

## **A novel inverse DEA model with application to allocate the CO<sub>2</sub> emissions quota to different regions in Chinese manufacturing industries**

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### **Abstract**

This paper aims to address the problem of allocating the CO<sub>2</sub> emissions quota set by government goal in Chinese manufacturing industries to different Chinese regions. The CO<sub>2</sub> emission reduction is conducted in a three-stage phases. The first stage is to obtain the total amount CO<sub>2</sub> emission reduction from the Chinese government goal as our total CO<sub>2</sub> emission quota to reduce. The second stage is to allocate the reduction quota to different two-digit level manufacturing industries in China. The third stage is to further allocate the reduction quota for each industry into different provinces. A new inverse data envelopment analysis (InvDEA) model is developed to achieve our goal to allocate CO<sub>2</sub> emission quota under several assumptions. At last we obtain the empirical results based on the real data from Chinese manufacturing industries.

**Keywords:** Data envelopment analysis (DEA); Inverse DEA; CO<sub>2</sub> emissions, Manufacturing Industries

### **1. Introduction**

Since the reform and opening up policy in 1978, China's economy has maintained long-term rapid development and made great achievements. As reported in the China Statistical Yearbook 2016, between 1978 and 2015 the China's nominal Gross Domestic Product (GDP) grew significantly from 367.87 to 68263.51 in billion RMB

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Yuan, an increase of about 186 times. Bian et al. (2015) also argued that China's nominal industrial GDP increased by 66.02 times between 1981 and 2009. At the same time, however, the contradiction between the rapid growth of economic development and the environmental problem has been increasingly prominent. The economic development brought about a severe pressure on the natural environment and resources in China, especially in recent several years. China Statistical Yearbook 2016 shows that in year 2015 the Total Waste Water Discharged and Common Industrial Solid Wastes Produced reach 7353.227 and 327.079 million tons, respectively. In particular, the number of Days of Air Quality Equal to or Above Grade II in China's Capital city Beijing is only 186 in the year 2015. Bian *et al.* (2015) also reported the total amount of industrial solid waste produced in 2009 was 5.42 times that of 1981. In 2007 the total consumption of energy in China in 2007 reaches 311, 442 in millions of standard coal equivalent (SCE), and the total consumption of energy in China grew from 57.144 in 1978 to 430.000 in 2015 in million tons of SCE, which is reported clearly in China Statistical Yearbook 2016.

To address the issues of environmental protection, especially reducing CO<sub>2</sub> emissions, China government has been searching the viable solutions to balance the economic growth and CO<sub>2</sub> emissions reduction. At June 30 2015, at the upcoming climate conference in France, Chinese Premier Li Keqiang announced China's latest voluntary reduction commitment: the CO<sub>2</sub> emissions in China will reach the peak at about 2030 and seek to reach it as early as possible.

The Chinese government goal motivates us to investigate the problem of allocating the CO<sub>2</sub> emissions quota in Chinese manufacturing industries to different Chinese regions. In this paper, we use a three-stage way to conduct the CO<sub>2</sub> emission reduction. Firstly, we obtain the total amount CO<sub>2</sub> emission reduction from the Chinese government goal as our total CO<sub>2</sub> emission quota to reduce in the first stage. Secondly, we allocate the reduction quota to different two-digit level manufacturing industries in China. Thirdly, we further allocate the reduction quota for each industry into different provinces. In the CO<sub>2</sub> emissions reduction process, we develop a new

inverse data envelopment analysis (InvDEA) model to achieve our goal to allocate CO<sub>2</sub> emission quota under several assumptions.

The remainder of the paper is organized as follows: Section 2 summarizes the existing literatures on CO<sub>2</sub> emissions and DEA models. Section 3 describes the dataset and input/output indicators of Chinese manufacturing industries in our study. Section 4 gives the detailed information on our proposed InvDEA method and the empirical results of CO<sub>2</sub> emission quota allocation. Section 5 concludes this paper and provides some remarks for future research.

## **2. Literature review on CO<sub>2</sub> emission and DEA**

In this section we provide latest development on measuring CO<sub>2</sub> emission using DEA models.

### **2.1. Literatures of using DEA for CO<sub>2</sub> emission**

Regarding the efficiency analysis with respect to CO<sub>2</sub> emissions, Murty et al. (2007) estimated the technical and environmental efficiency and firm-specific shadow prices of pollutants of some coal-fired thermal power plants in India based on directional output distance function with the given resources and technology. Mukherjee (2010), Riccardi et al. (2012) and Vlontzos et al. (2014) respectively examined the efficiency considering reduction of CO<sub>2</sub> emissions in Indian manufacturing sector, 21 industrialized countries and EU member state countries, using directional distance function or non-radial DEA model allowing for non-proportional adjustments of outputs.

Further, Molinos-Senante et al. (2014) who applied measured the efficiency of wastewater treatment plants and estimated the pure and mixed environmental performance indices for a sample of 60 Spanish wastewater treatment plants using DEA models. Sueyoshi and Goto (2014a) and Sueyoshi and Goto (2014b) applied a radial-based DEA model which is shaped by the Debreu-Farrell and Cui and Li (2015) proposed a new virtual frontier DEA model to measure unified environmental efficiency.

Currently, China has become one of the world's largest contributors of CO<sub>2</sub> emissions, so the environmental efficiency including CO<sub>2</sub> emission in Chinese industries has been a popular research topic. Some of previous studies on Chinese environmental efficiency have been reported Table 1.

**Table 1.** Previous studies on Chinese environmental efficiency.

Authors (year)	Research field and data	Methodological approaches	Major issues addressed
Zhang et al. (2015)	Province-level	Output-based CCR model with DDF+ ML index	Total-factor carbon emission performance of the Chinese transportation industry
Yang et al. (2015)	Province-level	Input-based CCR and super efficiency CCR	Regional environmental efficiencies in China
Wang et al. (2015)	City-level	Output-based BCC model with DDF	Environmental protection mechanisms and economic development of 211 cities in China
Fan et al. (2015)	Industrial sub-sectors of Shanghai	Output-based CCR model with DDF+ ML index	Industrial total factor CO <sub>2</sub> emission performance
Bian et al. (2015)	Regional-level data	Two-stage SBM DEA	Chinese regional industrial systems efficiency
An et al. (2015)	Plant-level	Enhanced Russell measure DEA	Environmental efficiency evaluation of thermal power enterprises
Zhu et al. (2014)	Pesticide-level	Input-based two-stage DEA	Eco-efficiency of Pesticides
Zhou et al. (2014)	Plant-level	CCR, BCC, NIRS	Energy efficiency performance of China's transport sector
Zhang et al. (2014)	Province-level	CCR model with DDF	Sustainability performance for China
Yin et al. (2014)	City-level	Input-based CCR	Eco-efficiency of Chinese cities
Wu et al. (2014)	Regional-level	Input-based fixed sum output DEA	Environmental efficiency evaluation of industry in China
Mahdiloo et al. (2014)	Regional-level	Output-based network DEA	Environmental quality efficiency
Huang et al. (2014)	Regional-level	SBM model with DDF	Regional eco-efficiency in China
Hou et al. (2014)	Agricultural systems-level	Input-based CCR with DDF	Sustainable value of degraded soils
Du et al. (2014)	Province-level	Output-based CCR with DDF+ML index	Measurement of the sources of economic growth
Bi et al. (2014a)	Thermal power sector	SBM model with DDF	Environmental regulation affect energy efficiency in China's thermal power generation
Long et al. (2013)	Chinese provinces		Environmental regulatory cost
Wang et al. (2013a)	Province-level	SFA model	Energy and CO <sub>2</sub> performance
He et al. (2013)	Iron and steel firm	Output-based CCR with DDF + ML index	Traditional energy efficiency, productivity, and environmentally sensitive productivity growth
Yang and Wang (2013)	Province-level	BCC model with DDF	Environmental efficiency and regulatory cost
Yuan et al. (2013)	Prefecture-level	Output-based BCC model with DDF	Environmental efficiency and determinants
Zhang and Choi (2013a)	Plant-level	CCR model with DDF + ML index	Total-factor carbon emission change
Zhang and Choi (2013b)	Plant-level	CCR model with DDF + ML index	Pure CO <sub>2</sub> emission change
Zhang and Choi (2013c)	Regional-level	SBM model	Environmental energy efficiency of China's regional economies
Wu et al. (2012)	Regional industrial sector	Input-based CCR with DDF + ML index	Total-factor energy efficiency change
Zhang et al. (2011)	Province-level	Output-based CCR with DDF + ML index	Environmentally sensitive productivity growth and environmental regulatory cost
Chang and Hu (2010)	Chinese provinces	CCR with DDF + ML index	Energy productivity growth
Kaneko et al. (2010)	Thermal power sector	Output-based CCR with DDF	Shadow price of SO <sub>2</sub>
Watanabe and Tanaka (2007)	Province-level industry	Output-based BCC with DDF	Efficiency with SO <sub>2</sub> and determinants
Kaneko and Managi (2004)	Province-level	Output-based CCR with DDF + ML index	Environmentally sensitive productivity growth

**Note:** (1) ML index denotes Malmquist–Luenberger productivity index; (2) SBM denotes slack-based measure

## 2.2. Inverse DEA

This section briefly reviews the origin and development of the inverse DEA methodology. The origin of inverse DEA is inverse optimization. Unlike normal optimization where the objective is finding an optimal solution, in an inverse optimization a feasible solution, which is not necessarily optimal, is given and the objective is to perturb the original data as less as possible in order to make that solution optimal (Ahuja and Orlin, 2001). Burton and Toint (1992) first studied an inverse problem in network flows specifically for the shortest path problems. Since then inverse optimization has been continuously enriched by new applications and a variety of inverse optimization problems in combinatorial optimization have been studied by researchers in the operations research community (Jiang et al. 2011; Pibernik et al. 2011; Ruiz et al. 2013; Wang et al. 2014). However, there are few articles about inverse continuous optimization like inverse linear programming and inverse DEA. Zhang and Liu (1996) investigated the first inverse linear programming model in the literature. Further research studies on inverse linear programming problems are given in Zhang and Liu (1999) and Huang and Liu (1999). One of the few applications of inverse linear programming in the literature is for predicting more accurate forecasting parameters developed in Amin and Emrouznejad (2007). The first inverse DEA methodology as a special case of the general inverse linear programming suggested in Wei et al. (2000) and further developed in Yan et al. (2002). Unlike the standard DEA whose objective is to find the efficiency score, the InvDEA assumes the efficiency given and aims to find the levels of inputs and outputs that are required to realize the desired efficiency score. Despite the potential applicability of the standard DEA in different contexts, there are few applications of inverse DEA that are reported in the literature such as application in resource allocation suggested in Hadi-Vencheh et al. (2008). Further recent of inverse DEA studies can be found in Jahanshahloo et al. (2015), Ghobadi and Jahangiri (2015), Ghiyasi (2017) and Amin et al. (2017a). In addition, Zhang and Cui (2016) discussed an extension of the inverse DEA model and Lim (2016) addressed the frontier change for setting a new product target using a new inverse DEA method. Gattoufi et al. (2014) extended the concept of inverse DEA to the context of mergers and acquisitions (M&A). The proposed inverse DEA in

Gattoufi et al. (2014) determines the optimal levels of inputs and outputs that are required from merging decision making units (DMUs) in order to allow the merged entity to realize a predefined efficiency target. More recently, Amin and Al-Muharrami (2016) addressed new inverse DEA models for mergers with negative data. Moreover, the potential of the inverse DEA has been used in Amin et al. (2017b) to anticipate whether a given restructuring between a group of DMUs makes a minor or a major consolidation. The successful result of the inverse DEA in M&A shows the potential power of this methodology in other sectors. In this paper we introduce an inverse DEA for allocation of CO<sub>2</sub> emissions reduction goal into different two-digit manufacturing industries and different regions.

### **3. Dataset and indicators**

The country level data of Chinese manufacturing industries in 2012 used in this study is mainly derived from China Statistical Yearbook 2013 and China Energy Statistical Yearbook 2013. The province level data is from 31 statistical yearbooks of each province in 2013 respectively. We select the two-digit manufacturing industries in China as the DMUs. According to the new standard on Industrial Classification for National Economic Activities (GB/T4754-2011) enforced by National Bureau of Statistics of China (NBS) from 2012, the number of two-digit manufacturing industries changed to 31. See the following Table 2. The industry statistics cover all industries above designated size, which is 20 million yuan of annual revenue from primary business.

In this paper, we use three indicators including Labor, Asset and Energy as the inputs and two indicators as the outputs, including Gross Industrial Output Value (GIOV) as the desirable output and CO<sub>2</sub> emissions as the undesirable one.

**Table 2.** The two-digit manufacturing industries in China.

No.	Two-digit manufacturing
1	Processing of Food from Agricultural Products
2	Manufacture of Foods
3	Manufacture of Liquor, Beverages and Refined Tea
4	Manufacture of Tobacco
5	Manufacture of Textile
6	Manufacture of Textile, Wearing Apparel and Accessories
7	Manufacture of Leather, Fur, Feather and Related Products and Footwear
8	Processing of Timber, Manufacture of Wood, Bamboo, Rattan, Palm and Straw Products
9	Manufacture of Furniture
10	Manufacture of Paper and Paper Products
11	Printing and Reproduction of Recording Media
12	Manufacture of Articles for Culture, Education, Arts and Crafts, Sport and Entertainment Activities
13	Processing of Petroleum, Coking and Processing of Nuclear Fuel
14	Manufacture of Raw Chemical Materials and Chemical Products
15	Manufacture of Medicines
16	Manufacture of Chemical Fibres
17	Manufacture of Rubber and Plastics Products
18	Manufacture of Non-metallic Mineral Products
19	Smelting and Pressing of Ferrous Metals
20	Smelting and Pressing of Non-ferrous Metals
21	Manufacture of Metal Products
22	Manufacture of General Purpose Machinery
23	Manufacture of Special Purpose Machinery
24	Manufacture of Automobiles
25	Manufacture of Railway, Ship, Aerospace and Other Transport Equipment
26	Manufacture of Electrical Machinery and Apparatus
27	Manufacture of Computers, Communication and Other Electronic Equipment
28	Manufacture of Measuring Instruments and Machinery
29	Other Manufacture
30	Utilization of Waste Resources
31	Repair Service of Metal Products, Machinery and Equipment

The variables used in this study are as follows: (1) Labor refers to the amount of labors in Chinese manufacturing industries. Due to the mobility of Labor, the amount of labor variable is different at different time in one year, so the number of annual average employed persons is taken as the indicator. (2) Asset refers to the amount of total assets. Data on this indicator are obtained by the year-end figures of total assets in the Assets and Liability Table of accounting



records of enterprises. (3) Energy refers to the total consumption of energy of various kinds by the production sectors in the country in a given period of time. (4) In this paper the GIOV is used as a desirable output. This variable has been estimated by dividing Industrial Sales Output Value (ISOV) to Sales Ratio of Products (SRP), as both variables are available for each sub-level manufacturing industry for the year 2013. (5) The CO<sub>2</sub> emission is the undesirable output in our study, which is also estimated based on the consumption of different types of energy. For details on data collection please see Emrouznejad and Yang (2016a, 2016b). The descriptive statistics for the country level dataset can be found also in Emrouznejad and Yang (2016a, 2016b).

#### **4 Methodology and empirical results**

Our main idea in this paper is to conduct the CO<sub>2</sub> emission reduction in a three-stage way. The first stage is to obtain the total amount CO<sub>2</sub> emission reduction from the Chinese government goal, denoted by  $CO2_{total}$ , as our total CO<sub>2</sub> emission quota to reduce. The second stage is to allocate the reduction quota  $CO2_{total}$  to different manufacturing industries, denoted by  $CO2_i$ , where  $i$  denotes different two-digit Chinese manufacturing industries, which satisfy  $\sum_i CO2_i = CO2_{total}$ . The third stage is to further allocate the reduction quota for each industry  $CO2_i$  into different provinces, denoted by  $CO2_{ij}$ , where  $j$  denotes different provinces and the following formula holds:  $\sum_j CO2_{ij} = CO2_i$ .

##### **4.1 Determining the total amount of CO<sub>2</sub> emission in Chinese manufacturing industries**

In manufacturing industries, the Gross Industrial Output Value (GIOV) plays the same role as GDP for the country. Chinese State Council released officially the "National Climate Change Plan (2014-2020)" in the September 2014 and announced China's CO<sub>2</sub> emissions to gross domestic product in 2020 would be reduced by 40% to 45% on the basis of 2005. At the world climate conference in France in June 2015, Chinese Premier Li Keqiang announced China's latest voluntary reduction commitment: China government aim to cut its greenhouse gas emissions intensity by 60-65% (per unit of

gross domestic product) from 2005 levels. Based on the above goal, we can propose CO<sub>2</sub> reduction goal as CO<sub>2</sub> emission/GIOV decrease 60% to 65% based on the level of 2005. The CO<sub>2</sub> emission/GIOV in China from 2004 to 2012 is listed in the following Table 3.

**Table 3.** The CO<sub>2</sub> emission/GIOV in China from 2004 to 2012.

Year	CO <sub>2</sub> emission (10 000 tons)	Gross Industrial Output Value (current prices-2010) (100 million yuan)	CO <sub>2</sub> emission /GIOV	Consumer Price Index (CPI) of China
2004	232270.3895	193961.0561	1.1975	81.8313
2005	253527.1366	217835.7400	1.1638	85.0227
2006	275441.6447	274571.6700	1.0032	86.5673
2007	293235.3426	353630.8400	0.8292	87.8369
2008	325151.5258	441358.3600	0.7367	92.0238
2009	341118.8413	479199.7200	0.7119	97.4532
2010	370079.7298	609558.5000	0.6071	96.7834
2011	395088.9957	733984.0100	0.5383	100.0000
2012	413471.1638	809255.1324	0.5109	105.4706

\*Source: China Statistical Yearbooks 2005 - 2013, China Energy Statistical Yearbook (Note: According to OECD statistics, we set Index 2010=100)

As it is been explained in Emrouznejad and Yang (2016a, 2016b) the value of GIOV transform to constant price in 2010 using the Consumer Price Index (CPI) of China, as shown in the last column of Table 3. This transformation approach is used in many other researches, *e.g.* Oh and Heshmati (2010). The CPI data is derived from OECD (2010).

Therefore in this paper we set the goal to decrease 60% to 65% of the level of CO<sub>2</sub> emission/GIOV in 2012 based on that in 2005. Thus CO<sub>2</sub> emission/GIOV in 2012 should be in the range of [0.4073, 0.4655]. However the real ratio of CO<sub>2</sub> emission/GIOV reaches 0.5109. If Chinese government achieves the goal of the CO<sub>2</sub> emission in 2012, the CO<sub>2</sub> emission in 2012 should be [329646.7686, 376739.1641]. However the real amount of CO<sub>2</sub> emission in manufacturing industries in China is 413471.1638 (10,000 tons). Thus the CO<sub>2</sub> emission reduction gap should be [36731.9997, 83824.3952] in the unit of 10,000 tons. As the CO<sub>2</sub> emission reduction

of Chinese government is an interval, we use the lower bound, which is 36731.9997 (unit: 10 thousand tons), as the minimal CO<sub>2</sub> reduction goal in this paper.

#### 4.2. A new InvDEA model for CO<sub>2</sub> emission quota allocation

Assume that there are  $n$  DMUs where the  $j^{th}$  DMU use  $M$  inputs  $x_{ij}$  ( $i = 1, \dots, M$ ) and produces  $R$  good outputs  $y_{rj}^g$  ( $r = 1, \dots, R$ ) and  $P$  undesirable or bad outputs  $y_{pj}^b$  ( $p = 1, \dots, P$ ), for each  $j = 1, \dots, n$ . Let  $L$  be the set of selected DMUs for reducing undesirable outputs. Generally in our modeling, we assume that  $L \subseteq \{1, \dots, n\}$  and reducing undesirable outputs from all DMUs means that  $L = \{1, \dots, n\}$ . Assume all the DMUs in  $L$  would keep their efficiency scores at least the same as before reducing bad outputs. Moreover, let  $\alpha_{ik}$ ,  $\beta_{rk}$ ,  $\gamma_{pk}$  be the levels of the  $i^{th}$  input,  $r^{th}$  good output and  $p^{th}$  bad output of the  $k^{th}$  DMU, respectively, after reducing the bad outputs (for each  $i = 1, \dots, M$ ,  $r = 1, \dots, R$ ,  $p = 1, \dots, P$  and every  $k \in L$ ).

First, we propose the following assumptions for the CO<sub>2</sub> emission reduction in our paper:

**Assumption 1.** *The efficient frontier will remain constant in the process of CO<sub>2</sub> emissions reduction.*

Based on this assumption, we assume  $F$  be the set of all efficient DMUs identified by the following model (1).

$$\begin{aligned} \vec{D}_{DDF,v}^G(X_k, Y_k, B_k, g_Y, g_B) = \max \vec{\beta}_k \\ s. t. \begin{cases} \sum_{j=1}^n \lambda_j X_j \leq X_k \\ \sum_{j=1}^n \lambda_j Y_j \geq (1 + \vec{\beta}_k) Y_k \\ \sum_{j=1}^n \lambda_j B_j = (1 - \vec{\beta}_k) B_k \\ \sum_{j=1}^n \lambda_j = 1 \\ \lambda_j \geq 0, j = 1, \dots, n \end{cases} \end{aligned} \quad (1)$$

Based on the results from model (1), we can have the inefficient DMUs as the targets of our CO<sub>2</sub> emission reduction.

The proposed inverse DEA method in this paper is the first attempt in the literature to

determine optimal allocation of CO<sub>2</sub> emissions. The base DEA model for the inverse problem can be any DEA model developed for undesirable output. In this paper, we consider the directional distance DEA model (1) as the base model simply because it is more relevant to the application.

It should be noted that there is enough space for CO<sub>2</sub> emission reduction goal of Chinese government using the inefficient DMUs as the reduction targets in this paper. Therefore, we can assume the *Assumption 1* holds. Otherwise, if we cannot achieve the government goal of reducing CO<sub>2</sub> emissions by inefficient DMUs only, we need to consider to reduce CO<sub>2</sub> emissions from efficient DMUs, which means the efficient frontiers will shift towards the direction of more desirable output(s) and less undesirable output(s). In such case, the problem will be more complex. A possible solution is to assume all the DMUs reduce further the same proportion of CO<sub>2</sub> emissions to achieve this goal, which technically means the frontiers shift in an average way.

*Assumption 2: The efficiencies of all DMUs will not decrease in the process of CO<sub>2</sub> emissions reduction.*

This assumption indicates the CO<sub>2</sub> emissions reduction will not damage the DMUs' efficiencies including both efficient and inefficient ones. Thus, the efficiency of none of DMUs will be deteriorated after the CO<sub>2</sub> emissions reduction.

*Assumption 3: There exist the possible policy thresholds for certain input or output.*

In the real scenario of policy making, the policy makers often need to consider some policy thresholds for certain input or output indicators. For example, in China, it is very difficult to fire too much employee in the manufacturing industries. Furthermore, the gross industrial output value (GIOV) cannot be reduced too much, because the Chinese government needs to keep the growth rate of gross domestic product (GDP) at a certain level. Therefore, in our model we consider such types of policy thresholds to make our model more reasonable and flexible.

Based on the above three assumptions, we propose the following InvDEA model for

allocation the given amount of bad outputs reductions to different DMUs.

**Remark 1.** In certain case, we have to shift the efficient frontier in the process of CO<sub>2</sub> emissions reduction to meet the CO<sub>2</sub> emission reduction targets. We will discuss this issue in the following subsection 4.4.

$$\begin{aligned}
& \min \sum_{k \in L} \sum_{i=1}^m \alpha_{ik} - \sum_{k \in L} \sum_{r=1}^R \beta_{rk} \\
& \text{s.t.} \\
& \sum_{j \in F} \lambda_j^k x_{ij} + \sum_{j \in L} \alpha_{ij} \lambda_{kj} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g + \sum_{j \in L} \beta_{rj} \lambda_{kj} - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b + \sum_{j \in L} \gamma_{pj} \lambda_{kj} - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k + \sum_{j \in L} \lambda_{kj} = 1, \quad \forall k \in L \tag{2} \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F_k, k \in L \\
& \lambda_{kj} \geq 0, \quad k, j \in L
\end{aligned}$$

The objective of the InvDEA model (2) is to minimize the sum of the amount of the inputs that should be kept and minimizing the amount of good outputs that should be dropped from each DMU in  $L$  in a way that the amount of  $a_p$  from the  $p^{th}$  ( $p = 1, \dots, P$ ) bad output of DMUs in  $L$  should be reduced. There is also limitation on the amount of reduction of good outputs shown by the constraints  $(1 - c_{rk}) y_{rk}^g \leq \beta_{rk} \leq y_{rk}^g$  ( $\forall k \in L, r = 1, \dots, R$ ) where  $c_{rk}$  is a constant given by decision makers. For instance, a policy of reducing at most 5% of good outputs in order to reduce a given amount of bad outputs, if feasible, can be employed by considering  $c_{rk} = 0.05$ . Furthermore,  $\hat{\beta}_k$  is a parameter that guarantees the efficiency scores of DMUs in  $L$  would not be decreased after bad outputs reduction since  $0 \leq \hat{\beta}_k \leq \overline{\beta}_k^*$ , where  $\overline{\beta}_k^*$  is the optimal value of DEA model (1).

It should be noted that the Assumptions 1-3 are given to simplify the implementation of the suggested inverse DEA model (2). In fact, the non-linear model (2) can be simplified to a linear programming problem (3). Assumption 1 guarantees that there would be no frontier change after CO<sub>2</sub> emission reduction and this would simplify the non-linear model to a linear model. The following theorem shows the possibility of this relaxation.

**Theorem 1:** The NLP InvDEA model (2) can be simplified to the following relaxed LP InvDEA model.

$$\begin{aligned}
& \min \sum_{k \in L} \sum_{i=1}^m \alpha_{ik} - \sum_{k \in L} \sum_{r=1}^R \beta_{rk} \\
& s.t. \\
& \sum_{j \in F} \lambda_j^k x_{ij} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k = 1, \quad \forall k \in L \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F, k \in L
\end{aligned} \tag{3}$$

**Proof:** We first assume that  $L$  contains only inefficient DMUs. This means that none of the DMUs in  $L$  can be a benchmark for itself and/or other DMUs, implying that  $\lambda_j^{k*} = 0$  for all  $k, j \in L$  in any optimal solution of the InvDEA model (2). The NLP InvDEA Model (2) can be similarly relaxed to model (3) even if some of the inefficient DMUs in  $L$  targeted to be fully efficient after reducing bad outputs, or equivalently  $\hat{\beta}_k = 0$  for some  $k \in L$ . In fact, these new efficient DMUs fall on the efficiency frontier and therefore can be presented in terms of the a convex combination of the existing efficient DMUs.

Now, consider a case when reducing bad outputs from an efficient DMU<sub>k</sub> is at concern or equivalently  $k \in L$ . According to the assumption we have  $0 \leq \hat{\beta}_k \leq \overrightarrow{\beta}_k^* = 0$ , and so  $\hat{\beta}_k = 0$ . Therefore, DMU<sub>k</sub> is efficient before and after reducing bad outputs and therefore can be presented in terms of DMU<sub>k</sub> itself. This concludes that

$$y_{pk}^b - \gamma_{pk}^* = 0, p = 1, \dots, P$$

Or equivalently reducing bad outputs from an efficient DMU would be zero. It worth noting that this would be the case if we wouldn't change the efficiency frontier. This completes the proof. ■

It should be noted that in certain situations, there may be the cases that model (3) will not have feasible solutions because of the setting of policy thresholds. For example, as we mentioned above, there is a limitation on the amount of reduction of good outputs shown by the constraints  $(1 - c_{rk})y_{rk}^g \leq \beta_{rk} \leq y_{rk}^g$  ( $\forall k \in L, r = 1, \dots, R$ ) where  $c_{rk}$  is a constant given by decision makers. Those policy thresholds may not provide enough space for CO<sub>2</sub> emission reduction. Thus, we suggest to decide the lower bound of those thresholds  $C_r^*$  using the following model (4) as the parameters in model (3), which mean the decision makers have to allow to reduce the good outputs at least to the level of  $(1 - C_r^*), r = 1, \dots, R$ .

$$\begin{aligned}
& \min \sum_{r=1}^R C_r \\
& s. t. \\
& \sum_{j \in F} \lambda_j^k x_{ij} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k = 1, \quad \forall k \in L \tag{4} \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& c_{rk} \geq C_r, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F, k \in L
\end{aligned}$$

Therefore, we use the following procedure to conduct the allocation the CO<sub>2</sub> emission reduction among designated DMUs.

**Procedure 1.**

**Step 1:** Use model (1) to divide all DMUs into two sets of efficient and inefficient DMUs respectively, which are denoted as  $F$  and  $L$  respectively.

**Step 2:** Select all inefficient DMUs in the set  $L$  as the targets for CO<sub>2</sub> reduction.

**Step 3:** Set policy thresholds for certain input or output for CO<sub>2</sub> reduction.

**Step 4:** Use model (3) to allocate CO<sub>2</sub> emission reduction into inefficient DMUs in the set  $L$ .

**Table 4.** The results of model (1) and two sets.



DMUs		$\bar{\beta}_k^*$	Sets
DMU <sub>1</sub>	Processing of Food from Agricultural Products	0.0000	<i>F</i>
DMU <sub>2</sub>	Manufacture of Foods	0.4274	<i>L</i>
DMU <sub>3</sub>	Manufacture of Liquor, Beverages and Refined Tea	0.4269	<i>L</i>
DMU <sub>4</sub>	Manufacture of Tobacco	0.0000	<i>F</i>
DMU <sub>5</sub>	Manufacture of Textile	0.3688	<i>L</i>
DMU <sub>6</sub>	Manufacture of Textile, Wearing Apparel and Accessories	0.0699	<i>L</i>
DMU <sub>7</sub>	Manufacture of Leather, Fur, Feather and Related Products and Footwear	0.0000	<i>F</i>
DMU <sub>8</sub>	Processing of Timber, Manufacture of Wood, Bamboo, Rattan, Palm and Straw Products	0.0000	<i>F</i>
DMU <sub>9</sub>	Manufacture of Furniture	0.1795	<i>L</i>
DMU <sub>10</sub>	Manufacture of Paper and Paper Products	0.8209	<i>L</i>
DMU <sub>11</sub>	Printing and Reproduction of Recording Media	0.3142	<i>L</i>
DMU <sub>12</sub>	Manufacture of Articles for Culture, Education, Arts and Crafts, Sport and Entertainment Activities	0.0000	<i>F</i>
DMU <sub>13</sub>	Processing of Petroleum, Coking and Processing of Nuclear Fuel	0.0000	<i>F</i>
DMU <sub>14</sub>	Manufacture of Raw Chemical Materials and Chemical Products	0.0000	<i>F</i>
DMU <sub>15</sub>	Manufacture of Medicines	0.4288	<i>L</i>
DMU <sub>16</sub>	Manufacture of Chemical Fibres	0.3603	<i>L</i>
DMU <sub>17</sub>	Manufacture of Rubber and Plastics Products	0.2527	<i>L</i>
DMU <sub>18</sub>	Manufacture of Non-metallic Mineral Products	0.3148	<i>L</i>
DMU <sub>19</sub>	Smelting and Pressing of Ferrous Metals	0.0000	<i>F</i>
DMU <sub>20</sub>	Smelting and Pressing of Non-ferrous Metals	0.0000	<i>F</i>
DMU <sub>21</sub>	Manufacture of Metal Products	0.0771	<i>L</i>
DMU <sub>22</sub>	Manufacture of General Purpose Machinery	0.0491	<i>L</i>
DMU <sub>23</sub>	Manufacture of Special Purpose Machinery	0.2183	<i>L</i>
DMU <sub>24</sub>	Manufacture of Automobiles	0.0000	<i>F</i>
DMU <sub>25</sub>	Manufacture of Railway, Ship, Aerospace and Other Transport Equipment	0.3776	<i>L</i>
DMU <sub>26</sub>	Manufacture of Electrical Machinery and Apparatus	0.0000	<i>F</i>
DMU <sub>27</sub>	Manufacture of Computers, Communication and Other Electronic Equipment	0.0000	<i>F</i>
DMU <sub>28</sub>	Manufacture of Measuring Instruments and Machinery	0.0664	<i>L</i>
DMU <sub>29</sub>	Other Manufacture	0.7736	<i>L</i>
DMU <sub>30</sub>	Utilization of Waste Resources	0.0000	<i>F</i>
DMU <sub>31</sub>	Repair Service of Metal Products, Machinery and Equipment	0.0000	<i>F</i>

### 4.3 Allocate the CO<sub>2</sub> emission reduction to different two-digit Chinese manufacturing industries

As we discussed in subsection 4.1, we use the lower bound of CO<sub>2</sub> reduction interval, which is 36732(unit: 10 thousand tons), as the minimal CO<sub>2</sub> reduction goal in this paper. We use the above **Procedure 1** to conduct the allocation the CO<sub>2</sub> emission reduction among different two-digit Chinese manufacturing industries.

**Step 1:** Two sets of efficient and inefficient two-digit Chinese manufacturing industries are as follows (See Table 4):

**Step 2:** We select all inefficient DMUs in set  $L$  as the targets for CO<sub>2</sub> reduction in the following Table 5. Also we assume the parameter  $\hat{\beta}_k$  that guarantees the efficiency scores of DMUs in  $L$  wouldn't be decreased after CO<sub>2</sub> emission reduction.

**Table 5.** The inefficient DMUs in set  $L$ .

Inefficient DMUs		$\overrightarrow{\beta}_k^*$ in model (1)	Sets	$\hat{\beta}_k$ (case 1)	$\hat{\beta}_k$ (case 2)
DMU <sub>2</sub>	Manufacture of Foods	0.4274	$L$	0.4274	0.3847
DMU <sub>3</sub>	Manufacture of Liquor, Beverages and Refined Tea	0.4269	$L$	0.4269	0.3842
DMU <sub>5</sub>	Manufacture of Textile	0.3688	$L$	0.3688	0.3319
DMU <sub>6</sub>	Manufacture of Textile, Wearing Apparel and Accessories	0.0699	$L$	0.0699	0.0629
DMU <sub>9</sub>	Manufacture of Furniture	0.1795	$L$	0.1795	0.1616
DMU <sub>10</sub>	Manufacture of Paper and Paper Products	0.8209	$L$	0.8209	0.7388
DMU <sub>11</sub>	Printing and Reproduction of Recording Media	0.3142	$L$	0.3142	0.2828
DMU <sub>15</sub>	Manufacture of Medicines	0.4288	$L$	0.4288	0.3859
DMU <sub>16</sub>	Manufacture of Chemical Fibres	0.3603	$L$	0.3603	0.3243
DMU <sub>17</sub>	Manufacture of Rubber and Plastics Products	0.2527	$L$	0.2527	0.2274
DMU <sub>18</sub>	Manufacture of Non-metallic Mineral Products	0.3148	$L$	0.3148	0.2833
DMU <sub>21</sub>	Manufacture of Metal Products	0.0771	$L$	0.0771	0.0694
DMU <sub>22</sub>	Manufacture of General Purpose Machinery	0.0491	$L$	0.0491	0.0442
DMU <sub>23</sub>	Manufacture of Special Purpose Machinery	0.2183	$L$	0.2183	0.1965
DMU <sub>25</sub>	Manufacture of Railway, Ship, Aerospace and Other Transport Equipment	0.3776	$L$	0.3776	0.3398
DMU <sub>28</sub>	Manufacture of Measuring Instruments and Machinery	0.0664	$L$	0.0664	0.0598
DMU <sub>29</sub>	Other Manufacture	0.7736	$L$	0.7736	0.6962

We propose two ways to determine the parameter  $\hat{\beta}_k$ :

**Case 1:** The first one is to keep the  $\hat{\beta}_k$  as the value of  $\overrightarrow{\beta}_k^*$  in model (1), which

means all inefficient DMUs keep their efficiencies in the process of reducing CO<sub>2</sub> emission.

**Case 2:** The second one is to improve the directional distance  $\overrightarrow{\beta}_k^*$  by 10%, which means that we define  $\hat{\beta}_k = 90\% \times \overrightarrow{\beta}_k^*$  for each  $k \in L$ .

**Step 3:** We set the policy threshold for at least 95% of the good output GIOV should be kept. Thus we have the following constraints in model (3):

$$(1 - 0.05) y_{rk}^g \leq \beta_{rk}, \forall k \in L, r = 1, \dots, R$$

**Step 4:** We use model (3) to allocate CO<sub>2</sub> emission reduction into inefficient DMUs in the set L. See Table 6.

**Table 6.** The CO<sub>2</sub> emission allocation. (unit: 10 thousand tons)

Inefficient DMUs		CO <sub>2</sub> emission allocation	
		$(\hat{\beta}_k\text{-Case 1})$	$(\hat{\beta}_k\text{-Case 2})$
DMU <sub>2</sub>	Manufacture of Foods	0.000	175.515
DMU <sub>3</sub>	Manufacture of Liquor, Beverages and Refined Tea	0.000	0.000
DMU <sub>5</sub>	Manufacture of Textile	0.000	0.000
DMU <sub>6</sub>	Manufacture of Textile, Wearing Apparel and Accessories	0.000	0.000
DMU <sub>9</sub>	Manufacture of Furniture	3.299	4.988
DMU <sub>10</sub>	Manufacture of Paper and Paper Products	1380.958	1109.357
DMU <sub>11</sub>	Printing and Reproduction of Recording Media	0.000	0.000
DMU <sub>15</sub>	Manufacture of Medicines	0.000	0.000
DMU <sub>16</sub>	Manufacture of Chemical Fibres	758.759	0.000
DMU <sub>17</sub>	Manufacture of Rubber and Plastics Products	0.000	0.000
DMU <sub>18</sub>	Manufacture of Non-metallic Mineral Products	34016.294	34740.234
DMU <sub>21</sub>	Manufacture of Metal Products	0.000	0.000
DMU <sub>22</sub>	Manufacture of General Purpose Machinery	0.000	0.000
DMU <sub>23</sub>	Manufacture of Special Purpose Machinery	0.000	0.000
DMU <sub>25</sub>	Manufacture of Railway, Ship, Aerospace and Other Transport Equipment	0.000	0.000
DMU <sub>28</sub>	Manufacture of Measuring Instruments and Machinery	0.000	0.000
DMU <sub>29</sub>	Other Manufacture	572.689	701.904
Total		36731.999	36731.999

#### 4.4 Allocate the CO<sub>2</sub> emission reduction to different regions

Without loss of generality, we assume that we select the Case 2 in subsection 4.3 as the results for the further allocation of the CO<sub>2</sub> emission reduction to different regions.

That means we assume that we aim to improve the directional distance  $\overline{\beta}_k^*$  by 10%, i.e.,  $\hat{\beta}_k = 90\% \times \overline{\beta}_k^*$  for each  $k \in L$ . Therefore we use the following procedure to conduct the second stage allocation of CO<sub>2</sub> emission reduction.

**Step 1.** We first select the DMUs for the second stage of allocating the CO<sub>2</sub> emission reduction to different regions in China. Based on the results in the above Table 6, we have the following Table 7 for the further allocation of CO<sub>2</sub> emission reduction.

**Table 7.** The DMUs to be further allocated. (unit: 10 thousand tons)

Inefficient DMUs		CO <sub>2</sub> emission allocation ( $\hat{\beta}_k$ -Case 2)
DMU <sub>2</sub>	Manufacture of Foods	175.515
DMU <sub>9</sub>	Manufacture of Furniture	4.988
DMU <sub>10</sub>	Manufacture of Paper and Paper Products	1109.357
DMU <sub>18</sub>	Manufacture of Non-metallic Mineral Products	34740.234
DMU <sub>29</sub>	Other Manufacture	701.904

**Step 2.** We conduct the similar procedure to Procedure 1 in subsection 4.3 where we substitute the Chinese manufacturing in Procedure 1 for the 31 different provinces of China. Furthermore, we also assume that we aim to improve the directional distance, which is obtained from model (1) when applied to the 31 different provinces, by 10%. We repeat this process for DMU<sub>2</sub>, DMU<sub>9</sub>, DMU<sub>10</sub>, DMU<sub>18</sub>, and DMU<sub>29</sub>. Thus we have the final results as follows (See Table 8):

It should be noted here that for the Manufacture of Foods, Manufacture of Furniture, Manufacture of Paper and Paper Products, and Other Manufacture, the policy thresholds for good output reduction are all 5%, which provides enough space for CO<sub>2</sub> emissions reduction. However for the Manufacture of Non-metallic Mineral Products, model (3) cannot find feasible solution for CO<sub>2</sub> emission reduction with the constraints of the policy thresholds for good output reduction are all 5%. Therefore,

we first use model (4) to find the lower bound of thresholds on GIOV as  $C^* = 37.73\%$  using model (4), which mean the decision makers have to allow to reduce the GIOV at least to the level of  $1 - C^* = 62.27\%$ .

**Table 8.** The CO<sub>2</sub> emission allocation in the second stage. (unit: 10 thousand tons)

<b>Regions</b>	<b>DMU<sub>2</sub></b>	<b>DMU<sub>9</sub></b>	<b>DMU<sub>10</sub></b>	<b>DMU<sub>18</sub></b>	<b>DMU<sub>29</sub></b>
Beijing	0.000	0.000	0.000	309.717	0.550
Tianjin	0.000	0.000	0.000	137.314	0.439
Hebei	0.000	0.000	0.000	2146.731	3.054
Shanghai	0.000	0.000	83.583	774.586	8.095
Jiangsu	0.000	0.000	34.813	3081.870	34.393
Zhejiang	0.000	2.151	0.000	1759.959	46.269
Fujian	0.000	0.000	0.000	1469.718	15.602
Shandong	0.000	0.000	0.000	0.000	17.738
Guangdong	0.000	0.000	49.630	3787.145	87.358
Hainan	0.000	0.000	0.000	0.000	17.074
Liaoning	0.000	0.000	0.000	0.000	25.137
Jilin	0.000	0.000	31.932	0.000	1.011
Helongjiang	0.000	0.000	0.000	0.000	0.286
Anhui	0.000	0.000	0.000	2547.221	1.023
Jianxi	0.000	0.000	0.000	0.000	4.067
Henan	0.000	0.000	265.378	1665.366	2.624
Hubei	41.888	0.000	0.000	1606.724	4.764
Hunan	0.000	0.000	131.797	1654.607	409.330
Shanxi	0.000	0.000	0.000	0.000	4.228
Inner Mongolia	0.000	0.000	0.000	0.000	2.672
Guangxi	0.000	2.837	97.441	2148.201	1.579
Chongqing	0.000	0.000	71.824	0.000	4.129
Sichuan	119.836	0.000	206.662	3572.914	1.423
Guizhou	0.000	0.000	3.763	1636.424	0.837
Yunnan	0.000	0.000	59.840	2100.445	2.933
Tibet	0.000	0.000	0.000	0.000	5.291
Shaanxi	13.790	0.000	8.627	1387.635	0.550
Gansu	0.000	0.000	0.000	991.986	0.439
Qinghai	0.000	0.000	0.000	198.552	3.054
Ningxia	0.000	0.000	54.753	364.846	8.095
Xinjiang	0.000	0.000	9.315	1398.272	34.393
<b>Total CO<sub>2</sub> emission reduction allocation</b>	<b>175.515</b>	<b>4.988</b>	<b>1109.357</b>	<b>34740.234</b>	<b>701.904</b>

For Other Manufacture, another case happens. We first use model (1) to find the efficient regions and inefficient regions. Here we list the inefficient DMUs in set  $L$  as follows (See Table 9):

**Table 9.** Total CO<sub>2</sub> emission in inefficient regions of Other Manufacture (unit: 10 thousand tons)

Inefficient regions	Sets	CO <sub>2</sub> emission	CO <sub>2</sub> emission of projections of inefficient regions on the frontier of model (1)
Hebei	$L$	3.250	2.606
Shanghai	$L$	8.394	3.965
Guangdong	$L$	93.845	85.944
Liaoning	$L$	17.743	8.855
Henan	$L$	4.090	0.309
Inner Mongolia	$L$	4.420	2.552
Chongqing	$L$	1.704	1.664
Sichuan	$L$	4.424	3.908
<b>Total</b>		<b>137.870</b>	<b>109.803</b>

From the above Table 9, we can see that the total CO<sub>2</sub> emission is 137.870. However the CO<sub>2</sub> emission reduction quota for Other Manufacture is 701.904, which means using only inefficient regions as the reduction targets cannot meet the requirements. Here we have to use the efficient regions as the CO<sub>2</sub> emission reduction targets also. See Column 3 in Table 9. As mentioned in Remark 1, in certain cases we need to replace the Assumption 1-3 as the following Assumption 4:

***Assumption 4.** The efficient frontier can be shift in the process of CO<sub>2</sub> emissions reduction using an average way, which means the existing technology need to be improved by reducing the same proportion of CO<sub>2</sub> emission for each DMU.*

From this assumption, we can see that, for Other Manufacture, we first find the amount of CO<sub>2</sub> emission of projections of inefficient regions on the frontier of model (1). See Column 4 in Table 9. Therefore, we can see that if we fix the efficient frontier in model (1), the maximum amount of CO<sub>2</sub> emission reduction is 28.068. There is still a big gap between our CO<sub>2</sub> emission reduction target 701.904, which is  $701.904 - 28.068 = 673.836$ . Thus we allocate this 673.836 CO<sub>2</sub> emission to all regions using a proportional way and we can have the final allocation results as shown in the

Column 6 of Table 8. That means there is a strong need for Other Manufacture to improve its technology to meet the CO<sub>2</sub> emission reduction targets.

The conventional and inverse DEA are two different methods in nature. They solve two different type of problems. Being completely two different methods, the results are not really comparable. The conventional DEA focuses on the data and finds the efficiency score while the inverse DEA focuses on the efficiency and finds the data point.

## **5. Concluding Remarks**

In this paper we tried to tackle the problem of allocating the CO<sub>2</sub> emissions quota set by government goal in Chinese manufacturing industries to different Chinese regions. This objective is implemented using a three-stage way based on several assumptions. In the first stage, we obtained the total amount CO<sub>2</sub> emission reduction from the Chinese government goal as our total CO<sub>2</sub> emission quota to allocate to different regions to reduce. Based on this, we further allocate the reduction quota to different two-digit level manufacturing industries in China in the second stage. In the last stage we allocate the CO<sub>2</sub> emissions reduction quota for each industry into different provinces. The empirical results can provide an alternative solution for the allocation of CO<sub>2</sub> emissions reduction in China for policy making.

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