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A numerical algorithm of tooth profile of non-circular cylindrical gear

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Abstract. Non-circular cylindrical gear (NCCG) is a common form of non-circular gear. Different from the circular gear, the tooth profile equation of NCCG cannot be obtained. So it is necessary to use a numerical algorithm to calculate the tooth profile of NCCG. For this reason, this paper presents a simple and highly efficient numerical algorithm to obtain the tooth profile of NCCG. Firstly, the mathematical model of tooth profile envelope of NCCG is established based on the principle of gear shaping, and the tooth profile envelope of NCCG is obtained. Secondly, the polar radius and polar angle of shaper cutter tooth profile are chosen as the criterions, by which the points of NCCG tooth cogging can be screened out. Finally, the boundary of tooth cogging points is extracted by a distance criterion and correspondingly the tooth profile of NCCG is obtained.

INTRODUCTION

Gear is a kind of key mechanical component which can be used to transmit motion and force. It can be divided into circular gear and non-circular gear according to its shape of pitch curve. Circular gear has a fixed transmission ratio for its circular pitch curve and it is usually manufactured by rotary forging [1-6]. Non-circular gear has a variable transmission ratio for its non-circular pitch curve. Non-circular gears are widely used in various industrial fields because they can meet special variable transmission requirements [7]. For example, non-circular gear planetary system of skipping resistance differential in vehicle, elliptic cylindrical gear in the liquid flowmeter, gear rack with variable ratio in steering system and so on. Non-circular cylindrical gear (NCCG) is a common type of non-circular gear which has been widely used in aeronautics and astronautics instruments, track plotter and some optical instruments [8].

Because the pitch curve of NCCG is non-circular and complicated, NCCG has no accurate tooth profile equation, which brings the great difficulty in the accurate design and modeling of NCCG. This paper presents a simple and highly efficient numerical algorithm to obtain the tooth profile of NCCG.

A NUMERICAL ALGORITHM FOR NCCG TOOTH PROFILE

Principle of Shaping NCCG

According to the gear shaping principle, the NCCG tooth profile can be formed by the kinematic motion of pure rolling between the shaper cutter and the pitch curve of NCCG. According to the kinematic relationship of shaper cutter when shaping the NCCG, the coordinate systems are established in Fig. 1. 11

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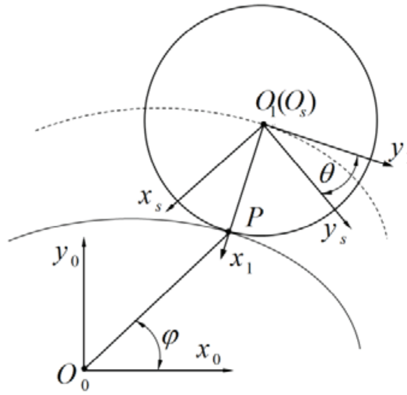

FIGURE 1. Coordinate position relation between NCCG and shaper cutter.

The pitch curve equation of NCCG is $r(\varphi)$, φ is the polar angle of point P, rectangular coordinate system $O_0x_0y_0$ is established at the center of the NCCG pitch curve, and the following coordinate system $O_1x_1y_1$ is established on the normal equidistant line of NCCG pitch curve, whose direction is from the shaper cutter center to the tangent point P. Because the shaper cutter not only moves on the pitch curve, but also rotates around its center, the coordinate system $O_sx_sy_s$ is needed to be fixed on the center of shaper cutter.

In order to obtained the NCCG tooth profile, the coordinate transformation matrix M_{1s} from the coordinate system $O_sx_sy_s$ to the coordinate system $O_1x_1y_1$, and the coordinate transformation matrix M_{01} from the coordinate $O_1x_1y_1$ system to the coordinate system $O_0x_0y_0$ are needed. M_{1s} and M_{01} can be calculated by the coordinate relations of the coordinate systems in Fig. 1.

The envelope of NCCG can be calculated by Eq. (1).

$$r_{k1}(\varphi_1) = M_{01}(\varphi_1)M_{1s}(\varphi_1)r(\varphi_1) \tag{1}$$

In this paper, the first order elliptic cylindrical gear is taken as an example, whose pitch curve equation is $r(\varphi_1)=64/(5-3\cos\varphi_1)$, tooth number is $z_1 = 25$. The parameters of shaper cutter are as follows: tooth number is $z_0 = 12$, pressure angle is $\alpha=20^\circ$.

According to the above mathematical model and parameters, the tooth profile envelope of NCCG is obtained in MATLAB programming, as shown in Fig. 2.

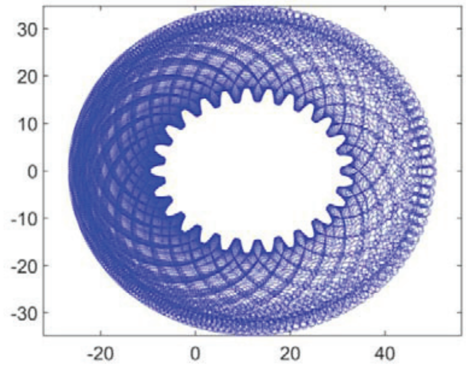

FIGURE 2. Tooth profile envelope of NCCG.

The tooth profile envelope reproduces the shaping process of the NCCG, which is the basis of the numerical algorithm of NCCG tooth profile.

A Numerical Screening Algorithm for NCCG Tooth Profile based on the

Normal Equidistant Curves of Pitch Curve of NCCG

The following conclusions can be drawn from the shaping principle of NCCG by using the shaper cutter: When the tooth of the shaper cutter continuously cuts out the NCCG blank, the equidistant curves within shaper cutter tooth entity will be removed and form the part of NCCG tooth cogging, while the remains form the part of the NCCG tooth.

In this paper, we can consider the equivalent inverse process of shaping NCCG: NCCG blank envelopes the shaper cutter, the part of NCCG within the shaper cutter tooth is removed and the NCCG tooth cogging is formed.

The mathematical model of NCCG blank can be expressed as the data points of the normal equidistant curves of NCCG pitch curve. Take n normal equidistant curves which can be expressed by $r_i(\varphi), (i = 1, 2, \dots \dots n)$, h_a is the tooth addendum of the NCCG, h_{ap} as the tooth addendum of the shaper cutter, $h = h_a + h_{ap}$ is the whole depth of the NCCG tooth. The rectangular coordinate equation of the i th equidistant curve can be expressed by Eq. (2).

$$\begin{cases} x_i(\varphi) = r_i(\varphi)\cos\varphi + h_i(r_i'(\varphi)\sin\varphi + r_i(\varphi)\cos\varphi)(r_i^2(\varphi) + r_i'^2(\varphi))^{-1/2} \\ y_i(\varphi) = r_i(\varphi)\sin\varphi - h_i(r_i'(\varphi)\cos\varphi - r_i(\varphi)\sin\varphi)(r_i^2(\varphi) + r_i'^2(\varphi))^{-1/2} \end{cases} \tag{2}$$

where h_i is the distance between the NCCG pitch curve and the i th equidistant curve: $h_i = i \times h / n - h_{ap}$.

Take m points on each equidistant curve, the rectangular coordinate equation of the j th point $R_{ij}, (j = 1, 2, \dots \dots m)$ can be expressed by Eq. (3).

$$\begin{cases} x_{ij} = r_i(\varphi_j)\cos\varphi_j + h_i(r_i'(\varphi_j)\sin\varphi_j + r_i(\varphi_j)\cos\varphi_j)(r_i^2(\varphi_j) + r_i'^2(\varphi_j))^{-1/2} \\ y_{ij} = r_i(\varphi_j)\sin\varphi_j - h_i(r_i'(\varphi_j)\cos\varphi_j - r_i(\varphi_j)\sin\varphi_j)(r_i^2(\varphi_j) + r_i'^2(\varphi_j))^{-1/2} \end{cases} \tag{3}$$

Transform point R_{ij} to point P_{ij} of coordinate system of shaper cutter by the coordinate transmission. The position of point P_{ij} is calculated to judge if the point P_{ij} locates within the tooth of shaper cutter. The tooth position of the shaper cutter can be calculated by the parameters of shaper cutter, the range of tooth and cogging polar angle of shaper cutter is shown in Fig. 3.

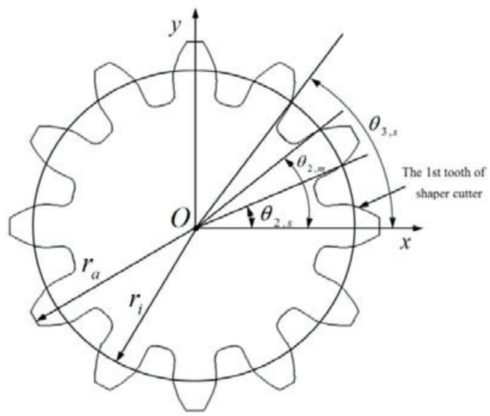

FIGURE 3. Range of tooth and cogging polar angle of shaper cutter

In Fig. 5, r_a is the addendum radius of shaper cutter, r_i is the polar radius of point P_{ij} , $\theta_{2,s}$ is the polar angle of right tooth profile of second tooth under the polar radius r_i, $\theta_{2,m}$ is the polar angle of left tooth profile of second tooth under the polar radius r_i, $\theta_{3,s}$ is the polar angle of right tooth profile of third tooth under the polar radius r_i. The polar angle range of t th tooth can be expressed as $(\theta_{t,s}, \theta_{t,m})$ by Eq. (4).

$$\begin{cases} \theta_{t,s} = -0.5\varphi_0 + t\dfrac{2\pi}{z_0} \\ \theta_{t,m} = 0.5\varphi_0 + t\dfrac{2\pi}{z_0} \\ t = 1,2,\ldots,z_0 \end{cases} \tag{4}$$

where $\theta_{t,s}$ is the polar angle of right tooth profile of t th tooth under the radius r_i , $\theta_{t,m}$ is the polar angle of left tooth profile of t th tooth under the radius r_i, φ_0 can be expressed by Eq. (5).

$$\begin{cases} \varphi_0 = \dfrac{s}{r_0} - 2(\tan\alpha_i - \alpha_i - \tan\alpha + \alpha) \\ \alpha_i = \arccos\dfrac{r_b}{r_i} \end{cases} \tag{5}$$

where S is the tooth thickness of pitch circle, r_0 is the radius of pitch circle, α_i is the pressure angle of tooth profile under the polar radius r_i

From above analysis, we can conclude that $(\theta_{t,s}, \theta_{t,m})$ is the polar angle range of t th tooth of shaper cutter under the polar radius r_i. Take this range as the criterion, by which all the data points that located within NCCG cogging at arbitrary processing moment can be screened out.

Judge all the points of equidistant curves by this screening algorithm in MATLAB, the points located within gear cogging is shown in Fig. 4.

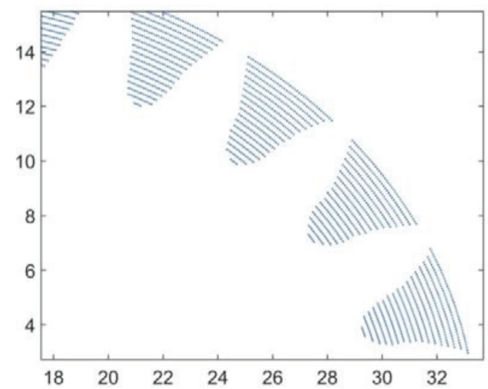

FIGURE 4. Cogging points of NCCG

Extraction of Tooth Profile of NCCG

As can be seen from Fig. 5, the blue points are tooth cogging points, while the red dots are the boundary points of the tooth cogging, namely the tooth profile points. The distance between two red points on the same equidistant curve is distinctly different with other two adjacent blue points on the same equidistant curve.

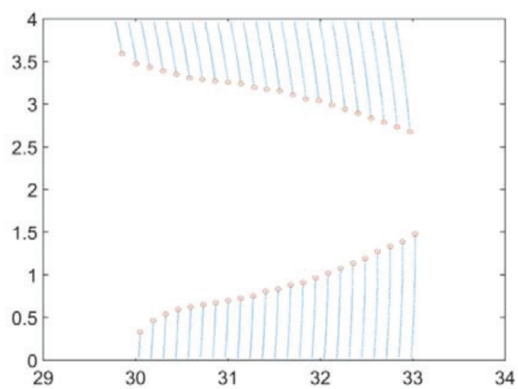

FIGURE 5. Boundary points of cogging points

CONCLUSION

1. Coordinate transformation relation is obtained according to the principle of gear shaping. According to the coordinate transformation matrix and the tooth profile equation of the shaper cutter, the envelope of the

NCCG tooth profile is obtained by programing in MATLAB. The tooth profile envelope reproduces the shaping process of the NCCG, which is the basis of the numerical algorithm of tooth profile.

2. Based on the process of shaping NCCG, a simple and highly efficient numerical screening algorithm of NCCG tooth profile is proposed. The range of polar radius and polar angle of shaper cutter tooth profile is set as the screening criterion, by which the tooth profile points of NCCG can be obtained and the three-dimensional model of NCCG can be established correspondingly.

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