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A Novel Quantum Inspired Particle Swarm Optimization Algorithm for Electromagnetic Applications

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ABSTRACT Quantum inspired particle swarm optimization (QPSO) stimulated by perceptions from particle swarm optimization and quantum mechanics is a stochastic optimization method. Although, it has shown good performance in finding the optimal solution to many electromagnetic problems. However, sometimes it falls to local optima when dealing with hard optimization problems. Thus, to preserve a good balance between local and global searches to avoid premature convergence in quantum particle swarm optimization, this paper proposed three enhancements to the original QPSO method, the proposed method is called modified quantum particle swarm optimization (MQPSO) algorithm. Firstly, a novel selection technique is introduced that will choose the best particle among the population within the search domain to achieve a high-performance exploration. Secondly, a new mutation method is used to preserve the easiness of available QPSOs. Also, a dynamic parameter strategy is proposed for further facilitating the algorithm and tradeoff between exploration and exploitation searches. The experimental results obtained by solving standard benchmark functions and an electromagnetic design problem which is the superconducting magnetic energy storage (SMES) system available in both three parameters and eight parameters problems are reported to showcase the usefulness of the proposed approach.

INDEX TERMS Electromagnetic application, optimization design, particle swarm optimizer, quantum mechanics, mutation mechanism.

I. INTRODUCTION

In the global optimization world, when one desires to solve the engineering optimization problems rising from electromagnetics than more devotion will be paid to stochastic techniques. This is because many of the design problems include objective function with more than one optimum and by the existence of the stochastic elements, the stochastic techniques will reach the global optimum with certainty under mild condition. Recent stochastic approaches used are simulated annealing, evolutionary algorithms, tabu search method and particle swarm optimization. One of the deficiencies of these kinds of methods is the slow convergence behavior or more computational load. Thus, to relieve the unnecessary computational load and develop the robustness of the method, current research on these techniques focuses on the refinements of the methods to increase their efficiency and to create a good balance between precision, reliability and computational loads. In this regard, many stochastic methods and their variants have been developed as recorded in the following paragraph.

An adaptive null-steering beamformer based on a bat algorithm was proposed for uniform linear array antennas to suppress the interference [1]. A fast-numerical optimization

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algorithm was proposed for the design and optimization of radome-enclosed antenna arrays [2]. A new brainstorm method with multi-information relations was presented to solve optimization problems [3]. In [4], a deep feature fusion technique was proposed for extracting degradation features. A new super resolution scattering center extraction method was introduced for the dimension reduced optimization problems [5]. A novel beamforming technique was presented for controlling the sidelobes and the nulling level [6]. In [7], a new ant colony-based optimization method for numerous standard applications has been proposed. A cockroach swarm optimization was presented in [8] to determine paths with the shortest travel time. A whale optimization approach has applied to renewable energy impact on sustainable development [9]. In [10], the fruit fly method was successfully applied to the optimization of antenna design problems. However, according to no free lunch theorem, there is no global technique that can be successfully applied to all optimization design problems. Thus, it is required to search and developed a new global method for the study of electromagnetic applications.

Moreover, compared to other evolutionary algorithms, particle swarm method is an addition to the evolutionary world. PSO is very easy in perceptions and implementations. Since it has been applied effectively in solving a broad range of engineering problems. However, as an emerging methodology, the PSO algorithm has still many issues. For example, the PSO method may encounter premature convergence when looking for the global optimum of difficult optimization problems due to its inadequacy in maintaining a balance between local and global searches that result in a stagnation probably occurs and the algorithm trapped to local minima. To address such inadequacy in traditional PSO, a quantum inspired version of the particle swarm optimization (QPSO) was proposed in [11]. However, there are still open issues in QPSO that need to be addressed.

In this regard, a novel selection strategy is introduced to pick up the fittest particle within the swarm that will further take part in the exploration process. Also, a new mutation mechanism is used with student t probability distribution method, and some parameter updating rule is proposed as reported in this work to enhance the QPSO performance and strengthen the improvements in its global searching capabilities. Numerical results of the proposed MQPSO method on well-known test functions and a workshop TEAM problem 22 are also presented to showcase the applicability of the proposed MQPSO method.

II. QUANTUM PARTICLE SWARM OPTIMIZER

The flock of bird's target a promising food location can be model by simple rules of exchange of information between individual birds. Such attributes motivate Kennedy and Eberhart to originate PSO [12] as a technique for global optimization.

In a PSO, each individual represents a potential solution to the problem, the particle's position is influenced by the best position found by itself and the best position found by its neighbor. The best position is called the global best when the neighbor is a complete swarm, and the algorithm is called *gbest* PSO (global best). But when a smaller neighbor is used it is called lbest PSO (local best). Each individual performance can be measured by a fitness solution.

Let us consider a group of individuals, each evolving in D dimensional region with its coordinates representing a possible solution to a problem. In the process of evolution, each individual move with a velocity within the search domain and keeps the best position ever achieved in its memory. The velocity v and position x of the i^{th} particle is updated by the following equations.

$$v_i(t+1) = w \times v_i(t) + c_1 r_1 \times (p_i(t) - x_i(t)) + c_2 r_2 \times (p_g(t) - x_i(t)) \quad (1)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (2)$$

where v_i is the velocity of the i^{th} particle, x_i represent the position of the i^{th} particle, the p_i is called the previous best position of the i^{th} particle, p_g is known as the best particle found by all particles.

The w is inertia weight that controls the moment of the particles, r_1 and r_2 are the two uniform random numbers within the interval [0, 1], c_1 and c_2 are the two learning factors and t indicates the number of iterations (generations).

The trajectory analysis [11] illustrate that the PSO convergence behavior can be definite if each particle converges to its local attractor $p_i = (p_{i,1}, p_{i,2}, \ldots\ldots\ldots\ldots, p_{i,d})$, of which the coordinates are

$$p_i(t) = (c_1 p_i(t) + c_2 p_g(t))/(c_1 + c_2) \quad (3)$$

or

$$p_i(t) = \varphi . p_i(t) + (1-\varphi) . p_g(t) \quad (4)$$

where $\varphi = c_1 r_1/(c_1 r_1 + c_2 r_2)$. It has been shown that the local attractor particle i lies in a hyper rectangle with p_i and p_g are the two ends of its diagonal.

In [11], the parameter $L(t)$ is evaluated as

$$L(t) = 2.\beta . |p_i(t) - x_i(t)| \quad (5)$$

As the control method of parameter $L(t)$ is important to the convergence behavior and algorithm performance. Furthermore, the mean best position is introduced to evaluate $L(t)$, the Mainstream thought or mean best position is defined as the center of personal best position of the population. i.e.

$$m(t) = (m_1(t), m_2(t), \ldots\ldots, m_d(t)) = \left(\frac{1}{N}\sum_{i=1}^{N} p_{i,1}(t), \frac{1}{N}\sum_{i=1}^{N} p_{i,2}(t), \ldots\ldots, \frac{1}{N}\sum_{i=1}^{N} p_{i,d}(t) \right) \quad (6)$$

where N represents the population size. Thus, parameter L will become,

$$L(t) = 2.\beta . |m(t) - x_i(t)| \quad (7)$$

where β is known as the contraction expansion coefficient, it is used to control the convergence behavior of the optimizer and is given by,

$$\beta = 0.5 + (1.0 - 0.5)(Maxiter - t)/Maxiter \tag{8}$$

Thus, the particle's position will be updated accordingly,

$$x_i(t+1) = p_i(t) \pm \beta \times |m(t) - x_i(t)| \times \ln(1/u) \tag{9}$$

where u is a uniform random number within the interval [0,1].

The "Eq. (9)" is known as the position updated equation of QPSO.

III. RELATED WORK

To facilitate the understanding of the proposed MQPSO method, this section will review some latest related works of the researchers.

An improved quantum based PSO with elitist breeding strategy was proposed for unconstrained optimization problems [13]. A dynamic cooperative quantum-based particle swarm algorithm was proposed in [14], the proposed method incorporates a new method for dynamically updating the context vector. A decentralized quantum behaved particle swarm method with a cellular structured population has been proposed to keep the diversity high and maintain a balance with local and global searches [15]. An improved quantum-based particle swarm optimization was presented by employing a chaotic search method to promote the quality of initial populations [16]. A quantum inspired particle swarm optimization method with an enhanced strategy was proposed for constrained optimization problems in [17].

In this context, many QPSO variants have been proposed. However, there are still many open issues in QPSO. Thus, for this purpose in this work three enhancement to the original QPSO is proposed for the optimizations of electromagnetic application.

IV. THE PROPOSED MQPSO APPROACH

The main deficiency of QPSO and other evolutionary approaches while dealing with complex engineering optimization problems is premature convergence that consequences in great efficiency loss and sub optimal solutions. In QPSO, the exchange of information is fast between the individuals because of its collectiveness and the diversity of the population decreases swiftly which will make the QPSO algorithm in great difficulties to avoid from local optima. However, in QPSO the search area of each particle is the complete feasible solution region of the problem, still diversity loss of the swarm occurs because of clustering.

To avoid such problems and enrich the QPSO performance, this work proposed three enhancements to the original QPSO and proposes a new method called MQPSO, this will avoid the population from clustering, intensify the diversity, and facilitate the convergence behavior of the proposed approach.

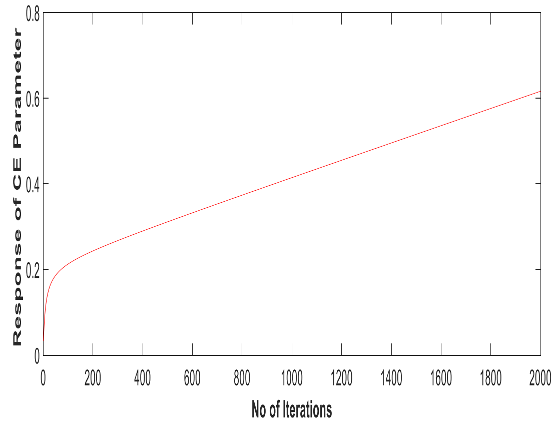

FIGURE 1. Distribution of β (CE parameter) with no of iterations.

A. SELECTION OF BEST PARTICLE

Firstly, a new particle called P_{best2} is randomly produced in the current search area by adopting the following methodology

$$P_{best2}(t) = L_U(t) - E_1 \times (L_U(t) - L_L(t)) \tag{10}$$

where L_U and L_L are the two boundary limits referred to as upper and lower limits set for the decision parameters and E_1 is a random number generated with exponential distribution method within a specified interval, the value of E_1 is varied according to the limits set by the decision parameters.

The P_{best2} will then be compared with the previous best particle p_i in the current swarm, If the P_{best2} is better (it has better fitness value) than the p_i, the P_{best2} will be replaced by the p_i, otherwise, the p_i will persevere in the same position for the next generations of a cycle.

The proposed selection strategy is chosen, because during the exploration process the diversity of the swarm is initially high but later on it decreases quickly, this is because the distance between the *Mbest* and current particle $|Mbest - x_i(t)|$ is very small for the particle to avoid from local minima. Hence, the proposed method will extend the distance between current particles and *Mbest* consequently, it would make the particles explode temporarily.

B. INTRODUCTION OF A MUTATION MECHANISM

Secondly, a new mutation method is used to help the algorithm in escaping from local minima and will achieve an optimum solution.

In this method, firstly, the random numbers are produced using the student t probability distribution within a specified interval. Then, this new method combined with the mutation operator is given as follows.

$$p_g = (st_1 p_i(t) + st_2 p_g(t))/2 \tag{11}$$

where st_1 and st_2 are the two random numbers generated with student t probability distribution method, p_i is the personal best position of particle and p_g is the global best position of a particle.

TABLE 1. Standard test functions.

Expression	Search Limit
$f_1(x)=\sum_{i=1}^{D}(100.(z_{i+1}-z_i{}^2)^2+(z_i-1)^2)+$ $f\_bias, z=x-o+1,$	$x\in[-100,100]^D$
$f_2(x)=\frac{1}{4000}\sum_{i=1}^{D} z_i{}^2-\prod_{i=1}^{D}\cos(\frac{z_i}{\sqrt{i}})+1+$ $f\_bias, z=x-o,$	$x\in[-600,600]^D$
$f_3(x)=\sum_{i=1}^{D} x_i{}^2$	$x\in[-100,100]^D$
$f_4(x)=\sum_{i=1}^{D}(100.(x_{i+1}-x_i{}^2)^2+(x_i-1)^2)$	$x\in[-100,100]^D$
$f_5(x)=\sum_{i=1}^{D} ix_i{}^4$	$x\in[-100,100]^D$
$f_6(x)=\frac{1}{4000}\sum_{i=1}^{n} x_i{}^2-\prod_{i=1}^{D}\cos(\frac{xi}{\sqrt{i}})+1$	$x\in[-100,100]^D$
$f_7(x)=\sum_{i=1}^{D}\lvert x_i\rvert+\prod_{i=1}^{D}\lvert x_i\rvert$	$x\in[-100,100]^D$
$f_8(x)=\sum_{i=1}^{D}[x_i^2-10\cos(2\pi x_i)+10]$	$x\in[-5.12,5.12]^D$
$f_9(x)=-a\exp\left(-b\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i{}^2}\right)-\exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(c\cdot x_i)\right)$ $+a+\exp(1)$	$x\in[-32,32]^D$
$f_{10}(x)=\sum_{i=1}^{D} z_i^2+f\_bias, z=x-o,$ $x=[x_1,x_2,\ldots,x_D]$	$x\in[-100,100]^D$
$f_{11}(x)=\sum_{i=1}^{D}(\prod_{j=1}^{i} z_j)^2+f\_bias, z=x-o,$ $x=[x_1,x_2,\ldots,x_D]$	$x\in[-100,100]^D$
$f_{12}(x)=\sum_{i=1}^{D}(10^6)^{\frac{i-1}{D-1}}x_i^2$	$x\in[-100,100]^D$

The proposed mutation method will bring good cooperation between the possibility of having a large number with small amplitudes around the current solution and a small possibility of having higher amplitudes that may permit the individuals to move away from the current solution and avoid the local optima. This is because when the proposed mutation is applied, the global best particle will intensify the average distance of the personal best particle from its mean best position. This will increase the distance between the current particles and mean best, which will extend the search scope of a particle.

C. PARAMETER UPDATING FORMULAE

Thirdly, as the contraction expansion coefficient β is a vital parameter and is used for tuning the optimizer performance and play an important role to bring balance between the exploration and exploitation searches. Hence, if the value of β is constant, the balance between exploration and exploitation will be disturbed and the individual could not find the global optima for complex optimization problems. Also, the local and global searches require a minimum and maximum value for the contraction coefficient β parameter. Because a constant value will encounter premature convergence. Thus, it is clear that without proper adjustment of the β parameter will result in the optimizer stuck into local minima.

Therefore, many researchers have proposed different methodologies for β parameter to control the convergence behavior of the optimizer as stated in [18], [19].

Thus, to avoid trapping to local minima and bring a balance between local and global searches, in this work some strategy for parameter updating is proposed as

$$\beta = \frac{\sin(1-rand)}{(1+t)^Z} - (Maxiter - t)/Maxiter \tag{12}$$

$$Z = \exp(1-\log(t)) \tag{13}$$

where $rand$ is a uniform random number within the interval [0,1], $Maxiter$ represents the maximum number of iterations and t is the current iteration.

As shown in Fig 1, the value of β is set initially low, this is because as when the particle is far away from mean best position, then one expects a small β to help it come back while if the particle is just close to mean best then a large β is preferred to force it to bounce away and tradeoff between exploration and exploitation searches.

TABLE 2. Comparison of different optimal methods on standard benchmark functions.

Test functions		SQPSO	GQPSO	LTQPSO	MQPSO
f_1	Mean	0.1376	8.9316	2.7624	4.8216×10^{-3}
	Variance	3.8629×10^{-2}	0.1308	0.2642	2.0317×10^{-6}
f_2	Mean	3.0716×10^{-2}	1.8028×10^{-1}	1.7624×10^{-2}	2.5408×10^{-3}
	Variance	4.8620×10^{-3}	1.2303×10^{-2}	2.8936×10^{-4}	5.2107×10^{-7}
f_3	Mean	1.1087×10^{-29}	1.9489×10^{-24}	1.4891×10^{-40}	1.6749×10^{-153}
	Variance	2.9477×10^{-39}	6.8768×10^{-35}	1.3858×10^{-53}	3.932×10^{-213}
f_4	Mean	35.114	27.041	28.975	2.107
	Variance	22.106	2.2833×10^{-1}	0.1783	3.674×10^{-3}
f_5	Mean	4.1673×10^{-38}	4.7732×10^{-33}	3.2064×10^{-65}	1.6749×10^{-213}
	Variance	4.893×10^{-51}	1.5431×10^{-47}	4.9640×10^{-76}	5.8464×10^{-375}
f_6	Mean	9.6730×10^{-3}	9.1645×10^{-5}	5.9300×10^{-2}	7.4015×10^{-18}
	Variance	6.9431×10^{-5}	2.8513×10^{-7}	2.9463×10^{-3}	6.9251×10^{-29}
f_7	Mean	1.0345×10^{-6}	6.2876×10^{-13}	2.2365×10^{-37}	4.9407×10^{-223}
	Variance	2.8362×10^{-9}	3.9412×10^{-19}	3.8351×10^{-51}	2.9635×10^{-378}
f_8	Mean	21.893	2.0289×10^{-2}	7.6020	9.7700×10^{-16}
	Variance	5.6098	5.3850×10^{-3}	3.4911	2.9463×10^{-25}
f_9	Mean	3.4639×10^{-14}	8.6509×10^{-14}	6.9025×10^{-14}	5.9253×10^{-17}
	Variance	8.9432×10^{-19}	1.7535×10^{-19}	1.5935×10^{-19}	1.9352×10^{-29}
f_{10}	Mean	3.6732×10^{-4}	1.8311	1.3167	2.5674×10^{-3}
	Variance	4.9465×10^{-7}	8.0815×10^{-1}	6.2054×10^{-1}	7.1819×10^{-6}
f_{11}	Mean	9.0494×10^{-6}	6.3451	10.8134	2.6092×10^{-5}
	Variance	5.6372×10^{-9}	1.2379	9.2064	4.7251×10^{-8}
f_{12}	Mean	1.8463×10^{-2}	1.9463	2.8463	1.9463×10^{-6}
	Variance	2.9537×10^{-3}	0.4532	1.2543	3.7264×10^{-11}

V. NUMERICAL RESULTS

In this section, a set of benchmark functions taken from [20], are used to validate the applicability of the proposed MQPSO. The details of the functions are reported in Table 1.

For a fair comparison, this case study is solved using the proposed MQPSO, SQPSO [11], LTQPSO [21] and GQPSO [22] approach. In this case study, the parameters are set as the swarm size used is 40 with 30-dimensional problems for the corresponding number of generations is 2000. In the numerical studies, each experiment has run 30 trials and the final outcomes of the optimal algorithms are recorded in table 2.

The outcomes of table 2 reveals that the proposed MQPSO has improved its global searching capability as compared to other optimal methods on most of the tested functions except at f_{10} and f_{11}, where the standard QPSO significantly improved its performance. The proposed MQPSO beats the GQPSO and LTQPSO on all the tested functions on f_1 to f_{12}. However, the GQPSO and LTQPSO completely fails and could not produce improved outcomes on the shifted problems to avoid trapping into local optima.

Moreover, Figure 2 to 7, reveals the convergence process of different optimal methods (with 25 times runs) in a logarithmic scale of best objective function value on standard test functions, using a population size of 40, a number of iterations is 2000 for corresponding dimensions of 30. In this regard, the proposed MQPSO approach found an appropriate mean behavior in approximately initial iterations on most tested functions during the evolution process while all other optimal methods trapped into local minima.

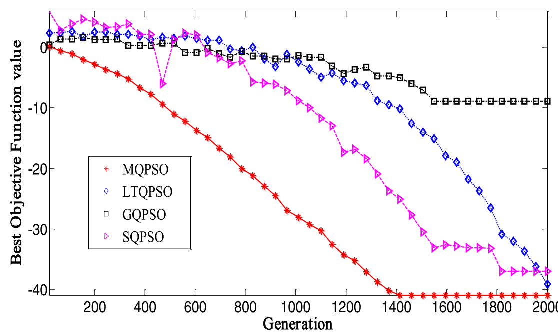

FIGURE 2. Convergence plots comparison of different optimizers on f_1.

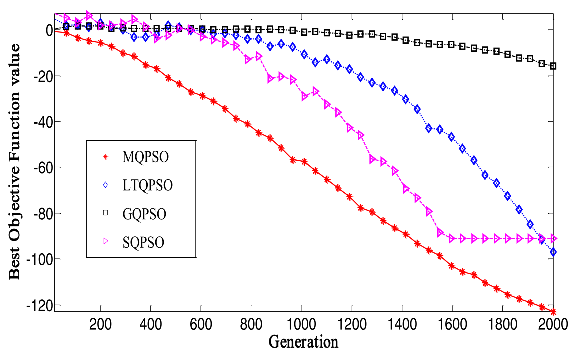

FIGURE 3. Convergence plot comparison of different optimizers on f_2.

The statistical analysis also illustrates that the convergence behavior of the proposed MQPSO is fast and the MQPSO method is a global optimizer on many tested functions.

One can reveal from the results of table 2, that the proposed MQPSO has outstanding performance on most tested functions as compared to other well-designed stochastic

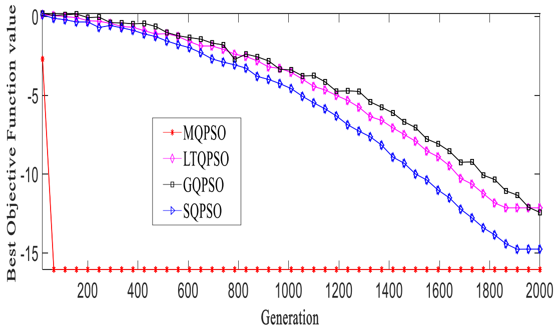

FIGURE 4. Convergence plots comparison of different optimizers on f_3.

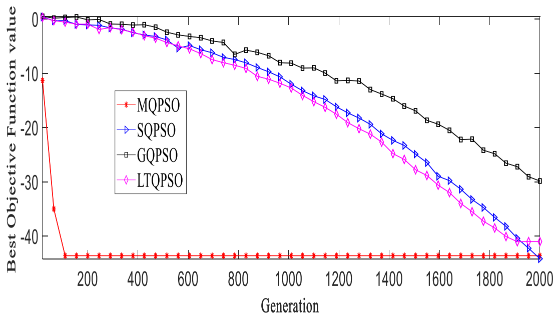

FIGURE 5. Convergence plots comparison of different optimizers on f_4.

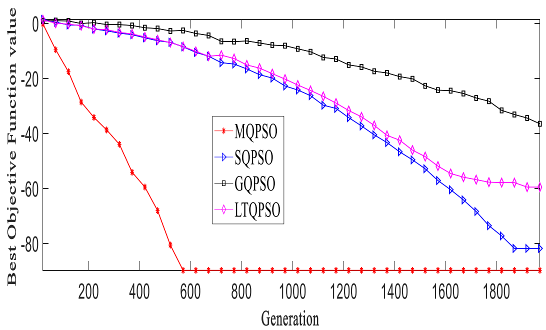

FIGURE 6. Convergence plots comparison of different optimizers on f_5.

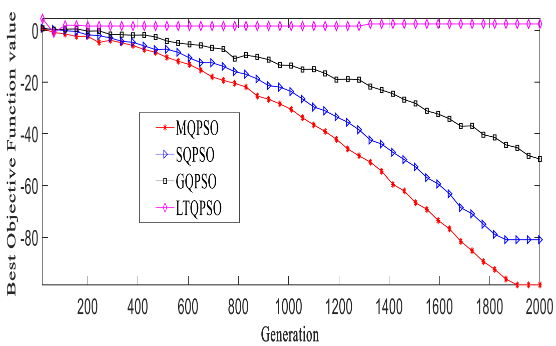

FIGURE 7. Convergence plots comparison of different optimizers on f_6.

optimizers and the proposed approach could hit the optimal solution with high accuracy and faster convergence speed.

It can be summarized, from the above discussions and statistical analysis that the proposed enhancements to the original QPSO method can efficiently extend the solution quality and convergence behavior of the proposed MQPSO.

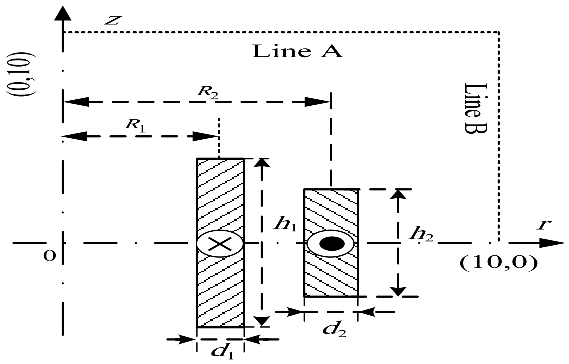

FIGURE 8. SMES configuration.

VI. ELECTROMAGNETIC APPLICATION

A standard electromagnetic design problem is the Team Workshop problem 22 of a superconducting magnetic energy storage (SMES) configuration with three parameters as stated in [23]–[25], is then solved using the proposed approach. As shown in Fig 8, the system contains two concentric coils. They are the inner solenoid and an outer solenoid for reducing the stray field. The current directions in the coils are opposite to each other. The goal of the design is to acquire the desired store energy with negligible stray field and the design should fulfill:

1) The energy stored in the device is 180MJ.
2) The generated magnetic field inside the solenoids must not violate certain physical conditions to ensured superconductivity.
3) The mean stray field at 21 measurement points along lines A and B at a distance of 10 meters should be as small as possible.

To ensure the superconductivity of the conductors the constraint equation defines the current density of the two solenoids and their magnetic flux densities as follows:

$$J_i \leq (-6.4\,|(B_{\max})_i| + 54)(A/mm^2)(i = 1, 2) \tag{14}$$

where J_i and $|B_{\max}|_i$ are respectively, the current density and maximal magnetic flux density in the i^{th} coil.

In the three parameters problem, the inner solenoid is fixed at are optimized as, $r_1 = 2m$, $h_1/2 = 0.8m$, $d_1 = 0.27m$. The dimensional parameters of the outer solenoid are optimized under the following constraint conditions of $2.6m < r_2 < 3.4m$, $0.204m < h_2/2 < 1.1m$, $0.1m < d_2 < 0.4m$. Hereafter, we present the eight-parameters design problem (R_1, R_2, h_1, h_2, d_1, d_2, J_1, J_2) carried out by using the proposed MQPSO algorithm.

Moreover, the current densities of the coils are set to be $22.5A/mm^2$. However, for the convenience of mathematical implementation, equation (15) is simplified to $B_{\max}| \leq 4.92T$. Under these simplifications, the optimization problem is formalized as doubt

$$\min f = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|Energy - E_{ref}|}{E_{ref}} \quad \text{subject to } |B_{\max} \leq 4.92T \tag{15}$$

TABLE 3. Comparison of different optimizer on team problem 22 (three parameter).

Algorithms	Min (Best)	Mean	Standard Dev	Maximum	r_2	$h_2/2$	d_2
LTQPSO	0.1103	0.1211	0.01314	0.1396	3.1446	0.2419	0.3795
GQPSO	0.1222	0.1282	0.01397	0.1472	3.1723	0.2319	0.3892
SQPSO	0.1077	0.1189	0.01284	0.1348	3.0786	0.2414	0.3795
IQPSO	0.0796	-	-	-	3.1407	0.3149	0.2886
MQPSO	0.0789	0.0874	0.00346	0.1017	3.1003	0.2417	0.3874

TABLE 4. Comparison of optimal methods on team problem 22 (eight parameter).

Parameter	QEA	ABC	MQPSO
$R_1(m)$	1.5704	1.5702	1.5693
$R_2(m)$	2.1012	2.1018	2.0991
$h_1/2(m)$	0.7844	0.7844	0.7912
$h_2/2(m)$	1.4191	1.4201	1.4202
$d_1(m)$	0.6001	0.6002	0.5974
$d_2(m)$	0.2570	0.2574	0.2583
$J_1(A/mm^2)$	17.3358	17.3401	17.3401
$J_2(A/mm^2)$	-12.9658	-12.9653	-12.7924
F_{min}	6.6239×10^{-3}	6.7238×10^{-3}	6.0573×10^{-3}

TABLE 5. Comparison of different stochastic methods with different parameters for team problem 22.

Algorithms	$f_{\min}$	$B^2_{stray}\times10^{-7}$	Energy (MJ)	$r_2(m)$	$h_2/2(m)$	$d_2(m)$	No of Iteration
SQPSO	0.107	8.249	180.19	3.078	0.2414	0.3795	1578
GQPSO	0.122	7.946	179.21	3.172	0.2319	0.3892	1704
LTQPSO	0.110	8.437	181.06	3.144	0.2419	0.3795	1639
NTS	0.089	7.588	179.23	3.080	0.254	0.370	1800
PBIL	0.101	8.970	179.73	3.110	0.241	0.421	3278
MTS	0.086	7.757	180.00	3.090	0.249	0.375	1324
MQPSO	0.078	7.258	179.91	3.100	0.2417	0.3874	1309

where, $B_{norm} = 3 \times 10^{-3}T$, *Energy* is the stored energy in the SMES device, $(B_{\max})_i (i = 1, 2)$ is the maximum field in the i^{th} coil, B^2_{stray} is a measure of the stray fields evaluated along 22 equidistance points of lines A and B, using the following equation.

$$B^2_{stray} = \sum_{i=1}^{22} B^2_{stray,i} \Big/ 22 \tag{16}$$

In the numerical study, the performance parameters as required by (15) and (16) are evaluated based on a 2-dimensional finite element approach.

To compare performances, the proposed MQPSO method, standard QPSO [11], LTQPSO [21], GQPSO [22] and IQPSO [26] approaches are used to solve this case study. Table 3 summarizes the final optimal outcomes using three parameters model of different stochastic approaches with 10 independent runs. However, Table 4 presents the final outcomes on eight parameters design of the proposed MQPSO algorithm with QEA [27] and ABC [28]. Moreover, table 5 presents the optimal results for proposed MQPSO method with the results obtained by three other evolutionary techniques, a new tabu search (NTS) [29], a population based incremental learning (PBIL) method [30] and a modified tabu search (MTS) method [31] have taken from the literature for comparisons.

Since the iterative number is an appropriate parameter to evaluate the computational time, one can compute the computational efficiency using this parameter.

Moreover, from the outcomes of table 3 to 5, it can be illustrated that the optimal values of the decision parameters found by the proposed MQPSO method have significantly improved as compared to other tested methods. This positively proves the robustness and efficiency of the proposed MQPSO method for electromagnetic applications.

Hence, the above discussion and convergence analysis reveal the merit of the proposed MQPSO approach on other tested optimizers in terms of both the final solution searched (objective functions) and convergence behavior (number of iterations).

VII. CONCLUSION

In this work, a new version of particle swarm optimization called MQPSO is proposed and tested. The experimental outcomes on the case studies demonstrate that the proposed method can significantly improve as compared to other well-designed stochastic approaches. Furthermore,there is only one parameter that required tuning. A simple and effective optimizer for the study of electromagnetic applications is therefore reported. For future studies, it is suggested that other potential well stochastic approaches should be investigated and their performance should be evaluated and reported.

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