A Novel Pulse Compression Scheme Based on Minimum Mean-Square Error Reiteration[1]

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Abstract — **This paper presents an alternative approach to pulse compression that is based upon iterative Minimum Mean-Square Error (MMSE) estimation. It is similar to the well-known least squares (LS) approach but does not suffer from the adverse effects caused by scatterers closer than some nominal range. This results in a more robust estimate of the radar returns while maintaining nearly the same sidelobe level. Furthermore, the proposed pulse compression scheme is highly parallelizable in range and therefore can be computed efficiently.**

I. INTRODUCTION

Pulse compression allows a radar to obtain the range resolution of a short pulse without the need for very high peak transmit power. This is accomplished by transmitting a long pulse that is phase or frequency modulated to generate a wideband signal. The wideband signal is reflected back to the radar by scatterers in the beam of the radar which can be viewed as the convolution of the wideband signal with a channel that is representative of the range profile illuminated by the radar. The purpose of pulse compression is then to estimate the radar channel based on the known transmitted signal and the received radar return signal.

The classical approach to pulse compression is known as matched filtering [1] which has been shown to maximize the received signal-to-noise ratio (SNR) and is accomplished by convolving the transmitted signal with the received radar return signal. One can represent matched filtering in the digital domain as the filtering operation

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H \tilde{\mathbf{y}}(\ell), \tag{1}$$

where $\hat{x}_{MF}(\ell)$, for $\ell = 0,\cdots,L-1$, is the estimate of the ℓ^{th} delayed sample (range gate index), $\mathbf{s} = [s_1\ s_2\ \cdots\ s_N]^T$ is the length-N sampled version of the wideband transmitted waveform, $\tilde{\mathbf{y}}(\ell) = [y(\ell)\ \ y(\ell+1)\ \cdots\ y(\ell+N-1)]^T$ is the length-N vector of received radar return samples, and $(\bullet)^H$ and $(\bullet)^T$ are the complex conjugate (or Hermitian) and transpose operations, respectively. Each individual radar return sample can be expressed as

$$y(\ell) = \tilde{\mathbf{x}}^T(\ell)\,\mathbf{s} + v(\ell)\,, \tag{2}$$

where $\tilde{\mathbf{x}}(\ell) = [x(\ell)\ \ x(\ell-1)\ \cdots\ x(\ell-N+1)]^T$ consists of samples of the true radar channel and $v(\ell)$ is additive noise. The matched filter output can therefore be written as

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H [\mathbf{A}(\ell)]^T \mathbf{s} + \mathbf{s}^H \mathbf{v}(\ell)\,, \tag{3}$$

where $\mathbf{v}(\ell) = [v(\ell)\quad v(\ell+1)\quad \cdots\quad v(\ell+N-1)]^T$ and

$$[\mathbf{A}(\ell)] = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+N-1) \\ x(\ell-1) & x(\ell) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(\ell+1) \\ x(\ell-N+1) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix} \tag{4}$$

is a collection of sample-shifted snapshots of the radar channel.

From (4), it is obvious that estimation via matched filtering will suffer from range sidelobes due to the influence from neighboring radar channel coefficients (*i.e.* the off-diagonal elements of $[\mathbf{A}(\ell)]$). To alleviate this well-known effect, least-squares (LS) solutions have been proposed [2]-[6] that decouple neighboring channel coefficients which have been smeared by the transmitted waveform. The LS solution models the length-(L+N-1) received radar return as

$$\mathbf{y} = [\mathbf{S}]\,\mathbf{x} + \mathbf{v}\,, \tag{5}$$

where $\mathbf{x} = [x(0)\ \ x(1)\ \cdots\ x(L-1)]^T$ are the L true radar channel coefficients, $\mathbf{v} = [v(0)\ \ v(1)\ \cdots\ v(L+N-2)]^T$ are additive noise samples, and the convolution of the transmitted waveform with the radar channel is approximated as the matrix multiplication

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$$[\mathbf{S}]\,\mathbf{x} = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ \vdots & s_1 & & \vdots \\ s_N & \vdots & \ddots & 0 \\ 0 & s_N & & s_1 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & s_N \end{bmatrix} \mathbf{x}\,. \tag{6}$$

The general form of the LS solution is

$$\hat{\mathbf{x}}_{LS} = \left(\mathbf{S}^H \mathbf{S}\right)^{-1} \mathbf{S}^H \mathbf{y}\,. \tag{7}$$

For the received signal model of (5), it can be shown that the LS solution of (7) is optimal in the mean-square error (MSE) sense when the additive noise is white. However, upon further inspection one finds that the LS received signal model does not completely characterize the received radar returns because it does not account for convolutional components involving samples of $x(\ell)$ prior to $\ell = 0$. These samples fold over into the desired window of ranges due to the temporal (and hence spatial) extent of the transmitted waveform. The result is that the presence of a significant scatter within N-1 range gate samples prior to $x(0)$ can have a significant deleterious effect on the estimation of a large portion of the range window.

To compensate for this shortcoming of the LS estimator we first revisit the matched filter and note that when $\ell = 0$ in (4), the matched filter does account for the samples just prior to the beginning of the range window. It is therefore necessary to obtain a middle ground between the matched filter and the LS estimate. The remainder of this paper develops the Minimum Mean-Square Error (MMSE) estimator [7] for pulse compression that provides a robust estimate of the radar channel while maintaining range sidelobes at or very near the level attained by the LS estimator.

II. MMSE Pulse Compression

Minimum Mean-Square Error estimation is a Bayesian estimation approach that employs prior information in order to improve estimation accuracy. The exact form that the prior information will take for pulse compression will be addressed shortly. First, however, the signal model must be constructed. From (3), we see that the collection of N samples of the received radar return can be expressed as

$$\tilde{\mathbf{y}}(\ell) = [\mathbf{A}(\ell)]^T\,\mathbf{s} + \mathbf{v}(\ell)\,. \tag{8}$$

This is the same received signal model used by the matched filter and takes into account all the necessary radar channel components for a given received return sample. To develop the MMSE filter, the matched filter $\mathbf{s}^H$ in (3) is replaced with the MMSE filter, denoted $\mathbf{w}^H(\ell)$, in which the exact form of the MMSE filter is dependent upon the swath of range under consideration. Thereafter, the MMSE cost function

$$J(\ell) = E\left[\left|x(\ell) - \mathbf{w}^H(\ell)\,\tilde{\mathbf{y}}(\ell)\right|^2\right] \tag{9}$$

is solved for each range gate $\ell = 0, \cdots, L-1$, where $E[\bullet]$ denotes expectation. This is done by differentiating with respect to $\mathbf{w}^H(\ell)$ and then setting the result to zero. The MMSE filter is found to take the form

$$\mathbf{w}(\ell) = \left(E\left[\tilde{\mathbf{y}}(\ell)\,\tilde{\mathbf{y}}^H(\ell)\right]\right)^{-1} E\left[\tilde{\mathbf{y}}(\ell)\,x^*(\ell)\right], \tag{10}$$

where $(\bullet)^*$ is the complex conjugate operation. After substituting for $\tilde{\mathbf{y}}(\ell)$ from (8) and assuming that the radar channel range gates are, in general, uncorrelated with one another and are also uncorrelated with the noise, one obtains

$$\mathbf{w}(\ell) = \rho(\ell)\,\left([\mathbf{C}(\ell)] + [\mathbf{R}]\right)^{-1}\mathbf{s}\,, \tag{11}$$

where $\rho(\ell) = E[\,|x(\ell)|^2\,]$ is the expected power of $x(\ell)$, and $[\mathbf{R}] = E[\mathbf{v}(\ell)\;\mathbf{v}^H(\ell)]$ is the noise covariance matrix. On assuming neighboring range cells are uncorrelated, the $(i, j)^{th}$ element of the matrix $[\mathbf{C}(\ell)]$ is

$$[\mathbf{C}(\ell)]_{i,j} = E\left[\sum_{n=\kappa_L}^{\kappa_U} \rho(\ell - n + i - 1)\; s(n)\; s^*(n - i + j)\right] \tag{12}$$

in which $\kappa_L = \max\{0, i-j\}$ is the summation lower bound and $\kappa_U = \min\{N-1, N-1+i-j\}$ is the upper bound. Also, any prior information regarding the noise can be employed via the noise covariance matrix $[\mathbf{R}]$. For instance, for a white noise assumption, $[\mathbf{R}]$ is diagonal.

Obviously, in its current state the MMSE pulse compression filter is a function of the powers of the radar returns in surrounding range gates, which in practice are unavailable. This lack of prior knowledge can be taken into account by setting all the initial radar channel estimates equal. Therefore, the initial MMSE pulse compression filter reduces to the form

$$\tilde{\mathbf{w}} \cong \left[\tilde{\mathbf{C}}\right]^{-1}\mathbf{s} \tag{13}$$

where the noise term is assumed negligible and

$$\left[\tilde{\mathbf{C}}\right]_{i,j} = \left[\sum_{n=\kappa_L}^{\kappa_U} s(n)\; s^*(n - i + j)\right] \tag{14}$$

is invariant to the range gate delay ℓ. The initial MMSE pulse compression filter can be pre-computed and then implemented in the same way as the traditional matched filter. The inclusion of the matrix $[\tilde{\mathbf{C}}]$ serves to provide a "local" LS initial guess.

Figure 1 illustrates the MMSE pulse compression algorithm for three iterations. In general, the reiterative MMSE filter operates as follows:

1) Collect the received samples $y(-(M-1)(N-1)),\cdots,$ $y(L+M(N-1))$, which comprise the length-L radar channel window along with the $(M-1)(N-1)$ samples prior to the window and the $M(N-1)$ samples after the window.

2) Apply the initial MMSE pulse compression filter from (13) to obtain the initial radar channel estimates $\hat{x}_1(-(M-2)(N-1)),\cdots,\hat{x}_1(L+(M-1)(N-1))$.

3) Compute the initial power estimates $\hat{\rho}_1(\ell)=|\hat{x}_1(\ell)|^2$ for $\ell=-(M-2)(N-1),\cdots,L+(M-1)(N-1)$ which are used to compute the range-dependent filters $\mathbf{w}_1(\ell)$ as in (11), and then apply to $y(\ell)$ to obtain $\hat{x}_2(\ell)$.

4) Repeat **2**) and **3**), changing the indices where appropriate, until the desired length-L range window is reached.

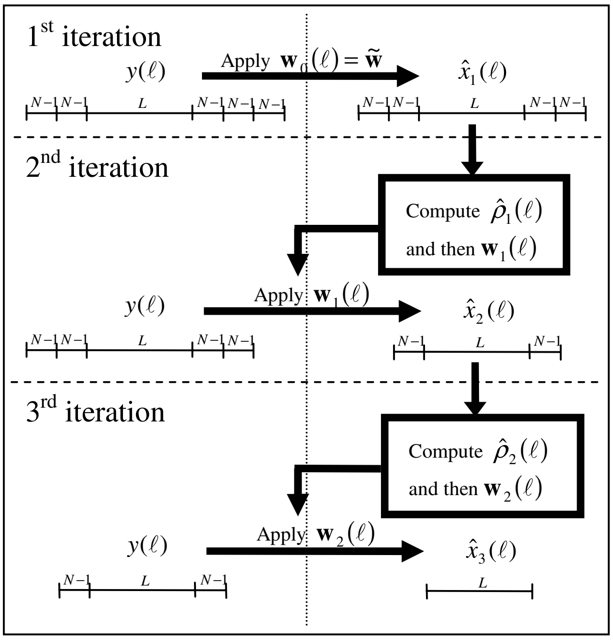

Fig. 1. Operation of reiterative MMSE pulse compression filter

The initial guess of the radar channel found by applying the MMSE filter can be used as *a priori* information to reiterate the MMSE filter and improve performance. This is done by employing the MMSE filter formulation from (11) in which the respective powers of the radar returns are taken from the current estimate. It has been found that two or three reiteration steps allow the MMSE pulse compression filter to exclude the effects of scatterers prior to the range window, as well as to suppress the range sidelobes very close to the level of the noise floor. The MMSE filter does especially well when the radar channel is somewhat sparsely parameterized (*i.e.* highly spiky), as is the case with high range resolution radar.

Note that each reiteration step will reduce the number of range gate estimates by $2(N-1)$. To counteract this, it is necessary to increase the size of the range window by $2M(N-1)$ samples, where M is the number of reiteration steps. Typically, however, $L >> N$ so that this reduction in range window size is negligible.

III. Implementation

One may immediately notice that in order to reiterate the MMSE pulse compression filter it is necessary to invert an $N\times N$ matrix for every range gate which requires on the order of N^3 operations for each inverse. This can cause a computational burden whenever either N or L is large. However, the inversion of the roughly L matrices can be divided into small groups and computed in parallel. For this reason, the computational complexity of the MMSE filter scales with the number of parallel processors and hence can be computationally feasible given enough processors.

Another important factor regarding practical implementation is the non-singularity of the $N \times N$ matrix $([\mathbf{C}(\ell)]+[\mathbf{R}])$. This can be addressed by instituting a nominal level for which the estimated returns are not allowed to go below. An alternative to this would be to re-estimate only those range gates that are above some threshold since the small-valued range gates most likely do not contain a target and are therefore of little interest.

IV. Simulation Results

To demonstrate the performance of the MMSE pulse compression filter we examine two cases. The first case is typical of the scenario often addressed for LS estimation techniques and consists of a radar channel with a single large target return in noise. The other case we address is when there is a second large target that resides in a range just prior to the range window. In this case it is expected that the LS estimator will substantially degrade since this important region is not accounted for in the received signal model. The waveform selected is the length N = 30 polyphase modulated Lewis-Kretschmer P3 code [8], which on receive (after down-conversion to baseband) is defined as

$$s(n)=\exp\left(\frac{j\pi n^2}{N}\right), \qquad n=0,1,\cdots,N-1. \qquad (15)$$

The noise and clutter are modeled as zero-mean Gaussian with powers set to -50 dB and -30 dB, respectively. For both cases we perform two iterations of the MMSE filter and then compare the MMSE-estimated radar channel with the true radar channel, as well as with the results obtained from using LS and the matched filter.

The radar channel for the first case is depicted in Fig. 2 for a single target present in the range window which consists of 200 range gates. Figure 3 illustrates the results from the different estimation techniques in which the matched filter experiences significant range sidelobes while both LS and MMSE have suppressed the range sidelobes so that the true target location is evident.

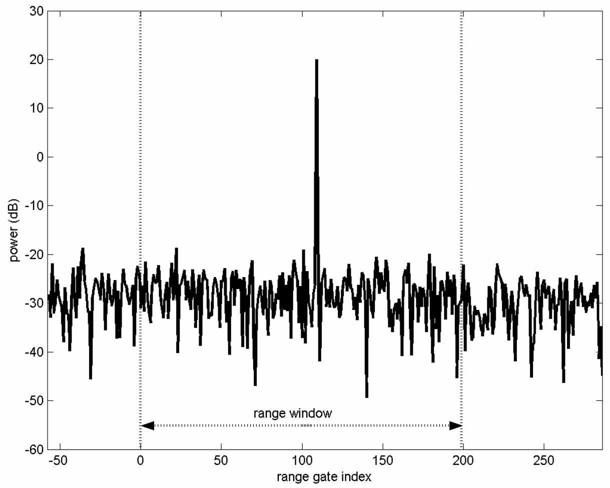

Fig. 2. Radar channel with single target in the range window

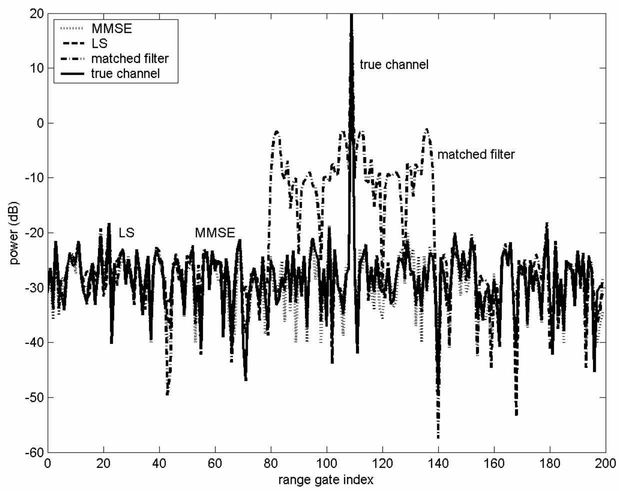

Fig. 3. Performance of MMSE, LS, & matched filter for single-target channel

For the scenario just described, the LS and MMSE estimators perform almost identically. However, when there is a significant scatterer present just prior to the range window, as depicted in Fig. 4, the LS estimator is expected to degrade substantially. From Fig. 5, the LS estimator truly does suffer from severe mis-estimation when a significant scatterer cannot be expressed in the model. However, the performance of the MMSE estimator is indistinguishable from the previous case where only a single target was present. This is obviously due to the fact that the MMSE estimator takes all necessary range regions into account when estimating the radar channel.

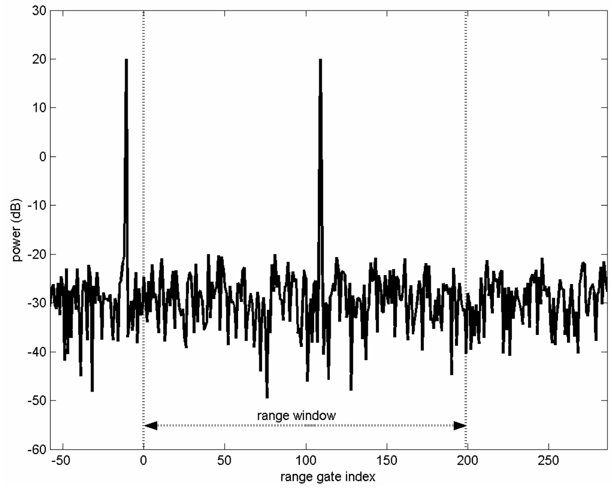

Fig. 4. Radar channel with one target in range window and one target just before range window

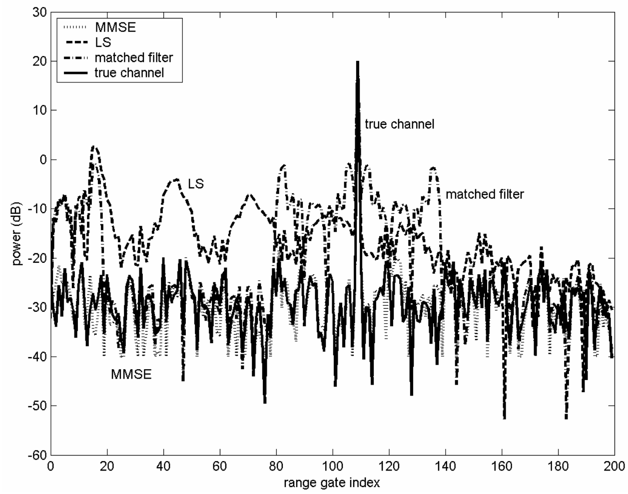

Fig. 5. Performance of MMSE, LS, & matched filter for target before range window

Table 1 presents the Mean Squared-Error (MSE) for both radar channels discussed using LS, MMSE, and normalized matched filtering. The MSE is averaged for all 200 range gates and over 100 runs using the same target(s), with the clutter and noise distributed according to a zero-mean Gaussian distribution for each run independently.

Table 1. MSE performance comparison

	Norm. MF	LS	MMSE
Channel 1	-12.3 dB	-40.9 dB	-35.8 dB
Channel 2	-11.2 dB	-11.3 dB	-34.7 dB

For channel 1 (single target), the normalized matched filter has a large MSE due to range sidelobes, while LS and MMSE perform nearly the same with LS just marginally better. However, for channel 2 (target just prior to the range window), the MSE attained by LS degrades to nearly that of the normalized matched filter, while the MMSE maintains roughly the same MSE as in the previous case.

V. CONCLUSION

Least-Squares pulse-compression estimation of the radar channel is a well-known approach that has been shown to be an optimal estimator for the assumed radar channel model. However, scatterers may exist prior to the range window of interest which would inherently invalidate the LS radar channel model. To alleviate this problem, the MMSE estimator is proposed that reiteratively finds a "local" least-squares solution and in so doing takes all necessary radar channel information into account. The result is an estimator that substantially reduces range sidelobes compared to matched filtering, as well as provides robustness to scatterers that may lie outside the range window of interest. Furthermore, reiterative MMSE estimation is not limited to pulse compression alone. It may also find use in range profiling, image recognition for SAR and ISAR, or any other application requiring robust deconvolution of a known waveform with an unknown impulse response.

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