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Abstract—A method for determining the dielectric properties of infinite half-space of generally lossy dielectric materials is described. This method utilizes the measurement of the admittance of a rectangular waveguide radiating into such dielectrics. It is shown that the real part of the admittance is relatively insensitive to the variations of the imaginary part of the dielectric constant. Subsequently, a numerical procedure is initiated which provides a simple and fast-converging approach for calculation of the dielectric properties of the material. This numerical procedure renders itself to implementation by personal computers—a major advantage over the existing computational schemes. The theoretical formulation for the expression of the admittance of an open-ended waveguide and the numerical procedure to calculate the dielectric properties are discussed in detail. Results of several measurements of free-space and lossy dielectric samples (rubber with carbon black) to verify the theory and the numerical scheme are given as well. The results of this technique were also compared with other measurement schemes, and very good agreement was obtained. Comments on the accuracy of the results are also provided.

I. INTRODUCTION

EVALUATING the dielectric properties of materials is an important topic in microwave-related fields. Consequently, numerous techniques have been developed for measuring the dielectric properties of materials at microwave frequencies. Among the most common techniques are the partially filled transmission line, cavity method, open-ended transmission lines, and free-space method [1]–[13]. Some of these techniques require tedious sample preparation (destructive), while others offer techniques for which no sample preparation is required (nondestructive). However, in some industrial and medical applications, nondestructive measurement is the only available option. Open-ended coaxial lines and waveguides are widely used for these types of measurements [14]–[16], [1], [8]. In these cases an open-ended transmission line radiates into the dielectric medium, and its properties are extracted via admittance or reflection coefficient measurement. Open-ended waveguide techniques may offer less complicated measurement setups and may be more adapted to on-line industrial applications than open-ended coaxial line schemes. Also, there are several simple techniques available for admittance measurement of open-ended waveguides (via reflection coefficient properties) using slotted lines, whereas, for open-ended

coaxial lines usually a more complex measurement apparatus is needed.

Open-ended transmission line techniques render simple and fast measurement procedures. However, their involved electromagnetic analysis usually requires complicated computational algorithms. Thus, most of what is reported in the literature requires a large computer, is insufficient in its detailed derivations for replication purposes or is given for a limited set of parameters (e.g., frequency). In this paper we describe an open-ended rectangular waveguide technique with a *simple and fast numerical procedure* which can be programmed on a personal computer. This is an attractive scheme which eliminates the use of large computers and yet allows quick extraction of the dielectric properties of an infinite half-space dielectric once the admittance of the open-ended waveguide in the presence of the dielectric (or the reflection properties) is measured. This method is quite suitable for lossy dielectric materials as the infinite half-space condition can be achieved more readily than with medium- or low-loss dielectrics.

II. THEORETICAL BACKGROUND

Fig. 1 shows an open-ended rectangular waveguide aperture radiating into a homogeneous dielectric sample with relative complex dielectric constant of $\epsilon_r = \epsilon_r - j\epsilon_r''$. The theoretical foundation of an aperture in an infinite ground plane radiating into a dielectric medium is well known [15]–[17]. A look at the results of these theories reveals that the admittance of such aperture is stationary with respect to small variations of the aperture electric field. In other words, an approximation of the electric field expression still yields a good estimate for the aperture admittance expression. This is to say that consideration of dominant mode alone results in a reasonably good estimation of the aperture admittance. The validity of this approximation has been tested and will be shown in the results section.

The expression for the admittance of a rectangular waveguide aperture radiating into a half-space dielectric medium is given by Lewin [17]:

$$Y = G + jB = \frac{2j}{\pi a b k_1} \int_0^a \int_0^b (b - x)$$

$$\cdot \left[K_2(a - y) \cos \frac{\pi y}{a} + \frac{a}{\pi} K_1 \sin \frac{\pi y}{a} \right] G_e \, dy \, dx.$$
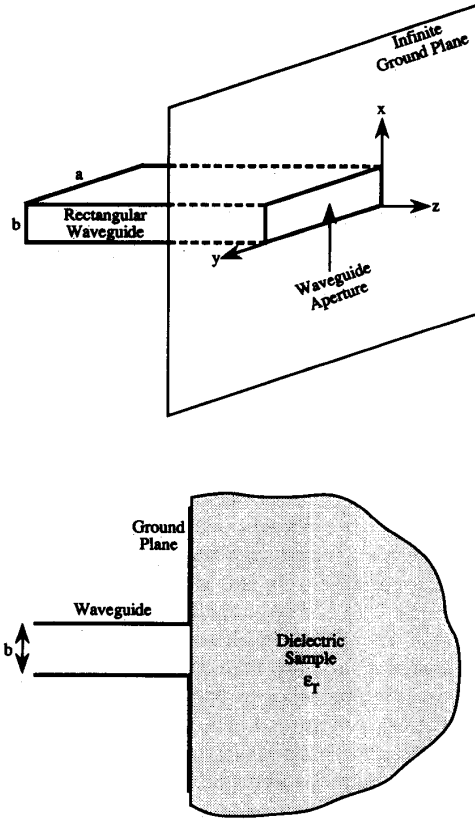
$$(1)$$

Fig. 1. Open-ended waveguide radiating into infinite half-space with dielectric constant ϵ_r.

G and B are the conductance and the susceptance of the aperture, a and b are the broad and narrow dimensions of the waveguide cross section respectively, and k_1 is the dominant mode wavenumber given by

$$k_1 = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

$$k_0 = \frac{2\pi}{\lambda_0}$$

and k_0 is the free-space wavenumber. $K_{1,2}$ are given by:

$$K_1 = k_r^2 + \left(\frac{\pi}{a}\right)^2$$

$$K_2 = k_r^2 - \left(\frac{\pi}{a}\right)^2$$

where

$$k_r = k_0\sqrt{\epsilon_r}.$$

G_e is given by

$$G_e = \frac{1}{r} e^{-jk_r r}$$

where r is the distance between the source point on the aperture and a field point inside the dielectric on the aperture interface and is given by

$$r = \sqrt{x^2 + y^2}.$$

Equation 1 can be manipulated upon normalization and introduction of the following parameters:

$$x = b\xi, \quad y = a\eta, \quad s = \frac{b}{\lambda_0}, \quad t = \frac{a}{\lambda_0}, \quad \sqrt{\epsilon_r} = e_r + je_i$$

so that it can be decomposed into a real and an imaginary part which will give the conductance and the susceptance of the open-ended waveguide respectively. Thus, G and B can be expressed as

$$G = H \int_0^1 \int_0^1 (1 - \xi)(AF + CE) \, d\xi \, d\eta \qquad (2)$$

$$B = H \int_0^1 \int_0^1 (1 - \xi)(AE - CF) \, d\xi \, d\eta \qquad (3)$$

where

$$H = \frac{2s}{\sqrt{4t^2 - 1}}$$

$$A = S_2(1 - \eta) \cos(\pi\eta) + \frac{1}{\pi} S \sin(\pi\eta)$$

$$S_1 = 4t^2(e_r^2 - e_i^2) + 1$$

$$S_2 = 4t^2(e_r^2 - e_i^2) - 1$$

$$C = -8e_r e_i t^2 \left[(1 - \eta) \cos(\pi\eta) + \frac{1}{\pi} \sin(\pi\eta) \right]$$

$$E = E_1 \frac{\cos(2\pi r_s e_r)}{r_s}$$

$$F = E_1 \frac{\sin(2\pi r_s e_r)}{r_s}$$

$$E_1 = e^{2\pi r_s e_i}$$

$$r_s = \sqrt{s^2\xi^2 + t^2\eta^2}.$$

III. CALCULATION PROCEDURE

To calculate admittance of the waveguide aperture we have to solve for a system of two integral equations describing G and B. This indeed is not a simple task, and in some cases may be time and computer resource prohibitive. The time required to solve these equations is dictated by (2) and (3). To perform these integrations, we have used a numerical technique based on the Gauss–Legendre integration [18]. It is worth noting that the integrals will experience singularities at $\xi = \eta = 0$. However, the integration method used here is insensitive to these singularities due to the fact that these integrations are not performed at the exact values of the upper and lower bounds but very close to them. This integration technique pro-
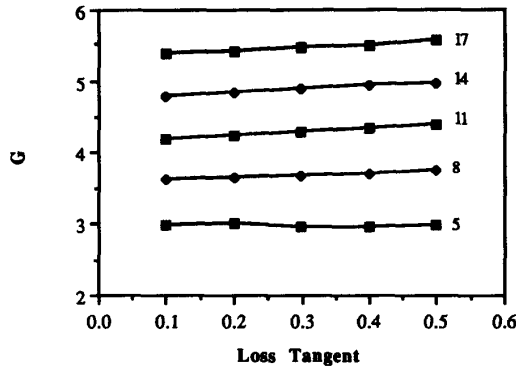
Fig. 2. Dependence of the real part of the admittance to loss tangent for different values ϵ_r'.

duced very good results (mainly due to the smooth behavior of the integrals) which were closely compared to the other available techniques. It is important to mention that for higher dielectric constant materials the number of Gauss points needed for the integration should be higher than that needed for the lower dielectric case.

The analysis showed the G is nearly *invariant* as a function of the imaginary part of the dielectric constant (ϵ_r''). Fig. 2 shows the dependence of G on loss tangent (tan $\delta = \epsilon_r''/\epsilon_r'$) for various values of ϵ_r' at 10 GHz (the variation of G with respect to ϵ_r'' is less than 4.9% for this range). Thus, we can make a strong argument that ϵ_r' can be very closely approximated from G independent of the value of ϵ_r''. Considering this, we can calculate a fairly good starting value for ϵ_r' using some estimated value for ϵ_r''. In the event that we have no clue as to what a reasonable estimate for ϵ_r'' is, we may start with $\epsilon_r'' = 0$. This will only increase the calculation time slightly (which may not be important as this is a very fast converging procedure). There will only exist one root such that the results fall in the range of physically meaningful values. Thus, a simple root-finding scheme may be utilized. Once the first approximations for the real and the imaginary parts of the dielectric constant are obtained, these values can be used to repeat the process, so that more accurate values of the dielectric constant are obtained (by calculating G and B values that match their measured counterparts very closely). The process may then be terminated as a prescribed limit of accuracy is reached. Fig. 3 shows the flow chart for this procedure.

Through extensive analysis, this process proved to be very fast converging for all cases. Furthermore, the entire approach was programmed on a personal computer. This renders itself as a versatile technique where many parameters such as waveguide aperture size, frequency, etc. can be changed, and results may be obtained very quickly.

The admittance of the open-ended waveguide is related to the complex reflection coefficient by

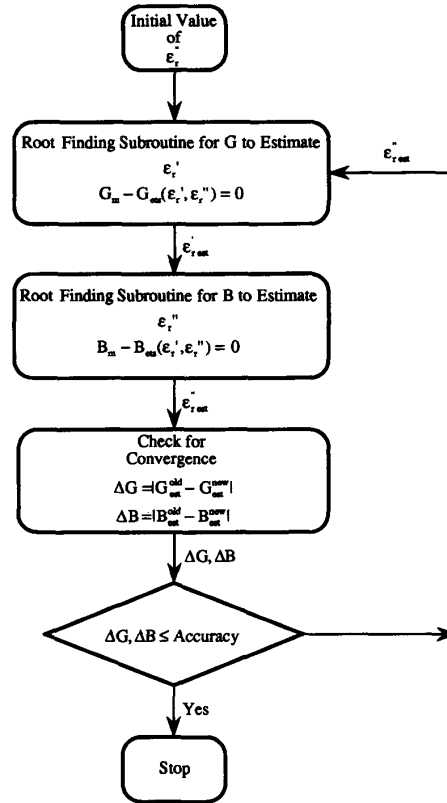$$Y = \frac{1 - |\Gamma| e^{j\phi}}{1 + |\Gamma| e^{j\phi}}.$$

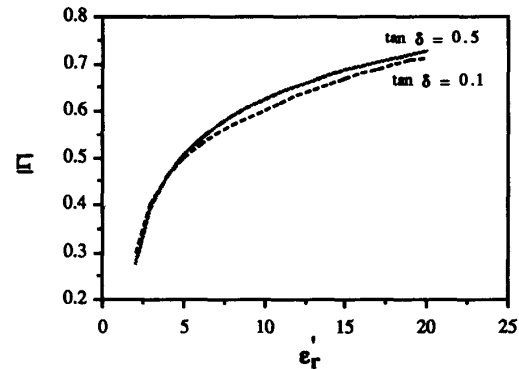

Fig. 3. Flow chart of the numerical procedure.

Fig. 4. Theoretical results of the magnitude of the reflection coefficient as a function of ϵ_r' for two values of loss tangent at 10 GHz.

Thus, once the magnitude and phase of Γ are obtained, B and G can be calculated, and using the procedure described above ϵ_r' and ϵ_r'' are determined. Fig. 4 shows the variation of the magnitude of the reflection coefficient as a function of ϵ_r for two values of tan δ at 10 GHz. This shows that ϵ_r' can be closely approximated from the measurement of the magnitude of the reflection coefficient alone for generally lossy dielectrics. The accuracy associated with this approximation (about 10%) gets better for smaller values of ϵ_r'.
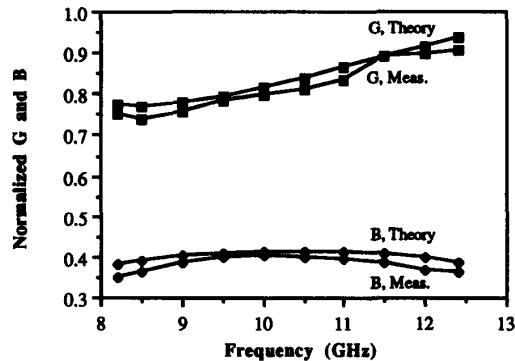
Fig. 5. Comparison between the theoretical and measured admittance of an X-band waveguide radiating into free-space.

TABLE I
DIELECTRIC PROPERTIES OF TWO RUBBER SAMPLES OBTAINED USING THE OUTLINED PROCEDURE

Sample	Frequency (GHz)	ϵ_r'	ϵ_r''
1	9	7.0 ± 0.3	1.67 ± 0.14
1	10	7.1 ± 0.3	1.72 ± 0.14
2	9	11.2 ± 0.3	2.30 ± 0.16
2	10	11.3 ± 0.3	2.33 ± 0.16

IV. RESULTS

To check the validity of the theoretical predictions, we measured the real and imaginary parts of the admittance of an X-band waveguide radiating into free-space. These measurements were conducted using a precision slotted line and measuring the standing wave ratio and the shift of the location of minima relative to a shorted slotted line (the detailed description of this well-known technique is not discussed here). Fig. 5 shows the measured values of G and B and those predicted by the theory (1). It is evident from Fig. 5 that there is a good agreement between theory and measurement. These measured values of G and B were also compared to those obtained by other investigators, and the results were very much in concert [19]. This confirms the argument that considering only the dominant mode in our theoretical solution (approximation of the fields) is quite an acceptable approximation for most practical cases.

We also measured G and B (via the measurement of the reflection coefficient) for two different rubber samples. Sample number 1 had a carbon concentration of roughly 33%, and the other sample contained higher carbon concentration (exact value was not known). Having calculated G and B, the samples' dielectric properties were calculated using our numerical procedure. Table I shows the results of these measurements. Standard deviations shown in Table I are calculated by feeding partial measurement errors of the reflection coefficient into the numerical procedure. Thus, we estimate our accuracy (for these rubber samples) to be better than 4.3% for ϵ_r' and 8.4% for ϵ_r''.

The results of the dielectric properties of the second sample at 10 GHz were checked using a partially filled waveguide method [20]. A 1-m thick sheet of rubber was placed inside an X-band slotted waveguide and using the procedure outlined in [19] the dielectric properties of the sample were measured to be $\epsilon_r' = 11.1 \pm 0.3$ and $\epsilon_r'' = 2.4 \pm 0.15$, which are close to the values obtained by our procedure. This comparison confirms the validity of our technique.

V. CONCLUSIONS

A powerful numerical approach for determining the dielectric properties of materials using an open-ended rectangular waveguide was described. This technique offered a nondestructive, fast-converging, and simple numerical procedure which can be implemented on a personal computer. This numerical procedure is based on the measurement of the admittance of an open-ended waveguide terminated by an infinite half-space dielectric. The insensitivity of the real part of the admittance to the imaginary part of the dielectric constant is used to initiate our numerical technique. The theoretical development for the expression of the admittance only considers the dominant mode of the waveguide. The validity of this consideration was tested by measuring the admittance of a waveguide radiating into the free-space. The measurement and theoretical results were in very good agreement. Dielectric properties of two different samples of rubber (containing carbon black) were also measured. The results of one of these samples were compared with those obtained using another scheme. Very good agreement between these results was obtained as well.

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