d(k)]}{\Phi[H(k)]} \right\| \\ &= \|\Gamma(k)\|^2 \cdot \|\mathrm{E}(k)\|^2 \cdot \left\| H(k)^T \right\|^2 \\ &\quad + \frac{2\|\Gamma(k)\| \cdot \|\mathrm{E}(k)\|^2 \cdot \left\| H(k)^T \right\|}{\|\Phi[H(k)]\|} \\ &\quad + \frac{2\|\Gamma(k)\| \cdot \|E(k)\| \cdot \left\| H(k)^T \right\| \cdot \left\| W_d^*\Phi[H_d(k)] \right\|}{\|\Phi[H(k)]\|} \\ \Delta L_k &\le \zeta(k) \cdot \|\mathrm{E}(k)\|^2 + \delta(k) \cdot \left\| W_d^*\Phi[H_d(k)] \right\| \\ \Delta L_k &\le \zeta(k) \cdot \|\mathrm{E}(k)\|^2 + \delta(k) \cdot \|\mathrm{E}(k)\| \cdot \|\mu_d\| \quad (34) \end{aligned}$$

where $\zeta(k)$ and $\delta(k)$ are defined as,

$$\zeta(k) = \|\Gamma(k)\|^2 \cdot \left\|H(k)^T\right\|^2 + \frac{2\|\Gamma(k)\| \cdot \left\|H(k)^T\right\|}{\|\Phi[H(k)]\|}$$

$$\delta(k) = \frac{2\|\Gamma(k)\| \cdot \left\|H(k)^T\right\|}{\|\Phi[H(k)]\|}$$

Because,

$$n \min\left(\tilde{w}_i^2\right) \leq L_k \leq n \max\left(\tilde{w}_i^2\right)$$

where $n \min\left(\tilde{w}_i^2\right)$ and $n \max\left(\tilde{w}_i^2\right)$ are $\mathcal{K}_\infty - functions$, and $\zeta(k) \cdot \|\mathrm{E}(k)\|^2$ is a $\mathcal{K}_\infty - function$, $\delta(k) \cdot \|\mu_d\|$ is a $\mathcal{K} - function$. So, L_k admits an ISS (input-state stability) Lyapunov function [10], the dynamic of the identification error is input-to-state stable.

From (26) and (32) we know L_k is the function of $E(k)$ and $W_d^* \Phi[H_d(k)]$. The "INPUT" and the "STATE" correspond to both terms of (34). However, usually, $\Phi[H_d(k)] \ll \Phi[H(k)]$.

Because the "INPUT" is bounded and the dynamic is ISS, therefore the "STATE" $\mathrm{E}(k)$ is bounded.

Applying the bounded conditions for $W_N(k+1)$ and $\Gamma_N(k+1)$, equation (34), from 1 up to T and using $0 < L_T$ and L_1 is a constant, we obtain,

$$\zeta_N(k) \cdot \left(\sum_{k=1}^{T} \|E_N(k)\|^2\right) + \delta_N(k) \cdot \left(\sum_{k=1}^{T} \|E_N(k)\|\right) \cdot \bar{\mu}_d \leq L_T - L_1$$

$$\zeta_N(k) = \|\Gamma_N(k)\|^2 \cdot \left\|H(k)^T\right\|^2 + \frac{2\|\Gamma_N(k)\| \cdot \left\|H(k)^T\right\|}{\|\Phi[H(k)]\|}$$

$$\delta_N(k) = \frac{2\|\Gamma_N(k)\| \cdot \left\|H(k)^T\right\|}{\|\Phi[H(k)]\|}$$

$$\zeta_N(k) \cdot \left(\sum_{k=1}^{T} \|E_N(k)\|^2\right) \leq L_T - L_1 - \delta_N(k) \cdot \left(\sum_{k=1}^{T} \|E_N(k)\|\right) \cdot \bar{\mu}_d$$

(29) is established.

Remark 1: It is not easy to obtain high modeling accuracy for the classical fuzzy neural networks, because the hyper-parameters of the fuzzy neural systems are difficult to be decided. But our fuzzy system with adaptive neurons has less hyper-parameters to be chosen. And we prove that the modeling error converges to the zone $\bar{\mu}_d$.

Remark 2: If the fuzzy system (2) could match the nonlinear plant (1) exactly ($\mu_d(k) = 0$), i.e., we could find the best membership function μ_H and W^* such that the nonlinear system could be written as $Y(k) = W^* \Phi[\mu_H]$, the thee same learning law makes the identified error $\|\mathrm{E}(k)\|$ asymptotically stable

$$\lim_{K \to \infty} \|\mathrm{E}(k)\| = 0 \tag{35}$$

Remark 3: The normalization of the learning rates in (26) and (27), are time-varying in order to insure the stability of identification error. The learning rates are easier to be reached than [10], [11], where they select $\gamma = 1$. Because the initial condition does not need any previous information, the time-varying learning rates usually are robust.

V. FUZZY SYSTEM FOR EARTHQUAKE MODELING

The experimental data of the seismological accelerograms [32], [35], are provided by the NSS-IG-UNAM (National Seismological System of the Institute of Geophysics of the National Autonomous University of Mexico). The data come from seven seismological stations, located in the southeast of the Mexican Republic. They are in Huatulco, Oaxaca (HUIG), Yosondúa, Oaxaca (YOIG), Fresnillo de Trujano, Huajuapan,Tehuacán, Puebla (TPIG), Yautepec, Morelos (YAIG), to Popocatépetl station, Mexico State (PPIG).

We will compare our fuzzy model, named "AN fuzzy", with the pouplar ANFIS model [13], [14].

To perform the modeling of the earthquake registered on September 8, 2017, for the seismic accelerograms of the east-west, north-south, up-down components, a second order filter with a cutoff frequency of 30 KHz was designed, after the resulting vectors were obtained, which are the inputs and outputs to the fuzzy system.

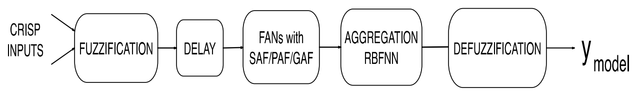


FIGURE 3. Fuzzy system for earthquake modeling.

The input-output mapping of the fuzzy system is shown in Fig. 3. There is the fuzzification (fuzzy rules) of the inputs, a time delay, the FANs with activation function, and an aggregation of type RBFNN, then the defuzzification (fuzzy rules) of the output.

From (2), the resultant of the three components of seismic accelerograms can be modeled by,

$$\begin{aligned} y_{model\ i}(k) = {} & w_{FAN-R\ \ 11} \cdot \mu_{PAF1}(k) \cdot c_1 + w_{FAN-RBF21} \\ & \cdot \mu_{GAF2}(k) \cdot c_2 + w_{FA\ \ 31} \cdot \mu_{SAF3}(k) \cdot c_3 \\ & + w_{FAN-RBF41} \cdot \mu_{GAF4}(k) \cdot c_4 + w_{FAN-R\ \ 51} \\ & \cdot \mu_{GAF5}(k) \cdot c_5 \end{aligned} \tag{36}$$

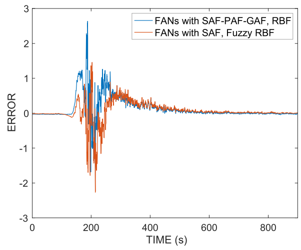
where $i = FTIG, YAIG$.

$$\begin{aligned} y_{model\ i}(k) = {} & w_{FAN-RBF11} \cdot \mu_{PAF1}(k) \cdot c_1 + w_{FAN-RBF21} \\ & \cdot \mu_{GAF2}(k) \cdot c_2 + w_{FAN-RBF31} \cdot \mu_{SAF3}(k) \\ & \cdot c_3 + w_{FAN-RBF41} \cdot \mu_{GAF4}(k) \cdot c_{4a} + c_{4b} \\ & + w_{FAN-RBF51} \cdot \mu_{GAF5}(k) \cdot c_5 \end{aligned} \tag{37}$$

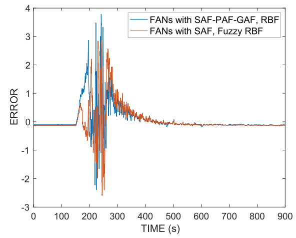
where $i = YOIG, HLIG, TPIG, PPIG$.

We use our novel fuzzy system to model the resulting seismic accelerograms based on the data of the seven seismological stations. The fuzzy model has two inputs,

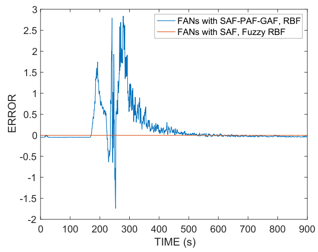
$$z_{in\ FAN_{ji1}}(k) = z_{in\ classic\ RBFNN_{ji1}}(k) = z_{HUIG}\left(k - \Delta k_j\right)$$



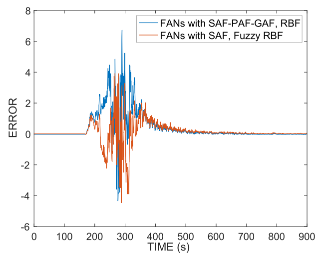
(a) YOIG



(b) TPIG



(c) YAIG



(d) PPIG

FIGURE 4. Modeling errors.

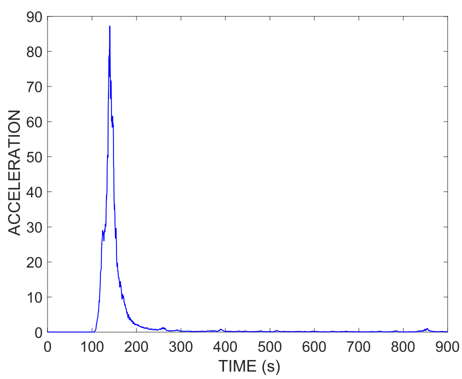


FIGURE 5. HUIG real data.

The initial conditions are

- Weights $\epsilon[0, 1]$, $w_{inFAN-SAF_{ij}}(k) = w_{inFAN-PAF_{ij}}(k) = w_{inFAN-GAF_{ij}}(k) = 0$; $i = 1, \cdots, 5$; $j = 1$.
- Weights ϵ $[-1, 1]$, $w_{FAN-RBFNN_{ij}}(k) = 0$; $i = 1, \cdots, 5$; $j = 1$.

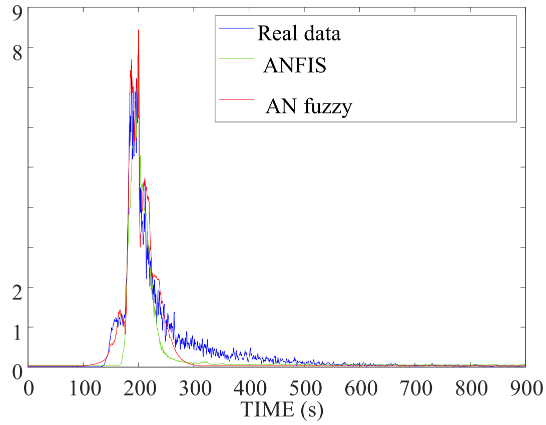


FIGURE 6. Modeling YOIG.

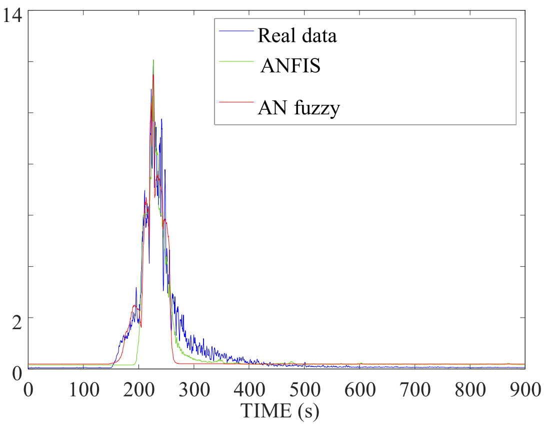


FIGURE 7. Modeling TPIG.

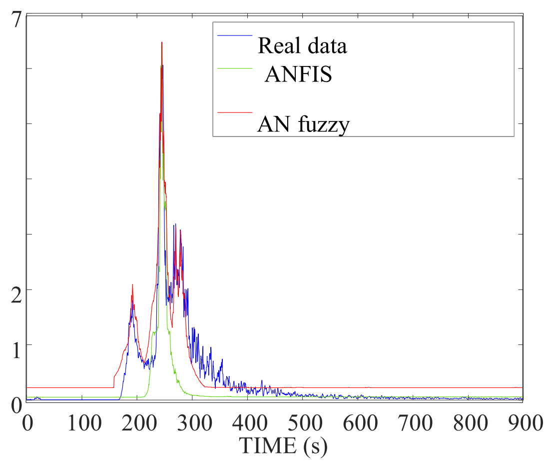


FIGURE 8. Modeling YAIG.

- Learning factors, fixed values, $\gamma_{FANi}(k) = 1$.
- Thresholds, fixed values, $V_{thresholdFAN}(k) = 0$.
- Inputs, $z_{injli1}(k) = z_{HUIG}(k - \Delta k_j)$, $z_{inYOIG_{SAF31}}(k) = z_{HUIG}(k - \Delta k_j + 8.5)$, $z_{inHLIG_{SAF31}}(k) = z_{HUIG}(k - \Delta k_j + 8.5)$, $z_{inPPIG_{GAF41}}(k) = z_{HUIG}(k - \Delta k_j - 8.5)$.

$$l = SAF, PAF, GAF; \quad i = 1, \cdots, 5;$$

$$j = \max(Y_{refYOIG}), \max(Y_{refFTIG}), \max(Y_{refHLIG}), \max(Y_{refTPIG}), \max(Y_{refYAIG}), \max(Y_{refPPIG}).$$

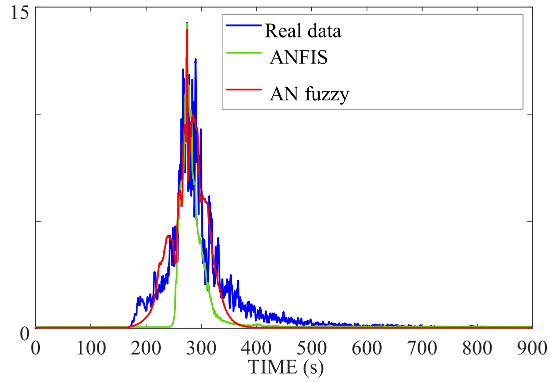


FIGURE 9. Fig. 9. Modeling PPIG.

TABLE 1. Modeling errors.

$J(N)$	FANs-classical RBFNN	FANs-fuzzy RBFNN
YOIG	0.0648	0.0418
FTIG	0.4107	9.81e-8
HLIG	0.0256	0.0117
TPIG	0.1231	0.1171
YAIG	0.1229	4.1e-8
PPIG	0.51	0.23

- Reference outputs $\tilde{y}_{refi}(k) = y(k)$.
- $r_e = 0.01$.
- Ideal values of the weights are unknown.
- Sampling period is $K_{sample} = 0.01 second$.
- Proposed values, $c_{YOIG1} = 2.7664e-4, c_{YOIG2} = 0.1, c_{YOIG3} = 0.08, c_{YOIG4a} = 0.1, c_{YOIG4b} = -0.013, c_{YOIG5} = 0.1. c_{FTIG1} = 2.7664e-4, c_{FTIG2} = 0.02, c_{FTIG3} = 0.08, c_{FTIG4} = 0.01, c_{FTIG5} = 0.1. c_{HLIG1} = 2.7664e-4, c_{HLIG2} = 0.1, c_{HLIG3} = 0.08, c_{HLIG4a} = 0.1, c_{HLIG4b} = -0.013, c_{HLIG5} = 0.1. c_{TPIG1} = 2.7664e-4, c_{TPIG2} = 0.2, c_{TPIG3} = 0.08, c_{TPIG4a} = 0.2, c_{TPIG4b} = -0.014, c_{TPIG5} = 0.1. c_{YAIG1} = 2.7664e-4, c_{YAIG2} = 0.02, c_{YAIG3} = 0.08, c_{YAIG4} = 0.01, c_{YAIG5} = 0.1. c_{PPIG1} = 4.4262e-4, c_{PPIG2} = 0.15, c_{PPIG3} = 0.104, c_{PPIG4a} = 0.155, c_{PPIG4b} = -0.02, c_{PPIG5} = 0.1$.
- Fixed values, $\delta_{iPAF1} = 0.001, \sigma_{iPAF1} = 0.0025. \delta_{iGAF2} = 0.3, \sigma_{iGAF2} = 0.1. a_{iSAF3} = 20, b_{iSAF3} = 10, c_{iSAF3} = 9. \delta_{iGAF4} = 0.2, \sigma_{iGAF4} = 0.1. \delta_{iGAF5} = 0.9, \sigma_{iGAF5} = 0.1$. $i = YOIG, FTIG, HLIG, TPIG, YAIG, PPIG$.

The weights are updated after ten episodes of training. Seismic accelerograms were filtered and scaled to the interval [0,1]. The modeling errors of these 4 data sets are shown in Table 1, (38) and Fig. 4. So, our novel fuzzy system works well for the seismic accelerograms modeling. Defining the mean squared error for finite time,

$$J(N) = \frac{1}{2N} \sum_{k=1}^{N} e^2(k) \tag{38}$$

The real data of HUIG is shown in Fig.5. The comparison results with ANFIS for the four data sets are shown in Fig.6-Fig.9.

VI. CONCLUSION

In this paper, a novel fuzzy model is proposed. This fuzzy model is based on the adaptive neurons. It can be interpreted as a simple neural network. We design a simple training method for this fuzzy model. Stability of the proposed training method is given. We apply this novel model for the seismic accelerograms modeling. The results show that the new model has better performance that the classical fuzzy neural networks for nonlinear system identification.

Multiple applications can be carried out applying this novel fuzzy system, such as systems identification, control and automation of systems [8], low-scale unmanned aerial vehicles (UAVs) [27], and optimization of manufacturing processes.

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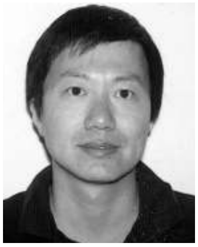

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