A Novel Method to Estimate the Maximum Power for a Photovoltaic Inverter System

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***Abstract*—This paper describes a novel method to approximate the maximum power for a photovoltaic inverter system for solar distributed generation. It is designed for power systems applications and utilities. The proposed method takes in consideration the interaction between solar panels, photovoltaic inverter, Maximum Power Point Tracking (MPPT) control, solar panel dc side dynamic model and the effective intensity of light over the solar panel. The new method has the advantage to provide a new simple way to approximate the optimal voltage or rated voltage (Vop), the optimal or rated current (Iop) and maximum power rating (Pmax) produced by a solar panel and the photovoltaic inverter. Furthermore, this straightforward method will be named Linear Reoriented Coordinates Method (LRCM) with the advantage that Pmax and Vop can be approximated using the same variables as the dynamic model without using complicate approximations or Taylor series. Finally, some simulations results are presented.**

***Keywords*— Photovoltaic power systems, solar energy, solar power generation**

I. INTR DUCTI N

The use of renewable and green energies (i.e. solar energy, wind energy, geothermic energy, etc.) is growing in many countries and the contribution to reduce the global warming and protection of the environment is increasingly important. Since the last thirty years the interest to use the solar energy in applications of power distribution are growing very fast. Applications for the solar energy are in urban areas, electric drives, satellites, etc. Today, solar energy is considered as a real alternative resource of energy to be used for production of electrical energy.

To convert solar energy into electrical energy, it is required the basic components such that solar panels, inverters, control (i.e. Maximum Point Power Tracking) and sensors, to produce electric energy. The PV arrays are linked in series to achieve the sufficient dc voltage for generating an ac output voltage at the inverter. Unfortunately, the dynamic models used to describe, the interaction of solar panels, control and PV inverter are too complicated, with a lot of required parameters and the models cannot produce a symbolic solution to obtain the optimal voltage and maximum power produced by the PV inverter giving the necessity to use long and tedious iterations. Also, these models are not very practical for straightforward power flow analysis. To solve this problem, this paper proposes a novel mathematical dynamic model for utility applications, using the dc side dynamic equations for a solar panel and the linear reoriented coordinates method.

This dynamic model has the advantage to take in consideration the behavior of the solar panels for different intensities of light and initial conditions of the current. It describes the power and current produced by PV inverter, the operations of the MPPT with the interaction of the inverter and the effects of the intensity of light Also, an approximate symbolic solution is given to determine the optimal voltage of operation and the maximum power using MPPT. The method to approximate the optimal voltage (Vop) and maximum power (Pmax) is named Linear Reoriented Coordinates Method (LRCM). The LRCM is a simple method which it uses the same variables as the proposed dynamic model and it is a time saver to calculate Vop and Pmax reducing the long and tedious iterations. In addition, the simulated results will show that the proposed technique is very effective giving a small error between the real values and estimated values, inclusive if the effective intensity of light is changing over the solar panel.

II. DC SIDE D NAMIC E UATI NS F R A PV INVERTER S STEM

A PV Inverter System is a series connection of solar panels or photovoltaic modules with a dc-ac power electronics inverter circuit, and novel generation control circuit, (MPPT) control [5] to generate ac voltage from a solar source. The MPPT can compensate for the reduction in output power caused by the shadow covering the photovoltaic modules [3]. Also, the MPPT controls the inverter to produce the maximum power and the ac power to be connected to the load or utility grid. Fig. 1 shows a PV inverter system with the three principal stages of operation.

The proposed I-V characteristic model equation (1) takes into consideration the percentage of effective intensity of the light over the solar panels, the characteristic constant for the I-V curves, a shading linear factor, the short-circuit current rating and the open-circuit voltage for each panel [1]. Now to obtain the dynamic equation of the current with respect to the voltage, lets differentiate (1) with respect to the voltage will produce the dynamic model equation for the current (2) with respect to the voltage.

$$I(V) = \alpha \times I_{\max} - \alpha \times I_{\max} \times \exp\left(\frac{V}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} - \frac{1}{b}\right) \quad (1)$$

$$\frac{dI(V)}{dV} = \frac{I(V) - \alpha \times I_{\max}}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} \quad (2)$$

(1) is multiplied by V to obtain the P-V characteristic equation as shown in (3). Now, (3) is differentiated with respect to the voltage giving (4). The dynamic equation of the Power with respect to the voltage is obtained (4). Fig. 2 shows the I-V and P-V characteristic curves for different percentages of effective intensities of light over the solar panels. If we want the dynamic equations for P(V) and I(V) in time, we just need to multiply the equations (2) and (4) by dV/dt.

$$P = V \times I(V) = \alpha \times V \times I_{\max} - \alpha \times V \times I_{\max} \times \exp\left(\frac{V}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} - \frac{1}{b}\right) \quad (3)$$

$$\frac{dP}{dV} = i + \frac{P \times (1 - \alpha \times I_{\max}/i)}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} \quad (4)$$

The meaning for the variable, P is the PV inverter total output power. V is the voltage of operation for the PV inverter system. I(V) is the PV inverter total output current. I(0) is the short circuit current rating and V_{max} is the open-circuit voltage rating of the solar panels array, for an effective intensity of light of 100 over the solar panels. I_{max} depends on I(0), their relationship is in (5). I_{max} is obtained by solving (1), when $V = 0$ using the experimental I-V characteristic curve (100) as illustrated in Fig. 3. (5) shows the relationship between I_{max} and the short-circuit current for the solar panel.

The experimental I-V characteristic curve is used to calculate α, b, γ, V_{min} and V_{max} as shown in Fig. 3. α is the percentage/100 of effective intensity of the light over the solar panels (i.e. 100 of intensity if light over the solar panels is $\alpha = 1$). b is the exponential I-V characteristic constant. The characteristic constant, b, is calculated from the (6). The voltage (V_b) is approximated, when the current is 0.6321 I(0) using the experimental I-V characteristic curve (100) as illustrated in Fig. 3. γ is the shading linear factor depending of V_{max}. The shading linear factor for V_{max}, γ, is defined as the percent of voltage (V_{max}) loss from a maximum intensity of light to a minimum intensity of light as indicated in (7). V_{min} is the open-circuit voltage rating of the solar panels array for an effective intensity of light less than 20 over the solar panels.

From Fig. 2, it can be observed that at any particular intensity of light, there is a unique point for the maximum power; this value is named the MPP. The MPP is calculated exactly by differentiating (3) and solving for the optimal voltage (V_{op}), as indicated in (8). Unfortunately, it is not possible to find a symbolic solution hence the only way to solve (8) is numerically and this solution requires long and tedious iterations, making the solution not practical.

$$I_{\max} = \frac{I(0)}{\alpha - \alpha \times \exp\left(\frac{-1}{b}\right)} \quad (5)$$

$$b = 1 - V_b / ((\gamma \times \alpha + 1 - \gamma) \times V_{\max}) \quad (6)$$

$$\gamma = 1 - V_{\min} / V_{\max} \quad (7)$$

$$\alpha \times I_{\max} + \exp\left(\frac{V_{op}}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} - \frac{1}{b}\right) \times \left(-\alpha \times I_{\max} - \frac{\alpha \times V_{op} \times I_{\max}}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}}\right) = 0 \quad (8)$$

A novel method called Linear Reoriented Coordinates Method (LRCM) is proposed in the paper, to solve (8) and then to find an approximate symbolic solution for the Pmax calculated by the MPP.

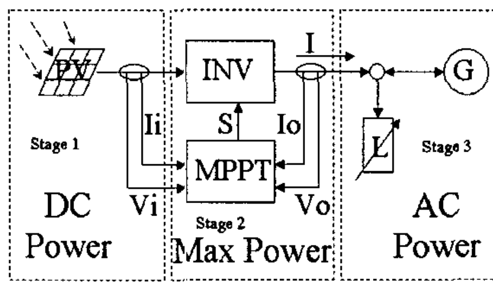

Fig. 1. PV Inverter System for utility applications

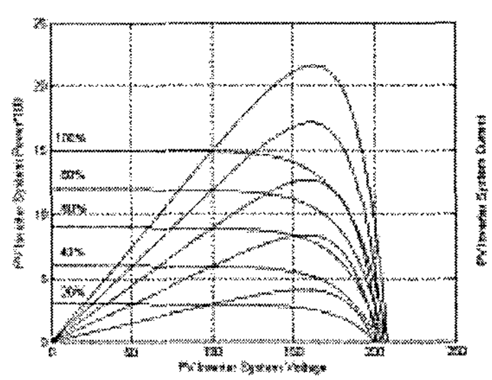

Fig. 2. P-V and I-V Characteristics for different intensities of light

III. LINEAR RE RIENTED C RDINATES METH D

The main idea for the LRCM is to find the I-V curve knee point, sees Fig. 4. The I-V curve knee point is the optimal current (I_{op}) and the optimal voltage (V_{op}) that produces P_{max}. Using the I-V Curve, see Fig. 4, a linear current equation can be determined from the initial and final values, (9). The slope of the I-V Curve at the knee point is approximated by the slope of the linear current equation, (10). From this approximation, we will approximate V_{op} with V_{ap}. The current equation, (1) and the linear current equation, (9) are differentiated ((11) and (12)) and set equal to each other to solve for V then the solution is V_{ap},. The equations of V_{ap} are given in (14) and (15). V_{ap} is substituted in (1) to obtain I_{ap},.

$$IL(V) = \alpha \times I(0) \times (1 - V/V_{\max}) \tag{9}$$

$$\frac{dIL(V)}{dV} \cong \frac{dI(V)}{dV} \tag{10}$$

$$\frac{dI}{dV} = \left(-\frac{\alpha \times I_{\max}}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}}\right) \times \exp\left(\frac{V}{b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max}} - \frac{1}{b}\right) \tag{11}$$

$$\frac{dIL}{dV} = \frac{-\alpha \times I(0)}{V_{\max}} \tag{12}$$

$$V_{ap} = b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max} \times \left(\frac{1}{b} + \ln\left(\frac{(\gamma \times \alpha + 1 - \gamma) \times b \times I(0)}{\alpha \times I_{\max}}\right)\right) \tag{13}$$

After substituting (5) into (13), a simplified V_{ap} is given by (15).

$$V_{ap} = (\gamma \times \alpha + 1 - \gamma) \times V_{\max} + b \times (\gamma \times \alpha + 1 - \gamma) \times V_{\max} \times \ln\left[b \times (\gamma \times \alpha + 1 - \gamma) \times \left(1 - \exp\left(\frac{-1}{b}\right)\right)\right] \tag{14}$$

Now, lets substitute (15) into (1) to obtain I_{ap}.

$$I_{ap} = \alpha \times I_{\max} \times \left[1 - b \times (\gamma \times \alpha + 1 - \gamma) \times \left(1 - \exp\left(\frac{-1}{b}\right)\right)\right] \tag{15}$$

$$P_{\max} = V_{op} \times I_{op} \cong V_{ap} \times I_{ap} \tag{16}$$

An approximated P_{max}, (16) is given by the multiplication of (14) and (15). Finally, if V_{ap} solves (8) hence we found the exact solutions for P_{max} , I_{op} and V_{op}. To prove how good it is the range of our approximation for P_{max}, lets use Fig. 4 to do a geometric analysis and lets define some physical constraints for the different constants. As a note, for the value b = 0, is for an ideal solar panels array without no losses, (22).

$$P \in A \text{ where A is } \{P \in \Re \;\; 0 \le P \le P_{\max}\} \tag{17}$$

$$V \in B \text{ where is } \{V \in \Re \;\; 0 \le V \le V_{\max}\} \tag{18}$$

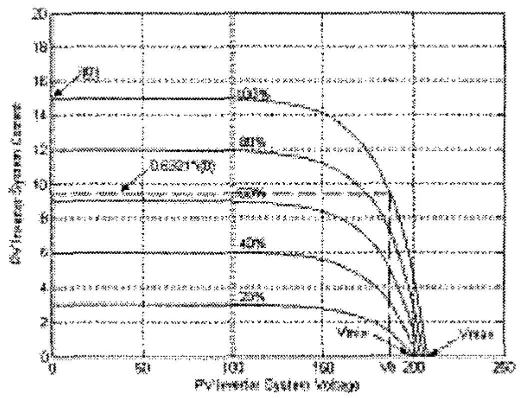

Fig. 3. Experimental I-V Characteristics

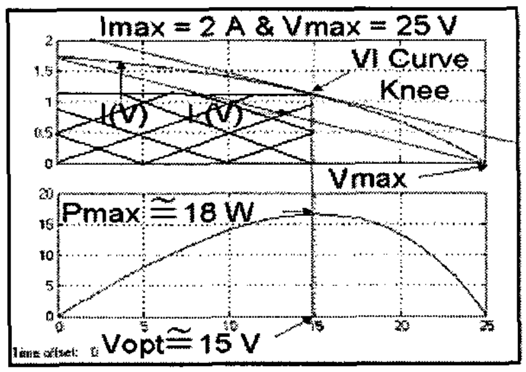

Fig. 4. P-V and I-V Curves with LRCM

$$I \in C \text{ where C is } \{I \in \Re \;\; 0 \le I \le I(0)\} \tag{19}$$

$$\alpha \in D \text{ where D is } \{\alpha \in \Re \;\; 0 < \alpha\} \tag{20}$$

$$b \in E \text{ where E is } \{b \in \Re \;\; 0 < b < 1\} \tag{21}$$

$$P_{\max} = I(0) \times V_{\max} \text{ if and only if } b = 0 \tag{22}$$

y geometric analysis, we can obtain the following two inequalities (23) and (24) for P_{max}, where P_{max} can be seen as the maximum rectangular area inside of the curve produced by (1). It is trivial that P_{max} is less than the total area current curve, (23). Using the fact that the P-V Characteristic Curve has a unique maximum point, we know that P_{max} is more or equal than our approximate P_{max}. To prove the last part of (24), the author did some computer simulations presented in the Fig. 5 where it can be seen that the estimated P_{max} is always more than (22) multiplied by 0.315. Now, (23) and (24) can give us the range of our approximation for P_{max}.

$$\int_0^{V_{\max}} I(s)ds > I(V_{op}) \times V_{op} = P_{\max} \tag{23}$$

$$P_{\max} \ge I_{ap} \times V_{ap} > 0.315 \times I(0) \times V_{\max} \tag{24}$$

Finally, the LRCM is a simple method where, instead of calculating the optimal voltage (rated voltage) and maximum power solutions using the power equations, the solutions are obtained using the current equation and the linear current equation to obtain V_{ap} and I_{ap}, P_{max} is then estimated. Also, the LRCM has the advantage of giving an approximated symbolic solution for V_{op}. The LRCM can produce the same results as the old models without the use of Taylor series, continuous fraction expansion or other approximations and it is more practical for simulations and power flow analysis providing symbolic solutions. The following results will prove that the proposed technique is very effective, giving a small error between the actual values and estimated values, inclusive if the effective intensity of light is changing over the solar panels.

IV. LRCM RESULTS F R A PV INVERTER S STEM WITH MPPT

The main advantage of the LRCM is that only the characteristic constants of the VI curve (i.e. b, α , γ, V_{max}) are required to estimate P_{max}. Also, approximate symbolic solutions for P_{max},, I_{op} and V_{op} are given when (8) has no symbolic solution to calculate P_{max}. If V_{ap} is a solution for (8) then we found the exact solution for for P_{max},, I_{op} and V_{op}. Figs. 6-7 show the simulation results for a PV Inverter System with the estimated curve for P_{max} and the knee points. The parameters for the simulation results are, I_{max} is 15 A, V_{max} is 208 V, γ is 0.05 and α is 1.

Fig. 6 shows the P-V Curve for different characteristic constants and the estimated curve for P_{max}. It is very close to P_{max}. The characteristic constant will determine the initial current and the location of the knee point. The P_{max} will be more for small characteristic constant, hence an I-V curve with b equal to 0.1 produces a bigger P_{max} than a I-V curve with b equal to 0.5. Fig. 7 shows the P-V Curve for different characteristic constants and the estimated curve for Pmax. It is very close to Pmax. The maximum error using the LRCM to estimate P_{max} in the normalized form was approximately to 0.3 as illustrated in Fig.8.

The power and current dynamics equations were given to describe a PV Inverter System for utility applications. The proposed P-V and I-V dynamic equations have the advantage of being simple, but at the same time powerful, describing the interaction between solar cells, a photovoltaic inverter, Maximum Power Point Tracking (MPPT) control, and the effective intensity of light over the solar cell. The LRCM has the advantage that it is guaranteed that an approximate symbolic solution will be found for exponential functions without symbolic solutions. Finally, it has been proved that the LRCM has a maximum error for the estimation of P_{max} near to 0.3 changing b to obtain different V-I characteristic curve. These results prove that the LRCM is valid for different values of I_{max}, V_{max}, γ and α.

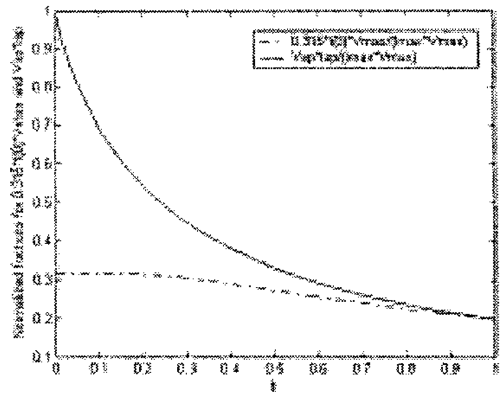

Fig. 5. Simulation to prove $I_{ap} \times V_{ap} > 0.315 \times I(0) \times V_{max}$

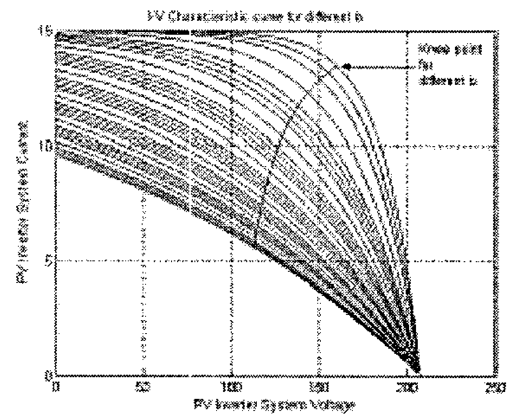

Fig. 6. I-V Curves with estimated knee points

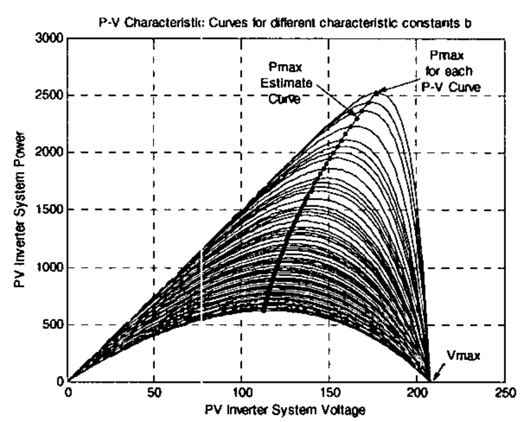

Fig. 7. P-V Curves with estimated Pmax curve

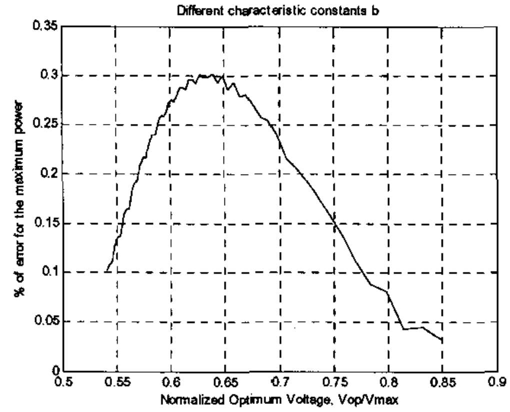

Fig. 8. Error Curve for Pmax and Pmax estimated

V. C NCLUSI NS

A novel simple method named Linear Reoriented Coordinates Method (LRCM) is used to estimate P_{max} ,V_{op} and I_{op} for a PV Inverter System. The LRCM is very effective, to solve exponential functions without the homeomorphism property. Finally, an approximate symbolic and numerical solution is found for a Photovoltaic Inverter System.

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