

A NUMERICAL EXPERIMENT ON NONLINEAR IMAGE RECONSTRUCTION FROM FIRST-ARRIVAL TIMES FOR TWO-DIMENSIONAL ISLAND ARC STRUCTURE

Ichiro NAKANISHI and Kaoru YAMAGUCHI

*Department of Geophysics, Faculty of Science,
Hokkaido University, Sapporo, Japan*

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Many inversion studies of first P-arrival times have been made for the three-dimensional upper mantle structure beneath the Japanese Islands by using the linear damped least-squares method (AKI and LEE, 1976). One of the most recent contributions relying on this method is that by HASEMI *et al.* (1984) on the structure beneath the Tohoku region. In the linear inversion it is assumed that the ray paths in the inverted model are the same as those in the initial model. The ray tracing experiments attempted in the early 1970's (JACOB, 1970) showed that the ray paths are strongly guided by the high velocity slabs descending beneath the island arcs.

The aim of this study is to examine the performance of a nonlinear inversion of travel times of first-arrivals for the island arc structure. We apply the Dijkstra method (DIJKSTRA, 1959) of network theory in the forward calculation of ray paths and travel times instead of the shooting or bending method (JULIAN and GUBBINS, 1977). We subdivide a two-dimensional space of 100 km \times 100 km into a number of blocks of 10 km \times 10 km as shown in Fig. 1. We assume an average layered velocity profile of the area under consideration a priori. We make the velocity profile by taking the value of the JMA standard P-wave structure (HAMADA, 1984) for each 10 km starting from a depth of 5 km.

The algorithm proposed by DIJKSTRA (1959) is considered to be the fastest of the algorithms which find the shortest paths connecting a node and the other all nodes of a network (IRI and KOBAYASHI, 1976). For the algorithm, one should consult DIJKSTRA (1959) or IRI and KOBAYASHI (1976). We seek the shortest time paths connecting a node n_s and the other all nodes $n_j(j \neq s)$ of the network. When the node n_s is assigned to an event, and the nodes $n_j(j \neq s)$ include station positions, we are able to obtain the shortest time paths from the event to all the stations. When the node n_s indicates a station and all the event locations are included in $n_j(j \neq s)$, the algorithm gives us the shortest time paths from all the events to the station. In this study we shall put the nodes on the boundaries of the blocks.

Figure 1 shows a laterally heterogeneous model that corresponds to a vertical section of the uppermost mantle perpendicular to the strike of island arcs. The

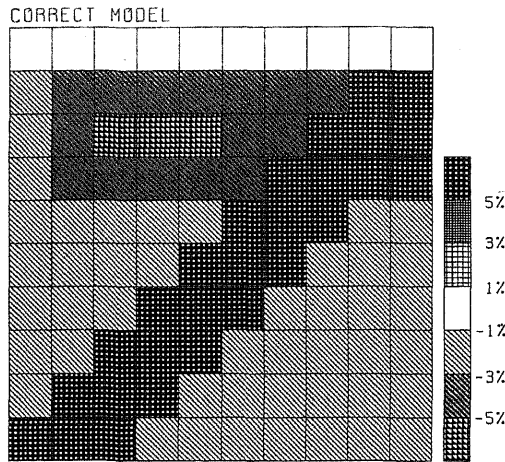


Fig. 1. A model for the island arc P-wave velocity structure. The difference from the JMA model is shown. The model is used to synthesize the data for the numerical experiment. The symbol convention is indicated in the figure.

model is characterized by an inclined high velocity plate. The P-wave velocity of the plate is 6% higher than that of the JMA model. The surrounding mantle, which consists of three parts, is 2%, 4%, or 6% lower in velocity than the JMA model.

Figures 2(a) and 2(b) show the ray paths obtained for the laterally homogeneous and laterally heterogeneous model, respectively. We notice that the rays from deep events to the stations on the oceanic side (the right-hand side in Fig. 2) are severely bent in the heterogeneous model.

The interval of the nodes on the block boundaries is 1 km in the above examples and the following inversion experiment. It is important to examine the accuracy of the travel times obtained from this node interval. We calculated the travel times and ray paths for node intervals of 0.5 km and 0.2 km by using the heterogeneous model of Fig. 1. The average and the standard deviation of the travel time differences between the node intervals of 1.0 km and 0.5 km are 1.9% and 1.7% of the standard deviation of the travel time residuals. Those for the node intervals of 1.0 km and 0.2 km are 2.5% and 1.9%. Thus we should expect about 2% noise in the calculated travel times.

To solve the nonlinear problem we take the approach of an iterative use of a linear inversion. A more direct approach to the nonlinear inverse problem is given by TARANTOLA and VALETTE (1982). However, it may be too expensive to apply their method to a tomographic inversion of much data, say, 10^4 – 10^5 arrival times. We use ART (Algebraic Reconstruction Technique) (HERMAN, 1980) for the linear inversion. The convergence property of the ART solution and its relation to the generalized inverse solution is investigated by TANABE (1971). The same algorithm as used by NAKANISHI (1985) is adopted here. We made a numerical experiment of

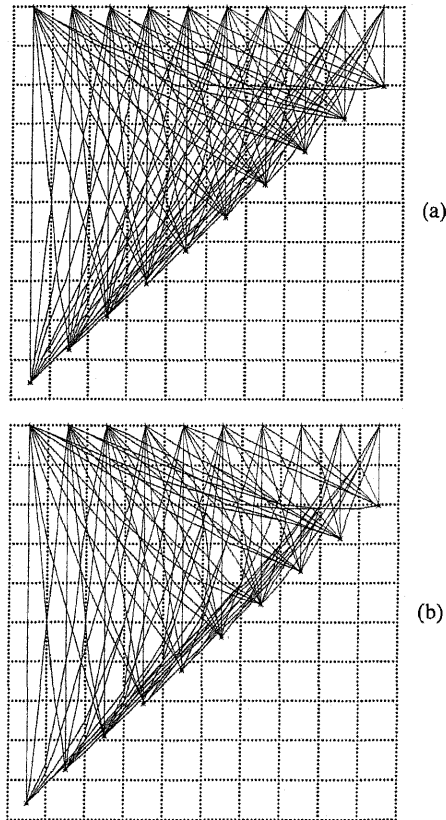


Fig. 2. Ray paths obtained by the Dijkstra method. The nodes used in the calculation are also shown in the figure. (a) Laterally homogeneous model (JMA P-velocity structure). (b) Laterally heterogeneous model (island arc model) (Fig. 1).

the linear inversion by using ART and the damped least squares method. The result showed that both the methods provide us with an almost identical solution.

In the iterative nonlinear inversion the $(i+1)$ -th model obtained by the i -th linear inversion does not necessarily support the data, because the ray paths of this model are different from those of the i -th model used in the i -th inversion. It is necessary to make a "trick" which ensures a monotonic decrease of the travel time residuals. We choose a relaxation factor in ART (HERMAN, 1980) in such a way that $S^{(i+1)} < S^{(i)}$ is satisfied, where $S^{(i)}$ is the sum of the squares of the travel time residuals for the i -th model.

We made an experiment using the model shown in Fig. 1 and an ideal data set without noise in order to test the ability of our iterative algorithm. The data set consists of observations of 10 events made at 10 stations, as shown in Fig. 2(b). We assumed that the locations of the events and stations were known exactly. We

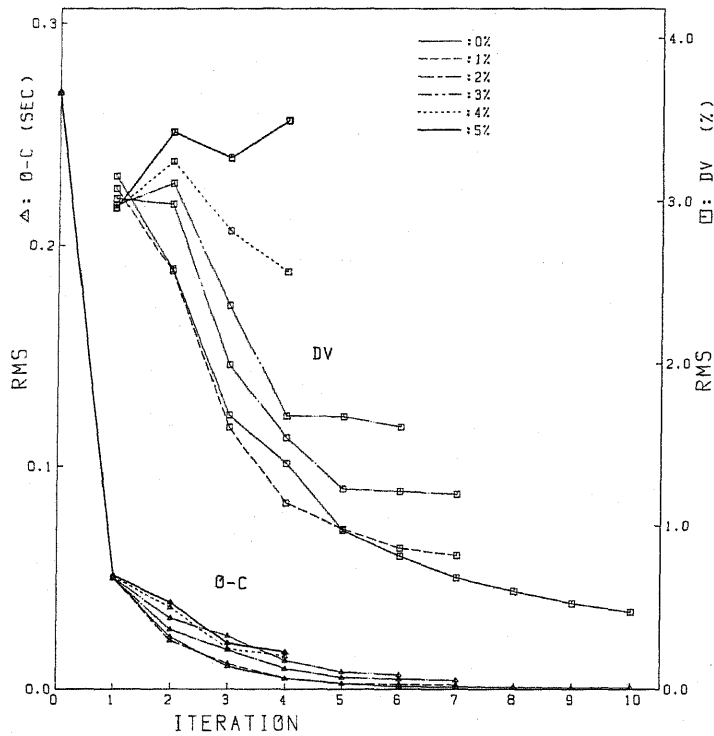


Fig. 3. RMS of the travel time residuals (O-C) and the reconstruction errors (DV) as functions of the iteration number. The first iteration corresponds to the linear inversion. The amounts of the random noise (%) added in the calculated travel times are indicated in the figure.

used the JMA model as the starting model. Thus the initial and correct ray paths are those presented in Figs. 2 (a) and 2 (b), respectively. We must choose the relaxation factor f and the number of global iterations K in ART (NAKANISHI, 1985). Taking into account results of a preliminary experiment using ART, we chose $f=0.5$ and $K=400$ in the following experiment. The relaxation factor f must be divided by 2 unless $S^{(i+1)} < S^{(i)}$ is satisfied.

Figure 3 shows the RMS of the travel time residuals and the reconstruction errors as functions of the iteration number. The RMS of the travel time residuals is 2.69×10^{-1} s for the starting model, which is reduced to 5.00×10^{-2} , 2.24×10^{-3} , and 6.50×10^{-4} s for the models obtained at the first, fifth, and tenth iteration in the noise free case. As compared with the linear inversion (first iteration) we obtained a significant improvement in the travel time residuals after 10 iterations. The RMS of the reconstruction errors of the models is 3.1, 0.9, and 0.4% for the first, fifth, and tenth iteration. The convergence seems to be slow at least in this test. The accuracy examination of the travel time calculation presented above may suggest that this was caused by the node interval of 1 km.

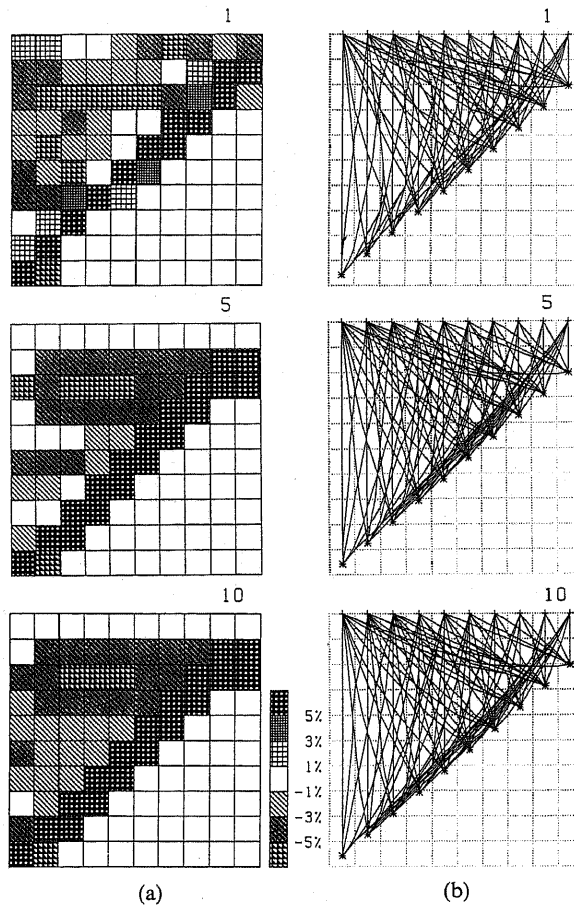


Fig. 4. Inversion results for the noise free case. The symbol convention is indicated in the figure. The number on the right shoulder counts the iteration. (a) Models obtained by the nonlinear inversion. (b) Ray paths calculated for the inverted models.

Figure 4(a) shows the inverted models for the first, fifth, and tenth iteration. The model improvement attained by the nonlinear inversion over the linear inversion is significant for the noise free case. Figure 4(b) shows the ray paths for the corresponding models. We notice important changes in the ray paths near the boundary between the high velocity and low velocity regions.

We examined effects of random noise in travel times upon the improvement by the iterative inversion. For this purpose we added the normally distributed random noises to the calculated travel times, and applied the same inversion algorithm to the noisy data as well as to the ideal data. The results of the experiment are summarized in Fig. 3. Figure 5 shows the retrieved models at four iteration steps in the 2% noise case. Figure 3 shows a strong effect of the random noise on the reconstruction

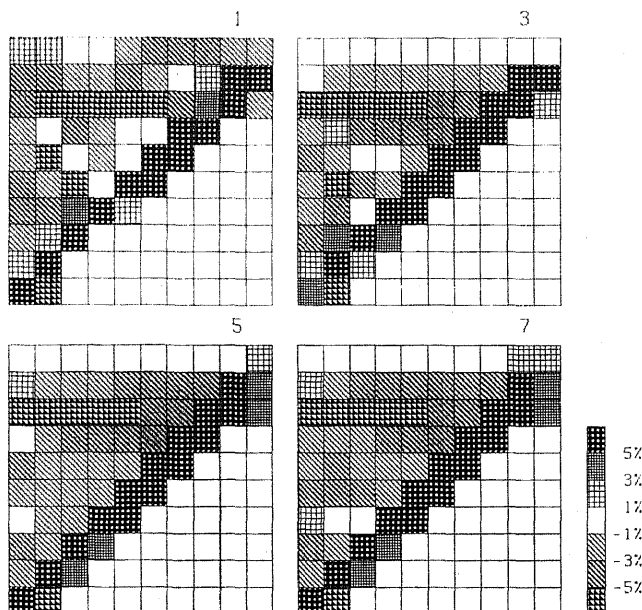


Fig. 5. Models obtained from 2% noise data. The convention is the same as in Fig. 4(a).

errors. The iteration no longer improves the reconstructed image upon the linear inversion (first iteration) for the travel times with 5% data noise. If we consider the travel time errors introduced by the node interval of 1 km, we should raise the value up to 7%. This figure corresponds to a travel time variation of 0.02 s.

In this study, we solved the nonlinear inversion of travel times of first-arrivals by the iterative use of linear inversion. The experiment using the data with random noise shows that the iterative inversion improves the linear solution if the random noise does not exceed 5–7% of the travel time residuals.

We used the HITAC M-280H at the Hokkaido University Computing Center and the NEC MS190 at the Research Center for Earthquake Prediction.

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