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RESEARCH ARTICLE

A novel internal waves generator

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Abstract We present a new kind of generator of internal waves which has been designed for three purposes. First, the oscillating boundary conditions force the fluid particles to travel in the preferred direction of the wave ray, hence reducing the mixing due to forcing. Second, only one ray tube is produced so that all of the energy is in the beam of interest. Third, temporal and spatial frequency studies emphasize the high quality for temporal and spatial monochromaticity of the emitted beam. The greatest strength of this technique is therefore the ability to produce a large monochromatic and unidirectional beam.

1 Introduction

Stratified fluids are nowadays a field of paramount importance in fluid mechanics and are carefully studied theoretically, experimentally and numerically by a large and always increasing scientific community. The underlying reason is, of course, the identification of many crucial phenomena appearing in the oceanic and atmospheric environments, where the stratification cannot be overlooked. The possibility to reproduce in a laboratory experiment conditions close to theoretical hypotheses is, of course, a very important way to validate theoretical ideas developed in the last decades. Indeed, if one will always rely on in situ observations for the final decisive tests, new fundamental ideas are usually derived within an oversimplified framework that cannot be reproduced in a real environment. Among many difficulties, let us mention just a few. The linear stratification hypothesis is almost always used in theoretical works despite the complicated oceanic or atmospheric stratification (Thorpe 2005). Internal waves are also often considered as monochromatic plane waves, while oceanic measurements usually report a beam-like structure, with a width smaller than a wavelength (Lam et al. 2004; Gerkema et al. 2004). The paradox of internal waves impinging onto topography (Dauxois and Young 1999) was solved for a linear oblique slope which can only be a poor description of a real seamount (Eriksen 1998). It is not satisfactory to test this last theoretical example by considering a *beam* impinging onto a *curved* topography in a fluid with a *real and complicated* stratification (exponential, thermocline, mixed layer,...). Indeed, possible disagreements between theoretical and experimental results might always be attributed to one of these three hypothesis which are not fulfilled in the experimental test. Controlled experiments in a simplified framework are therefore of paramount importance.

In a laboratory experiment, one can identify three main difficulties: the preparation of a stratified fluid with a controlled *stratification*, the *generation* and the

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visualization of internal waves propagating within this fluid. The first difficulty was rapidly solved thanks to the well-known double bucket method and its improved versions (Oster 1965; Hill 2002; Benielli and Sommeria 1998). The visualization difficulty has been nicely solved in the last decade by two different techniques: the Schlieren method, on the one hand, later improved into the modern quantitative synthetic Schlieren method (Dalziel et al. 2000), gives time resolved density fields; on the other hand, the particle image velocimetry (PIV) technique provides time resolved velocity fields (Fincham and Delerce 2000). However, the generation of monochromatic plane waves remains a crucial problem from the experimental point of view. We propose in this article a new experimental set-up that is able to provide a single monochromatic and unidirectional large beam.

The paper is organized as follows. In Sect. 2, we briefly review existing generation methods, with a special emphasis on their limitations. In Sect. 3, we present the new internal waves generator. In Sect. 4, we analyze the internal waves which are generated with a particular emphasis on its temporal, and spatial properties. Section 5 concludes the paper.

2 Review of existing generation methods

2.1 Important properties of internal gravity waves

Internal waves in a continuously stratified fluid are shear waves that propagate with a phase velocity perpendicular to their group velocity. The density stratification allows the propagation of such waves by generating a restoring force. This latter is characterized by the Brunt–Väisälä frequency N, which corresponds to the vertical oscillation frequency of water masses in a local density gradient. It is defined as $N^2 = -(g/\rho)(\mathrm{d}\rho/\mathrm{d}z)$ where g is the gravity, ρ the density and z the vertical coordinate. The frequency N will be kept constant in the remainder of the paper.

In such a stratified fluid, it is well-known that the dispersion relation for linear internal gravity waves is

$$\omega = N \sin\theta, \quad (1)$$

where ω is the temporal frequency and θ the angle of the wave vector with the vertical direction. This dispersion relation emphasizes that for a fixed frequency, the direction in which the shear propagates is fixed and is denoted σ. This direction is also found to be orthogonal to the direction of propagation of the energy, denoted s.

The study of internal waves generation must therefore take into account these very unusual and restrictive geometrical conditions. Moreover, it is very important to notice that the spatial wavelength of the wave does not appear in the dispersion relation (1). There is no mechanism of wavelength selection other than the physical boundary conditions imposed by the forcing technique. This last point will be of first importance in the following.

2.2 Oscillating bodies

The simplest experiments on internal waves emission use oscillating bodies as radiation sources and, more precisely, in a two-dimensional set-up, a vertically oscillating cylinder (see the sketch in Fig. 1a). The first experiment was performed by Görtler (1943) in a two-dimensional setup. Nonetheless, as this experiment is often forgotten, one usually refers to the seminal experiment by Mowbray and Rarity (1967). This is nowadays a simple laboratory set-up, often used in laboratory tutorials at the undergraduate level. In the simplified two-dimensional set-up, these generators emit four beams of internal waves, making a constant angle with the horizontal. A typical resulting pattern is presented in Fig. 1b. Recent three-dimensional experiments with an oscillating sphere have reported striking double cones (Peacock and Weidman 2005).

As the wavelength does not enter the dispersion relation (1) of internal waves, it is absolutely non trivial, a priori, to predict the spatial structure of the beam. This question has been at the origin of several theoretical studies (Thomas and Stevenson 1972; Hurley 1997; Hurley and Keady 1997; Hurley and Hood 2001; Voisin 2003) which have proposed a non-trivial scaling of the beam spatial structure by the size of the object, the viscosity of the fluid and the emission frequency. In all cases, the beam envelope does not contain a full wavelength. Therefore, it is difficult to use these sources to study spatial properties of internal waves.

2.3 Paddle-like generators

It has also been proposed to generate directly a shear motion in the stratified fluid by using a multi-bladed folding paddle (McEwan 1973; Cacchione and Wunsch 1973; Teoh et al. 1997; Silva et al. 1997; Ivey et al. 2000; Gostiaux et al. 2006): see the sketch in Fig. 1c. The flow generated by such a device corresponds to the superposition of two shear waves propagating in opposite directions along the zig-zag paddle, each one being independently solution of the internal waves' equations. Such a set-up thus generates two beams propagating symmetrically leftwards and rightwards (resp. upwards and downwards) when the paddle is set horizontally (resp. vertically) as in Teoh et al. (1997). Locating a vertical wall close to the paddle (see vertical black thick line in Fig. 1d) allows to avoid a second beam: moreover, because of the reflection of the leftward beam on this wall, one may increase artificially the width of the rightward beam.

The energy transmission is proportional to the sine of the angle between the paddle and the along-direction of the beam. To gain in amplitude, one might therefore propose to tilt the paddle orthogonally to the desired beam. One realizes immediately, however, that the generated shear is no more the superposition of two internal wave beams. Consequently it will excite strong nonlinearities. Intermediate position between the horizontal and this inclination can be proposed, this method, however, still generates higher harmonics. As shown in Fig. 1d (see Gostiaux et al. 2006 for additional details), the main part of the energy is transmitted to a beam of internal wave of frequency ω, while higher harmonics $n\omega$ are also excited by the oscillating paddle. Their frequencies being higher, their propagation angles are also larger (see Fig. 1d) and a screen can therefore be located appropriately to remove most harmonics (Gostiaux et al. 2006). Fortunately, in Ivey et al. (2000), this effect was avoided since the excitation frequency ω was larger than half of the Brunt–Väisälä. In conclusion, as avoiding harmonics generation is of primary importance for energy fluxes measurements and comparisons with theoretical predictions, especially for low frequency ratios ω/N, such a method is not ideal.

2.4 Tidal-like excitation

Finally, a third way of generating internal waves has been proposed and realized in a form of a global excitation of internal waves. A vertical periodic motion of the fluid container (see the sketch in Fig. 1e) acts as a perturbation of the gravity force on the whole fluid. Tidal-like excitations by barotropic forcing (Ivey and Nokes 1989; Gostiaux and Dauxois 2006) also excite globally the fluid. Resonant modes are selected and can be focused on internal waves attractors. This excitation

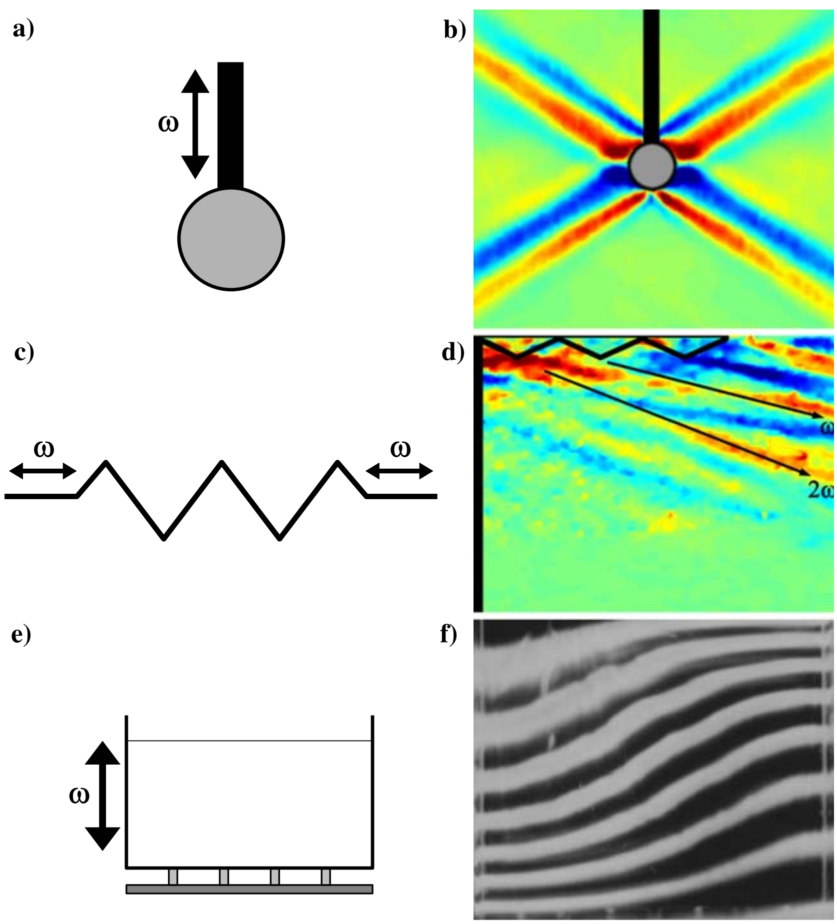

Fig. 1 Principles and internal waves patterns for the three previous internal gravity wave generators. **a** Presents the sketch of a vertically oscillating cylinder. **b** Shows the resulting four rays as captured with a synthetic Schlieren technique. **c** Presents the paddle-like generator, while **d** shows a typical instantaneous PIV measurement. Note harmonic waves propagating in different directions. **e** Presents the parametric excitation principle while **f** shows the resulting internal wave pattern (Benielli 1995)

is highly dependent of the size and shape of the fluid domain, and relies on resonances of the enclosed geometry itself (Maas et al. 1997) as shown in Fig. 1f.

The first very limitation is that oscillating vertically a large tank is, of course, very difficult and cannot be easily performed. In addition, the whole stratified fluid is thus excited so that it is not possible to discern how internal waves, generated in one part of the tank, might encounter independently a topography, or another internal wave, in another part of the tank.

3 The new solution for internal waves generation

As briefly motivated in the previous section, a new principle for internal wave generation has to be proposed, if a *monochromatic plane wave* is necessary for laboratory experiments. The key idea is to create physical boundary conditions that effectively propagate within the fluid interior, instead of a stationary forcing.

3.1 Mechanism of the generator

The generator consists of a pile of 24 expanded PVC sheets ($2 \times 36 \times 150$ cm) enclosed in a parallelepiped half-opened box and free to slip one over the other. The plates are weighted with thin lead sheets so that they are buoyantly neutral in water; it minimizes friction forces between plates. They are thus separated by 2.2 mm one from the other. Two rectangular holes in each plates allow two identical camshafts (see Fig. 2a)

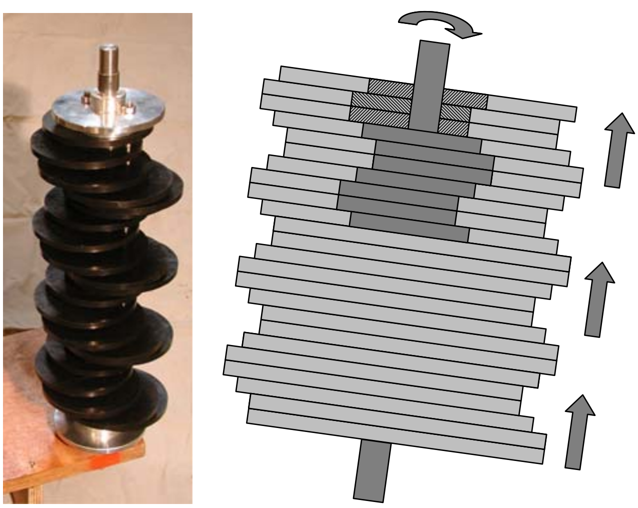

Fig. 2 Internal wave generator mechanism. The *left panel* presents one of the two identical camshafts, while the *right* one shows the cross section of the generator. One clearly identify the plates (*light grey*) and one of the camshafts (*dark grey*)

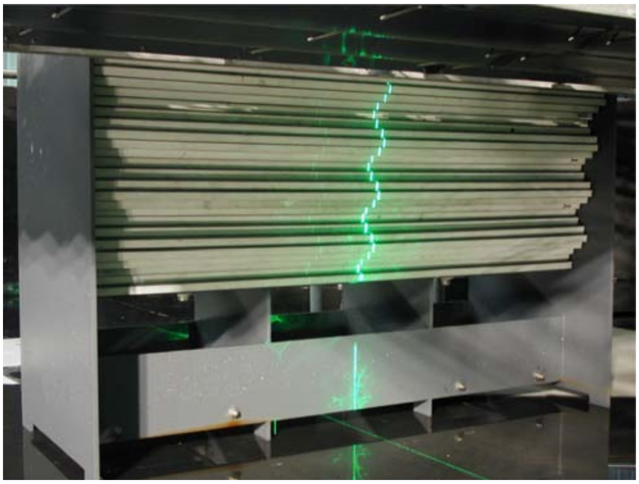

Fig. 3 Perspective view of the wave generator before the filling of the tank. The *thin vertical line* in the *middle* of the plates is the impact of the laser sheet, used for the PIV visualization. We can see the two *vertical* sidewalls of the generator that hinder to view the field very close to the generator

to go through the pile, imposing the relative position of the plates. At rest, the plates are sinusoidally shifted, due to the helicoidal repartition of the cams (see Fig. 2b). The rotation of the camshafts applies a periodic motion to the plates which propagates upward (resp. downward) for a clockwise (resp. anti-clockwise) rotation. The eccentricity of the camshafts defines the amplitude of oscillation of the plates, namely 6 cm peak to peak. A perspective view of the generator can be seen in Fig. 3.

To optimize the emission process, the generator is slightly tilted in direction of the emitted beam, i.e. the direction of shear propagation. In the following, we generate a downward propagating beam (defined by the direction of the group velocity) corresponding to an upward propagating shear (direction of the phase velocity). As anticipated, the device can easily generate upward propagating beams by inverting the sign of rotation of both camshaft.

3.2 Flow visualization

We performed experiments in the 13 m diameter circular tank of the Coriolis Plateform in Grenoble, filled with 90 cm of linearly stratified fluid at 3% corresponding to a Brunt–Väisälä period of $T_{BV} = 11.7 \pm 0.2$ s. A vertical laser sheet illuminates the wavemaker (visible in Fig. 3) and the 400 μ diameter particles polystyrene beads seeded in the stratified fluid. PIV measurements of the velocity field were performed in order to characterize the waves generated by the new device. All experiments discussed in

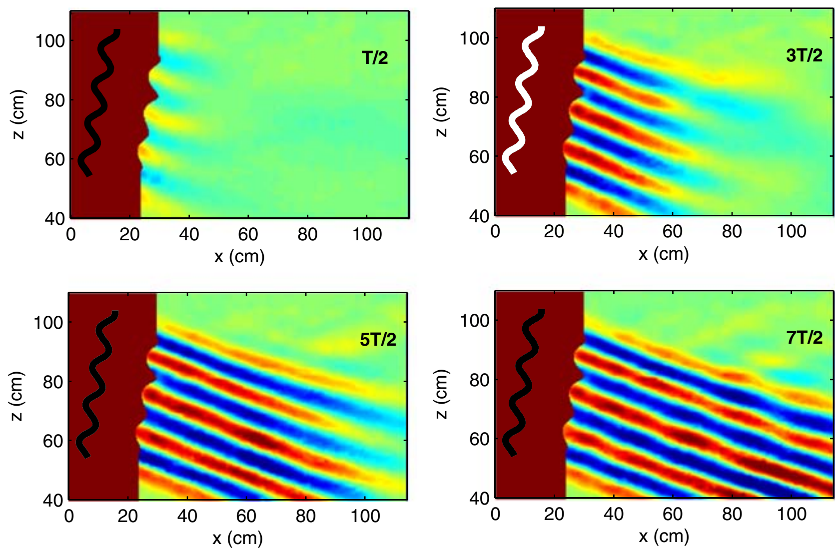

Fig. 4 Horizontal velocity pattern in the vertical plane after 1, 2, 3 and 4 periods T of excitation. The position of the wavemaker is indicated by the oscillating and *tilted white line*. On the *left* of each panel, the *shadowed region* corresponds to the invisible region hidden by the sidewalls of the generator

this paper were carried out with a period of excitation $T = 36$ s. It corresponds to an angle of emission $\theta = 19°$. Snapshots of the flow every 600 ms were recorded using a 12 bits 1,024 × 1,024 pixels camera. Subsequently, we compiled the CIV correlation algorithms (Fincham and Delerce 2000) between successive frames to get the velocity field induced by the wavemaker. Results are reported and analyzed in the next section.

4 Analysis of the emanating internal waves

4.1 Qualitative analysis

Figure 4 present four successive images of the emitted wave. Colors show the horizontal velocity which is the strongest component in this configuration since the angle of propagation is rather small. As can be seen, only one downward propagating beam is emitted: this is the first important improvement of this new generator. These four pictures clearly emphasize the generation of the internal plane waves propagating towards the fluid interior.

The transversal wave structure perfectly fits also the profile of the wavemaker, with four wavelengths emitted. It is the largest beam ever generated in a laboratory experiment. It is however important to stress that there is no theoretical limitation to have a larger beam. This is of crucial importance for experiments designed to test plane waves properties.

Figure 5 shows the velocity field. As expected, it is parallel to the direction of emission, emphasizing nicely the pure shear structure. Moreover, it clearly attests that iso-phase lines are propagating perpendicularly to the energy propagation. The maximal velocity value measured in that case is 1.8 mm/s, while amplitude displacements are of the order of 1 cm.

4.2 Temporal chromaticity

As stated in Sect. 1, *monochromatic* internal gravity waves are highly desirable for studying, for example, the role of nonlinearities involved in collisions with topography (Ivey and Nokes 1989; Dauxois and Young 1999; Gostiaux and Dauxois 2006) or with other internal waves (Teoh et al. 1997): the chromaticity quality of the incident beam is therefore of primary importance. With the present device, as the forcing is itself solution of the wave equation, no harmonics are measurable in the flow. To attest this result, Fig. 6a shows a temporal evolution of the horizontal velocity at a point situated in the centerline of the beam, 29 cm away from the generator.

First, note that the signal is vanishingly small before starting to increase at $t = 30$ s. This remark allows to compute the group velocity of the beam. As the generation and measurement points are distant by 29 cm, one gets a group velocity $c_g \simeq 0.97$ cm/s. This value has to be compared with the theoretical one, $c_g = \lambda/(T \tan\theta)$, where T is the excitation period, λ the wavelength of the internal beam and θ the angle of propagation. With $\lambda = 13.2$ cm, $T = 36$ s and $\theta = 19°$, the expected value $c_g = 1.06$ cm/s compares well with the measurement.

Moreover, one can notice that the transient regime is almost invisible as the wave reaches the measurement area. This allows to consider the forcing of such a wave as a suddenly switched on sinusoid (sinus function convoluted with a Heaviside distribution), which

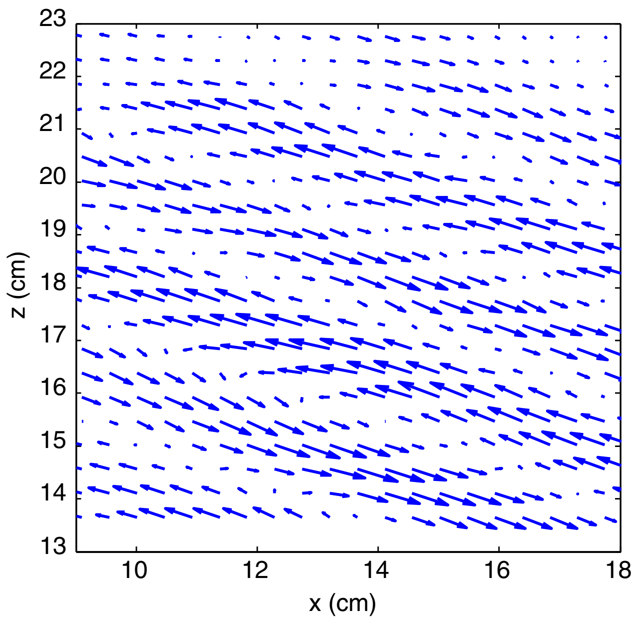

Fig. 5 Zoom of the velocity field generated by the generator

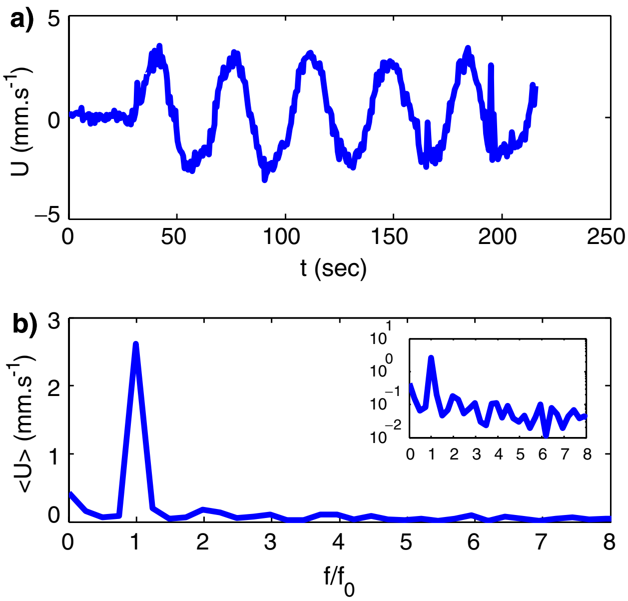

Fig. 6 Temporal analysis of the internal waves beam in a point located at 29 cm from the wavemaker and in the middle of the beam. **a** Presents the time evolution of the horizontal velocity field, while **b** shows the Fourier transform of this signal. Frequencies are renormalized with the excitation one, $f_0 = 1/T$. The *inset* in **b** presents the spectrum in logarithmic scale

allows fine comparisons with analytical results using such a forcing (Dauxois and Young 1999).

Once the beam reaches the measurement point, the velocity oscillates at a well defined frequency. The temporal spectrum of the wave, plotted in Fig. 6b, confirms that the frequency signal is, as expected, imposed by the generator. Moreover, it shows that amplitudes of higher harmonics (see the inset in logarithmic scale) are at least one order of magnitude smaller than the fundamental one. The emitted beam is thus highly temporally monochromatic. The latter characteristic allows to filter the velocity field at the excitation frequency in order to reduce measurement noise. The filtering technique is used in Figs. 7 and 8 which present spatial properties of the beam.

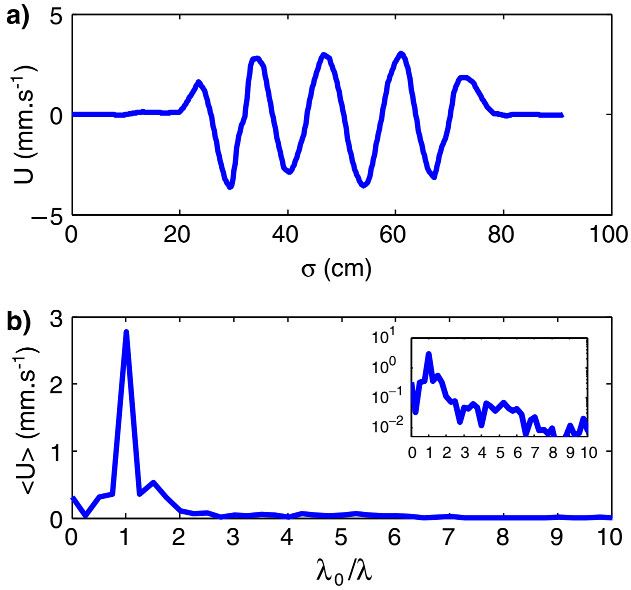

Fig. 7 Transversal structure of the internal waves beam, 29 cm from the wavemaker. **a** Presents the spatial evolution of the horizontal velocity field, while **b** shows the Fourier transform of this signal. Wavelengths are renormalized with the excitation one λ_0. The *inset* in **b** presents the spectrum in logarithmic scale

4.3 Spatial structure of the beam

Contrary to optical or acoustic waves, and owing to the unusual dispersion relation (1) of internal waves, temporal monochromatism does not imply any spatial monochromatism: it is necessary to study them separately. Figure 7a presents a cross section of the horizontal velocity field at 29 cm from the wavemaker. One may easily distinguish slightly more than four wavelengths. Its corresponding wavelength spectrum is shown in Fig. 7b. The spatial monochromatism of the produced beam is attested by the very well defined peak of the spectrum. Note that a careful study shows that end effects in the horizontal y-direction have very little, if no, consequences: one can only detect a very small transverse component of the velocity field.

Finally, Fig. 8 emphasizes that the viscous damping of the beam along its propagation is extremely weak.

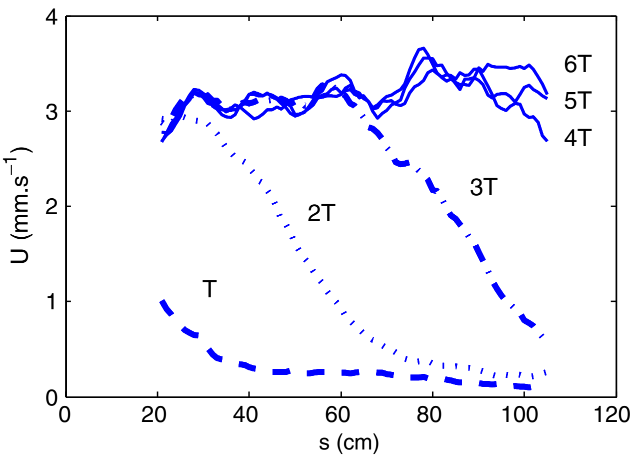

Fig. 8 Superposition of successive longitudinal profiles of the horizontal velocity after 1, 2, 3, 4, 5 and 6 excitation periods

One can notice the non-dispersive profile of the wave front: the beam does not disperse since it is spatially monochromatic. From this picture, it is possible to get another reliable estimate of the group velocity. Indeed, by measuring the location difference of the appearance of the velocity $U = 1$ mm/s or by trying to superpose the first three profiles, one gets $c_g \simeq 38$ cm per period i.e. 1.06 cm/s: the precise theoretical value derived previously.

This generator was used to excite waves up to 1.5 m away from the region of interest, no appreciable damping was measured at this point. The absence of nonlinear losses may explain this remarkable constancy of the wave amplitude.

5 Conclusion

We have reported the design of a new kind of internal waves generator that imposes a rigid boundary condition compatible with the equations governing the propagation of plane internal waves. By generating a propagating shear, we select a single direction of emission of the waves. The waves emitted are purely monochromatic, both spatially and temporally, which is of first importance for the study of nonlinear processes involved in internal waves dynamics.

Finally, it is important to emphasize that this experimental setup allows to generate any kind of wave form. Not only sinusoidal, but also gaussian beams or more complicated shapes can be easily generated, by modifying the camshafts eccentricity appropriately. This possibility opens new perspective for future studies of the dynamics of internal waves in stratified fluids.

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