Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

A Novel Mathematical Study on the Predictions of Volatile Price of Gold Using Grey Models

A. Manickam[1,3], S. Indrakala[2], Pushpendra Kumar[3*]

[1]School of Advanced Sciences & Languages, VIT Bhopal University, Kothri Kalan-466 114, Sehore, Madhya Pradesh, India
[2]Kunthavai Nacchaiyar Government Arts College for Women, Thanjavur 613007, Tamil Nadu, India
[3]Institute for the Future of Knowledge, University of Johannesburg, PO Box 524, Auckland Park 2006, South Africa
Email: kumarsaraswatpk@gmail.com

Received: 29 January 2023; **Revised:** 18 March 2023; **Accepted:** 14 April 2023

Abstract: In the history of the gold market, contemporary gold prices are higher than the previous values, and the current gold market is highly non-linear and unpredictable. Gold is the most popular precious metal for investment out of all the precious metals. The gold market, like other markets, is vulnerable to speculation and volatility. Gold has served as a secure base in several countries when compared to other precious metals used for investment. In this study, we suggest a time series model for predicting daily variations in the amount of gold per gram in Indian rupees. To increase the forecasting accuracy, we use a hybrid prediction method known as the Grey-Fourier Markov model which includes Grey models (GM), Fourier series, and Markov state transition. Here, we divide the forecasting process into three steps. The first step is to simulate the data of daily volatile price of gold using GM (1, 1), GM (2, 1), and Grey Verhulst models and also to calculate corresponding residual errors. In the second step, we utilize the residual error produced by the above grey models to predict the trend of the gold price with the help of the Fourier series and Markov Model. In the third step, we use hybrid grey models to improve the precision. Finally, we conclude that the proposed methodology outperforms the aforementioned strategies in terms of results.

***Keywords*:** GM model, Grey Verhulst models, Fourier series, Markov chain, forecasting models

MSC: 03C30, 62P20

1. Introduction

Gold worth prediction has become a major investigation topic around the world in recent years, and the price of gold has fluctuated dramatically in recent years. Shareholders, government officials, and scholars all over the world have been paying too much attention to gold price predictions. The grey prediction model is an important component of the grey system theory [1]. In [2], the authors provided a comparison between Grey model GM (1,1) and GM (2,1) model. The DGM (2, 1) model has been widely used in various fields. Moreover, many scholars have performed different aspects of research and improvement for the model and studying the predictive performance of the DGM (2, 1) model and its improved models [3].

Yao and Tan L [4] established a forecasting method for predicting stock market price indices by means of a neural

DOI: https://doi.org/10.37256/cm.4220232389

network with stochastic time real functions, and [5] defined trial evidence that a neural network is an appropriate technique for predicting foreign currency exchange rates. The complexity of incomplete information has occurred in a variety of situations. A grey prediction mode was proposed in [6] to solve such problems. Gold is a widely traded commodity, and projecting its price is both theoretical and practical. The restricted information in this study is thought to be useful to financial managers. In reality, knowing when gold will rise in price permits financial actors to make the greatest trading decisions. This study took into account the accuracy of gold price forecasting.

In [7], a Grey model was developed, which is a forecasting methodology that can solve the difficulty of tiny data or sample sizes, as well as inadequate data information. After calculating residual series from the original and forecasted series, the accuracy of the Grey Fourier forecasting model was enhanced. Using the discrete decomposition of the Fourier series, AGO's grey theory method and the regularity of the origin series reduce forecasting accuracy. Finally, by determining the transition probability of an object from one state to another in several stages, Markov Chains can be used to forecast the next state or condition.

The Grey-Fourier Markov (GFM) forecasting model [8] is a shared grey model with high exactness prediction, which integrated *GM* (1, 1), *GM* (2, 1) Grey Verhulst model, Fourier series and Markov chain. The purpose of that study will be investigated based on the daily data of gold price and to compares the results of Grey-Fourier Markov Grey-Fourier Markov (2, 1), and Grey Verhulst-Fourier Markov Model. The organization of this paper is as follows. Section 2 describes the Mathematical formulation and assumptions are found for Grey-Fourier Markov models. Section 3, an application of the proposed models with the experimentations of gold price forecasting is given and results are discussed. In Section 4, we conclude the result performance of Hybrid Grey Models. Based on the daily trend of gold price, the GM (1, 1)-Fourier Markov Model, GM (2, 1)-Fourier Markov Model, and GM Verhulst Fourier Markov Model will be examined in this study.

2. Materials and methods

The Grey Theory is a foundational model for generating input and output sequences. As a result, it is necessary to check whether the original sequences satisfy a quasi-smoothness and an order accumulated generating operation sequences satisfy the law of quasi-exponentially for predicting and forecasting the output sequences based on the associated sequences in order to predict and forecast the output sequences. Suppose that the original sequence is $g_{(0)}(x)$, $x = 1$ to n then the one-time collected generating operation (1-AGO) sequence is $g_{(1)}(x)$, $x = 1$ to n. smoothness proportion $R(W)$ can be dignified by the following formula [9]

$$R(W) = \frac{g_{(0)}(W)}{\sum_{x=1}^{k} g_{(0)}(W)} = \frac{g_{(0)}(W)}{g_{(1)}(W-1)},\ W = 3, 4, \ldots, n \tag{1}$$

at $W > 3$ and $R(W) < 0.5$;

The corresponding quasi-exponential of $g_{(1)}$ tested as follows [16]

$$Q_{(1)}(W) = \frac{g_{(1)}(W)}{g_{(1)}(W-1)},\ W = 3, 4, \ldots, n \tag{2}$$

at $W > 3$ and $Q_{(1)}(W) \in [1, 3/2]$;

Affording to the above examination, the real data $g_{(0)}$ are a positive quasi-smooth categorization, whereas the gathering systems $g_{(1)}$ follow a quasi-exponential rule.

2.1 *Grey likelihood (1, 1) model*

The mathematical design of the Grey prediction model (*GM* Model) has described as follows:

Assume there are n elements in an original non-negative sequence [6,10]

$$g_{(0)} = \left\{g_{(0)}(1), g_{(0)}(2), g_{(0)}(3), \ldots, g(W)\right\} \tag{3}$$

$g_{(0)}(W)$ is the cost at time $W = 1$ to n.

The news $g_{(1)}$ structure can be created by 1-AGO from the original system $g_{(0)}$, which is defined by [11-13]

$$g_{(1)} = \left\{g_{(1)}(1), g_{(1)}(2), g_{(1)}(3), \ldots, g_{(1)}(W), \ldots, g_{(1)}(n)\right\} \tag{4}$$

Here,

$$g_{(1)}(W) = \sum_{x=1}^{W} g_{(0)}(x),\ W = 1, 2, \ldots, n$$

The GM (1, 1) model has measured from the concept of first order Differential Equation with one variable defined as follows:

$$\frac{dg_{(1)}(W)}{dt} + \theta g_{(1)}(W) = \eta \tag{5}$$

As a result of means of the least square method, we single-minded these two coefficients defined as follows:

$$[\theta, \eta]^T = (M^T M)^{-1} M^T N$$

$$M = \begin{bmatrix} -\dfrac{g_{(1)}(1) + g_{(1)}(2)}{2} & 1 \\ -\dfrac{g_{(1)}(2) + g_{(1)}(3)}{2} & 1 \\ \vdots & \vdots \\ -\dfrac{g_{(1)}(n-1) + g_{(1)}(n)}{2} & 1 \end{bmatrix} \tag{6}$$

and $N = [g_{(0)}(2), g_{(0)}(3), \ldots, g_{(0)}(n)]^T$.

Resolving equation (4), we get the time reaction function of the GM (1, 1) as follows

$$\hat{g}_{(0)}(W) = \left(g_{(0)}(1) - \frac{\eta}{\theta}\right) e^{-\theta(W-1)} + \frac{\eta}{\theta},\ W = 2 \text{ to } n. \tag{7}$$

Based on 1-AGO, the predicted series $\hat{g}_{(0)}$ can be obtained as follows

$$\hat{g}_{(0)}(1) = \hat{g}_{(1)}(1)$$

$$\hat{g}_{(0)}(1) = \hat{g}_{(1)}(W) - \hat{g}_{(1)}(W-1),\ W = 2, 3, 4, \ldots, n.$$

Correspondingly, the second-order LDE of *GM* (2, 1) model can be follow-on

$$\frac{d^2 g_{(1)}(W)}{dt^2} + \theta \frac{dg_{(1)}(W)}{dt} = \eta \tag{8}$$

$$\hat{\theta} = [\theta, \eta]^T = (M^T M)^{-1} M^T N \tag{9}$$

$$M = \begin{bmatrix} -g_{(0)}(2) & 1 \\ -g_{(0)}(3) & 1 \\ \vdots & \vdots \\ -g_{(0)}(n) & 1 \end{bmatrix} \text{ and } N = \begin{bmatrix} g_{(0)}(2) - g_{(0)}(1) \\ g_{(0)}(3) - g_{(0)}(2) \\ \vdots \\ g_{(0)}(n) - g_{(0)}(n-1) \end{bmatrix} \tag{10}$$

Since equation (8), we have

$$\hat{g}_{(1)}(W+1) = \left(\frac{\theta}{\eta^2} - \frac{g_{(0)}(1)}{\theta}\right) e^{-\theta W} + \frac{\eta}{\theta}(W+1) + \left(g_{(0)}(1) - \frac{\eta}{\theta}\right)\left(\frac{1+\theta}{\theta}\right) \tag{11}$$

estimate values of output classification can be obtained by applying 1-AGO to $\hat{g}_{(1)}$ as follows

$$\hat{g}_{(0)}(1) = \hat{g}_{(1)}(1)$$

$$\hat{g}_{(0)}(W) = \hat{g}_{(1)}(W) - \hat{g}_{(1)}(W-1),\ W = 2, 3, 4, \ldots, n.$$

and the sequence of Grey Verhulst Model can be derived from the non-homogeneous first order differential equation as follows:

$$\frac{dg_{(1)}(W)}{dt} + \theta g_{(1)}(W) = \eta \left(g_{(1)}(W)\right)^2 \tag{12}$$

where θ is the increasing coefficient and η is the grey input coefficient. These can be assessed by least square method

$$[\theta, \eta]^T = (M^T M)^{-1} M^T N$$

$$M = \begin{bmatrix} -\frac{g_{(1)}(1)+g_{(1)}(2)}{2} & \left(\frac{g_{(1)}(1)+g_{(1)}(2)}{2}\right)^2 \\ -\frac{g_{(1)}(2)+g_{(1)}(3)}{2} & \left(\frac{g_{(1)}(2)+g_{(1)}(3)}{2}\right)^2 \\ \vdots & \vdots \\ -\frac{g_{(1)}(2)+g_{(1)}(3)}{2} & \left(\frac{g_{(1)}(n-1)+g_{(1)}(n)}{2}\right)^2 \end{bmatrix} \tag{13}$$

and $N = [g^{(0)}(2), g^{(0)}(3), \dots, g^{(0)}(n)]^T$.

By solving equation (12), the time response function of the GVM model is given by

$$\hat{g}_{(1)}(W) = \frac{\theta g_{(1)}(0)}{\eta g_{(1)}(0) + \left[\theta - \eta g_{(1)}(0)\right] e^{\theta W}}, \text{ where } W = 2 \text{ to } n. \tag{14}$$

$\hat{g}_{(0)}$ can be found as follows θ, η

$$\hat{g}_{(0)}(1) = \hat{g}_{(1)}(1)$$

$$\hat{g}_{(0)}(W) = \hat{g}_{(1)}(W) - \hat{g}_{(1)}(W-1), \ W = 2 \text{ to } n.$$

2.2 *Grey likelihood (1,1) fourier model*

GF models have been improved after modifying the residual series using Fourier Series. non-negative sequences with n entries are defined by

$$g_{(0)} = \left\{g_{(0)}(1), g_{(0)}(2), g_{(0)}(3), \dots, g_{(0)}(W)\right\}$$

and its predicted sequence under *GM* is

$$\hat{g}_{(0)} = \left\{\hat{g}_{(0)}(1), \hat{g}_{(0)}(2), \dots, \hat{g}_{(0)}(k), \dots, \hat{g}_{(0)}(n)\right\}$$

residual error series ε is well-defined as

$$e = \left\{e(1), e(2), \dots, e(W), \dots, e(n)\right\}, \ W = 1 \text{ to } n. \tag{15}$$

Now, let us consider a sub series

$$e = \left\{e(2), \dots, e(W), \dots, e(n)\right\}, \ W = 2 \text{ to } n. \tag{16}$$

The discrete decomposition of Fourier series $e(k)$ ise

$$\hat{e}(k) = \frac{1}{2}a_0 + \sum_{x=1}^{m}\left[a_x \cos\left(\frac{360°x}{n-1}W\right) + b_x \sin\left(\frac{360°x}{n-1}W\right)\right], \tag{17}$$

where $W = 2, 3, \dots, n$.

Then the above equation can be written as

$$\hat{e} = A.B \tag{18}$$

where

$$A = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{360° * 1}{n-1} * 2\right) & \sin\left(\frac{360° * 1}{n-1} * 2\right) & \cdots & \cos\left(\frac{360° * m}{n-1} * 2\right) & \sin\left(\frac{360° * m}{n-1} * 2\right) \\ \frac{1}{2} & \cos\left(\frac{360° * 1}{n-1} * 2\right) & \sin\left(\frac{360° * 1}{n-1} * 2\right) & \cdots & \cos\left(\frac{360° * m}{n-1} * 2\right) & \sin\left(\frac{360° * m}{n-1} * 2\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{2} & \cos\left(\frac{360° * 1}{n-1} * 2\right) & \sin\left(\frac{360° * 1}{n-1} * 2\right) & \cdots & \cos\left(\frac{360° * m}{n-1} * 2\right) & \sin\left(\frac{360° * m}{n-1} * 2\right) \end{bmatrix}$$

and $B = [a_0, a_1, b_1, a_2, b_2, \dots, a_m, b_m]^T$.

Here, a_0 is the mean value of the residual series,

$m = \frac{n-1}{2}$ is called minimum deployment frequently of Fourier series,

n is the total number of periods,

$$a_x = \frac{2}{n}\sum_{W=1}^{n}\left(e(W)\cos\left(\frac{360°x}{n-1}W\right)\right), x = 1, 2, 3, \dots, m \tag{19}$$

$$b_x = \frac{2}{n}\sum_{k=1}^{n}\left(e(W)\sin\left(\frac{360°x}{n-1}W\right)\right), x = 1, 2, 3, \dots, m \tag{20}$$

The predicted sequence of grey Fourier models which is determined by the Fourier residual modification of grey models are

$$\breve{g} = \{\breve{g}(1), \breve{g}(2), \breve{g}(3), \dots, \breve{g}(W), \dots, \breve{g}(n)\} \tag{21}$$

where

$$\begin{cases} \breve{g}(1) = \hat{g}(1) \\ \breve{g}(W) = \hat{g}(W) + \hat{e}(W) \end{cases}, W = 2, 3, \dots, n$$

2.3 *Grey likelihood (1, 1) Fourier Markov models*

Let g and $\breve{g}$ stand for the original n-entry sequence in addition to the anticipated order produced from GFM, respectively. A residual error sequence is defined as follows based on the predicted sequence $\breve{g}$:

$$\mathfrak{M} = \{\mathfrak{M}(2), \mathfrak{M}(2), \ldots, \mathfrak{M}(k), \ldots, \mathfrak{M}(n)\} \tag{22}$$

where $\mathfrak{M}(W) = \dfrac{g(W)}{\breve{g}(W)}$, $W = 2, 3, 4, \ldots, n$.

Subsequently, the predicted residual error value of the Grey Fourier models is then used to create Markov state transition matrices. Divide the m states as follows with those $\mathfrak{M}(W)$:

$$\text{State } 1 = \left[\min(\mathfrak{M}(W)),\ \min(\mathfrak{M}(W)) + \frac{\max(\mathfrak{M}(W)) - \min(\mathfrak{M}(W))}{m}\right]$$

$$\text{State } i = \left[\min(\mathfrak{M}(W)) + \frac{\max(\mathfrak{M}(W)) - \min(\mathfrak{M}(W))}{m} * (x-1),\ \min(\mathfrak{M}(W))\right.$$

$$\left. + \frac{\max(\mathfrak{M}(W)) - \min(\mathfrak{M}(W))}{m} * x\right], x = 1, 2, 3, \ldots, m-1$$

$$\text{Statem } m = \left[\min(\mathfrak{M}(W)) + \frac{\max(\mathfrak{M}(W)) - \min(\mathfrak{M}(W))}{m} \times (m-1),\ \max(\mathfrak{M}(W))\right],$$

The Transition Probability Martix (TPM) from state i to state j after m steps is well-defined by

$$P_{ij} = \frac{C_{ij}(m)}{C_i} \quad (i, j = 1 \text{ to } m) \tag{23}$$

m-step transition matrix is given by:

$$P(m) = \begin{bmatrix} P_{11}(m) & P_{12}(m) & \cdots & P_{1j}(m) \\ P_{21}(m) & P_{22}(m) & \cdots & P_{2j}(m) \\ \vdots & \vdots & \vdots & \vdots \\ P_{i1}(m) & P_{i2}(m) & \cdots & P_{ij}(m) \end{bmatrix} \tag{24}$$

$P(m)$ indicates the TPM among altered states besides the foundation of the forecasting model. After identifying the future transition states S (S = 1, 2, …, m), we will forecast the future value using Grey Fourier Markov Model as follows:

$$\breve{\mathfrak{M}}(W) = \hat{g}_{(0)}(W) \times \left(\min(\mathfrak{M}(W)) + \frac{\max(W) - \min(\mathfrak{M}(W))}{2m} \times (2s-1)\right) \tag{25}$$

3. Results and discussion

Consider the daily gold prices from Feb 27 2019 to Apr 26 2019 (https://investexcel.net/) smoothness and close of new real data sequence by means of follows [12-13]:

Quasi-Smoothness on $g_{(0)}$ Quasi-Exponentially on $g_{(1)}$

$$R(3)=\frac{g_{(0)}(3)}{g_{(1)}(2)}=\frac{2,951.00}{6,024.59}=0.49 \qquad Q_{(1)}(3)=\frac{g_{(1)}(3)}{g_{(1)}(2)}=\frac{8,975.59}{6,024.59}=1.49$$

$$R(4)=\frac{g_{(0)}(4)}{g_{(1)}(3)}=\frac{2,934.62}{8,975.59}=0.33 \qquad Q_{(1)}(4)=\frac{g_{(1)}(4)}{g_{(1)}(3)}=\frac{11,910.21}{8,975.59}=1.33$$

$$R(5)=\frac{g_{(0)}(5)}{g_{(1)}(4)}=\frac{2,919.57}{11,910.21}=0.25 \qquad Q_{(1)}(5)=\frac{g_{(1)}(5)}{g_{(1)}(4)}=\frac{19,829.78}{11,910.21}=1.25$$

$$\vdots \qquad \vdots$$

$$R(40)=\frac{g_{(0)}(40)}{g_{(1)}(39)}=\frac{2,874.37}{115,897.86}=0.03 \qquad Q_{(1)}(40)=\frac{g_{(1)}(40)}{g_{(1)}(39)}=\frac{115,897.86}{113,023.49}=1.03$$

$$R(41)=\frac{g_{(0)}(41)}{g_{(1)}(40)}=\frac{2,881.97}{118,779.83}=0.03 \qquad Q_{(1)}(41)=\frac{g_{(1)}(41)}{g_{(1)}(40)}=\frac{118,779.83}{115,897.86}=1.02$$

$$R(42)=\frac{g_{(0)}(42)}{g_{(1)}(41)}=\frac{2,888.53}{121,668.36}=0.024 \qquad Q_{(1)}(42)=\frac{g_{(1)}(42)}{g_{(1)}(41)}=\frac{12,688.36}{118,779.83}=1.02$$

Here, the smoothness relation lies in (0, 0.5) and it is decreasing and also the quasi exponent lies in (1, 1.5) when $W > 3$, so it satisfied the quasi smooth and exponent conditions.

3.1 *Hybrid GM (1, 1) model*

As of the trend of gold value, we can compute 1-AGO gold price value by put on the *GM* (1, 1) model, and we attained $[\theta, \eta]^T = (0.0017, 0.0726)^T$.

TRF is

$$g_{(1)}(x)=0.0726e^{-0.0017x} \tag{26}$$

The estimate value of gold value $\hat{g}_{(0)}(x)$ can be intended by performance an Inverse AGO on Eq. (7), and we get

$$\hat{g}_{(0)}(x+1)=0.0726e^{-0.0017x}(1-e^{0.0017x}) \tag{27}$$

To increase our prediction accuracy, we estimate the relative error between authentic and forecast values and we have applied a FRM method. In the direction of start, residuals are calculated using the equation (17) as follows:

$$e(W)=\{0,\ 59.3,\ 20.6,\ 6.2,\ -7.0,\ \ldots,\ -9.3,\ -7.1,\ 15.5,\ 25.0,\ 33.45\} \tag{28}$$

Subsequently the co-efficient matrix B is attained as

$$B = [0.18, 6.9, 4.26, 8.5, -0.102, -4.98, -2.42, \ldots, -2.64, 3.76, -1.37, 6.4, -4.72, -3.2]^T$$

Then, the predicted second residual $\hat{e}(k)$ is obtained according to the equation (19)

$$\hat{e}(W) = \{-21.2,\ -42.8,\ -28.7,\ -25.6,\ -39.3,\ -42.6,\ \ldots,\ -39.4,\ -28.9,\ -8.3,\ -41.5,\ -37.6,\ -33.7\}$$

Now estimate the values of gold price from GM (1, 1)-Fourier Model and the corresponding relative errors are deliberate by equation (24) as

$$\mathfrak{M}(W) = \{0.0, 7.21, 13.21, 13.57, 20.71, 14.25, 15.96, \ldots, 16.71, -0.19, -11.13, -27.44, -15.56, -33.32, -15.91$$

$$-17.81, 30.74\}$$

the state breaks of relative error are categorized into 4 portions, namely $S_1 = [-1.17, -0.61]$, $S_2 = [-0.61, -0.05]$, $S_3 = [-0.05, 0.51]$, $S_4 = [0.51, 1.1]$ and the corresponding occurrences are $C_1 = 6$, $C_2 = 13$, $C_3 = 11$ and $C_4 = 11$. The one-step TPM is obtained over the equation (26) as

$$A = \begin{bmatrix} 0 & 0.83 & 0 & 0.17 \\ 0.31 & 0.15 & 0.31 & 0.23 \\ 0 & 0.09 & 0.45 & 0.45 \\ 0.18 & 0.45 & 0.27 & 0.06 \end{bmatrix}$$

Finally, we predict the volatile of gold price by (27) which are given in the Table 1. The outputs of Hybrid GM (1, 1) model are plotted in Figure 1.

Table 1. Forecasting outcomes of Hybrid *GM* (1, 1) Model

Period	Gold Amount/ Rupees	GM (1, 1)	GM (1, 1)-Markov model	GM (1, 1)-Fourier Model	GM (1, 1) Fourier Markov Model
10-04-19	2915	2876	2894	2897	2887
11-04-19	2868	2874	2892	2875	2865
12-04-19	2870	2872	2890	2891	2881
15-04-19	2876	2870	2888	2859	2849
16-04-19	2854	2868	2886	2853	2844
17-04-19	2844	2867	2884	2855	2846
18-04-19	2846	2865	2882	2873	2863
22-04-19	2853	2863	2880	2869	2859
23-04-19	2854	2861	2879	2887	2877
24-04-19	2874	2859	2877	2890	2881
25-04-19	2882	2857	2874	2900	2890
26-04-19	2889	2855	2872	2858	2848

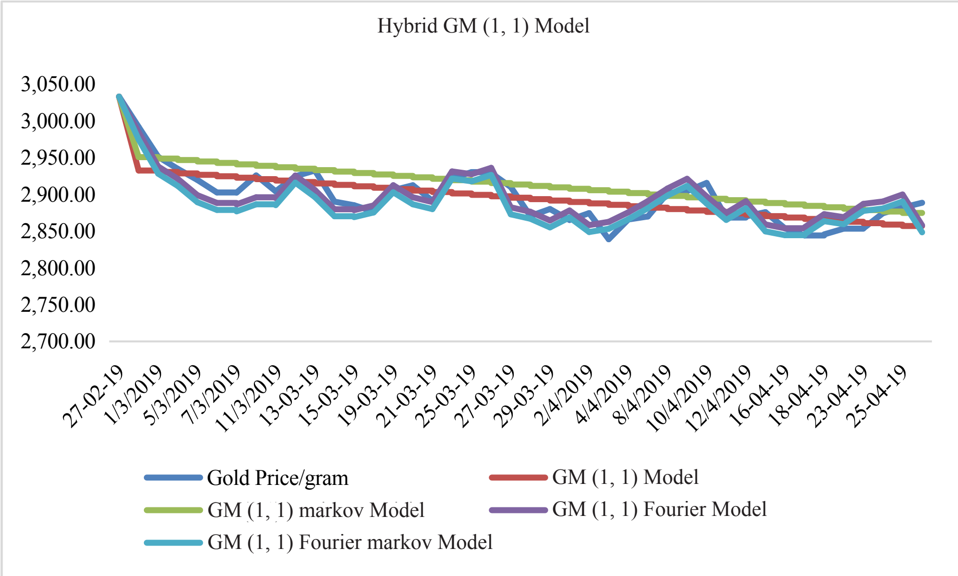

Figure 1. The interpretation of Hybrid GM (1, 1) Model

3.2 *Hybrid (2, 1) model*

Now we use G (2, 1) model to estimate the values of bounds as

$$\hat{\theta} = [\theta, \eta] = (0.0078, 19.1)$$

Then, the equivalent time reaction function is

$$\hat{g}_{(1)}(W+1) = (387954.08)e^{-0.0078W} + 2453.09(W+1) + 76039.66 \tag{29}$$

and the remaining error is

$$e(W) = \{0, -2.2, -34.3, 71.5, 83.4, 93.95, -106.6, \ldots, -41.94, -28.7, -26.9, -1.8, -9.2, 19.1\}$$

Next the co-efficient matrix B is achieved as

$$B = \{-26.1, 8.7, -8.2, 10.2, -7.2, 3.3, 7, \ldots, 1.9, -2.4, -2.3, -1.2, 0.2, 1.9, -4.3, 0.3, 0.71, 0.21\}$$

Then, the forecast residual $\hat{\varepsilon}(W)$ is obtained

$$\hat{e}(k) = \{-21.2, -42.8, -28.7, -25.6, -39.3, -42.6, \ldots, -39.4, -28.9, -48.3, -41.5, -37.6, -33.7\}$$

The calculated gold price using Fourier are intended which is given in Table 2. With esteem to the simulative values of *GM* (2, 1)-Fourier Model, the relative errors are intended by equation (24) as follows

$$\mathfrak{M}(W) = \{1, 0.99, 0.995, 0.99, 0.99, 0.987, 0.989, \ldots, 0.98, 0.986, 0.989, 0.978, 0.987, 0.987, 1.005\}$$

the state breaks of relative error are categorized into 4 portions namely S_1 = [0.97, 0.98], S_2 = [0.98, 0.993], S_3 = [0.991, 0.998], S_4 = [0.998, 1.004] and the equivalent occurrences are C_1 = 6, C_2 = 21, C_3 = 11 and C_4 = 3. The one-step TPM is obtained as follows

$$A = \begin{bmatrix} 0.33 & 0.33 & 0.33 & 0 \\ 0.19 & 0.48 & 0.19 & 0.14 \\ 0 & 0.64 & 0.36 & 0 \\ 0 & 0.67 & 0.33 & 0 \end{bmatrix}$$

Finally we predict the volatility of gold price by Hybrid Grey (2, 1) Model and given in Table 2. The outputs of Hybrid GM (2, 1) model are plotted in Figure 2.

Table 2. Forecasting results of Hybrid *GM* (2, 1)

Date	Gold Amount/ Rupees	GM (2,1)	GM (2,1)-Markov model	GM (2,1)-Fourier Model	GM (2,1) Fourier Markov Model
9-04-19	2904	2915	2916	2916	2902
10-04-19	2915	2911	2912	2912	2918
11-04-19	2868	2908	2878	2892	2859
12-04-19	2870	2904	2874	2888	2875
15-04-19	2876	2901	2871	2884	2871
16-04-19	2854	2897	2867	2877	2863
17-04-19	2844	2893.	2833	2888	2836
18-04-19	2846	2890	2860	2884	2852
22-04-19	2853	2886	2857	2883	2850
23-04-19	2853	2883	2853	2916	2863
24-04-19	2874	2880	2880	2912	2879
25-04-19	2881	2876	2876	2918	2884
26-04-19	2888	2873	2873	2872	2879

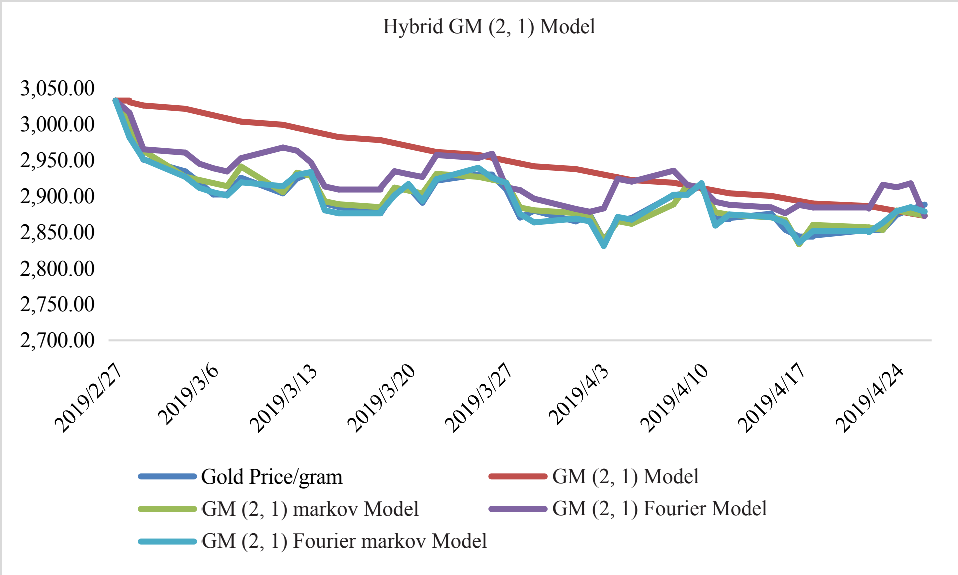

Figure 2. The interpretation of Hybrid GM (2, 1)

3.3 *Hybrid Grey Verhulst model*

Now take Grey Verhulst model [14] to estimate the values of parameters as

$$\hat{\theta} = [\theta, \eta]^T = (0.2, 0.00007)^T.$$

Then, the equivalent time response function is

$$\hat{g}_{(1)}(W) = \frac{-603.28}{-0.21 + 0.01e^{ak}} \text{ where } k \text{ various from 2 to } n \tag{30}$$

The equivalent residual error is

$$e(W) = \{0, -12.2, -29.4, -26.6, -26.5, -30.5, -21.1, \ldots, -34.3, -32.8, -24.8, -24.4, 3.8, 3.9, 1\}$$

Next the co-efficient matrix B is achieved as

$$B = \{-0{,}58, -15.2, 1.15, -0.3, -5.5, -0.23, 9.4, -6.14, \ldots, 0.49, 1.33, -0.8, -2.9, 3.7, -0.6, -1.38, -0.02\}$$

Then, the predicted residuals $\hat{\varepsilon}(k)$ are obtained as

$$\hat{e}(k) = \{-12.57, -28.05, -24.53, -27.36, -31.56, 15.86, -13.27, \ldots, -22.8, -31.6, -34.9, -23.6, -23.8, -3.6, 4.6,$$
$$10.8, 13.2\}$$

Table 3 shows the forecasted values, which were derived using Fourier techniques. The relative errors for the Grey Verhulst-Fourier model simulative values are calculated as follows:

$$\mathfrak{M}(W) = \{1, 1.001, 0.9, 0.99, 0.99, 0.99, 1.09, 0.99, \ldots, 0.99, 0.99, 0.98, 0.01, 1.001, 0.99, 0.99, 0.99, 0.99, 1.0\}$$

Then, the formal intervals of comparative error are classified into four parts, namely S_1 = [0.9935, 0.9982], S_2 = [0.9982, 1.0018], S_2 = [1.0029, 1.0057], S_4 = [1.0057, 1.001] and the corresponding occurrences are C_1 = 12, C_2 = 18, C_3 = 5 and C_4 = 5. The one-step TPM is obtained as

$$A = \begin{bmatrix} 0.15 & 0.53 & 0.15 & 0.15 \\ 0.28 & 0.56 & 0.17 & 0 \\ 0.6 & 0 & 0 & 0.4 \\ 0.6 & 0.4 & 0 & 0 \end{bmatrix}$$

Finally, we forecast the trend of gold price by Hybrid Grey Verhulst model and given in Table 3. The outputs of Hybrid Grey Verhulst model are plotted in Figure 3.

Table 3. The interpretation of forecasting results of Hybrid Grey Verhulst model

Date	Gold Amount/ Rupees	Grey Verhulst model	Grey Verhulst Markov model	Grey Verhulst Fourier model	Grey Verhulst Fourier Markov model
9-04-19	2904	2879	2894	2915	2900
10-04-19	2915	2878	2893	2882	2908
11-04-19	2868	2878	2915	2863	2876
12-04-19	2870	2878	2871	2881	2866
15-04-19	2876	2878	2871	2855	2881
16-04-19	2854	2878	2871	2848	2859
17-04-19	2844	2878	2849	2844	2843
18-04-19	2846	2878	2849	2855	2839
22-04-19	2853	2878	2849	2853	2852
23-04-19	2853	2878	2849	2874	2859
24-04-19	2874	2878.	2849	2882	2867
25-04-19	2881	2878	2871	2889	2888
26-04-19	2888	2878	2871	2865	2891

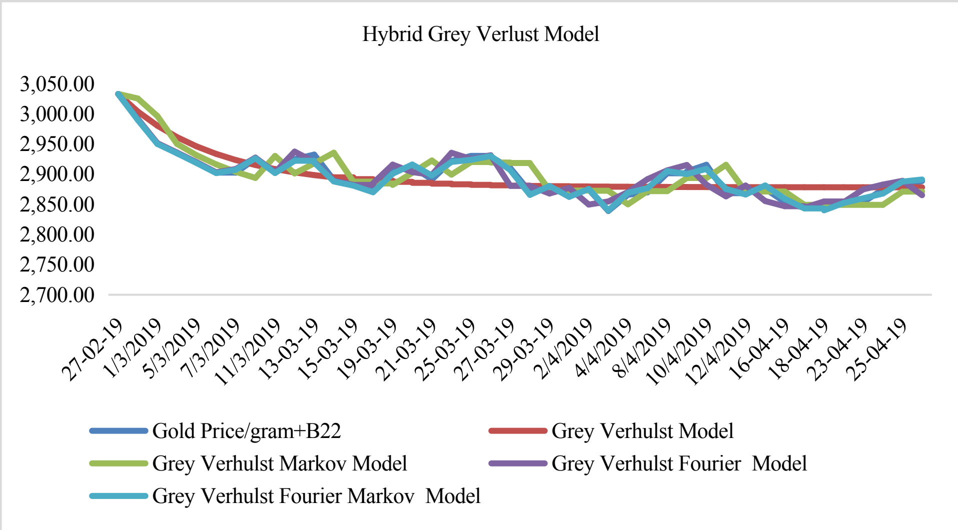

Figure 3. Hybrid Grey Verhulst model

Table 4. The interpretation of estimating performance of above 3 models

Model	MARP	MAD	TI
Hybrid GM (1, 1) model	0.006	16.5	0.6
Hybrid GM (2, 1) model	0.003	6.6	0.7
Hybrid Grey Verhulst model	0.001	4.7	0.4

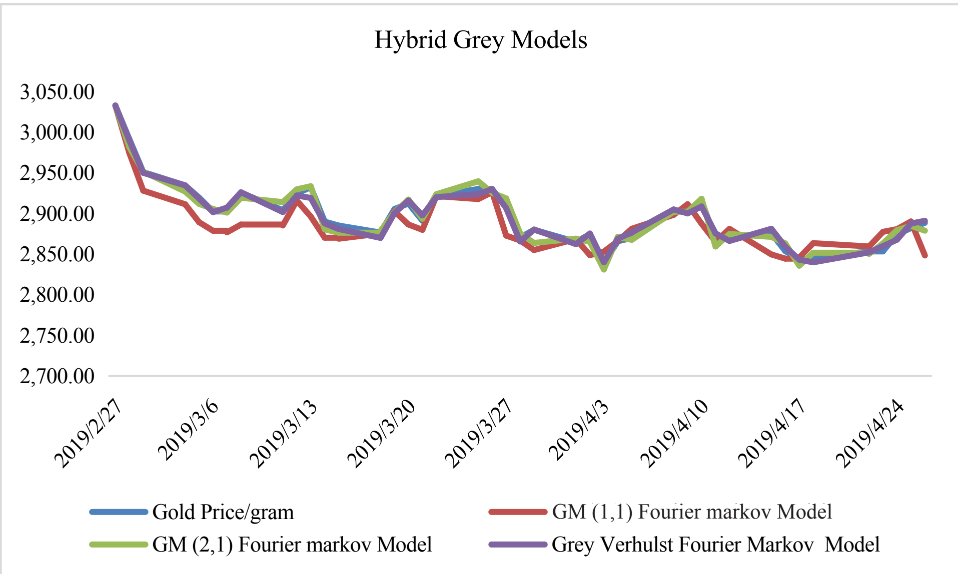

Figure 4. The interpretation of HG Models

In this study employs three metrics, mean absolute relative phase (MARP), minimum absolute difference (MAD), and time instance (TI) [15-16], to evaluate the concert and permanency of forecast methods [17-19], which are listed in Table 4. The outputs of HG models are plotted in Figure 4. Table one, two, and three demonstrate the outcomes of the expected $\hat{p}_k$ and the real p_k of the gold value using the suggested forecasting approach. Finally, we find that the Hybrid Grey Verhulst Model outperforms the other two models, demonstrating that the Hybrid Grey Verhulst model is sufficient for both accurateness and performance in estimating gold price explosiveness.

4. Conclusion

Predictions of gold prices are crucial in making economic decisions. Grey models are a common forecasting technique that has been utilized in a variety of fields. We constructed Grey theory based on Fourier theory and the Markov Model in this study. The Hybrid Grey Verhulst Model is thoroughly studied in the aforementioned analysis, and it improved prediction performance. Finally, we find that the suggested Hybrid Grey Verhulst Model outperforms the other Hybrid Grey Models in terms of results.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Kayacan E, Kaynak O, Ulutas B. Grey system theory-based models in time series prediction. *Expert Systems with Application*. 2010; 37: 1784-1789.

[2] Immawan LD, Ahdika A. Comparison of Grey-Markov (1,1), Grey-Markov (2,1), and moving average methods in forecasting small sized data of the unit price of materials in batam. *AIP Conference Proceedings.* 2018; 2021(1): 060019. Available from: https://doi.org/10.1063/1.5062783.

[3] Li JJ. The optimization DGM (2,1) model and its application. *J. Grey Syst*. 2012; 24: 181-186.

[4] Yao J, Tan CL. A case study on using neural networks to perform technical forecasting of forex. *Neurocomputing*. 2000; 34: 79-98.

[5] Wang J, Wang J, Fang W, Niu H. Financial time series prediction using elman recurrent random neural networks. *Computational Intelligence and Neuroscience*. 2016; 2016: 4742515. Available from: https://doi.org/10.1155/2016/4742515.

[6] Kayacan E, Ulutas B, Kaynak O. Grey system theory-based models in time series prediction. *Expert Systems with Applications*. 2010; 37(2): 1784-1789. Available from: https://doi.org/10.1016/j.eswa.2009.07.064.

[7] Deng JL. Introduction to grey system theory. *Journal of Grey System*. 1989; 1(1): 1-24.

[8] Ekpenyong EJ. Pseudo-additive (mixed) fourier series model of time series. *Asian Journal of Mathematics and Statistics*. 2008; 1(1): 63-68. Available from: https://doi.org/10.3923/ajms.2008.63.68.

[9] Huang KY, Jane CJ. A hybrid model for stock market forecasting and portfolio selection based on ARX, grey system and RS theories. *Expert Systems with Applications*. 2009; 36(3): 5387-5392.

[10] Zhou M, Zeng B, Zhou W. Hybrid grey prediction model for small oscillation sequence based on information decomposition. *Computational Methods for Modeling, Simulating, and Optimizing Complex Systems*. 2020; 2020: 5071267. Available from: https://doi.org/10.1155/2020/5071267.

[11] Hsu CC, Chen CY. Applications of improved grey prediction model for power demand forecasting. *Energy Conversion and Management*. 2003; 44: 2241-2249.

[12] Davis PD, Amankwah G, Fang Q. Predicting the rainfall of ghana using the grey prediction model GM (1,1) and the Grey verhulst model. *International Research Journal of Engineering and Technology (IRJET)*. 2019; 6(8): 1362-1372.

[13] Lin S, Wang G, Zhang S, Li J. Time series prediction based on support vector regression. *Information Technology Journal*. 2006; 5(2): 353-357. Available from: https://doi.org/10.3923/itj.2006.353.357.

[14] Uysal M. Comparison of ARIMA and RBFN models to predict the bank transactions. *Information Technology Journal*. 2007; 6: 475-477.
[15] Ismail Z, Jamaluddin F, Jamaluddin F. Time series regression model for forecasting malaysian electricity load demand. *Asian Journal of Mathematics & Statistics*. 2008; 1(3): 139-149.
[16] Li G, Wang T. A new model for information fusion based on Grey Theory. *Information Technology Journal*. 2011; 10: 189-194.
[17] Kung CY. Scheduling the criteria for outsourcing performances by grey decision-making method: A case study of electronic industry in Taiwan. *J. Applied Sci*. 2005; 5: 1022-1026.
[18] Li GD, Yamaguchi D, Nagai M. A Grey-based approach to suppliers selection problem. *Proceedings of the International Conference on Parallel and Distributed Processing Techniques and Applications & Conference on Real-Time Computing Systems and Applications.* PDPTA 2006, Las Vegas, Nevada, USA; 2006.
[19] Li GD, Yamaguchi D, Nagai M. A grey-based decision-making approach to the supplier selection problem. *Mathematical and Computer Modelling*. 2007; 46(3-4): 573-581. Available from: https://doi.org/10.1016/j.mcm.2006.11.021.