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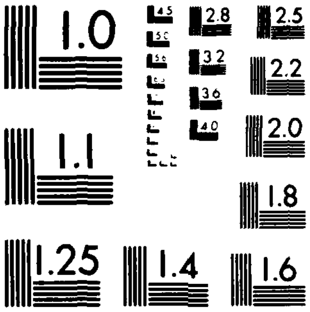
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A Numerical Model to Simulate Sediment Transport in the Vicinity of Coastal Structures

by

Marc Perlin and Robert G. Dean

MISCELLANEOUS REPORT NO. 83-10

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Prepared for

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An implicit finite-difference, n-line numerical model is developed to predict bathymetric changes in the vicinity of coastal structures. The wave field transformation includes refraction, shoaling, and diffraction. The model is capable of simulating one or more shore-perpendicular structures, movement of offshore disposal mounds, and beach fill evolution. The structure length and location, sediment properties, equilibrium beach profile, etc., are user-specified along with the wave climate.

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PREFACE

The purpose of this report is to provide coastal engineers and researchers with a numerical model which predicts sediment transport and the resulting bathymetry in the vicinity of coastal structures. The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Numerical Modeling of Shoreline Response to Coastal Structures work unit, Shore Protection and Restoration Program, Coastal Engineering Area of Civil Works Research and Development.

This report was written by Marc Perlin and Robert G. Dean, Coastal and Offshore Engineering and Research, Inc., under Contract No. DACW72-80-C-0030. The CERC contract monitor was Dr. F. Camfield, Chief, Coastal Design Branch, under the general supervision of Mr. N. Parker, Chief, Engineering Development Division.

Technical Director of CERC was Dr. Robert W. Whalin, P.E.

Comments on this publication are invited.

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For Billy D Best, LTC, CE
TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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FIGURES

CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1] To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: C = (5/9) (F -32).

To obtain Kelvin (K) readings, use formula: K = (5/9) (F -32) + 273.15.

A NUMERICAL MODEL TO SIMULATE SEDIMENT TRANSPORT IN THE VICINITY OF COASTAL STRUCTURES

by

Marc Perlin and Robert G. Dean

I. INTRODUCTION

1. General.

The need for reliable predictions of shoreline response to man-made or natural modifications is increasing due to environmental concerns and the rising cost of remedial measures. The capability of numerical modeling in addressing problems of shoreline response has advanced with improvements in wave climatology, programs to better understand sediment transport relationships, and improvements in numerical modeling. In-situ and remote sensing technology for the measurement of directional wave characteristics has been developed and applied, primarily within the last two decades. In addition to providing the necessary climatology, the resulting measurements have provided the basis for evaluation and refinement of directional wave prediction procedures. Studies such as the Channel Islands Harbor Longshore Sand Transport Study (Bruno, et al., 1981) and the Nearshore Sediment Transport Study (NSTS) (Gable, 1979) have yielded a better understanding of surf zone dynamics and the resulting sediment transport. The increased capacities of large computers and reduced computing costs combined with improved numerical modeling algorithms have resulted in an extremely promising potential for the numerical modeling of shoreline problems.

Although it is doubtful that numerical modeling will ever replace completely the use of movable-bed physical models, the former type offers many advantages. The modeling of shoreline response is somewhat analogous to the problem of simulating storm surges in the coastal zone in which the scale effects and measurement difficulties essentially preclude physical modeling. For shorelines, the scale effects inherent in modeling sediment are well recognized and the costs of representing a substantial length of shoreline may be prohibitive. The laboratory representation of a realistic wave climate is at the forefront of technology.

The investigation of shoreline response can best proceed by several approaches, with each approach selected for the particular strengths which it offers. Field programs are costly, usually because of the considerable equipment and the extensive time required, but these programs are essential for quantifying the values of constants or parameters, the forms of which may be available from laboratory measurements or theoretical considerations. Laboratory studies occupy a special niche by allowing the wave conditions and independent variables to be controlled readily, experiments to be repeated, and selected measurements to be conducted. Although, as noted before, scale effects are present in laboratory measurements of sediment transport, the physics governing the process should be the same. However, the relative magnitudes of suspended versus bedload transport in the laboratory and field may differ. Laboratory studies can also provide an excellent base for evaluating certain aspects of a numerical model, including wave refraction and diffraction and the resulting shoreline patterns due to, for example, the placement of a littoral barrier. Numerical modeling offers the capability to incorporate all the hydrodynamic wave-surf zone and sediment transport knowledge that is available from laboratory and field studies. Numerical modeling has the potential of providing accurate predictions of shoreline response to various structural and nourishment alternatives. Additionally, the possibility exists of employing numerical models and available field measurements to learn more about sediment transport mechanisms. In this latter mode, various candidate mechanisms or coefficients would be evaluated by determining the best match between measured and predicted shorelines and the bathymetry. Generally, this mode would require high-quality measurements of the forcing function (waves and nonwave-related currents) and the associated response (sediments) as well as the knowledge of appropriate conditions at the boundaries of the model.

The present report documents the development and application of an n-line numerical model to investigate bathymetric response to time-varying wave conditions and shoreline modification. The model includes both longshore and onshore-offshore sediment transport. Based on laboratory results, a new distribution of longshore sediment transport across the surf zone is used. The wave climate is specified on the model boundaries which do not need to extend to deep water. Efficient algorithms are employed for representing wave refraction and diffraction. The equation of sediment continuity and transport are solved by a completely implicit algorithm which allows a large time-step. Specified sediment transport values or specified contour positions can be accommodated at the model boundaries. The model is suitable for investigating the shoreline response to a variety of modifications such as one or more groins, terminal structures, structures with variable permeability, and beach nourishment with or without terminal structures.

2. Study Objectives.

The objectives of the present study include (a) the documentation of state-of-the-art models, (b) the development and documentation of an improved model which includes the capability to represent n-contour lines and (c) the application of the model to several relevant coastal engineering problems.

II. BACKGROUND

This discussion describes significant contributions which either address numerical modeling of shorelines directly or provide improved capability for modeling.

1. Wave Refraction (Noda, 1972).

Noda developed an algorithm for solving the following steady state equation for wave refraction

$$\vec{\nabla} \times \vec{k} = 0 \qquad (1)$$

in which $\vec{\nabla}$, the horizontal vector differential operator, and $\vec{k}$, the wave number, are defined in terms of their components as

$$\vec{\nabla} = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} \qquad (2)$$

$$\vec{k} = \vec{i}\,k_x + \vec{j}\,k_y \qquad (3)$$

where $\vec{i}$ and $\vec{j}$ are the unit vectors in the x and y directions respectively. Equation (1) can be expressed as

$$\frac{\partial(k\sin\theta)}{\partial x} = \frac{\partial(k\cos\theta)}{\partial y} \qquad (4)$$

in which θ is the direction of the vector wave number relative to the x-axis and k denotes $|\vec{k}|$. Noda expanded Equation (4) to the following form

$$k\cos\theta\,\frac{\partial\theta}{\partial x} + \sin\theta\,\frac{\partial k}{\partial x} \doteq -k\sin\theta\,\frac{\partial\theta}{\partial y} + \cos\theta\,\frac{\partial k}{\partial y} \qquad (5)$$

Since $\frac{\partial k}{\partial x}$ and $\frac{\partial k}{\partial y}$ are known from the angular frequency σ, the water depth h, and the dispersion equation

$$\sigma^2 = g\,k\,\tanh kh \qquad (6)$$

Equation (5) can be solved numerically, although there are problems of directional stability. The primary advantage of Equation (5) is that it allows the wave direction θ to be determined on a specified grid, compared to unspecified locations that would be obtained by, for example, wave ray tracing.

2. Crenulate Bays (LeBlond, 1972).

LeBlond attempted to model the evaluation of an initially straight shoreline between two headlands into a crenulate bay. The model constitutes a one-line (shoreline) representation. The transport equation employed related the total sediment transport to total water transport in the surf zone as predicted by the formulation provided by Longuet-Higgins (1970). The initial shoreline patterns resemble crenulate bays in nature; however, the predictions were found to be unstable for reasonably long periods of computational time and did not approach a realistic planform.

3. Crenulate Bays (Rea and Komar, 1975).

Rea and Komar employed a rather ingenious system of orthogonal grid cells to provide a cell which locally is displaced perpendicular to the general shoreline orientation. A one-line representation was employed. A simple and approximate representation of wave diffraction was employed. Although the model yielded reasonable results for the examples presented, the unique coordinate system would not be suitable for a general model as the coordinate system must be "tailored" to some degree to conform to the expected shoreline configurations.

4. General One-line Shoreline Model (Price, Tomlinson, and Willis, 1972).

Price, Tomlinson, and Willis' formulation consists of the sediment continuity equation and the total sediment transport equation

$$Q_s = \frac{0.70 \; E_b \; (nC)_b \sin\alpha_b \; \cos\alpha_b}{\gamma_\omega \; (1 - p) \; (S_s - 1)} \tag{7}$$

in which E represents the wave energy density, (nC) the group velocity, α the angle between the breaking wave front and the shoreline, γ_ω the specific weight of water, p the in-place sediment porosity, and S_s the specific gravity of the sediment relative to the water in which it is immersed. The subscript "b" represents values at breaking.

Two formulations were presented by Price, Tomlimson, and Willis (1972). In the first, Equation (7) was substituted into the continuity equation and the results cast into a finite-difference form. In the second, the two equations were employed separately. The latter formulation was selected due to its simplicity and used for the results presented.

Computations were carried out for the case of beach response due to the placement of a long impermeable barrier. The total sediment transport equation by Komar (1969) was used and the planform was calculated at successive times. Refraction was apparently not accounted for in the numerical model. To verify the computations, a physical model study was carried out for the same conditions using crushed coal as the modeling material. The comparison was interpreted as good for up to 3 hours; however, for greater times, substantial differences occurred and these were interpreted as being due to wave refraction not being represented. The crushed coal was supplied to the model at the updrift end at a rate based on the Komar equation, and the results were interpreted as substantiating this relationship. However, the updrift end of the model beach receded substantially both in the numerical and physical models. In the physical model, this can only be interpreted as due to the Komar equation predicitions being less than the actual transport rate, possibly due to the low specific gravity (1.35) of the crushed coal. The predicted recession of the updrift beach is puzzling, although it could be due to a problem in properly representing the updrift boundary condition.

Other one-line models for shoreline changes in the vicinity of coastal structures were developed by LeMehaute and Soldate (1977) and Perlin (1978). Perlin also developed a two-line model formulation, with one-line representing the shoreline and the second the offshore. Dragos (1981) developed an n-line nodel for bathymetric changes due to the presence of a littoral barrier.

III. THE NUMERICAL MODEL

1. Description.

There are several methods of modeling bathymetric changes due to the presence of a littoral barrier. An attempt can be made to either model the complete hydrodynamics and the resulting sediment transport or model using a combination of analytical and empirical sediment transport equations. The second method was chosen due to past relative success.

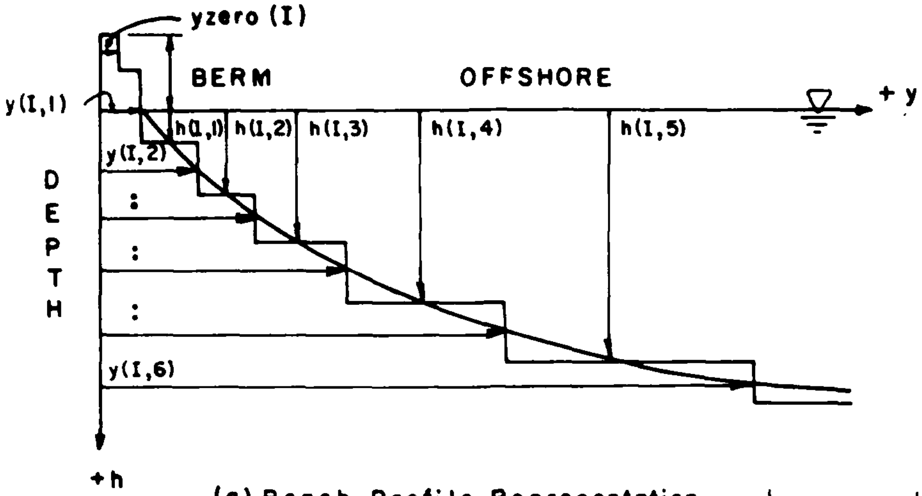
At least two methods of employing sediment transport equations exist: a fixed longshore and cross-shore grid system where the depth is allowed to vary or a fixed longshore and depth system where the cross-shore distance is allowed to change. Although it may seem somewhat awkward, the latter system was chosen for the model. This method allows the modeler to think of bathymetric changes due to a littoral barrier in terms of the effect on the contours; i.e., the contour realinement due to the structure's presence is observed. One limitation of this approach, at least as it was applied here, is that each depth contour must be single-valued; it is not possible to represent bars.

The next step in formulating the model was choosing the specific representation of the bathymetry. The model is an n-line representation of the surf zone in which the longshore direction x is divided into equal segments each Δx in length. The bathymetry is represented by n-contour lines, each a specified depth, which change in offshore location according to the equation of continuity. There are two components of sediment transport at each of the contour lines, a longshore component, Q_x, and an offshore component, Q_y. Figure 1 is a definition sketch showing the beach profile representation in a series of steps and the planform profile representation and notations used.

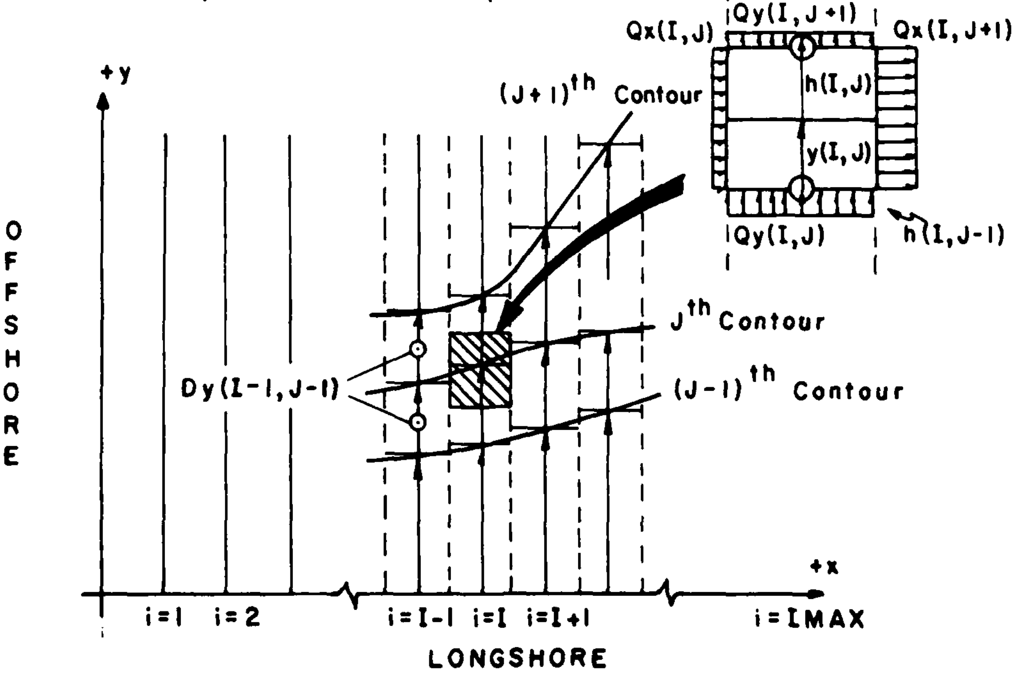
Implementation of the sediment transport equations requires knowledge of the wave field and the equilibrium offshore profile. A discussion of the refraction and diffraction schemes follows. The equilibrium profile is introduced when it is convenient. As an introduction to the logic used in the numerical model, a flow chart is presented in Figure 2.

2. Refraction.

A refraction scheme compatible with variable Δy's was required because of the variable distance to fixed depth contours (as opposed to the more usual fixed grid system where a grid center has a longshore and offshore coordinate with a variable depth). One of the benefits of the n-line model is the ease with which the response of the contours to a particular wave and structure condition can be visualized. A fixed grid system and an interpolation scheme could have been used to obtain the wave field; however, this would have reduced accuracy and increased computation time. The scheme developed also saves computation time because it does not use differential products terms.



(a) Beach Profile Representation



(b) Beach Planform Representation

Figure 1. Definition sketch.

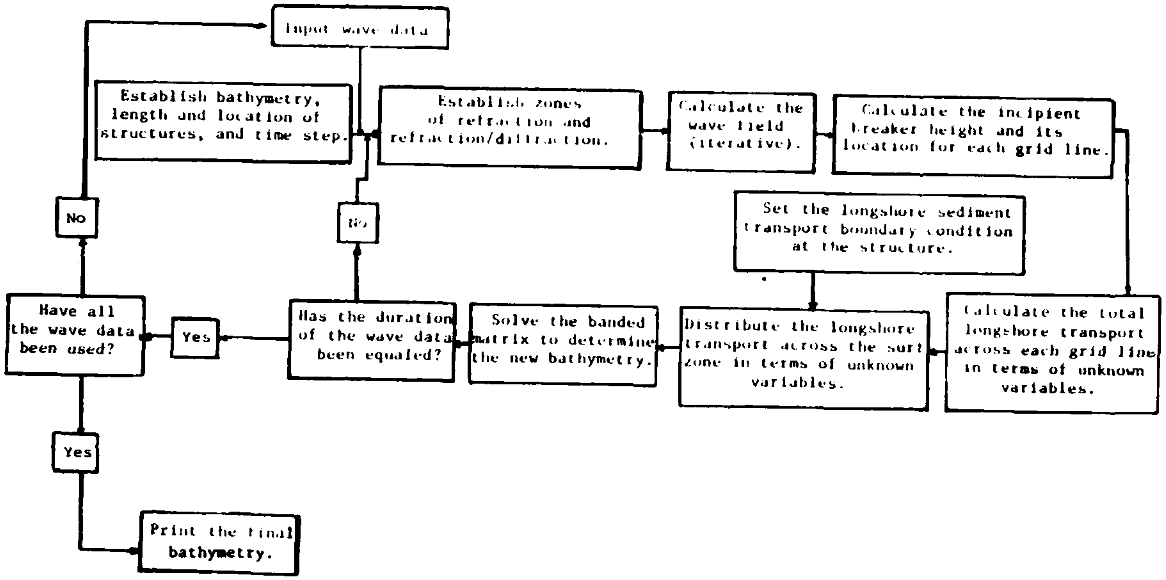


Figure 2. Flow chart.

The first of the governing equations used is the conservation of waves equation

$$\frac{d\sigma}{dt} + \vec{\nabla}_H \times \vec{k} = 0 \qquad (8)$$

where $\vec{\nabla}_H$ is the horizontal differential operator equal to $\vec{i}(\partial/\partial x) + \vec{j}(\partial/\partial y)$ in which $\vec{i}$ and $\vec{j}$ are the unit vectors in the x and y directions, respectively, and x is the longshore direction, with positive to the right when facing the water, y the offshore direction, with positive seaward, and z the vertical coordinate, with positive defined as upwards. For the steady-state case, equation (8) yields

$$\frac{\partial}{\partial x}(k_y) - \frac{\partial}{\partial y}(k_x) = 0 \qquad (9)$$

where k_x and k_y are the wave number projections in the respective directions. Defining as the angle k makes with the y-axis positive in the counterclockwise direction, the equation can be written in final form as

$$\frac{\partial}{\partial x}(k \cos \theta) = \frac{\partial}{\partial y}(k \sin \theta) \qquad (10)$$

where $\theta = \alpha + \pi$ (in radians). Noda (1972) and others have developed numerical solutions to expanded forms of equation (10). In the present study, equation (10) was initially central-differenced in the x-direction and forward-differenced in the y-direction with Snell's law used to specify the boundary conditions on the offshore boundary and one of the sides (i.e., the side of the wave angle approach). However, a numerical problem arose. The argument of the arcsine exceeded $\pm$ 1.0 for large $\Delta y/\Delta x$. To overcome this problem, a dissipative interface was used on the forward-difference term (after Abbott, 1979). The final finite-differenced form of equation (10) is

$$\theta^{n+1}_{i,j} = \sin^{-1}\left\{ \frac{1}{k_{i,j}} \left[\tau(k \sin \theta)_{i-1,j+1} + (1-2\tau)(k \sin \theta)_{i,j+1} \right.\right. \qquad (11)$$

$$\left.\left. + \tau(k \sin \theta)_{i+1,j+1} - \frac{\Delta y}{2\Delta x}\left((k \cos\theta)_{i-1,j} - (k \cos\theta)_{i-1,j}\right) \right] \right\}$$

where τ has been taken as 0.25. The past $\Theta^n_{i,j}$ and the present $\Theta^n_{i,j}$ wave angles are numerically averaged to give the $\bar{\Theta}_{i,j}$. Newton's method is used to compute the wave number via the linear wave theory dispersion relation. In addition, numerical smoothing is used at the conclusion of the wave field calculation. This approximates in an ad hoc manner diffractive effects (lateral transfer of wave energy along the wave) which exist in nature but have been omitted due to use of the equation for refraction (equation 8). The smoothing routine is

$$\theta_{i,j} = \frac{1}{4}\theta_{i-1,j} + \frac{1}{2}\theta_{i,j} + \frac{1}{4}\theta_{i+1,j} \qquad (12)$$

The second governing equation used in the refraction scheme is conservation of energy. Neglecting dissipation of energy due to friction, percolation, and turbulence, this equation can be expressed as

$$\vec{\nabla} \cdot (E\ \vec{C}_G) = 0 \tag{13}$$

where E is the average energy per unit surface area and $\vec{C}_G$ the group velocity of the wave train. Performing the operation indicated and replacing $\vec{C}_G$ by its components ($C_G \sin\theta$) and ($C_G \cos\theta$) results in the following:

$$\frac{\partial}{\partial x}\,(E\ C_G\ \sin\theta) + \frac{\partial}{\partial y}\,(E\ C\ \ \cos\theta) = 0 \tag{14}$$

Assuming linear theory,

$$E = \frac{\rho g H^2}{8} \tag{15a}$$

where ρ is the mass density of water, g the gravitational constant, and H the wave height. Dividing the equation by $\frac{\rho g}{8}$, finite-differencing and weighting the forward-differenced term as before, and solving for the wave height, results in the following:

$$H^{n+1}_{i,j} = \left\{ \frac{1}{(C_G \cos\theta)_{i,j}} \Big[(\tau)(H^2 C_G \cos\theta)_{i-1,j+1} + (1-2\tau)(H^2 C_G \cos\theta)_{i,j+1} + (\tau)(H^2 C_G \cos\theta)_{i+1,j+1} + \frac{\Delta y}{2\Delta x}\,[(H^2 C_G \sin\theta)_{i+1,j} - (H^2 C_G \sin\theta)_{i-1,j}] \Big] \right\}^{1/2} \tag{15b}$$

This equation is also solved by iterative techniques and the $H^{n+1}_{i,j}$ and $H^n{}_{i,j}$ are averaged at the conclusion of each iteration.

C_G is determined by the linear wave theory relationship

$$C_G = \frac{C}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{16}$$

where h is the water depth, k the wave number, and C the wave celerity. Wave height boundary conditions are input along the same boundaries as the wave angles using linear theory shoaling and refraction coefficients. The θ's have been previously determined. In both equations (11) and (15) for a variable grid system, the points (i + 1, j) and (i - 1, j) need to be determined (i.e., because the y coordinates are not fixed, adjacent values with the same subscripts can be farther or closer to shore, therefore interpolation must be used). The actual values are found by searching the (i + 1) and (i - 1) cross-shore lines, finding the adjacent values in the positive and negative y-direction, and interpolating to determine the value.

3. Diffraction.

The diffraction solution (in the lee of the structure) used in the model is based on the method of Penny and Price (1952). Assumptions used in this method include a semi-infinite breakwater, which is infinitesimally thin, linear wave theory and constant depth. A definition sketch for wave diffraction is shown in Figure 3. The quantity THETAO represents the angle of wave incidence relative to the jetty axis, ANGLE represents the angle from the jetty at the point where the diffraction coefficient is to be computed, and RAD is the radial distance. The radial distance is then cast into a dimensionless parameter, RHOND (= 2π RAD/L), where L is the wavelength. This is equivalent to multiplying the radial distance by the wave number k.

The diffraction coefficient AMP is expressed as the modulus of the diffracted wave

$$\text{AMP} = (\text{Sum } 1)^2 + (\text{Sum } 2)^2 \qquad (17)$$

where

$$\begin{aligned}
\text{Sum } 1 = {} & [\cos(\text{RHOND}(\cos(\text{ANGLE-THETAO}))) \cdot (\tfrac{1}{2}(1.0 + C_F + S))] + \\
& [\sin(\text{RHOND}(\cos(\text{ANGLE-THETAO}))) \cdot (-\tfrac{1}{2}(S - C_F))] + \\
& [\cos(\text{RHOND}(\cos(\text{ANGLE+THETAO}))) \cdot (\tfrac{1}{2}(1.0 + C_F + S))] + \\
& [\sin(\text{RHOND}(\cos(\text{ANGLE+THETAO}))) \cdot (\tfrac{1}{2} - (S - C_F))] \qquad (18)
\end{aligned}$$

$$\begin{aligned}
\text{Sum } 2 = {} & [\cos(\text{RHOND}(\cos(\text{ANGLE-THETAO}))) \cdot (-\tfrac{1}{2}(S - C_F))] + \\
& [\sin(\text{RHOND}(\cos(\text{ANGLE-THETAO}))) \cdot (\tfrac{1}{2}(1.0 + C_F + S))] + \\
& [\cos(\text{RHOND}(\cos(\text{ANGLE+THETAO}))) \cdot (-\tfrac{1}{2}(S - C_F))] + \\
& [\sin(\text{RHOND}(\cos(\text{ANGLE+THETAO}))) \cdot (\tfrac{1}{2}(1.0 + C_F + S))] \qquad (19)
\end{aligned}$$

In Equations (18) and (19), C_F and S represent Fresnel integrals and are computed in the model by means of an approximation after Abramowitz and Stegun (1965).

Having obtained AMP, the wave height at the location in question is simply the product of the specified partially refracted incident wave height and AMP. The angle of the wave crest is computed assuming a circular wave front along any radial; this angle is then refracted using Snell's law.

Throughout the refraction and diffraction schemes, the local wave heights are limited by the value, 0.78 x depth.

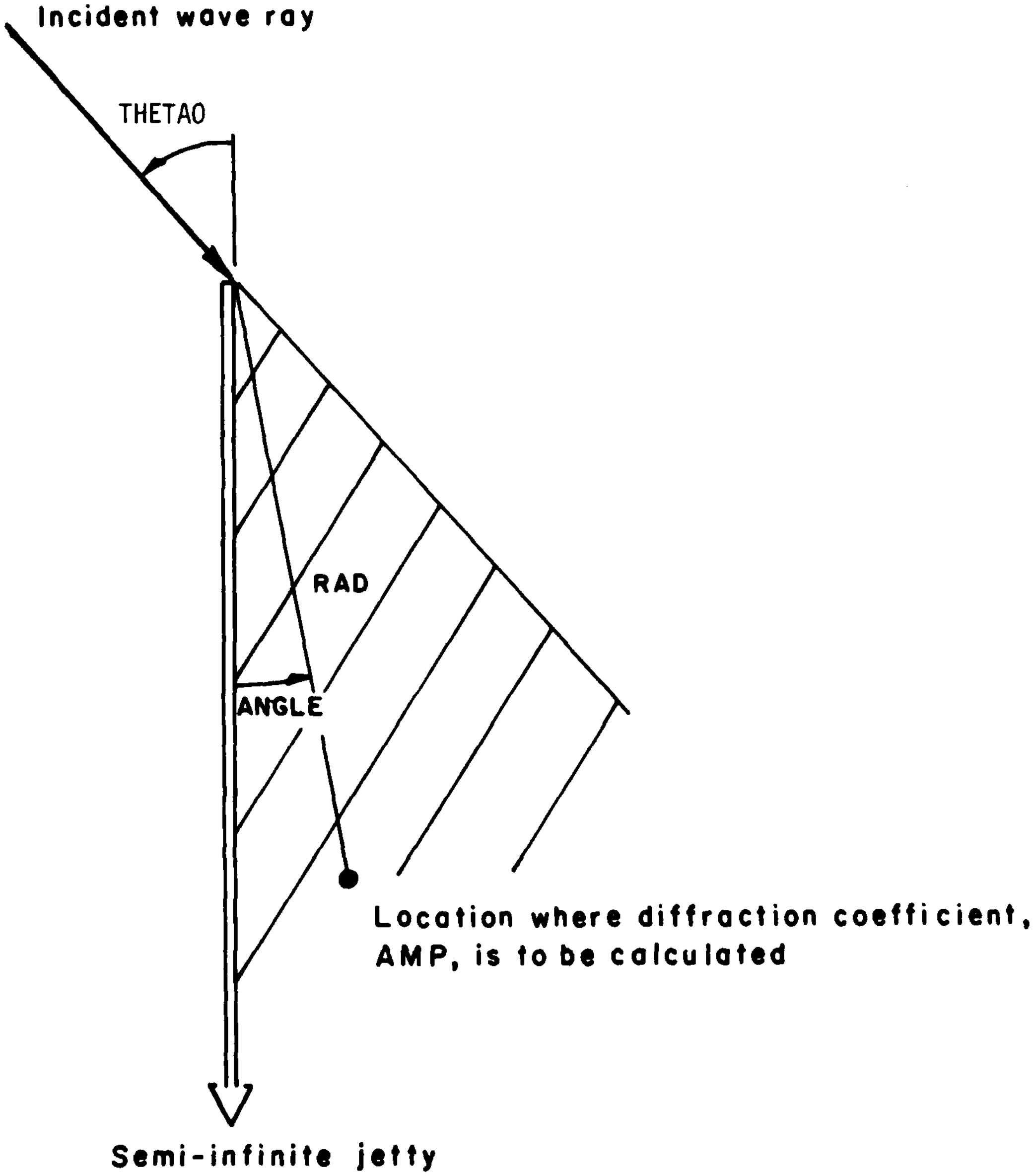


Figure 3. Definition sketch for wave diffraction.

4. Sand Transport Model.

a. Governing Equations. Three basic equations are used to simulate the sediment transport and bathymetry changes according to the wave field. The equation of continuity

$$\frac{\partial y}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \qquad (20)$$

requires as input, knowledge of the longshore and cross-shore components of sediment transport. The total transport alongshore has been measured by several investigators and many equations exist; however, the distribution of the transport across the surf zone is not well known. Fulford (1982) based on laboratory data from Savage (1959), developed a distribution of longshore sediment transport across the surf zone for the case of straight and parallel contours. Fulford's use of Savages experiment was based on two assumptions: 1) the structure must be a total littoral barrier and 2) onshore-offshore sediment transport could be neglected. Test 5-57 was chosen because the two criteria were nearly met. Savage reported that the groin acted as a total littoral barrier for the first 35 hours of the test (i.e., no bypassing occurred prior to 35 hours). This does not mean that no onshore-offshore transport occurred because as the profile steepens on the updrift side, onshore-offshore transport does occur. However, it was assumed to be negligible. In addition, the initial profile had been molded to an equilibrium profile via 150 hours of waves. Thus, the two criteria required to develop an inferred longshore distribution of sediment transport were nearly satisfied. This distribution is shown as a dashline in Figure 4. The smaller "maximum" is believed to be an extraneous effect of a groin downdrift from the location in the experiment where the data were taken. Therefore, this feature was replaced by a monotonically decreasing, smooth curve as shown by the "altered" curve. To analytically represent this distribution, a function of the following form was chosen

$$q_x\ (y) = (B)\ (y)^{n-1}\ e^{-(y)^n} \qquad (21)$$

This type of equation is convenient because it is easily integrable, and by properly choosing the constant, B, the integral of the equation from zero to infinity can be required to equal a particular value. This too is highly desirable because, as was done in the model, the integral is set equal to one and then multiplying by the value of the well-known longshore transport equation, the value of the transport at any location across the surf zone can be determined. Further investigation suggested a value of n = 3 to produce a curve similar to Fulford's curve. A more general form of the equation which allows more flexibility and curve fitting is

$$q_x(y) = B(y + a)^2\ e^{\left\{-\left[\frac{y + a}{cy_b}\right]\right\}^3} \qquad (22)$$

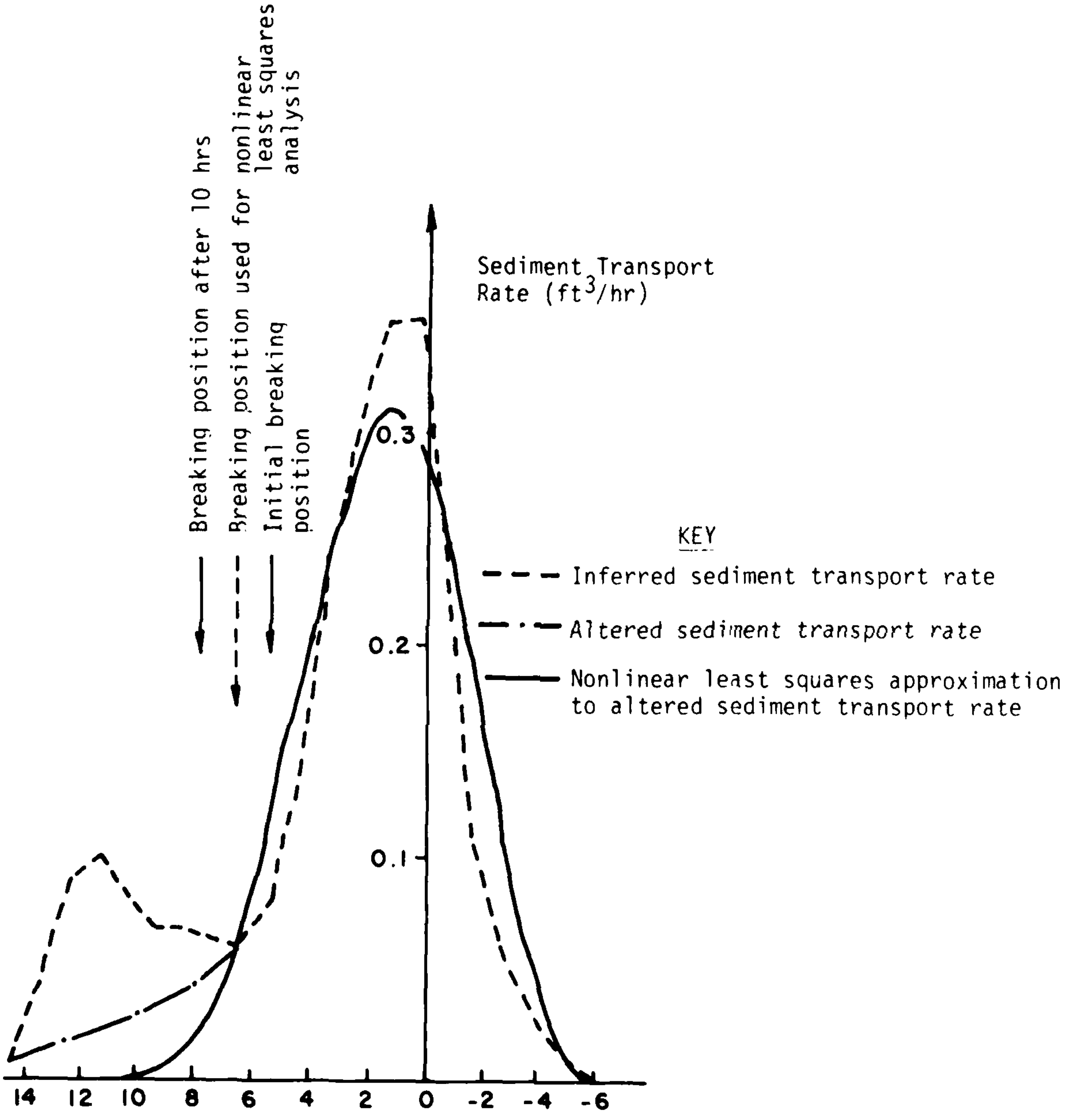


Figure 4. Distribution of sediment transport across the surf zone.

where y_b = distance to the point of breaking

a = constant to allow sediment transport above mean water line (MWL) (swash transport or transport in region of wave setup) to be represented

c = a constant establishing the width of the curve (to be determined)

$$B = \frac{3}{c^3 y_b^3} \qquad \left(\text{causes} \quad \int_0^\infty q_x(y)\, dy = 1.0\right)$$

Based on Fulford's (1982) results and considering a to be proportional to the breaking height divided by the beach slope, the constant of proportionality was determined to be unity; i.e., $a = h_b/(\partial h/\partial y)$. Using equation (22) and a digitized version of the curve shown in Figure 4, a nonlinear least squares regression was carried out to determine the value of c. A Taylor's series expansion of the form

$$f^{k+1}(c,y) = f^k(c,y) + \frac{\partial f}{\partial c}\, \Delta c \tag{23}$$

where k and k + 1 represent the number of the iteration carried out. Least squares regression minimizes the square of the difference between observed and predicted values with respect to a change in the parameter being computed, or

$$\frac{\partial}{\partial(\Delta c)} \left\{ \sum_{n=1}^{N} \left[f_{OBS} - \left(f^k(c,y) + \frac{\partial f}{\partial c}\, \Delta c \right) \right]^2 \right\} = 0 \tag{24}$$

where f_{OBS} represents the observed values, which in this case is $q_x(y)_{OBS}$. Carrying out the differentiation indicated and manipulating terms, Δc can be solved in terms of known quantities.

An iterative procedure was then used by updating the values of $f^k(c,y)$, $\partial f/\partial c$, and c until an acceptably small change in c results. For the data herein, the value of c was determined to be 1.25. The final form of sediment transport of a y location in the surf zone results for a shoreline with straight and parallel contours, as

$$q_x(y) = \frac{3}{(1.25)^3 (y_b)^3} (y + a)^2\, e^{-[(y + a)/(1.25\, y_b)]^3} \tag{25}$$

This equation, which is also presented in Figure 4, predicts the relative transport at point y. To obtain the fraction of transport between two y coordinates, the integral of equation (25), from y_1 to y_2, must be used.

$$Q_{x_{ND}} = Q_x \Big|_{y_1}^{y_2} = \int_{y_1}^{y_2} q_x(y)dy = e^{-[(y_1 + a)/(1.25\ y_b)]^3} - e^{-[(y_2 + a)/(1.25\ y_b)]^3} \qquad (26)$$

Q_x[ND] is dimensionless; therefore, to compute a value in, say, cubic feet per second, it must be multipled by the total transport along a perpendicular to the shoreline obtained from the total longshore transport equation used in the model

$$Q = C'\ H_b^{5/2} \sin(2\ \alpha_b) \qquad (27)$$

See Appendix A for a discussion of the constant C'. It is noted that the transformation of $q_x(y)$ to $q_x(h)$ can be effected by multiplying by the one-dimensional Jacobian ($\Delta y/\Delta h$). This latter form ($q_x(h)$) is more useful here because the present model simulates the changes in contour position (Δy) rather than changes by depth (Δh).

In the numerical model, Q_x (I,J) (see Fig. 1) is determined using equation (26) except for the shoreline contour, J=1, and the farthest offshore contour simulated, J = JMAX. The shoreline contour longshore transport, Q_x (I,1), in order to include swash transport, uses equation (16); however, the first term is set equal to 1.0. The seawardmost contour transport, Q_x (I,JMAX), in order to include any longshore transport not yet accounted for, neglects the second term of equation (26) (i.e., it accounts for transport from y(I,JMAX) to infinity). The dimensionless numbers are then multiplied by Q determined from equation (27). This method is based on parallel contours which may not exist. In order to compensate for the nonparallel nature of the contours (note that refraction does account for it as far as the wave field is concerned), the term sin ($2\alpha_b$) of equation (27) is replaced by sin ($2\alpha_L$) shoreward of the breakpoint, where α_L represents the angle between the "local" wave angle and the "local" contour. It can be argued that for a spilling breaker, the remaining surf zone at any point "sees" a total transport similar to equation (27), where α_b and H_b are the local values. The problem is that the constant of proportionality was determined for the entire surf zone and for nearly straight and parallel contours. This not being the case, the equation was altered on intuitive grounds to reflect the fact that the contours are no longer straight and parallel.

The second input required by the continuity equation to predict the bathymetric changes is the cross-shore sediment transport. The governing equation for onshore-offshore transport (after Bakker, 1968) is

$$Q_{y_{i,j}} = \Delta x \, C_{OFF_{i,j}} \left[y_{i,j-1} - y_{i,j} + W_{EQ_{i,j}} \right] \qquad (28)$$

where C_{OFF} is an activity factor (inside the surf zone = 10^{-5} feet per second for the prototype simulation herein, 10^{-4} feet per second for the physical model simulation) (see App. A. for a discussion) and $W_{EQ}(i,j)$ is the positive equilibrium profile distance between $y(i,j)$ and $y(i,j-1)$, determined from the equilibrium profile used in the numerical model $h = Ay^{2/3}$ (Dean, 1977). See Appendix A for discussion of the value of A. The physical interpretation of equation (28) is that as this profile steepens (flattens), sediment is transported offshore (onshore).

b. Methods of Solution. Three separate finite-difference techniques were used to solve the equations:

(1) Explicit longshore-continuity and explicit cross-shore continuity;

(2) Implicit longshore-continuity and explicit cross-shore continuity for half a time-step then vice versa; and

(3) Implicit longshore-cross-shore continuity.

An explicit formulation was first developed which used the refraction scheme, the distribution of longshore sediment transport across the surf zone, and the onshore-offshore sediment transport equation. Problems in addition to the usual ones which are encountered with explicit methods (e.g., computation time and cost) were immediately realized. In the explicit method, both transport computations are based on the former values of the contour locations and are completely uncoupled. Stability of an explicit scheme requires a small time-step. In addition, the noncoupled nature of the equations, in some cases, resulted in crossing of the contours due to the transport computed.

It is logical to assume that an implicit formulation of the longshore transport equation used as input to the continuity equation along with the explicit onshore-offshore transport component would help the numerical stability (on the other half time-step, the longshore component would be computed explicitly and the onshore-offshore transport equation would be solved implicitly with the continuity equation). Although this scheme would be superior to the explicit procedure, it still would be susceptible to crossing contours. It should be noted that the magnitude of the coefficient used in the onshore-offshore equation is very important to the extent that the simulation models natural phenomena. If the coefficient is very small or vanishes, sediment will not move offshore and contours will cross because of the variation in the distribution of longshore sediment transport across the surf zone. If the coefficient is too large, the onshore-offshore transport, may become large enough that on a particular time step, an offshore contour would move too far shoreward, thereby crossing an inshore contour or vice versa. Once the contours cross, not only does the bathymetry become unrealistic, but mathematically, the equation which computes the longshore distribution across the surf zone changes signs at some locations and the entire model becomes physically unrealistic.

To circumvent these problems, an implicit scheme that simultaneously solves the three governing equations, was developed. Utilizing equation (26), and the one-dimensional Jacobian ($\Delta y/\Delta h$) to convert to $Q_x(h)$, the total longshore transport equation (27), the following equation is obtained,

$$Q_{x_{i,j}} = \left\{ \left[\exp\left(-\left(\frac{(h_{i,j-1})^{3/2} + H_{b_i} A^{3/2}{}^{\,3}}{1.25\, h_{b_i}} \right) \right) - \exp\left(-\left(\frac{(h_{i,j}{}^{3/2} + H_{b_i} A^{3/2}{}^{\,3}}{1.25\, h_{b_i}} \right) \right) \right] \times \left(C' H_{b_{i,j}}^{5/2} \right) \right\} \times \sin(2\theta - 2\alpha_c) \qquad (29)$$

$Q_x(i,j)$ represents the sediment transport between depths $h(i,j)$ and $h(i,j-1)$ (see Fig. 1). The term in brackets represents the normalized distribution of longshore transport between $h(i,j)$ and $h(i,j-1)$; θ is the averaged wave angle at the location of $Q_x(i,j)$ and α_c is the local contour orientation angle. Defining everything except $\sin(2\theta - 2\alpha_c)$ as $\nu(i,j)$ and using a superscript to denote a time step, this equation can be written

$$Q_{x_{i,j}}^{n+1} = \nu_{i,j} \sin(2\theta - 2\alpha_c^{n+1}) \qquad (30)$$

The assumption has been made that the wave field (H and θ) do not vary during the bathymetric changes over the time-step. Using the following trigonometric identities,

$$\sin(2a - 2b) = \sin 2a \cos 2b - \cos 2a \sin 2b \qquad (31a)$$

$$\cos 2a = 2\cos^2 a - 1 \qquad (31b)$$

$$\sin 2a = 2 \sin a \cos a \qquad (31c)$$

and recognizing that the following expression is an approximation

$$\sin(\alpha_c^{n+1})_{i,j} \simeq \frac{\frac{1}{2}\left(y_{i,j}^{n+1} - y_{i-1,j}^{n+1} + y_{i,j}^{n} - y_{i-1,j}^{n}\right)}{\left((\Delta x)^2 + (y_{i,j} - y_{i-1,j})^2\right)^{1/2}} \qquad (32)$$

along with assuming that the change in the denominator is small for a reasonable time-step (the numerator has been averaged over the n^{th} and $n + 1^{th}$ time-steps), equation (30) results in

$$Q_{x_{i,j}}^{n+1} + (S3)_{i,j}\, y_{i,j}^{n+1} - (S3)_{i,j}\, y_{i-1,j}^{n+1} = (RHS1)_{i,j}^{n} \qquad (33)$$

where $$(S3)_{i,j} = \left(\tfrac{1}{2}\right)(\nu_{i,j}) \cos(2\theta)\,(2 \cos \alpha_c)\, \frac{1}{\left(\Delta x^2 + \Delta y^2\right)^{1/2}}$$

$$(RHS1)_{i,j}^{n} = (\nu_{i,j})\,(2 \sin\theta \cos\theta)(\cos^2\alpha_c - 1) - (S3)_{i,j}(y_{i,j}^{n} - y_{i-1,j}^{n})$$

Here it has also been assumed that $\cos^2\alpha_C$ does not change over the time step. Equation (33) is the final form of the longshore sediment transport equation prior to its use in conjunction with the other equations.

Averaging y values on the n^{th} and $(n+1)^{th}$ time-steps, equation (29) can be rewritten as

$$Q_{y_{i,j}} = \text{Const6}_{i,j} \left\{ \frac{1}{2} \left(y_{i,j-1}^{n+1} + y_{i,j-1}^{n} - y_{i,j}^{n+1} - y_{i,j}^{n} \right) + W_{EQ_{i,j}} \right\} \qquad (34)$$

where $\text{Const6}(i,j) = C_{OFF}(i,j)$. Δx. This is the final form on the onshore-offshore sediment transport equation.

The equation of continuity, finite-differenced for the n^{th} and $(n+1)^{th}$ time-steps, can be written as

$$\frac{y_{i,j}^{n+1} - y_{i,j}^{n}}{\Delta t} = \frac{1}{2\Delta x \Delta h} \left\{ Q_{x_{i,j}}^{n+1} + Q_{x_{i,j}}^{n} - Q_{x_{i+1,j}}^{n+1} - Q_{x_{i+1,j}}^{n} + Q_{y_{i,j}}^{n+1} + Q_{y_{i,j}}^{n} - Q_{y_{i,j+1}}^{n+1} - Q_{y_{i,j+1}}^{n} \right\} \qquad (35)$$

Defining $R_{i,j}$ as $1/(2\Delta x \Delta h)$, inserting equations (33) and (34) into equation (35), and transferring all known quantities for the n^{th} time-step to the right-hand side of the equation result in

$$\begin{aligned} & y_{i,j}^{n+1} + (\Delta t R_{i,j}) S3_{i,j}\, y_{i,j}^{n+1} - (\Delta t R_{i,j}) S3_{i,j}\, y_{i-1,j}^{n+1} - (\Delta t R_{i,j}) S3_{i+1,j}\, y_{i+1,j}^{n+1} \\ & + (\Delta t R_{i,j}) S3_{i+1,j}\, y_{i,j}^{n+1} - (\Delta t R_{i,j}\, \text{Const6}_{i,j}) \left(\frac{1}{2} \left[y_{i,j-1}^{n+1} - y_{i,j}^{n+1} \right] \right) \\ & + (\Delta t R_{i,j}\, \text{Const6}_{i,j+1}) \left(\frac{1}{2} \left[y_{i,j}^{n+1} - y_{i,j+1}^{n+1} \right] \right) = (AWARE)_{i,j} \qquad (36) \end{aligned}$$

Equation (36) can be rewritten as

$$(1 + U + V + Z1 + Z2)\, y^{n+1}_{i,j} - (U)y^{n+1}_{i-1,j} - (V)y^{n+1}_{i+1,j}$$

$$- (Z1)y^{n+1}_{i,j-1} - (Z2)y^{n+1}_{i,j+1} = (AWARE)_{i,j} \qquad (37)$$

where

$$U = \Delta t R_{i,j} S3_{i,j}$$

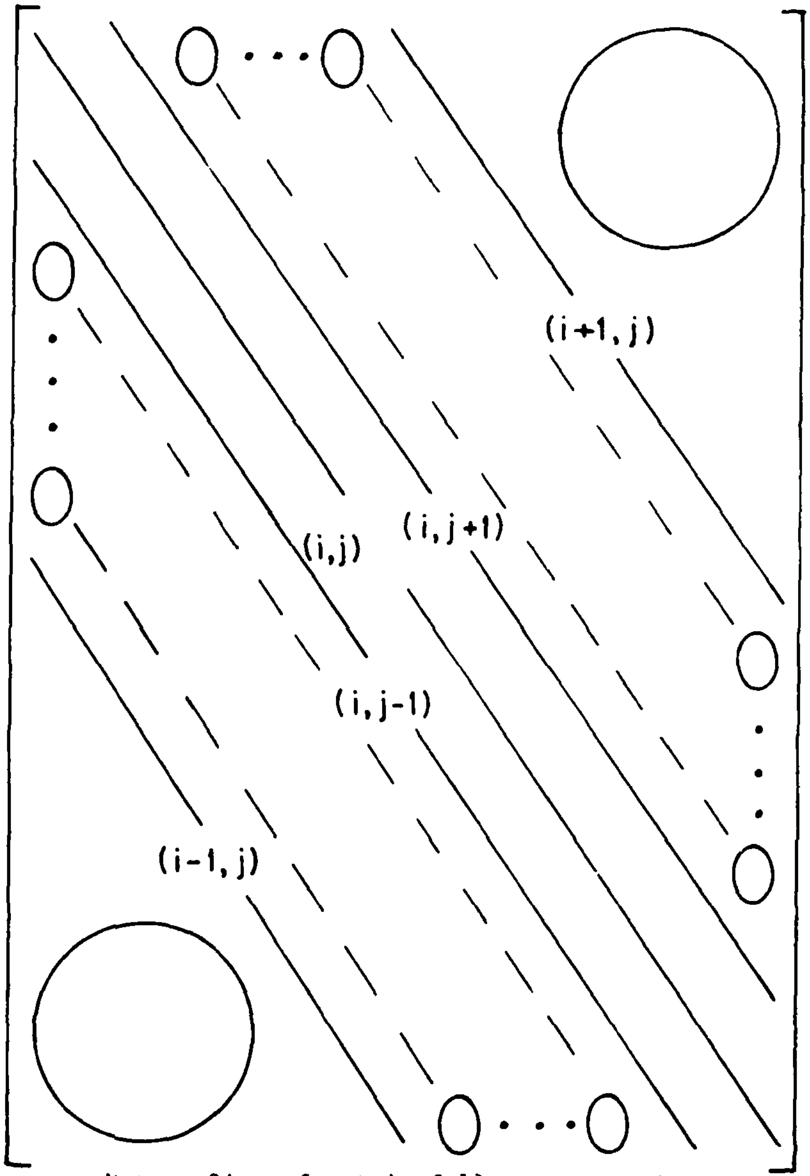
$$V = \Delta t R_{i,j} S3_{i+1,j}$$

$$Z1 = \left(\frac{\Delta t}{2}\right) R_{i,j} Const6_{i,j}$$

$$Z2 = \left(\frac{\Delta t}{2}\right) R_{i,j} Const6_{i,j+1}.$$

Equation (37) is a weighted, centered scheme in which $y^{n+1}_{i,j}$ is computed using a weighting of itself and its four adjacent grid "neighbors". The weighting factors (U, V, Z1, and Z2) are functions of the wave climate, the slope between contours, and the variables included in the original formulation. An investigation of a small gridded system demonstrated that by writing simultaneous equations, one for each $y_{i,j}$, a banded matrix results. This matrix can be solved by LEQT1B, one of the available routines from the International Math and Statistics Library (IMSL). A schematic representation of the matrix A which results from the matrix equation [A][y] = [B] is presented in Figure 5. In this schematic, the large zeros represent triangular corner sections of all zeros and the 0...0 represents bands of zeros, the number of which is dependent on the number of contours simulated (the number of zero bands between either remote nonzero bands and the tridiagonal nonzero bands equals two less than the number of contours modeled (in both the upper and lower codiagonals of the matrix)). An inspection of the subscripts in equation (29) yields the reason the zero bands are required. The more j values (contours) used, the more y grids there are along any perpendicular to shore. This causes zeros to appear in the matrix between bands as the weighting factors await being used to operate on $y^{n+1}(i-1,j)$ and $y^{n+1}(i+1,j)$. For this reason, the expense of simulating an increasing number of contours is exponential. The LEQT1B routine, utilizes banded storage and saves both storage and computation time; however, the routine has no special way of handling the interior zero bands. One refinement which would save computation time would be to develop an algorithm to solve and store the matrix by taking advantage of these inner zero bands; however, it is beyond the scope of this project.

Of course, the matrix requires boundary values on longshore extremities and on both onshore and offshore boundaries. The longshore boundary conditions are treated by modeling a sufficient stretch of shoreline so that effects of a structure's presence are minimal. The y values along these boundaries can therefore be fixed at their initial locations. In the onshore-offshore direction, boundaries are treated quite differently. The



Note: Size of matrix full storage mode
[(IMAX-2)(JMAX) x (IMAX-2)(JMAX)]

Size of matrix banded storage mode
[(IMAX-2)(JMAX) x (2JMAX + 1)]

Figure 5. Schematic representation of banded matrix if not stored in banded storage mode.

berm and beach face are assumed to move in conjunction with the shoreline position. The required sediment transport is then computed by the change in position of the shoreline. The two equations are

$$y^{n+1}_{i,0} = y^{n}_{i,0} + [y^{n+1}_{i,1} - y^{n}_{i,1}] \tag{38a}$$

$$Q^{n+1}_{y_{i,1}} = - [\frac{\text{Berm } \Delta x}{\Delta t}][y^{n+1}_{i,1} - y^{n}_{i,1}] \tag{38b}$$

The offshore boundary is treated by keeping y^{n+1}(i,jmax) (the contour beyond the last simulated contour) fixed, until the angle of repose is exceeded. Then, the y^{n+1}(i,jmax+1) is reset (at the conclusion of the n + 1 time-step) to a position such that the slope equals the angle of repose. Note that y^{n+1}(i,0) is represented in the program by $YZERO_i$.

There are also no-flow boundary conditions required at each of the structures being modeled. These are imposed on the adjacent y-grid points which are located downdrift (i.e., in the shadow zone) of the structure and shoreward of the structures' seaward extremities. They are imposed by setting $S3_{i,j}$ of equation (33) and $DISTR_{i,j}$ (the term in square brackets in equation (29) equal to zero, thereby causing Q_x(i,j) to be zero (i.e., the no-sediment flow condition). This boundary condition is imposed automatically for every shore-perpendicular structure.

It was found that even with the implicit formulation, high frequency oscillations occurred in the y values immediately updrift and downdrift of the structure. The solution did not "blow up"; however, on larger time-steps "sloshing" (oscillating) did occur. Part of this problem was due to the boundary condition at the structure which had been such that either no sand was allowed along a contour line or the sand determined by the equations was allowed to be transported. Because of the very large angle which existed around the tip of the structure when a contour first exceeded the length of the structure, very large amounts of sediment transport were predicted. In the nature where analog sand transport rather than digitized transport occurs, this does not happen. Therefore, the boundary condition was altered to constantly allow sand transport around the end of the structure in proportion to that part of the contour representation which exceeded the structure (i.e., the transport was calculated for the location at tip of the structure as if the structure was not there and then a proportion of this value was allowed to bypass). Although the transport around the tip of the structure is based on the values from the past time-step, it more closely simulated the natural phenomenon.

Additionally, a dissipative interface is used on the y values as follows:

$$y_{i,j} = (\tau)\, y_{i-1,j} + (1 - 2\tau)\, y_{i,j} + (\tau)\, y_{i+1,j} \tag{39}$$

where τ was again taken as 0.25. It is noted that only high frequency oscillations in y are affected by the use of equation (39); the total sum of y values is not affected. Also, in all the dissipative interface schemes used, if a boundary point is being computed, either a forward-difference or a backward-difference of equation (39) is used (after Abbott, 1979):

$$\text{Backward:} \quad y_{i,j} = (\tau)y_{i-1,j} + (1 - \tau)y_{i,j} \qquad (40a)$$

$$\text{Forward:} \quad y_{i,j} = (\tau)y_{i+1,j} + (1 - \tau)y_{i,j} \qquad (40b)$$

IV. SIMULATIONS AND VERIFICATION

Several simulations were run; two were attempts at verifying the numerical model, the others were run to gain insight. Because a complete data set does not exist, only the available data are compared. The first modeling effort was to simulate the physical model tests of Savage (1959). A second set of cases was run for shore-perpendicular structures. Next, an effort was made to model sediment transport in the vicinity of a hypothetical dredge disposal site in the 11- to 14-foot depths off Oregon Inlet. Finally, the Channel Islands Harbor Longshore Transport Study (Bruno, et al., 1981) was modeled. Bathymetric changes were closely monitored during this study; however, the wave climate (H, θ, T) used was determined from the Littoral Environmental Observation (LEO) data and uncertainties exist as to the accuracy of the data.

1. Simulation of Savage's Physical Model Tests.

The numerical model was used to simulate one of the physical model tests of Savage (1959). Test 5-57 was simulated numerically for a 10-hour period. In this physical model, the mean sediment size was 0.22 millimeters, the wave height averaged 0.25 feet, the wave period was 1.5 second, the wave angle was 30° (at a depth of 2.3 feet), and the groin was approximately 9.5 feet from still water to its seaward limit. C_{OFF} was held constant at 10^{-4} feet per second throughout the profile for this simulation. The offshore profile is presented in Savage (1959). Figure 6 represents three of the eight contours simulated. Note that the initial 0.3- and 0.5-foot-depth contours, in the numerical representation are too far seaward by approximately 2 feet. This is due to the $h = Ay^{2/3}$ equation as compared to the equilibrium physical model profile. Realizing this, it is the shape of the contour which must be used as an indication of the numerical model predictions. The general trend of the contours is similar, although the numerical model contours are displaced farther seaward as expected. The major differences are in the diffraction zone.

2. Several Runs Using Shore Perpendicular Structures to Demonstrate Effects of Altering Some of the Pertinent Parameters.

In the following simulations, the models were run until their near-equilibrium values were achieved. Coefficient C_{OFF} was not a function of depth (beyond the surf zone) but was held constant throughout the simulated area. Important variables are as shown in the figures. Only one wave condition (H_0 = 3 feet, T = 7 seconds, and a deepwater wave angle α_0

Note: Discrepancy between initial Savage contours and initial model contours is due to use of the $h = Ay^{2/3}$ profile.

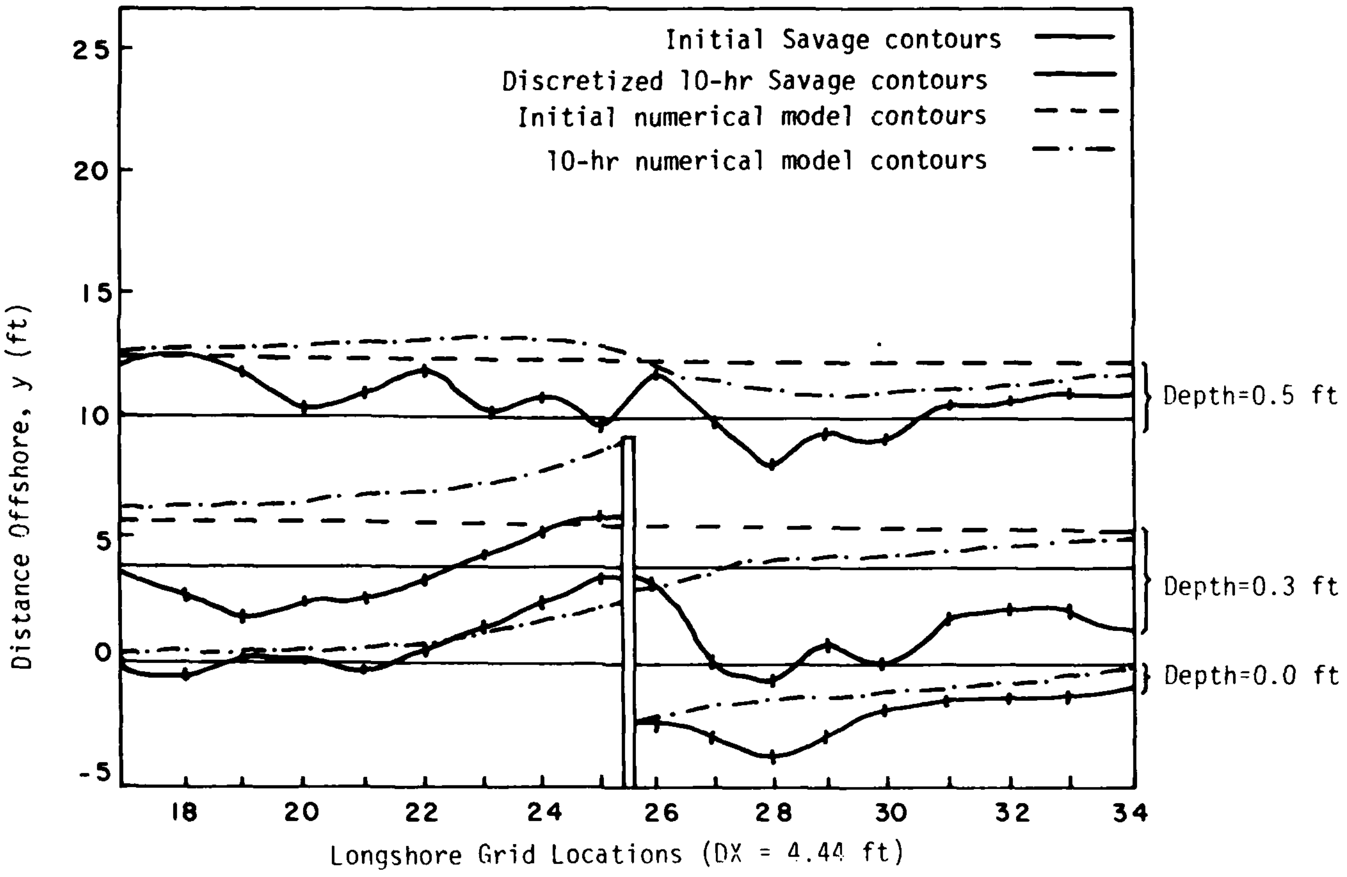


Figure 6. Simulation of the physical model of Savage (1959).

of 60°) was used as input for all four cases. Case 4.2a used an equilibrium shape factor A of 0.0899 and one groin. Case 4.2b was similar to 4.2a with the only modification being, that the A value was changed to 0.1486. In this way, a direct comparison was made based only on the shape of the equilibrium profile. Cases 4.2c and 4.2d used A-values of 0.0899 and 0.1486, respectively, but this time three shore-perpendicular, evenly spaced structures were simulated.

a. Comparison of Cases 4.2a and 4.2b. The most obvious difference between Figures 7 and 8 is the volume of sand impounded updrift and eroded downdrift. This is due to blockage of more of the active transport zone in the second case (i.e., a shorter groin is required for an equivalent performance on a steeper beach). The next obvious difference is the size of the perturbation which exists in the offshore contours. Clearly, case 4.2b is more perturbed and this is expected because larger offshore transports occur due to the steepening on the updrift side. Conversely, this means less sediment is initially bypassed (and along with the downdrift requirement for larger volumes of sand) causes larger erosional features in case 4.2b. Another interesting feature is the downdrift fillet which occurs in the third, fourth, and fifth contours. The fillet is due to the shape of the sixth contour which occurs because of the inability of the wave to transport more sediment (due to the reduction in wave height and angle in the diffraction shadow zone). The remaining difference is also due to the volume of sediment being impounded; i.e., the distance and extent of change the presence of the groin causes upcoast and downcoast.

b. Comparison of Cases 4.2c and 4.2d. The variations between cases 4.2c and 4.2d are very similar to the differences between cases 4.2a and 4.2b as would be expected with a groin field (here, three groins) as compared with a single groin (see Figs. 9 and 10). There is, however, one additional feature which can be attributed to the additional groins. Note that in the direction of littoral drift, the size of the fillet is decreasing. This is due to the updrift beach having an uninterrupted supply of sediment while the downdrift groin compartments are supplied sand at a rate determined by the bypassing. Part of this feature may also be due to the system not having attained complete equilibrium.

The effects of the fixed boundary conditions are evident on all cases run. In these example cases, the boundaries are clearly too close to the structure to provide a proper representation of the fillet contours.

3. Simulations of Sediment Transport of Dredge Disposal in the Vicinity of Oregon Inlet.

Hypothetical dredge disposal movement in the nearshore but beyond what is normally the surf zone at Oregon Inlet's adjacent beach to the south was modeled. In order to do these simulations, the program was altered such that for every nth iteration (time periods), the contours were shifted seaward to simulate the addition of dredged sediment disposal. The program presented in Appendix B does require slight modification to simulate this situation.

In general, the fifth and sixth contours were shifted seaward on a monthly basis to simulate the disposal of 121,000 cubic yards of sediment.

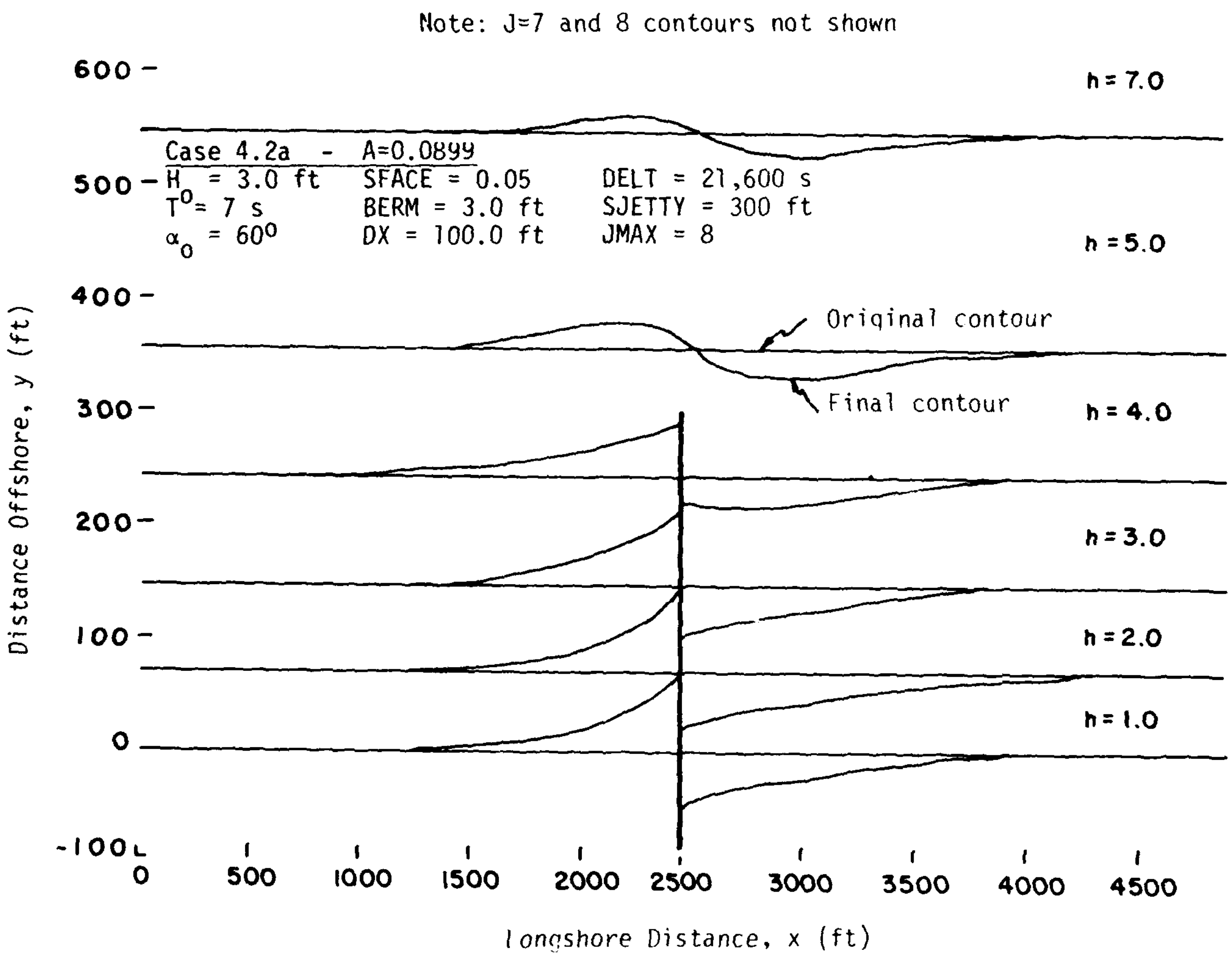


Figure 7. Equilibrium planform, case 4.2a.

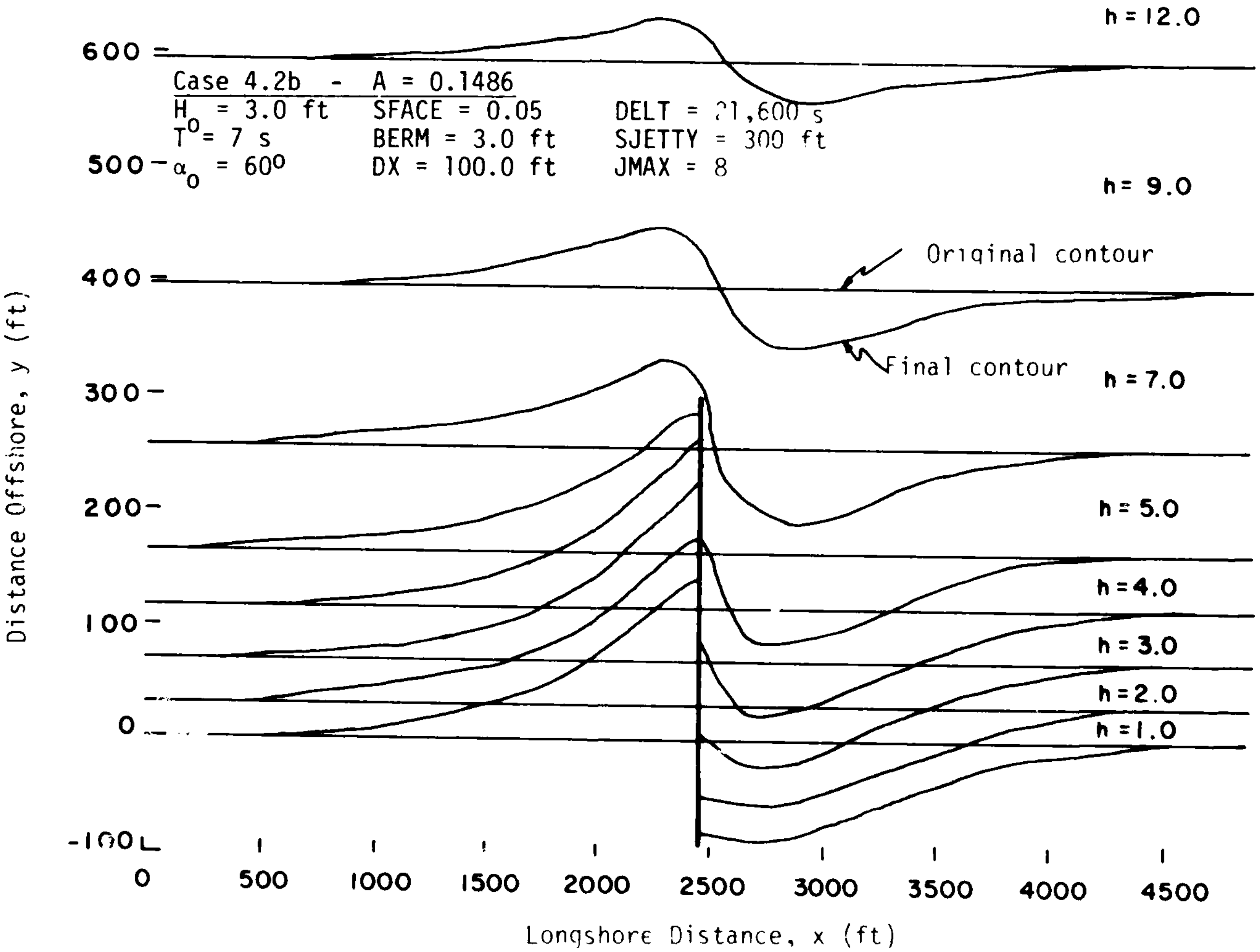


Figure 8. Equilibrium planform, case 4.2b.

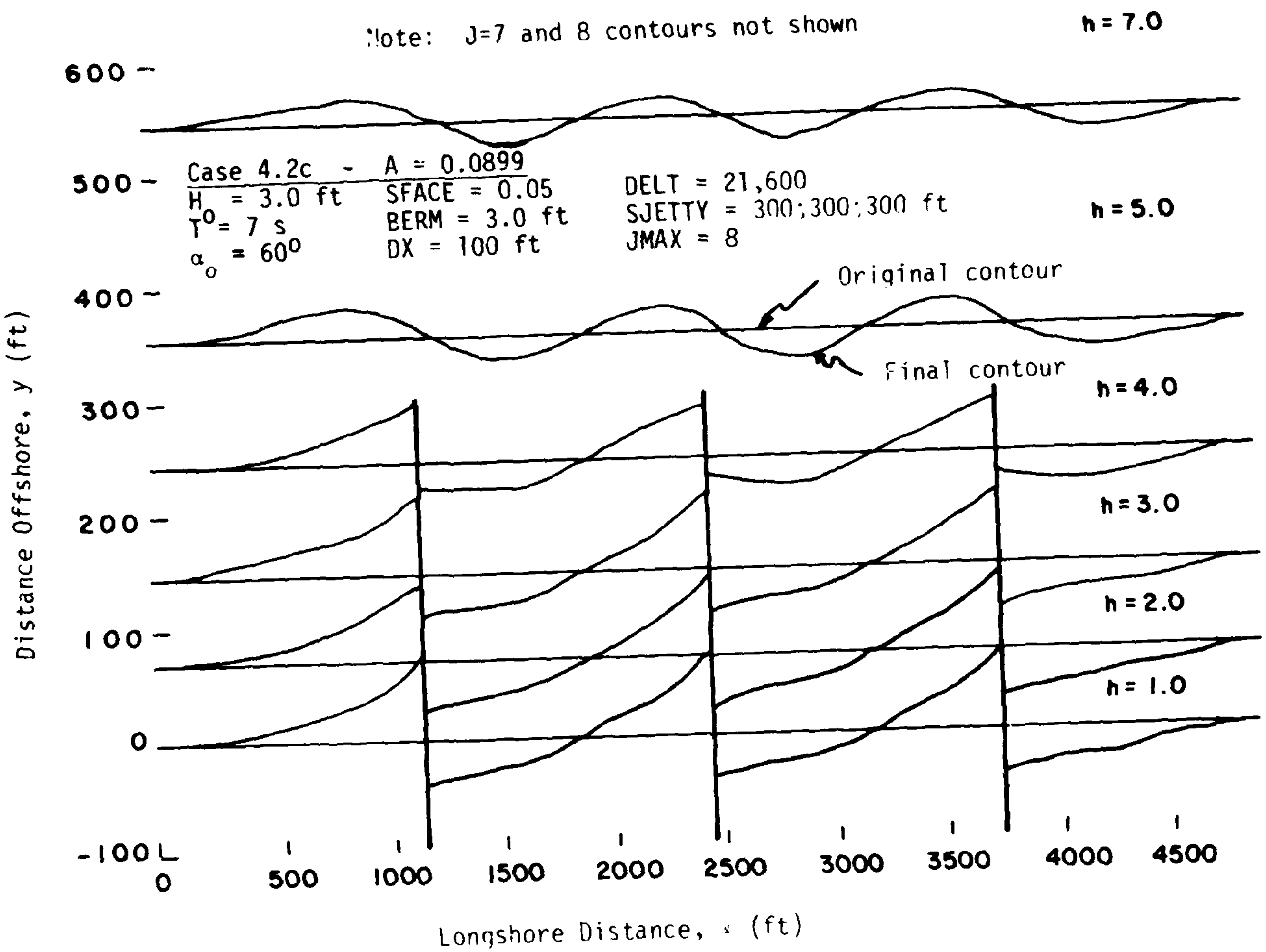


Figure 9. Equilibrium planform, case 4.2c.

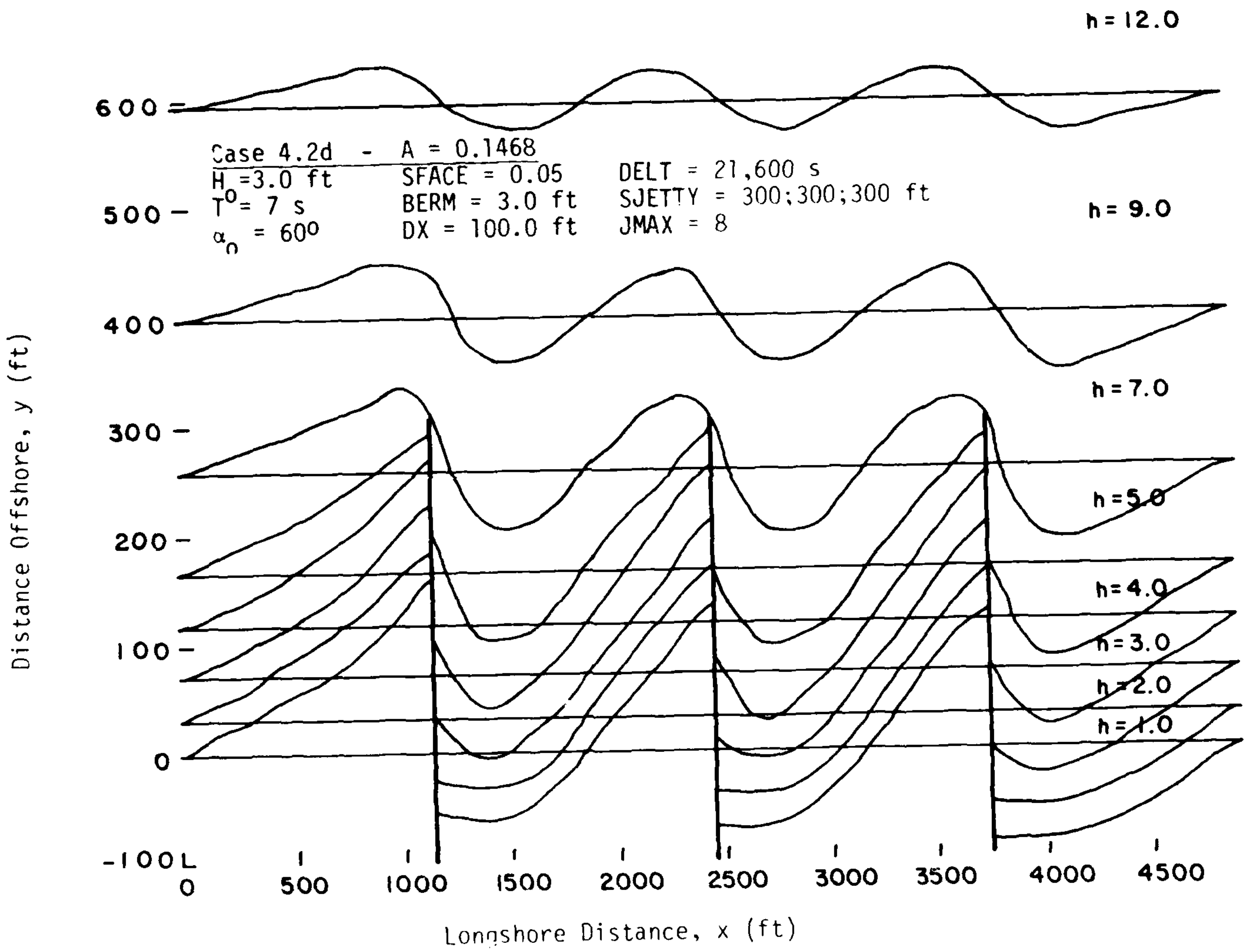


Figure 10. Equilibrium planform, case 4.2d.

In all these simulations, the following variables were held constant: (a) a time-step of 3 hours, (b) a shoreline length of 10,000 feet, (c) a longshore space-step of 200 feet, (d) an A value of 0.15 foot$^{1/3}$ for the equilibrium profile (see Fig. 11), (e) a berm height of 5.3 feet with a beach face slope of 0.05, and (f) a duration of 1 year. The wave climate was provided by the U.S. Army Engineer Waterways Experiment Station Wave Information Study (WIS) 1975 data and was initiated at different times of the year as indicated in the specific cases below. All simulations, prior to any addition of sediment, used the bathymetry shown in Figure 12. The shoreline (relative to mean low water, MLW) was scaled from a bathymetry-topography survey provided by the U. S. Army Engineer District, Wilmington. The initial offshore bathymetry was computed according to the equilibrium profile and the 0-foot contour; i.e., the profile was shifted seaward or landward, accordingly, (see App. C.) The boundary profiles were fixed throughout the simulations. The variation of COFF outside the surf zone was used because of the importance of the time rate of change in this simulation. Table 1 presents the percentage of sediment which moves out of the control volume (i.e., imaginary boundaries around the area where sediment was added) directly onshore and the percentage of sediment remaining in the control volume at the conclusion of the simulation for each of the cases. In addition, a seventh (case 3) and eighth (case 4) were modeled. In Case 3, the only difference was that sediment was placed at the 11- and 14-foot contours. Case 4, however, was quite different and will be described in detail later. It has a 20,000-foot shoreline, a longshore space-step of 400 feet, and sediment was added on a weekly basis. Also, the resolution in the profile was better.

a. Specific Cases.

(1) Case 2.a. In order to provide insight for the interpretation of the other modeling efforts, a simulation of the shoreline evolution using the January to December WIS time series, with no addition of sediment, was carried out. As expected, the contours almost attain an equilibrium planform shape (i.e., straight and parallel between the fixed end profiles; they do not, however, become aligned parallel to the base line because of the end conditions). Because of the scales involved, alongshore versus onshore-offshore, plotting the contours without distortion does not yield much information. Appendix C provides a listing of the final contours for all the cases modeled.

(2) Case 2.b. The only difference between cases 2.a and 2.b is the suppression of the WIS wave angle which was set equal to zero (i.e., wave crest approach is shore-parallel at the offshore boundary of the model). This does not cause the longshore sediment transport to vanish completely. There are still local gradients in the contours which cause refraction and relative angles between wave crest and contour, thereby driving the longshore sediment transport (even if refraction was not considered, the local angle between the wave crest and contour would cause sediment transport). Note the larger onshore transport (Table 1) for this case compared with Case 2.a. This is due to the reduction in longshore transport caused by the wave angle of 0°. The model still tries to smooth the contour lines; however, more of the smoothing for the present case must be done by onshore-offshore transport.

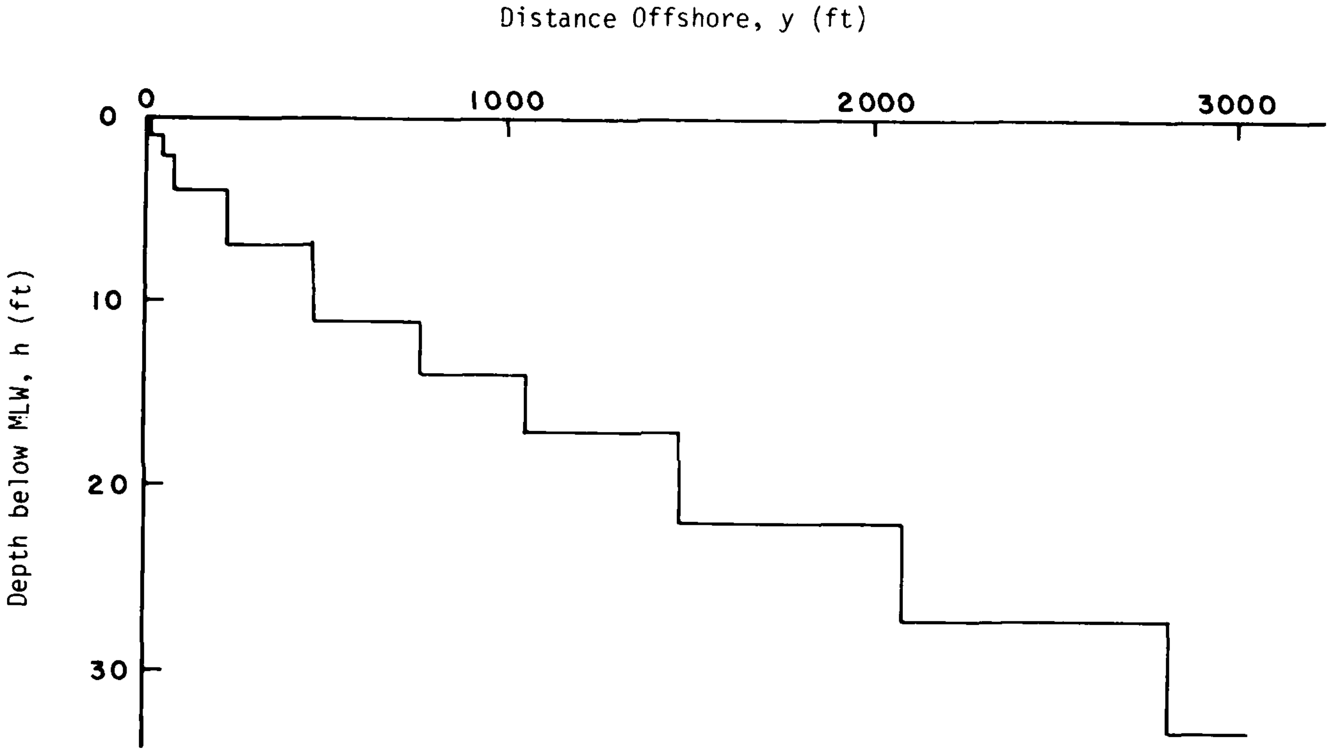


Figure 11. Stepped version of equilibrium profile used in the Oregon Inlet modeling, $h = Ay^{2\ 3}$ ($A = 0.15$ $\text{feet}^{1\ 3}$).

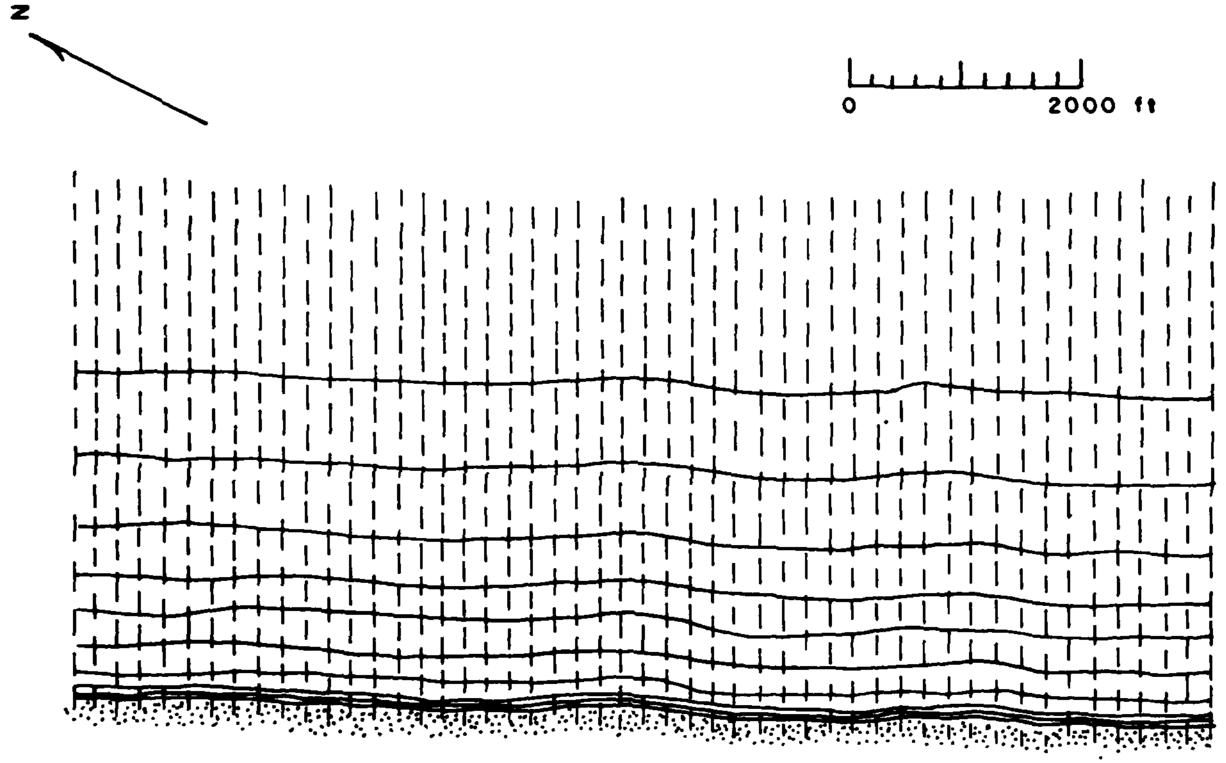


Figure 12. Initial contours used in the numerical model for all the Oregon Inlet simulations. (The scale for case 4 was twice the scale shown.)

Table 1. Summary of results at Oregon Inlet.

Case No.	Description	Pct Onshore out of control volume	Pct Remaining in control volume
2.a	No sediment added, WIS waves Jan. - Dec.	Onshore Movement (992 yd^3)	Increase (14,148 yd^3)
2.b	No sediment added, WIS waves ($\alpha = 0^\circ$) Jan. - Dec.	Onshore Movement (1624 yd^3)	Increase (9,356 yd^3)
2.c1	121,000 yd^3 added monthly, WIS waves Jan - Dec.	31.7 (460,264 yd^3)	38.6 (559,984 yd^3)
2.c2	121,000 yd^3 added monthly, WIS waves Apr. - Mar.	32.1 (466,160 yd^3)	36.9 (535,392 yd^3)
2.c3	121,000 yd^3 added monthly, WIS waves July - June.	28.6 (415,784 yd^3)	47.0 (682,088 yd^3)
2.c4	121,000 yd^3 added monthly, WIS waves Oct. - Sept.	27.2 (395,556 yd^3)	46.8 (670,848 yd^3)
3	121,000 yd^3 added monthly at the 11- and 14-foot contours WIS waves, Jan. - Dec.	8.9 * (32,164 yd^3)	78.0 (283,016 yd^3)
4	27,923 yd^3 added weekly on the 7-8-, 9-, and 10-foot contours, WIS waves Jan. - Dec.	19.0 (275,796 yd^3)	47.4 (687,525 yd^3)

* After 17 weeks, the addition of sand caused contours to cross. Prior sediment added was 363,000 yd^3. Problem was rectified; however, case was not rerun.

(3) Case 2.c1. In this simulation, sediment is added to the system each month. It was simulated by advancing the 7- and 11-foot contours on a monthly basis to represent 121,000 cubic yards per month. Specifically, the sand volumes were "tapered" starting at the center of the nourished area over a distance of ± 2,700 feet from the center. Table 2 presents the monthly Δy values for the blocks between the 7- to 11-foot contours and the 11- to-14 foot contours. Figure 13 shows the planform Δy values added monthly. WIS waves were used with the sequence being the normal calendar year, January through December.

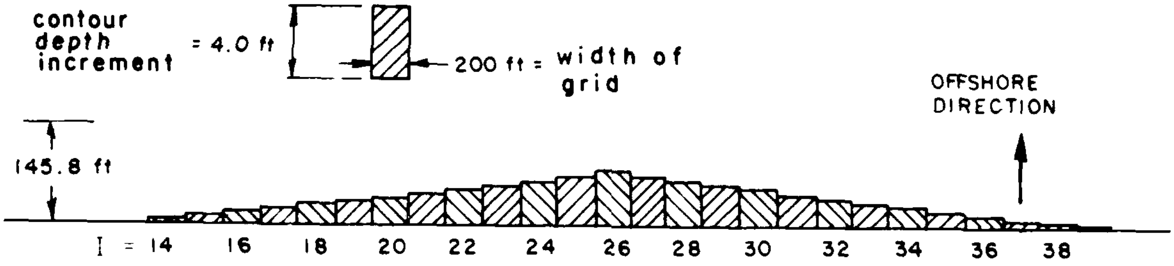


Figure 13. Monthly incremental values of Δy due to dredge disposal illustrated for the block between 7- and 11-foot contours.

The initial and final fifth and sixth contours have been plotted in Figures 14 and 15. The first figure has no distortion; the second is distorted 10 to 1. The simulation predicts that 31.7 percent of the dredge disposal will move shoreward out of the control volume. An additional 29.7 percent efflux occurs in the offshore and longshore directions, leaving only 38.6 percent of the total amount of sediment added remaining in the control volume. It is not clear what quantity of the sediment leaving in the longshore direction would reach shore. It is conceivable that most of this sediment would eventually reach the surf zone. The rate at which this material would move ashore would be expected to be slower than the rate at which the large mounds would move ashore because the deviation of the profile from equilibrium is much less.

(4) Cases 2.c2, 2.c3, and 2.c4. The next three simulations were the same as 2.c1 except the time series of wave events has been seasonally altered. Cases 2.c2, 2.c3 and 2.c4 use the 1975 wave climate from April through March, July through June, and October through September, respectively. The maximum variation is about 5 percent for the sediment volume moving onshore, and about 10 percent for the volume remaining. The variation in the

Table 2. Monthly values of Δy for the steps located between the 7- to 10-foot contours and the 11- to 14-foot contours.

Value of I	Monthly Δy value (ft) for steps between	
	7- and 11-foot contours	11- and 14-foot contours
26	145.8	194.4
25,27	135.4	180.5
24,28	125.0	166.6
23,29	114.6	152.7
22,30	104.1	138.9
21,31	93.7	125.0
20,32	83.3	111.1
19,33	72.9	97.2
18,34	62.5	83.3
17,35	52.1	69.4
16,36	41.7	55.5
15,37	31.2	41.7
14,38	20.8	27.8
13,39	10.4	13.9
All Others	0	0

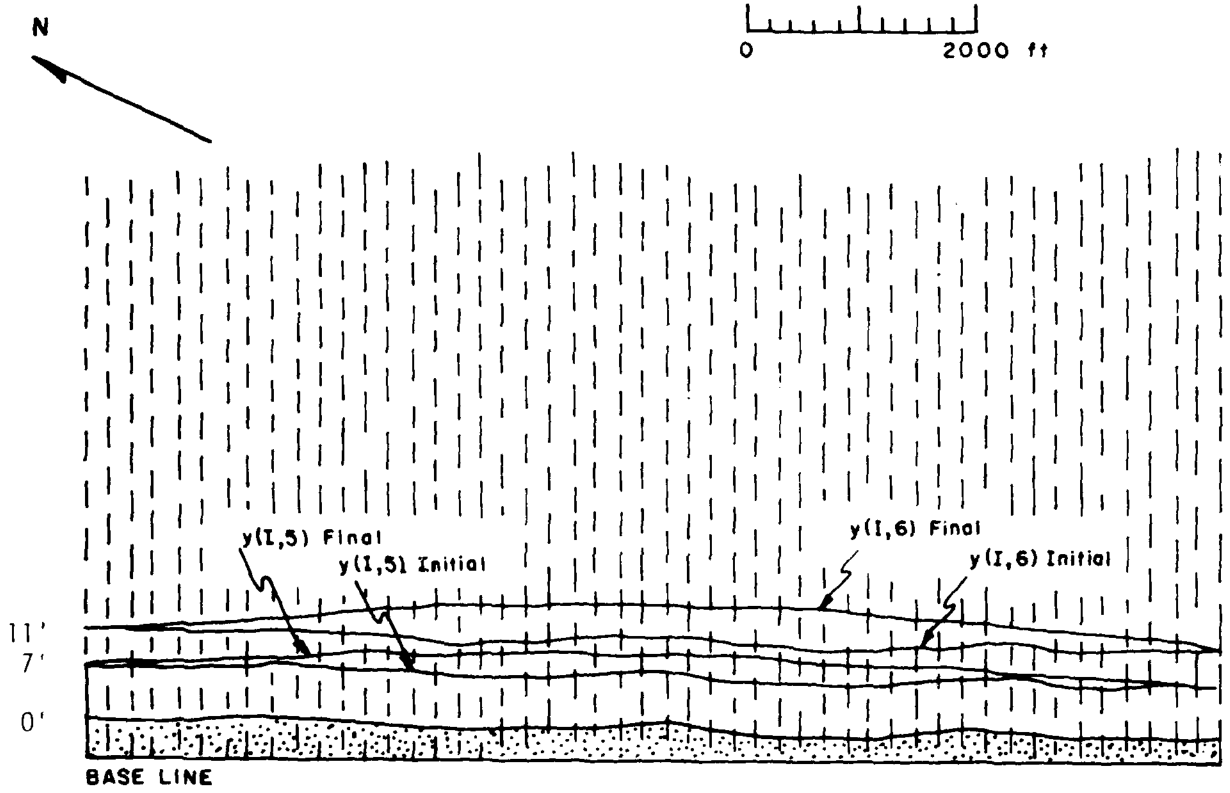


Figure 14. Initial and final 7- and 11-foot contours (no distortion).

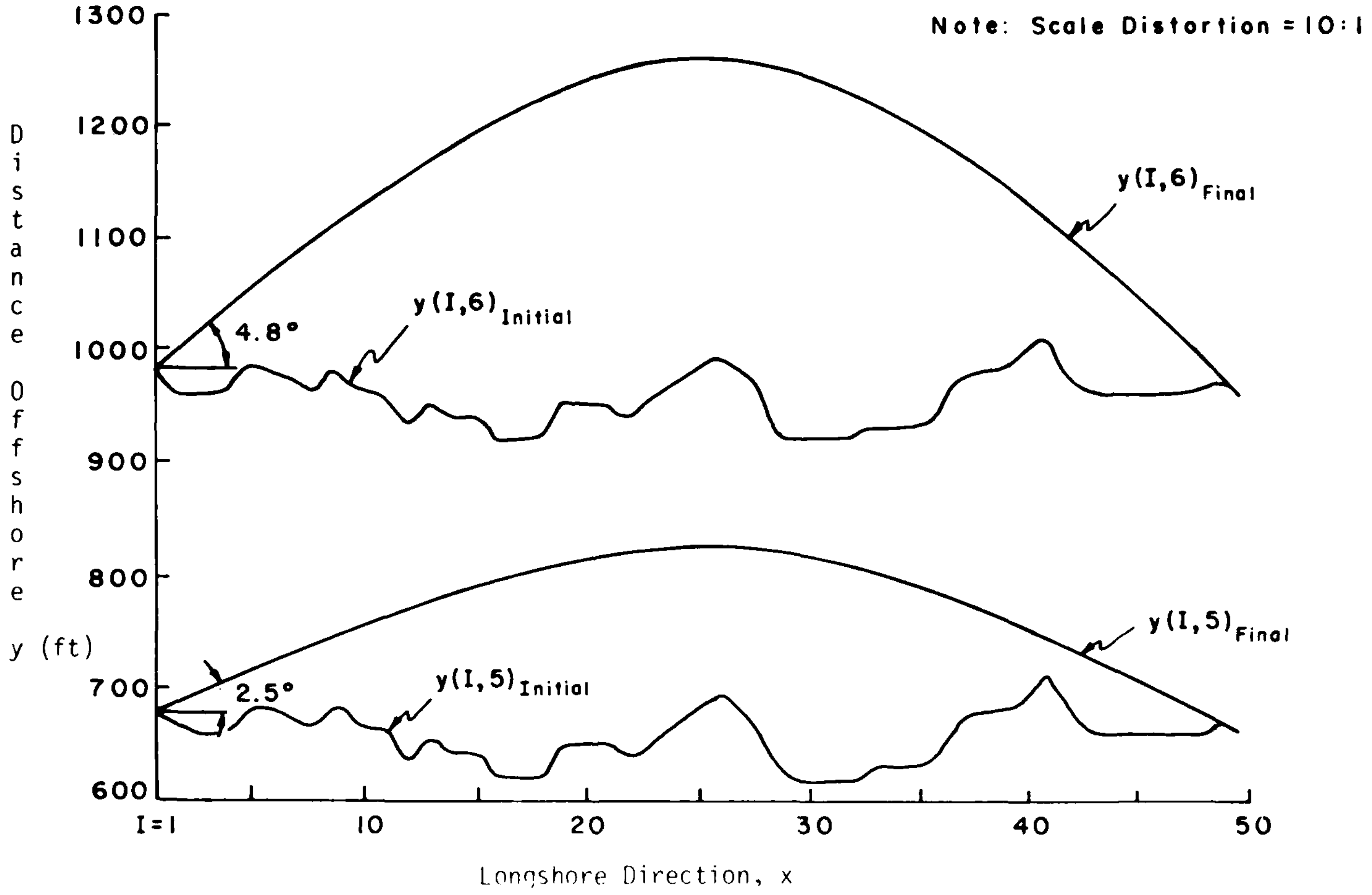


Figure 15. Initial and final contours for case 2.cl [y(I,5) and y(I,6)].

quantity moving onshore could be caused by waves that first tend to move more sediment longshore; then, the waves that transport more sediment onshore have a less out-of-equilibrium profile to cause movement upon. The variation in percentage remaining is due to the variation of the time series of the wave climate, with the last month in the simulation being especially important.

(5) Case 3. Instead of extending the 7- and 11-foot contours monthly to simulate the disposal of dredged sediments, the 11- and 14-foot contours were extended (194.4 feet each at the center of the disposal area). This case was modeled because the larger available dredge could not dump in more shallow water. The reduction and increase in the percent of onshore volume and the percent volume remaining (8.9 percent and 78.0 percent, respectively) demonstrate the sensitivity of the depths investigated. Qualitatively, these depths are the depths to which offshore bars occur along the Atlantic U.S. coast.

(6) Case 4. Further investigation of the disposal process demonstrated the need for an 11,000-foot shore-parallel disposal length with the sediment placed at the 11-foot contour building to about 7 feet. It was decided to model this physical situation also. The total shoreline length was changed to 20,000 feet, and the space step to 400 feet; the length of the disposal area in the longshore direction was increased to 10,800 feet. The resolution in the vicinity of the depths of the dump was improved by adding the additional contours and the profile is shown in Figure 16. As in the other seven cases, 1,452,000 cubic yards was added annually to the system; however, the addition was accomplished on a weekly basis (27,923 cubic yards per week). Sediment was still added by extending the contours seaward, but rather than placing one-fourth of the sediment at each of the four contours, the volumes were determined based on the trapezoidal cross section shown in Figure 17. This cross section more closely resembles the disposal mound formed by hopper dredging. The incremental values Figure 18 show, in planform, the extension of the contours to simulate the weekly sediment addition at the 8-foot contour.

A schematic illustration of the sediment transported from the nourished region is presented in Appendix C. Nineteen percent of the sediment added moved directly onshore out of the control volume.

b. Conclusions for the Movement of Disposed Sediment in the Vicinity of Oregon Inlet. The computer simulations, tempered with engineering judgment, demonstrate that between 15 and 35 percent of the material added to the 7- and 11-foot contours,or to the 7- 8- 9-, and 10-foot contours would be transported into the nearshore transport system during the first year. If the disposal process was continued, the system would approach steady state in terms of the volume of deposited material residing offshore.

For the case of sediment addition at the 11- and 14-foot contours, the computer simulations, tempered with engineering judgment, show that between 5 and 25 percent of the material added would be transported into the nearshore transport system during the first year.

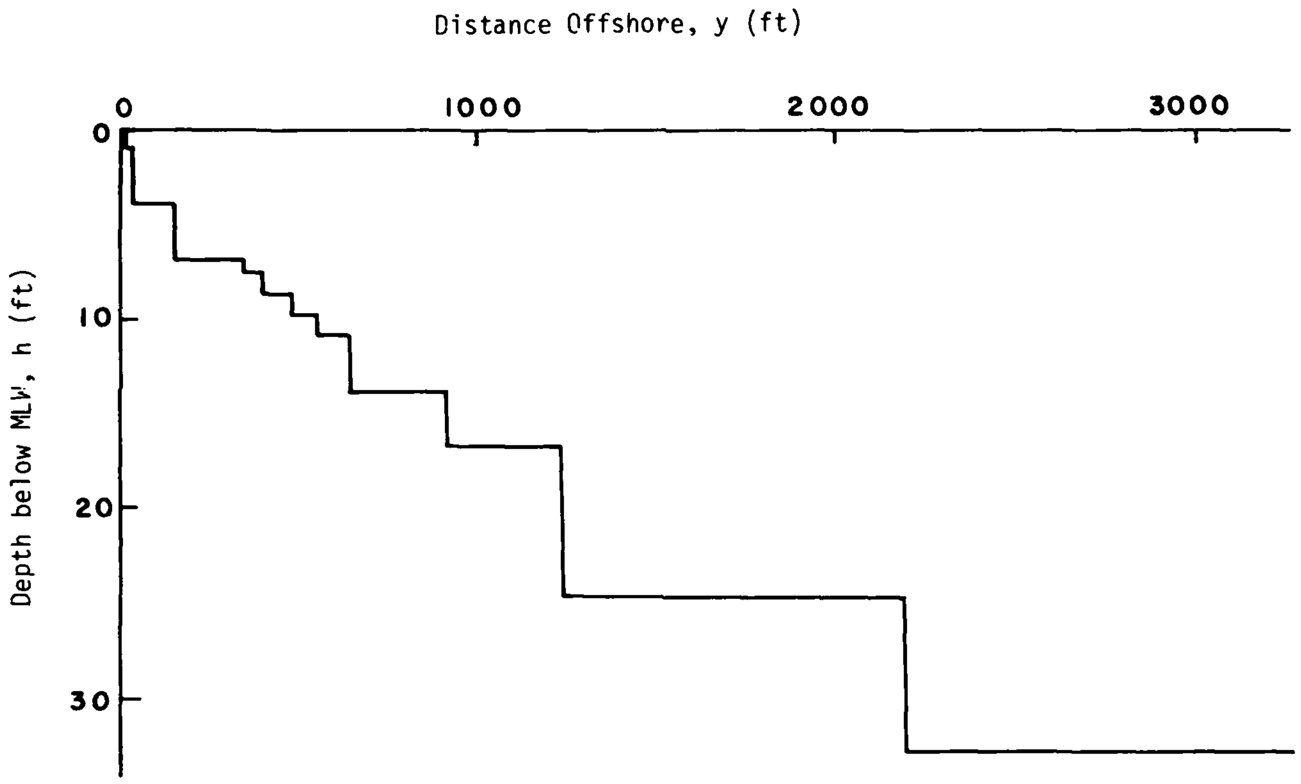


Figure 16. Stepped version of equilibrium profile used in the Oregon Inlet modeling, $h=Ay^{2/3}$ ($A=0.15$ $\text{feet}^{1/3}$), case 4. Note the resolution at 7, 8, 9, and 10 feet.

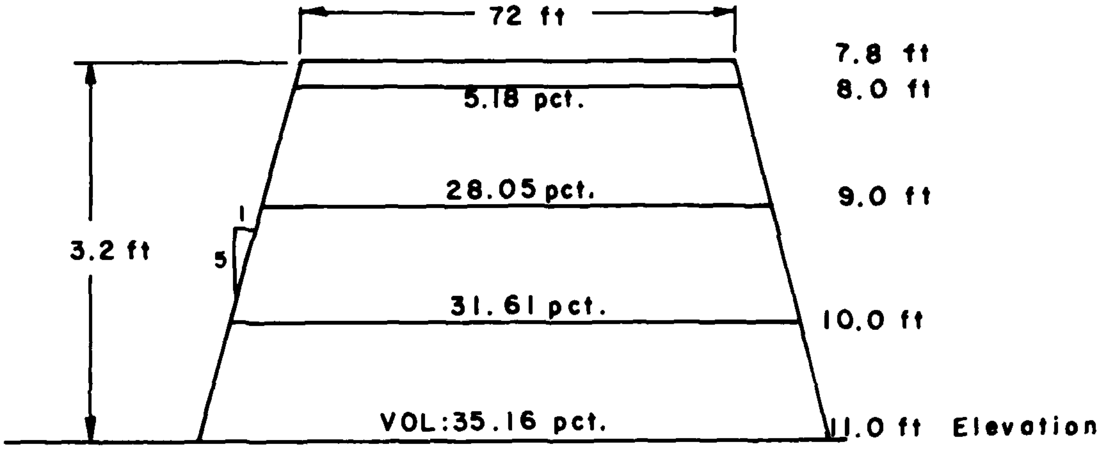


Figure 17. *Shore-perpendicular cross section of disposal* mound. The volumes represent the volume percentage of the trapezoidal section between contours and therefore, the quantity of sediment added to the 7-, 8-, 9-, and 10- foot contours.

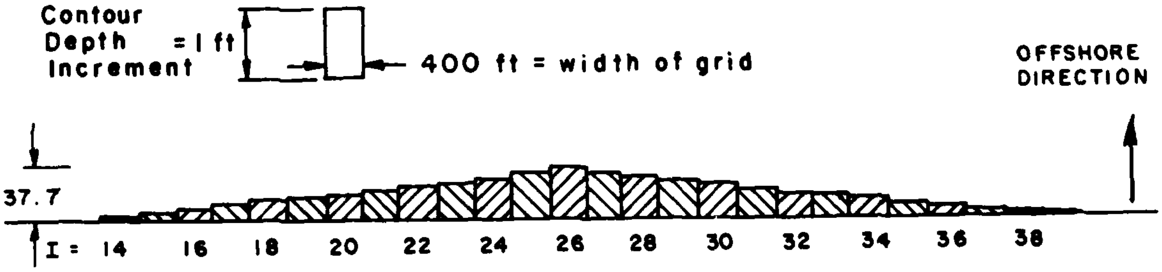


Figure 18. Incremental values of Δy due to dredge disposal, illustrated for the block between 8- and 9-foot contours (case 4).

4. Simulation of the Longshore Sand Transport Study at Channel Islands Harbor, California.

The CIH Longshore Sand Transport Study (Bruno, et al., 1981) was the only field study found suitable for verification purposes. Wave data collected included the LEO data and a two pressure-sensor gage array. Although the pressure gages were not in operation throughout the study, it was expected that the data they produced would be superior to that of the LEO data. However, these data were not available in a reduced form, so the LEO data were used. An adjustment of 11° was made to the breaker angle to orient the angle with respect to the base line, rather than to the local shoreline orientation angle. Observations had been taken twice daily at three locations; the middle location was used (observer No. 5714). Waves which approached the shoreline at angles too large to have originated in a depth of 10 meters, according to Snell's law, were set equal to 90^{0} at that depth (crest of wave perpendicular to the baseline). The 10-meter depth was chosen because it is the approximate depth at the tip of the offshore breakwater (for this reason, it was also chosen as the depth of the step beyond the y(I, JMAX + 2)$^{\underline{th}}$ contour). It was assumed that each of the two daily observations occurred for 12 hours and using a time-step of 6 hours, this meant two time-steps per wave. In cases where parts of the wave data (H_b, α_b, or T) were missed by the observer or were equal to zero, the data were ignored (no computations were made), but the time was included. Because the time rate of change is important for this simulation, the variation of C_{OFF} outside the break point was used.

The period chosen to model was 20 April through 1 December 1976. The initial survey was taken after dredging of the sediment trap and for this reason was known to be out of equilibrium. The bathymetric surveys were conducted using several methods, the most advanced being a Lighter Amphibious Resupply Cargo vessel (LARC) proceeding along shore-perpendicular lines (approximately in the vicinity of each survey station) taking fathometer readings every 10 seconds, with positioning systems trilaterating the vessel's position concurrently. These data were recorded on tape. The beach-face data were taken using standard surveying methods. Because the data fluctuated randomly about the stations, depending on the speed of the craft, the (x, y) coordinate positions had to be altered to fixed changes in x and y. This was accomplished using an interpolation routine. The x values were made to coincide with the stations used in the surveys, and the y values were determined at 100-foot intervals beginning from the base line. Stations 100+00 and 118+00 were located at the north jetty and termination of the detached breakwater, respectively (these correspond to I values of 16.5 and 34.5 in the model). See Figure 19.

Monotonic profiles of the form $h = A(y - ydel)^{2/3}$ were fit to the data along each station line. "ydel" represents the zero location of the fitted shoreline, the value of which was unknown. Because dredging had been done in the lee of the breakwater, there was no reason to expect the A value to correspond to the value upcoast where the influence of the structure and the dredging was negligible. For this reason, the profiles of Stations 122+00 through 134+00 were evaluated separately to determine an A value for the equilibrium profile to be used in the numerical model. For the detailed method used (LaGrange Multipliers and Newton-Raphson Method for nonlinear

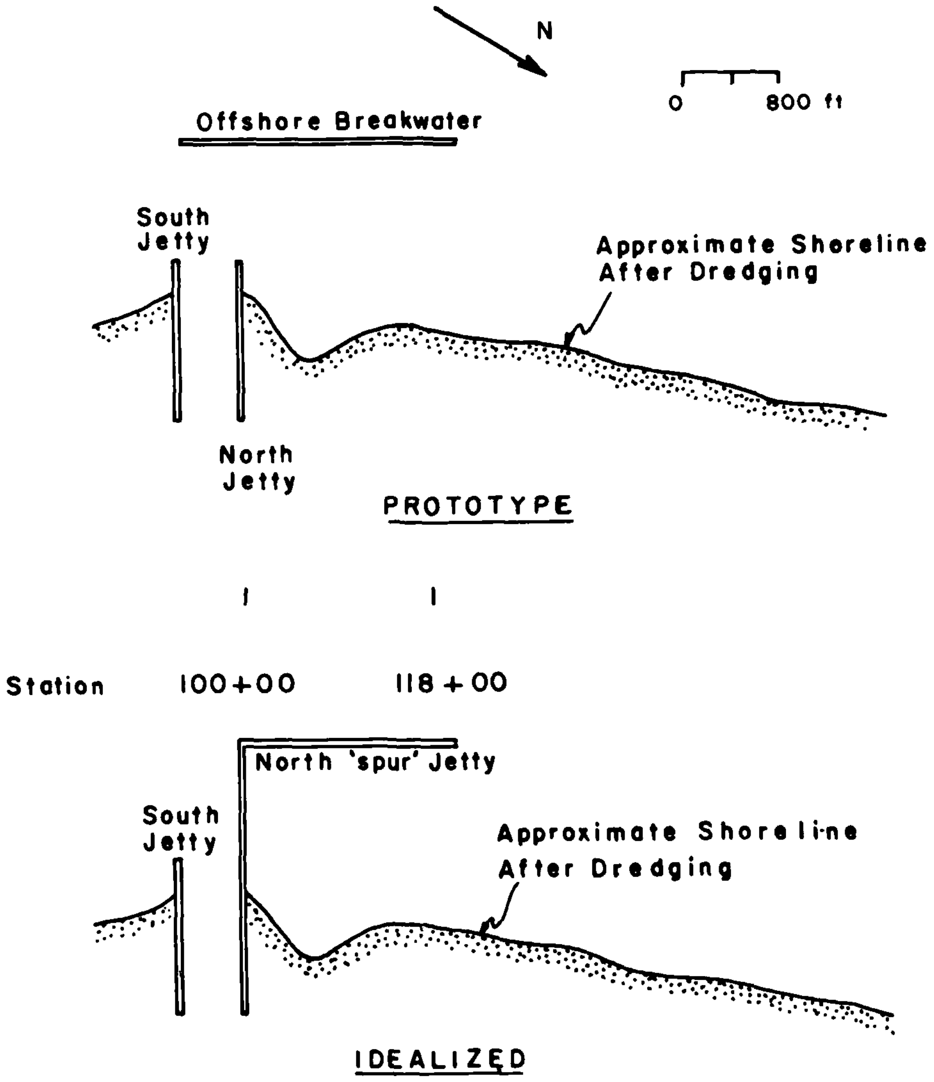


Figure 19. Idealized numerical model representation of offshore breakwater at Channel Islands Harbor, California.

equations) and the computer programs see Appendix D. The two values obtained for the surveys of 20 April and 1 December 1976 were averaged to obtain the value used in the model, A = 0.2606. Stations 101+00 through 121+00 were treated separately for the purpose of obtaining values with which to initialize those parts of the contours in the model and for comparison of the model predictions with the prototype values. Note that although the breakwater extends only to about Station 118+00, the influence of the structure and dredging extends beyond that location and so, although arbitrary, the 121+00 station was chosen as the dividing line. The initial and final values of the scaling parameter A for the profiles were 0.3233 and 0.3528, respectively. Because the initial shoreline is so irregular, a discontinuity between 121+00 and 122+00 is not evident.

One further idealization was made. The jetty-breakwater system was idealized as shown in Figure 19. This was required to simplify the physical situation, and although waves, currents, and sediment do pass through the opening in the prototype, it is hoped that they are of secondary importance.

The results of the numerical modeling of Channel Islands Harbor are presented in Figures 20 and 21. The first figure presents the shoreline contour (depth = 0); the second figure presents the farthest offshore, modeled contour. In both cases, the initial shoreline represents the model and prototype (after fitting of the profiles). The initial shoreline contour is further offshore along the section of beach beyond the end of the breakwater, while in the lee of the breakwater, as would be expected after dredging, the shoreline is closer to the base line. The final prototype contour has undergone erosion along the reach beyond the tip of the structure, and accretion in the lee.

The model's shoreline contour has undergone similar changes, and on the average, represents the final prototype contour quite well. The JMAXth contour has been displaced quite similarly to the shoreline contour with shoreward movement (erosion) along the reach beyond the tip of the breakwater and seaward movement (accretion) within. It appears that the final model's shoreline has predicted too much erosion and not enough accretion. Several parameters could be incorrect, with the onshore-offshore sediment transport rate coefficient, C_{OFF}, perhaps the most likely. Overall, the model seemed to predict reasonable values of the contours.

V. SUMMARY AND RECOMMENDATIONS

Some of the parameters that the model does not include are important and should be mentioned. As stated previously, the model does not include bar formation. This is precluded by an n-line system. There are no provisions for water level fluctuations or currents. Improvement to the model could also be facilitated with better longshore and cross-shore sediment transport relationships. A more reliable equation for distribution of sediment transport across the surf zone would also be helpful (or further testing and calibration of the equation proposed herein). Finally, combining refraction and diffraction using equations to predict their combined effect would improve the wave field. The program was constructed such that improvement

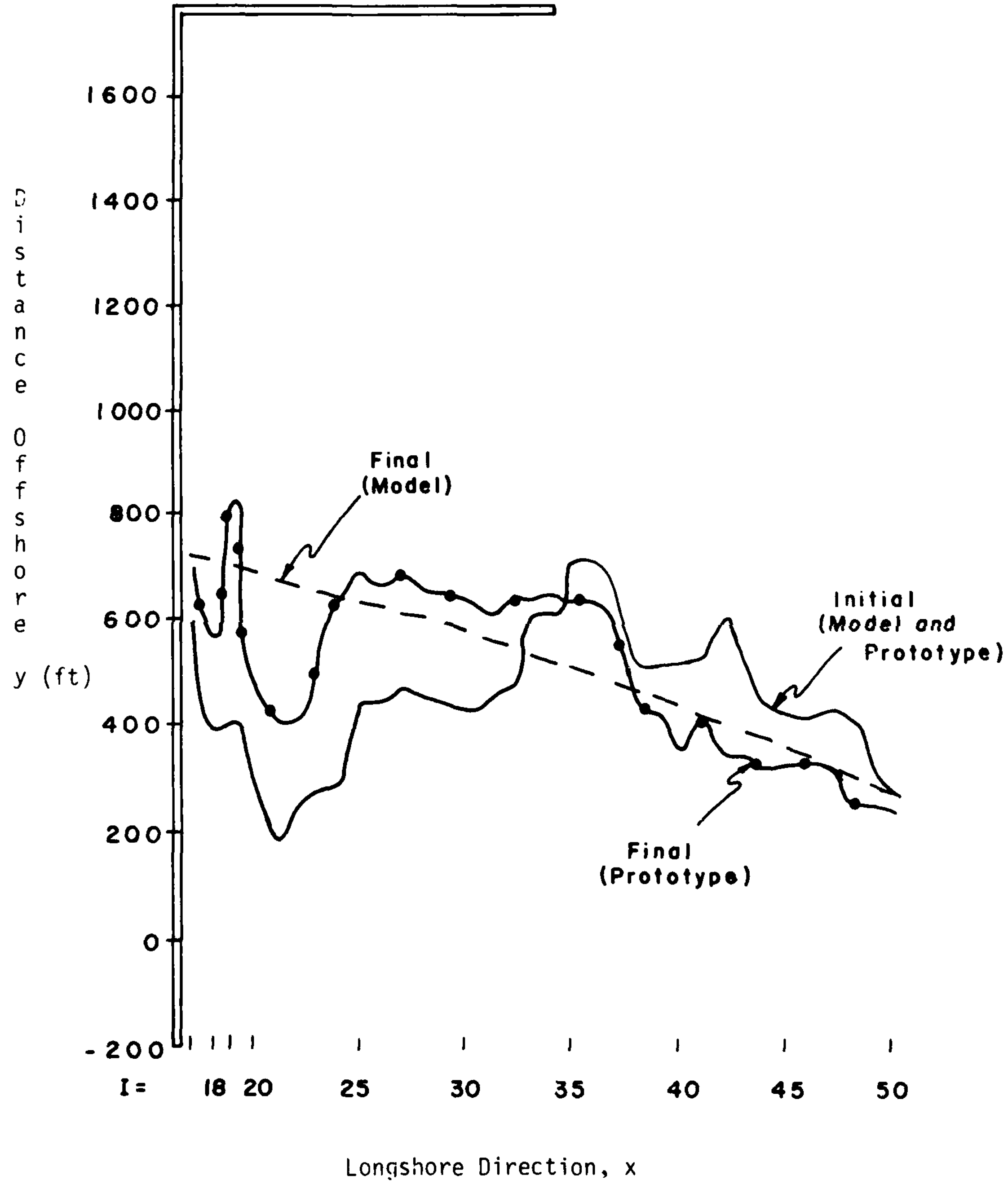


Figure 20. CIH simulation of shoreline contour, 20 April - 1 December 1976 (from LEO data).

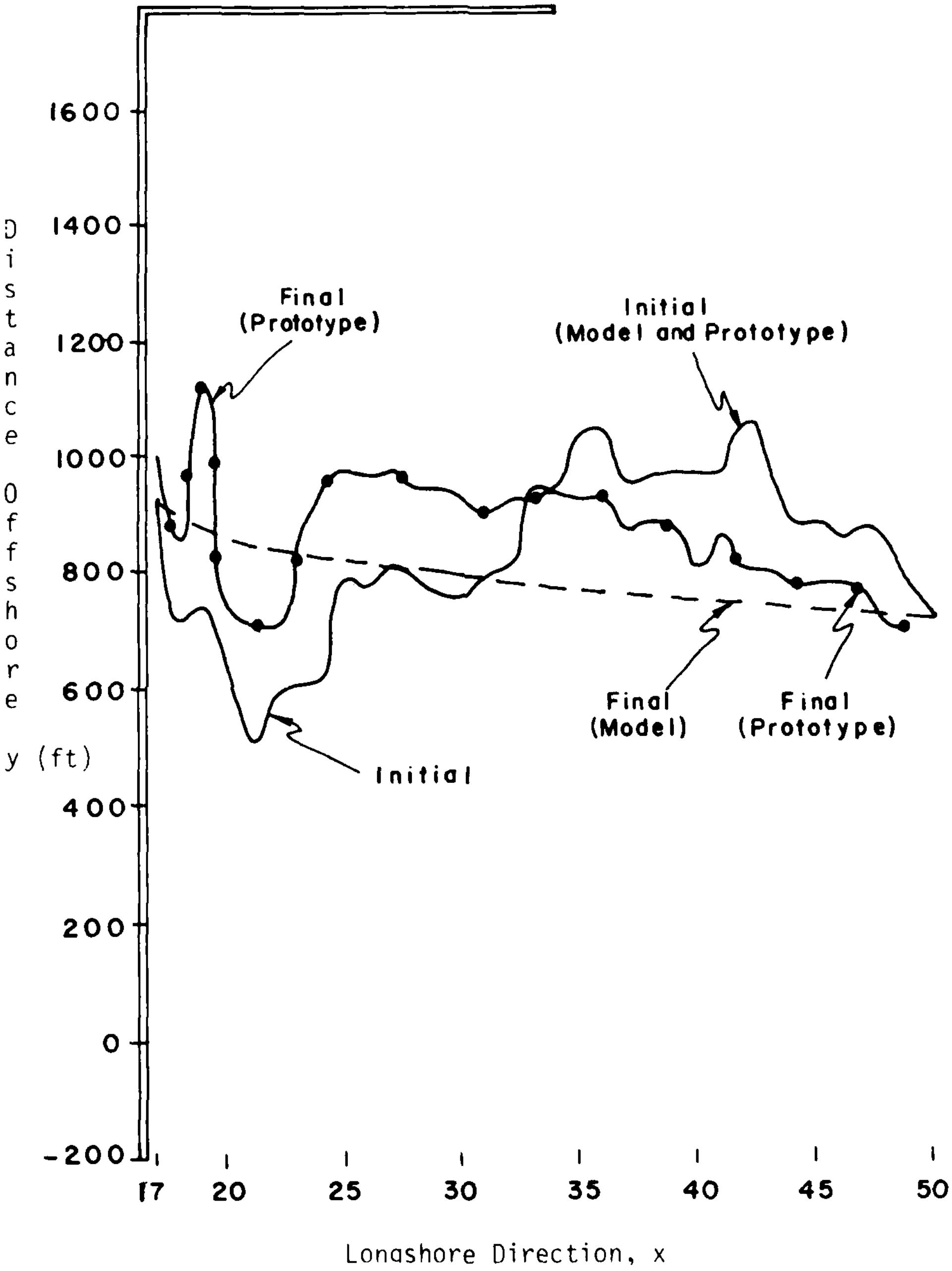


Figure 21. CIH simulation of $(JMAX)^{th}$ contour, 20 April - 1 December 1976 (from LEO data).

could be accomplished with minimum effort. Therefore, if a more suitable equation becomes available, the change of a subroutine should be sufficient for implementation of the equation.

Although the model is limited by the omission of the aforementioned parameters, it is reasonably correct. The ability to simulate various physical situations (shore-perpendicular structures, beach fills, breakwater and shore-perpendicular structures) has been demonstrated. In the CIH simulation where the data were first transformed to monotonically decreasing contours and LEO wave data were used, the model still predicts the prototype shoreline changes in a reasonable fashion.

Further research and model development should include exercising the model in a number of different situations. Several theoretical cases should be simulated, which if analyzed properly, would provide a tool for the coastal engineer. Combined refraction and diffraction should be included, if possible, along with any of the aforementioned parameters which have been omitted and for which relationships exist. Perhaps the most difficult problem to researchers working on modeling sediment transport in the vicinity of structures is the availability of field data. High-quality concurrent wave and bathymetric change data in the vicinity of coastal structures do not exist. One suggested field experiment is to monitor changes both updrift and downdrift of a jettied inlet which has a bypassing plant. Monitoring should begin immediately after bypassing, when the profiles are out of equilibrium. The recorded bathymetric and wave data would then provide data with which to calibrate, verify, and evaluate the existing models.

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APPENDIX A

DISCUSSION OF CONSTANTS AND SOME OF THE VARIABLES REQUIRED BY THE MODEL

Establishing the grid-contour system requires several variables. IMAX represents the number of cross-shore grid lines desired and JMAX the number of contours simulated. DX represents the spacing between the IMAX grid lines and DY the spacing between the contours. DX is a value which must be chosen along with IMAX and JMAX such that sufficient detail is obtained where necessary (e.g., in the shadow zone, if diffraction effects are believed to be very important, DX must be assigned a sufficiently small value so that at least some points lie within the shadow zone for the larger wave angles). DY is not a constant, but a dimensional array which is computed by the model according to the contour location. Once the depths of contours to be modeled are chosen, the initialization of DY and the y values are computed with the following equation after Dean, 1977

$$h = A\,y^{2/3} \tag{A-1}$$

where h is the depth, y is the offshore distance and A is the scaling parameter Dean gives values for A for several diameter sediments; however, if long-term beach profiles are available for the site being modeled, the modeler may want to choose a slightly different A value to more closely match the site-specific beach profile. Figure A-1 presents values of A versus diameter (after Moore, 1982). The model is programmed to input the h(I,J) values (depths as shown in Figure 1, called DEEP (I,J) in the program) read in the value of A (called ADEAN in program) and it then computes the y values. Also shown in Figure 1 is the height of the berm (BERM) and this value, along with the beach-face slope (SFACE), is required as program input and can be obtained from beach profile site data. Because the model does not include water level fluctuations such as tides, all values are to be referenced to a chosen datum. Other geometrical constants depending on the site include SJETTY (the length of the jetty), MMAX (the number of structures to be input), and IJET (M), M = 1,2,...MMAX (the smaller I value adjacent to the M^{th} structure's location). If no structure is required, as in a beach fill, the value of SJETTY must be entered as 0.0, with MMAX and IJET (M) entered as 1 and (IMAX/2), respectively. As set up presently, the groin locations must be equally spaced.

One constant used throughout the program is the breaking wave criteria (CAPPA in the program) equal to 0.78. It is required in several different computations and always governs the maximum wave height allowed according to the depth.

Another group of variables assigned values within the program is the sediment and fluid properties. These include fluid mass density, sediment mass density, porosity, and the angle of repose (e.g., RHO = 1.99, RHOS = 5.14, POROS = 0.40, and REPOSE = 32°, respectively). The values can easily be changed to reflect site conditions.

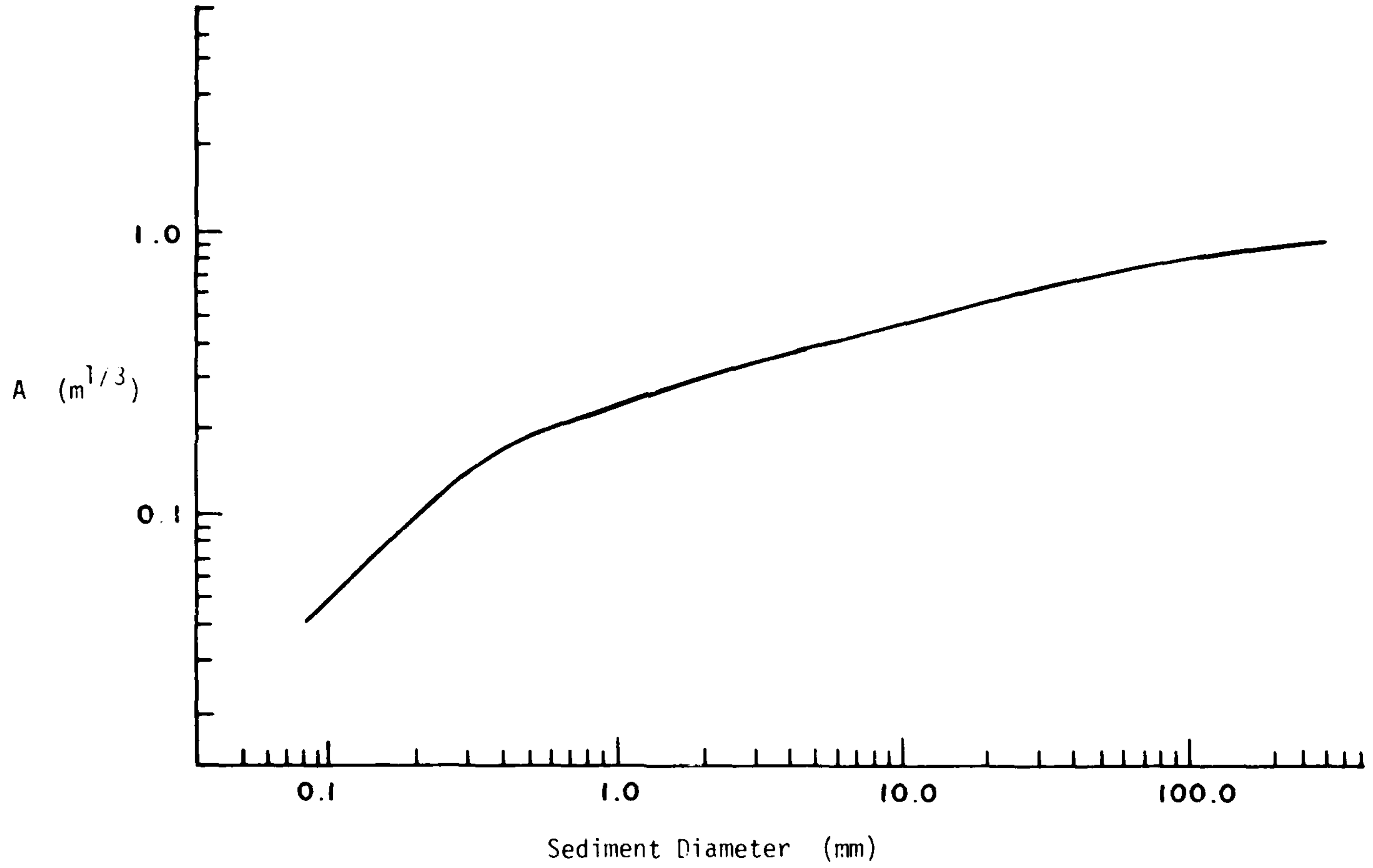

Figure A-1. A versus sediment diameter (after Moore, 1982).

Another very important set of constants is the constant chosen for the longshore and cross-shore components of sediment transport. Equation (27), the total longshore transport equation, contains the constant C' equal to

$$C' = \frac{K\rho \ (g)^{1/2}}{(\rho_s - \rho) \ (1 - p) \ (16) \ (\kappa)^{1/2}} \qquad \text{(A-2)}$$

where K = 0.77 (Komar and Inman, 1970)

g is the acceleration of gravity (32.17 ft/sec^2)

ρ_s and ρ are the mass densities of the sediment and the seawater (5.14 and 1.99 slugs per cubic feet, respectively

p is the porosity (0.40), and

κ is taken as 0.78.

Using these values to compute C' (TKSI in the program), a value of 0.325 is obtained. It is stressed that if any of these values are different for the site to be modeled, they should be changed and the program will compute another value for C'.

The parameter C_{OFF} is an "activity factor" which, based on earlier work primarily within the surf zone, was found to be

$$C_{OFF} = 10^{-5} \text{ ft/s}, \qquad h < h_b$$

To generalize this concept for transport seaward of the surf zone, the wave energy dissipation per unit volume was utilized as a measure of mobilization of the bottom sediment. Inside the surf zone, the dominant wave energy dissipation is caused by wave breaking; outside the surf zone, the dominant mode of wave energy dissipation is due to bottom friction. These two components will be denoted by D_1 and D_2, respectively.

(a) Energy Dissipation by Wave Breaking. The wave energy dissipation per unit volume by wave breaking, D_1, is

$$D_1 = \frac{1}{h} \frac{\partial}{\partial y} (E \ C_G) \qquad \text{(A-3)}$$

which, employing the spilling breaker assumption ($H = \kappa h$) within the surf zone, can be shown to be

$$D_1 = \frac{5}{16} \rho g^{3/2} \kappa^2 h^{1/2} \frac{\partial h}{\partial y} \tag{A-4}$$

or

$$D_1 = \frac{5}{24} \rho g^{3/2} \kappa^2 A^{3/2} \tag{A 5}$$

in which A is the scale parameter in the equilibrium beach profile

$$h\ (y) = Ay^{2/3} \tag{A-6}$$

(b) Energy Dissipation by Bottom Friction. The wave energy dissipation per unit volume due to bottom friction, D_2, is

$$D_2 = \frac{1}{h} \tau\, u_b = \frac{1}{h} \rho C_f \ \overline{|u_b|\, u_b^2} \tag{A-7}$$

in which C_f is a bottom friction coefficient, u_b is the bottom water particle velocity and the overbar indicates a time average. For linear waves, equation (A-7) can be reduced to

$$D_2 = \frac{1}{6\pi} \frac{\rho}{h} C_f \frac{H^3 \sigma^3}{\sinh^3 kh} \tag{A-8}$$

The activity coefficient C_{OFF}, outside the surf zone, is expressed as

$$C_{OFF} = \frac{1}{\Gamma} \frac{D_2}{D_1} \ \times \ 10^{-5} \text{ ft/s}, \qquad h > h_b \tag{A-9}$$

$$C_{OFF} = \frac{4}{5\Gamma} \frac{C_f \sigma^3}{g^{3/2} \kappa^2 A^{3/2} h} \left(\frac{H}{\sinh kh}\right)^3 \ \times \ 10^{-5} \tag{A-10}$$

in which Γ is a parameter relating the efficiency with which breaking wave energy (which occurs primarily near the water surface) mobilizes the sediment bottom ($0 < \Gamma \leq 1$). Herein, Γ is taken to be one.

Figure A-2 presents an example of the variation of the activity coefficient versus relative depth for a particular wave period and deep water wave height. It is seen that the activity coefficient reduces rapidly with increasing depth.

The value of C_{OFF} for the physical modeling of Savage's (1959) data was taken as 10^{-4} feet per second. Perlin (1978) presents some rationale for choosing a value of C_{OFF}; however, very little testing has been done and none is based on actual field measurement.

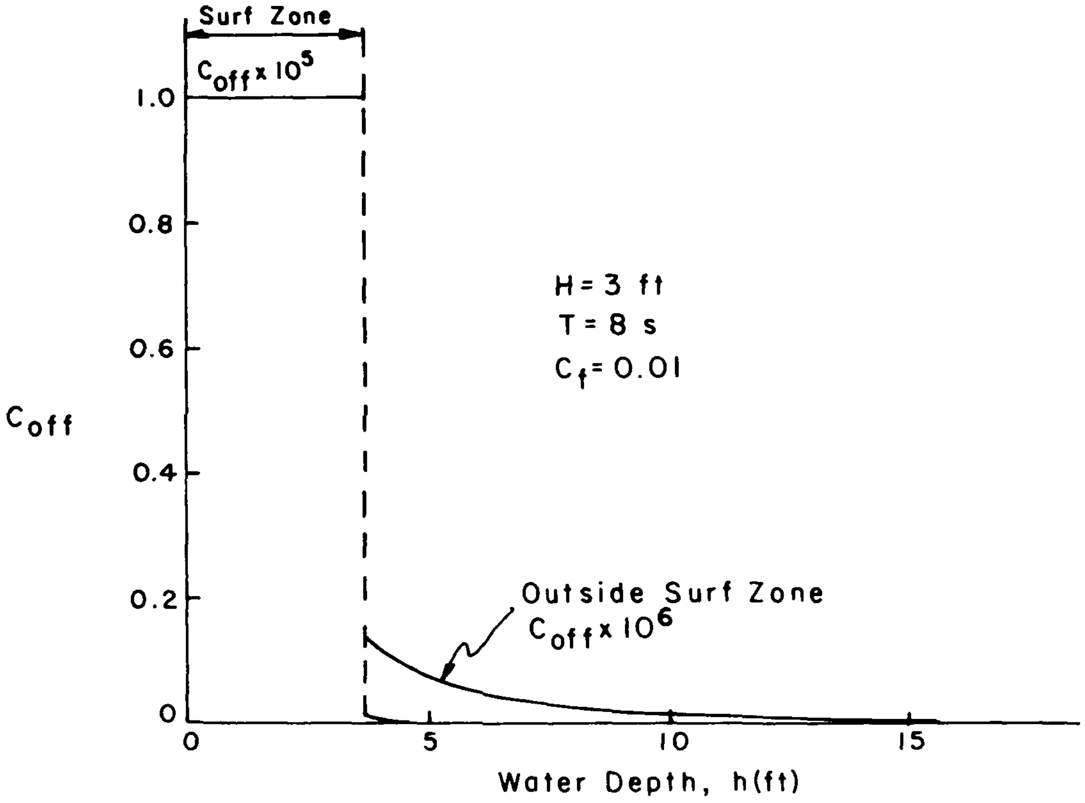

Figure A-2. Example of activity coefficient, C_{OFF} versus water depth, h, for particular wave conditions.

Finally, wave data are read into the program and the simulation begins. (For information regarding "Read Formats" for the various constants and variables, see Appendix E). Wave data required are wave height, wave period, wave angle relative to the x-axis of the model at a depth, WDEPTH and the duration of the wave climate (HS, T, ALPWIS, and a combination of NTIMES x DELT, respectively, in the model). As is always the case with numerical models, the time step and space steps are very important to both stability and accuracy. Time steps on the order of 3 to 6 hours (10,800 to 21,600 seconds) or less are recommended. However, the complexity of the bathymetry, variation and time series of the wave data, constants used (especially C_{OFF}) along with several other factors, greatly influences the stability and accuracy of the results.

Table A-1 lists several of the important variables in the computer program.

Table A-1. List of important variables in the program

ABAND	The input banded matrix which stores the values from equation (37)
ADEAN	The value of the scaling parameter in the equilibrium beach profile
ALPHAS	The angle a contour makes with the x-direction base line (counter-clockwise is positive)
ALPWIS	The angle (-90° to $+90^\circ$) the wave crest makes with the x-direction (counter-clockwise is positive)
AMP	The amplitude of the diffracted wave in the shadow zone
ANGGEN	The wave angle at a depth, WDEPTH
ANGLOC	The local contour orientation angle
AWARE	See equations (36) and (37)
BERM	The height of the berm above water level
BMATRX	The matrix which, upon solution of the banded matrix problem yields the new y values
C	The wave celerity
CAPPA	The breaking wave index
CC	Constant which establishes the width of the distribution of sediment transport across the surf zone
CG	The group velocity throughout the wave field
CGEN	The linear wave theory celerity at a depth, WDEPTH

CGGEN	The linear wave theory group velocity at a depth, WDEPTH
CO	The deepwater, linear wave theory wave celerity
COFF	The onshore-offshore transport rate coefficient within the surf zone
CONST	The constant in the longshore sediment transport relationship (0.77)
CONST6	The space step, DX, multiplied by the activity coefficient
DEEP	The water depth at any grid location
DEEPB	The initial breaking depth along each profile (between adjacent profiles)
DEEPBI	The initial breaking depth along each profile (at each profile, rather than between them)
DELT	The time-step in seconds (DELT x NTIMES = wave condition duration)
DIAM	The mean diameter of the sediment particles
DISTR	See equations (36) and (37)
DX	The alongshore space-step in the x-direction (distance between I values)
DY	The onshore-offshore space-step in the y-direction as defined by the stepped profile
ELO	The deepwater, linear wave theory wavelength
ELTIP	The wavelength at the tip of the structure
EPS	The change in the wave number which is acceptably small
G	The acceleration of gravity (32.17 feet/second^2)
GAMMA	The specific weight of seawater
H	The wave height throughout the wave field
HB	The maximum wave height which could exist throughout the wave field (where H = 0.78 * h)
HBI	The initial breaking wave height along any profile at the y values rather than between them
HBQ	The initial breaking wave height along any profile, between adjacent profiles

HGEN	Average wave height at a depth, WDEPTH
HS	The significant wave height input
I	The longshore grid location
IBREAK	The leeward side of the initial breaker location J value
IJET	Represents the lesser I value adjacent to the structure (these must be evenly spaced alongshore)
IMAX	The total number of grid points in the x-direction (alongshore)
J	The offshore contour location
JMAX	The value of the seawardmost contour simulated
JUSE	(JMAX + 2) the seawardmost contour at which the wave field is calculated
J1	Landward contour of refraction zone
J2	Seaward contour of refraction zone
J1REF	Landward J values of boundary of refraction zone
J2REF	Seaward J values of boundary of refraction zone
MMAX	The number of shore-perpendicular structures to be simulated (present maximum of 16)
NITER	The counterindex in the refraction routine
NTIME	The counterindex in the time simulation "DO" loop
NTIMES	The number of iterations of time-step, DELT, for which a particular wave is simulated
NUNIV	The total number of time-steps simulated at any time
PI	The value of π = 3.141592654
POROS	The porosity of the sediment
QX	The longshore sediment transport rate at a specific location
QXTOT	The total alongshore sediment transport rate due to the height and angle of the initial breaking wave
QY	The onshore-offshore sediment transport rate at a specific location
R	See equations (36) and (37)

REPOSE	The angle of repose of the sediment
RHO	The mass density of seawater
RHOND	The dimensionless distance from the tip of structure where diffraction is initiated
RHOS	The mass density of sediment
RK	The wave number
S3	See equations (36) and (37)
SFACE	The slope of the shoreface
SJETTY	The length of the shore-perpendicular structure (from the base line)
SIGMA	The wave radian frequency
T	The wave period
TAU	The dissipative interface parameter
THETA	The wave angle throughout the wave field
THEATO	The wave angle at the tip of the structure
TKSI	The longshore sediment transport rate coefficient
TWOPI	Twice the value of π
U	See equations (36) and (37)
UCRIT	The critical velocity required to move the sediment according to the Sheid's diagram
V	See equations (36) and (37)
WDEPTH	The depth of water in meters to which the input wave conditions are to be transformed
WEQ	The equilibrium profile distance between contours as defined by the stepped profile
XCOOR	The x-coordinate where the wave field is to be calculated. Together with YCOOR, they determine whether the position is within or beyond the diffraction shadow zone
XDISTN	The location of the structure along the shoreline in feet
Y	The distance offshore to the contours

YCOOR	The y-coordinate where the wave field is to be calculated. Together with XCOOR, they determine whether the position is within or beyond the diffraction shadow zone
YDISS	The value of y after the use of the dissipative interface
YOLD	The previous value of y
YZERO	The berm contour location
Z1	See equation (37)
Z2	See equation (37)

APPENDIX B

PROGRAM LISTING

```
C* *********** PROGRAM IMPLICIT SEDTRAN
C*THIS PROGRAM IS SET-UP TO HANDLE MULTIPLE GROINS(M<=10)
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/BB/WEQ(60,20)
      COMMON/B/ THETA(60,20),QXTOT(60), OLDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/N USED/JUSE,T,CO,CGEN,CGGEN,ANGGEN,DX,BERM,THETAO(10),MMAX
      COMMON/D/SIGMA,G,ELO,JMAX,IMAX,PI,TWOPI,PIO2,HGEN,IJET(10),SJETTY
      COMMON/F/ADEAN,REPOSE,DIAM
      COMMON/AAA/DELT,NTIMES
      COMMON/COUNT/NUNIV
      COMMON/EXPL/QYEXP(60,20),YIMP(60,20)
      DIMENSION CHANGE(20),HC(10),TC(10)
      DIMENSION YORIG(60,20),YZEROO(60),SANGLE(20)
      NUNIV=0
      JMAX=8
      JUSE=JMAX+2
      IMAX=50
      PI=3.141592654
      TWOPI=PI*2.
      PIO2=PI/2.0
      REPOSE=32.*TWOPI/360.
      WRITE(6,732)
  732 FORMAT('****************************************************')
      WRITE(6,733)
  733 FORMAT(2X,'TO WHAT DEPTH ARE THE WAVES TO BE TRANSFORMED')
C*WDEPTH MUST BE A DEPTH BEYOND THE END OF THE STRUCT, PREFERABLY AT
C**DEEP(JMAX) OR GREATER(OR ELSE SNELL'S LAW OR SHOAL COULD BLOWUP IN
C***DEEPER WATER. IT'S IN METERS HERE!
      READ(5,770)   WDEPTH
  770 FORMAT(10X,F10.3)
      WDEPTH=WDEPTH*3.28084
      WRITE(6,762)   WDEPTH
  762 FORMAT(2X,"THE DEPTH (IN FT) WAVES TRANSFORMED TO, WDEPTH= "
     *  F10.3)
      WRITE(6,732)
      WRITE(6,777)
  777 FORMAT(2X,"ITS TIME FOR SJETTY, BERM, SFACE, AND DIAM",/)
C*SJETTY MUST BE MUCH LESS THAN Y(I,JMAX).
      READ(5,776)   SJETTY,BERM,SFACE,DIAM
  776 FORMAT(2F10.3,F10.4,F10.3)
      WRITE(6,761)   SJETTY
  761 FORMAT(2X,'THE LENGTH OF THE STRUCTURE, SJETTY= ',F10.3)
      WRITE(6,740)   BERM
  740 FORMAT(2X,'THE HEIGHT OF THE BERM, BERM=  ,F10.3)
      WRITE(6,739)SFACE
  739 FORMAT(2X,'THE SLOPE OF THE BEACH FACE, SFACE= ',F10.4)
      WRITE(6,738)   DIAM
  738 FORMAT(2X,'THE SEDIMENT DIAMETER, DIAM= ',F10.3)
      WRITE(6,732)
  780 FORMAT(2X,'SUPPLY MMAX( THE NO. OF GROINS) AND THEIR I-LOC',/)
      UCRIT=16.3*SQRT(DIAM*0.00328)
C*THE NO. OF MULTIPLE GROINS,MMAX MUST BE GIVEN THEIR X LOCATIONS.
      READ(5,779)   MMAX
  779 FORMAT(I3)
      DO 760 M=1,MMAX
C*IJET REPS LESSER I-VALUE ADJACENT TO STRUCTURE.
  760 READ(5,779)   IJET(M)
      WRITE(6,759)  (M,IJET(M),M=1,MMAX)
  759 FORMAT(2X,'THE NUMBER',I5,' GROIN IS LOCATED AT GRID ,I5)
      WRITE(6,732)
C*CONVERT TO RADIANS
C*FIRST MUST GIVE Y COORS POSITIONS AND DEPTHS
C*FIRST, MUST SET UP ALL OF THE DEEP-VALUES
      WRITE(6,773)
  773 FORMAT(2X,"NOW ENTER THE VALUE OF ADEAN")
      READ(5,774)ADEAN
  774 FORMAT(F10.4)
      WRITE(6,749)   ADEAN
  749 FORMAT(2X,'THE VALUE OF ADEAN= ',F10.4,'  IN THE EQ. H=AY**2/3')
      WRITE(6,732)
```

```
      WRITE(6,772)
  772 FORMAT(2X,"READ IN THE SPACE STEP,TIMESTEP",/)
      READ(5,775)   DX,DELT
  775 FORMAT(2(F10.3))
      WRITE(6,737)   DX
  737 FORMAT(2X,'THE VALUE OF THE LONGSHORE SPACE-STEP, DX= ',F10.3)
      WRITE(6,736)   DELT
  736 FORMAT(2X,'THE TIME-STEP IN SECONDS, DELT= ',F10.3)
      DATA CHANGE/1.,2.,3.,5.,7.,11.,14.,17.,25.,32.808,10*0.0/
      DO 220 J=1,JMAX+2
      DO 220 I=1,IMAX
  220 DEEP(I,J)=CHANGE(J)
      DATA(HC(I),I=1,8)/1.87,0.5,0.35,.25,.21,.20,.19,.19/
      DATA(TC(I),I=1,8)/2.,3.,4.,6.,8.,10.,12.,14./
      DO 200 J=1,JMAX+2
      DO 200 I=1,IMAX
  200 Y(I,J+1)=(0.5*(DEEP(I,J+1)+DEEP(I,J))/ADEAN)**1.5+Y(I,1)
      WRITE(6,732)
C*************************
C*WE WILL ALWAYS REQUIRE Y(I,JMAX+2) TO COMPUTE DY AND YBAR.
C*WE WILL ALWAYS REQUIRE DEEP(I,JMAX+2) TO COMP SEDIMENT TRANSPORT
C*************************
      WRITE(6,734)
  734 FORMAT(2X,'THE BOUNDARY Y-VALUES, I=1,IMAX ARE AS FOLLOWS',/)
      WRITE(6,801)   (Y(1,J),J=1,JMAX+2)
      WRITE(6,801)   (Y(IMAX,J),J=1,JMAX+2)
      WRITE(6,732)
      WRITE(6,735)
  735 FORMAT(/,2X,'THE DEPTHS BETWEEN CONTOURS ARE AS FOLLOWS',/)
      WRITE(6,801)   (DEEP(1,J),J=1,JMAX+2)
      WRITE(6,732)
 801  FORMAT(2X,10(F8.2))
      DO 2 I=1,IMAX
  2   YZERO(I)=Y(I,1)-(BERM/SFACE)
C*WILL COMPUTE THE EQUIL WIDTH BETWEEN CONTOURS, HERE.
      DO 1 I=1,IMAX
      WEQ(I,1)=Y(I,1)-YZERO(I)
      DO 1 J=1,JMAX
      IF(J.NE.1)   GO TO 32
      YTEMP1=0.0
      GO TO 33
  32  YTEMP1=((0.5*(DEEP(I,J-1)+DEEP(I,J)))/ADEAN)**1.5
  33  YTEMP2=((0.5*(DEEP(I,J)+DEEP(I,J+1)))/ADEAN)**1.5
      WEQ(I,J+1)=YTEMP2-YTEMP1
   1  CONTINUE
C*LET'S STORE THE ORIG VALUES TO COMPUTE VOL CHANGES OVER CONTOURS,LATER
      DO 796 I=1,IMAX+1
      YZERO0(I)=YZERO(I)
      DO 796 J=1,JMAX+2
  796 YORIG(I,J)=Y(I,J)
C******************************************************************
C*READ THE DISK FILE AND TRANSFORM PARAMETERS INTO MY UNITS.
C********************!!!!!!!!!!!!!!!!!!!!! !!!!!!!!!!!!*******************
C*ALL ADJUSTMENTS TO WAVE ANGLE,HEIGHT,CELERITY,GROUP VEL, WILL BE MADE
C**HERE, AND THRUOUT THE REST  OF THE PROG, THEY WILL BE AS IF OCCURRED
C***AT WDEPTH!
  798 READ(5,799,END=1000)   HS,T,ALPWIS
  799 FORMAT(10X,3F6.1)
      NTIMES=1
      NCHECK=NUNIV+NTIMES
      HGEN=0.707107*HS
      SIGMA=TWOPI/T
      G=32.17
      CO=G*T/TWOPI
      ELO=CO*T
      IF(T.LE.2.0)   GO TO 797
      HCC=0.23
      DO 444 I=2,7
      T2=TC(I)
      IF(T.GT.T2)   GO TO 444
      T1=TC(I-1)
      DELTAT=T2-T1
```

```
      DT=(T-T1)/DELTAT
      DTT=(T2-T)/DELT
      HCC=HC(I)*DT+HC(I-1)*DTT
      GO TO 446
  444 CONTINUE
  446 CONTINUE
      IF(HGEN.LT.HCC)   GO TO 797
      ANGGEN=ALPWIS*TWOPI/360
C**************************************************************************
      CALL WVNUM(WDEPTH,T,DUMKK)
C*ANGGEN,HGEN,CGEN,CGGEN REPRESENT THE WAVE ANGLE,HEIGHT,CELERITY AND
C**GROUP VEL(RESPECT ) OF THE SPECIFIED WAVE INPUT AT A DEPTH, WDEPTH
      CALL WVNUM(11.0,T,DUMKKK)
      C11=TWOPI/(T*DUMKKK)
      CG11=0 5*C11*(1.+(2.*DUMKKK*11 0/SINH(2 *DUMKKK*11 0)))
      CGEN=TWOPI/(T*DUMKK)
      CGGEN=0 5*CGEN*(1 +(2 *DUMKK*WDEPTH/SINH(2.*DUMKK*WDEPTH)))
      CALL TRANS
  797 IF(NCHECK NE NUNIV)    NUNIV=NCHECK
  709 GO TO 798
 1000 CONTINUE
      STOP
      END
C**************************************************************************
      SUBROUTINE TRANS
C*THIS SUBROUTINE WILL COMPUTE SEDIMENT TRANSPORT
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/BB/WEQ(60,20)
      COMMON/B/ THETA(60,20),QXTOT(60), OLDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/N USED/JUSE,T,CO,CGEN,CGGEN,ANGGEN,DX,BERM,THETAO(10),MMAX
      COMMON/D/SIGMA,G,ELO,JMAX,IMAX,PI,TWOPI,PIO2,HGEN,IJET(10),SJETTY
      COMMON/E/RHO,RHOS,POROS,CONST,TKSI
      COMMON/F/ADEAN,REPOSE,DIAM
      COMMON/G/IBREAK(60),HNONBR(20)
      COMMON/P/HBQ(60),DEEPB(60)
      COMMON/ZZZ/NTIME
      COMMON/AAA/DELT,NTIMES
      COMMON/COUNT/NUNIV
      DIMENSION YOLD(60,20),R(60,20),S(60,20),HC(60,20),QY(60,20),YDISS(
     *   60,20)
      DIMENSION RHS1(60,20),S3(60,20),THETAB(60,20),ANGLOC(60,20)
      DIMENSION DISTR(60,20),AWARE(60,20
C**************************************************************************
C**************************************************************************
C***************  NOTE    : SIZE OF ABAND AND XL HAVE TO BE CHANGED
C***************            ACCORDING TO JMAX+1+JMAX AND JMAX+1,RESPECT
C***************            CHANGE REQ'D AT 7040 AND 18650
C**************************************************************************
     *   ),BMATRX(432),ABAND(432,19),QX(60,20),XL(432,10),CONST6(60,20)
      COMMON/MP/ RKB(60),HBI(60),DEEPBI(60)
      COMMON/EXPL/QYEXP(60,20),YIMP(60,20)
      DIMENSION SANGLE(20)
C*LET'S ZERO-OUT ALL OF THE DIMENSIONED MATRICES.
      DO 1000 J=1,JMAX+2
      SANGLE(J)=0.0
      DO 1000 I=1,IMAX+2
      YOLD(I,J)=0.0
      R(I,J)=0.0
      S(I,J)=0.0
      HC(I,J)=0.0
      QY(I,J)=0.0
      YDISS(I,J)=0.0
      RHS1(I,J)=0.0
      S3(I,J)=0.0
      THETAB(I,J)=0.0
      ANGLOC(I,J)=0.0
      DISTR(I,J)=0 0
      AWARE(I,J)=0 0
      QX(I,J)=0.0
      CONST6(I,J)=0 0
```

```
  1000 CONTINUE
       RHO=1 99
       RHOS=5 14
       POROS=0 40
       CONST=0 77
       CAPPA=0.78
       TAU=0 25
       TKS1=(CONST*RHO*SQRT(G))/((RHOS-RHO)*(1.0-POROS)*16.0*SQRT(CAPPA)
C*  QX(I,J) IS THE TRANSPORT BETWEEN THE (I,J+1) AND (I,J) CONTOURS.
C*THE 'DO 1 LOOP' SIMULATES TIME---TIME=DELT*NTIMES.
       COFF=0.00001
       GAMMA=RHO*G
       DO 1 NTIME=1,NTIMES
       NUNIV=NUNIV+1
C*THE MATRICES ABAND AND BMATRX  MUST BE "ZEROED OUT"
       K=0
       DO 26 I=2,IMAX-1
       DO 26 J=1,JMAX
       K=K+1
       BMATRX(K)=0 0
       DO 26 L=1,JMAX+1+JMAX
    26 ABAND(K,L)=0 0
       XNTIME=1 0*(NTIME)
       CALL PREDIF
C*SMOOTHING OF THE WAVE ANGLE,THETA, IS RE'D TO ACCT FOR DIFF EFFECTS.
       CALL SMOOTH(THETA,IMAX,JMAX,IJET,SJETTY,MMAX,Y)
       CALL QTRAN
C*FIRST THE LONGSHORE SEDIMENT TRANSPORT WILL BE DISTRIBUTED
C****ACROSS THE SURF ZONE....
       CC=1 25
C***QX(I,J) WILL BE DETERMINED BY SUBTRACTING FROM THE INTEGRAL
C**OF QX FROM DEEP(I,J-1) TO INFINITY, THE INTEGRAL OF QX FROM DEEP(I,J)
C***TO INFINITY. IN THIS WAY THE SEDIMENT TRANS FROM JMAX OUT GETS
C***INCLUDED IN QX(I,JMAX). TO INCLUDE THE SWASH TRANS, WHEN J=1
C*WE WILL SUBTRACT FROM 2 TO INFINITY FROM 1.0
C*LOOP FOR VALUES WHICH ARE HELD CONST AND STORED.
       THETAB(1,1)=0 5*(THETA(1,1)+0.0)
       R(1,1)=0.5/(DX*(DEEP(1,1)+BERM/2 ))
       DO 290 I=2,IMAX
       R(I,1)=0.5/(DX*(DEEP(I,1)+BERM/2.))
C*     THETAB(I,1)=0.25*(THETA(I,1)+THETA(I-1,1)+0.+0 )
       THETAB(I,1)=0.5*(THETA(I,1)+THETA(I-1,1))
C*NO NEED TO COMPUTE PROP ANGLE AT STRUCTS BECAUSE QX =0.0 AT IJET(M)+1
       ANGLOC(I,1)=ATAN((Y(I,1)-Y(I-1,1))/DX)
C*HBQ(IJET(M)+1) IS PROPERLY SET IN THE SUBROUTINE QTRAN.
       DISTR(I,1)=1.0-EXP(-((DEEP(I,1)**1.5+HBQ(I)*ADEAN**1 5)/
      *   (CC*DEEPB(I)**1 5))**3)
       DISTR(I,1)=DISTR(I,1)*TKSI*HBQ(I)**2.5
       DO 290 J=2,JMAX
       R(I,J)=0 5/(DX*(DEEP(I,J)-DEEP(I,J-1)))
       THETAB(I,J)=0 5*(THETA(I,J)+THETA(I 1,J))
       ANGLOC(I,J)=ATAN((Y(I,J)-Y(I-1,J))/DX)
       DISTR(I,J)=EXP(-((DEEP(I,J-1)**1.5+HBQ(I)*ADEAN**1.5)/(CC*DEEPB(I)
      *    **1 5))**3)-EXP(-((DEEP(I,J)**1.5+HBQ(I)*ADEAN**1.5)/(CC*
      *   DEEPB(I)**1 5))**3)
       DISTR(I,J)=DISTR(I,J)*TKSI*HBQ(I)**2.5
   290 CONTINUE
       DO 301 J=1,JMAX
       DO 301 I=2,IMAX
       AWARE(I,J)=DELT*R(I,J)*(QX(I,J)-QX(I+1,J)+QY(I,J)-QY(I,J+1))+Y(I,J
      *    )
       S1=2.*SIN(THETAB(I,J))*COS(THETAB(I,J))*(-1.+2.*(COS(
      *   ANGLOC(I,J)))**2)
       S2=COS(2.*THETAB(I,J))*COS(ANGLOC(I,J))/(SQRT(DX**2+
      *    (Y(I,J)-Y(I-1,J))**2))
       S3(I,J)=S2*DISTR(I,J)
       IF(SJETTY EQ 0.0)   GO TO 302
       DO 325 M=1,MMAX
       IF(I NE.IJET(M)+1)   GO TO 325
       IF(THETA0(M) GE 0.0)   ISIDE=IJET(M)
       IF(THETA0(M) LT 0.0)   ISIDE=IJET(M)+1
       YSEA=0.5*(Y(ISIDE,J)+Y(ISIDE,J+1))
       YSHORE=0.5*(Y(ISIDE,J)+Y(ISIDE,J-1))
```

```
      IF(YSEA GT SJETTY AND.YSHORE.GT SJETTY)   GO TO 302
      IF(YSEA GT SJETTY AND.YSHORE LE.SJETTY)   GO TO 298
C*BECAUSE A NO FLOW B C  IS USED ALONG THE STRUCT, NO ATTN WAS PAID
C**TO GETTING PROPER VALUES OF ANGLOC, THETAB,DISTR,ETC.
      S3(I,J)=0 0
      DISTR(I,J)=0 0
      GO TO 302
  325 CONTINUE
      GO TO 302
C***ABOVE, ALL PARAMETERS(I E .S1,S2,S3,THETAB,DISTR,ANGLOC)
C***ARE COMPUTED AS IF THE STRUCT IS NOT THERE  THE B C  AT THE
C***STRUCT TIP ASSUMES QX COMPUTED AS IF NO STRUCT PRESENT AND THEN
C***BYPASSES ACCORDING TO "RATIO".
  298 RATIO=(YSEA-SJETTY)/(YSEA-YSHORE)
      S3(I,J)=S3(I,J)*RATIO
      DISTR(I,J)=DISTR(I,J)*RATIO
  302 RHS1(I,J)=DISTR(I,J)*S1-S3(I,J)*(Y(I,J)-Y(I-1,J))
 301  CONTINUE
      CALL BREAK(IMAX,JMAX)
C*TO DETERMINE DECAY OF CONST6(I,J),NEED WAVE NO. AT BREAKING.
      DO 754 I=1,IMAX+1
  754 CALL WVNUM(DEEPBI(I),T,RKB(I))
C*USING SHIELD'S DIAG,Y AXIS=0 05 & (TAUO=RHO*C*U**2),GET UCRIT(FT/SEC)
      UCRIT=16 3*SQRT(DIAM*.00328)
      DO 750 I=1,IMAX+1
      CONST6(I,1)=COFF*DX
      DO 750 J=2,JMAX+2
C*CONST6(I,J) GOES W/ QY(I,J) WHICH IS ASSOC W/ DEEP(I,J-1)
      IF(DEEP(I,J-1) LE DEEPBI(I))   GO TO 751
C*HERE. MUST CAUSE COFF TO DECAY (WE'RE BEYOND SURF ZONE)
      UMAXB=HBI(I)*G*T*RKB(I)/(2.*TWOPI*COSH(RKB(I)*DEEPBI(I)))
      UMAX=H(I,J-1)*G*T*RK(I,J-1)/(2 *TWOPI*COSH(RK(I,J-1)*DEEP(I,J-1)))
      IF(UCRIT LT UMAX AND.UCRIT.LT.UMAXB)   GO TO 749
      CONST6(I,J)=0.0
      GO TO 750
  749 TOP=0 01*H(I,J-1)**3*SIGMA**3/(SINH(RK(I,J-1)*DEEP(I,J-1))**3)
      BOT=DEEP(I,J-1)*(0 625*TWOPI*G**1 5*0.78**2*ADEAN**1.5+
     *(0 01*HBI(I)**3*SIGMA**3/(DEEPBI(I)*(SINH(RKB(I)*DEEPBI(I)))**3)))
      CONST6(I,J)=DX*COFF*TOP/BOT
      GO TO 750
  751 CONST6(I,J)=COFF*DX
  750 CONTINUE
      K=0
C**PUT INTO BANDED FORM USING THE ALGORITHM  A(M,N)->B(M,NN) WHERE
C***NN=KB+1-M+N(KB IS THE NUMBER OF LOWER CODIAGONALS(=JMAX,HERE)).
      DO 304 I=2,IMAX-1
      DO 304 J=1,JMAX
      K=K+1
      AWARE(I,J)=AWARE(I,J)+DELT*RHS1(I,J)*R(I,J)-DELT*R(I,J)*RHS1(I+1,J
     *   )+DELT*R(I,J)*CONST6(I,J)*WEQ(I,J)-DELT*R(I,J)*CONST6(I,J+1)*
     *   WEQ(I,J+1)
      YDUM=YZERO(I)
      IF(J NE 1)   YDUM=Y(I,J-1)
      AWARE(I,J)=AWARE(I,J)+DELT*R(I,J)*CONST6(I,J)*0.5*(YDUM-Y(I,J))
     *    -DELT*R(I,J)*CONST6(I,J+1)*0.5*(Y(I,J)-Y(I,J+1))
      U=DELT*R(I,J)*S3(I,J)
      V=DELT*R(I,J)*S3(I+1,J)
      Z1=DELT*R(I,J)*CONST6(I,J)*0.5
      Z2=DELT*R(I,J)*CONST6(I,J+1)*0.5
C*NOW WILL SET UP THE MATRICES ABAND AND BMATRX.
      ABAND(K,JMAX+1)=1.0+U+V+Z1+Z2
      IF(I NE 2)   GO TO 305
      AWARE(I,J)=AWARE(I,J)+U*Y(I-1,J)
      GO TO 310
  305 ABAND(K,1)=-U
  310 IF(I.NE.IMAX-1)   GO TO 306
      AWARE(I,J)=AWARE(I,J)+V*Y(IMAX,J)
      GO TO 311
  306 ABAND(K,JMAX+1+JMAX)=-V
  311 IF(J NE 1)   GO TO 307
      ABAND(K,JMAX+1)=ABAND(K,JMAX+1)-Z1
      AWARE(I,1)=AWARE(I,1)+Z1*(YZERO(I)-Y(I,1))
      GO TO 312
```

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  307 ABAND(K,JMAX)=-Z1
  312 IF(J.NE.JMAX)   GO TO 308
      AWARE(I,J)=AWARE(I,J)+Z2*Y(I,JMAX+1)
      GO TO 309
  308 ABAND(K,JMAX+2)=-Z2
  309 BMATRX(K)=AWARE(I,J)
  304 CONTINUE
      KMAX=K
C**CALL IMSL ROUTINE LEQT1B TO SOLVE THE BANDED MATRIX.
      CALL LEQT1B(ABAND,KMAX,JMAX,JMAX,432,BMATRX,1,432,0,XL,IER)
C*NOW, GIVE Y'S THEIR NEW VALUES STORING OLD VALUES IN YOLD.
      K=0
      DO 315 I=2,IMAX-1
      YOLD(I,JMAX+1)=Y(I,JMAX+1)
      DO 315 J= 1,JMAX
      K=K+1
      YOLD(I,J)=Y(I,J)
      Y(I,J)=BMATRX(K)
  315 CONTINUE
      DO 320 J=1,JMAX+3
      YOLD(1,J)=Y(1,J)
  320 YOLD(IMAX,J)=Y(IMAX,J)
C*WILL USE ABBOTT'S DISSIPATIVE INTERFACE TO RID HIGH FREQ OSCILLATIONS
      DO 650 J=1,JMAX
      DO 650 I=2,IMAX-1
      YDISS(I,J)=TAU*Y(I-1,J)+(1.-2.*TAU)*Y(I,J)+TAU*Y(I+1,J)
      IF(SJETTY.EQ.0.0)   GO TO 650
      DO 649 M=1,MMAX
      IF(I.NE.IJET(M).AND.I.NE.IJET(M)+1)   GO TO 649
      IF(Y(IJET(M),J).GT.SJETTY.OR.Y(IJET(M)+1,J).GT.SJETTY)GO TO 649
      IF(I.EQ.IJET(M))YDISS(I,J)=TAU*Y(I-1,J)+(1.-TAU)*Y(I,J)
      IF(I.EQ.(IJET(M)+1))YDISS(I,J)=TAU*Y(I+1,J)+(1.-TAU)*Y(I,J)
  649 CONTINUE
  650 CONTINUE
      DO 651 J=1,JMAX
      DO 651 I=2,IMAX-1
  651 Y(I,J)=YDISS(I,J)
C*THIS LOOP WILL STORE THE IMPLICIT Y VALUES REQ'D TO COMP QY&QX
      DO 40 I=1,IMAX+1
      DO 40 J=1,JMAX+3
   40 YIMP(I,J)=Y(I,J)
C*THIS LOOP WILL EXPLICITLY MOVE CONTOURS SEAWARD IF REPOSE EXCEEDED.
      KOUNT=0
      SLOPEM=TAN(0.9*REPOSE)
      DO 48 I=1,IMAX
   43 KOUNT=KOUNT+1
      IF(KOUNT.GT.50000)   GO TO 41
C*LET US COMPUTE ALL THE SLOPES(PSLOP) FOR EACH CHANGE IN DEPTH.
      DO 47 J=1,JMAX+1
      DUM=-BERM/2.0
      IF(J.NE.1)   DUM=DEEP(I,J-1)
      DELH=0.5*(DEEP(I,J+1)+DEEP(I,J))-0.5*(DEEP(I,J)+DUM)
      PSLOP=DELH/(Y(I,J+1)-Y(I,J))
   47 SANGLE(J)=ATAN(PSLOP)
C*FIND THE MIN NEG SLOPE ANGLE OR THEN THE POS SLOPE>REPOSE OR FORGET IT
      ASLOPM=-1.0E50
      ASLOPP=0.0
      DO 46 J=1,JMAX+1
      IF(SANGLE(J).GT.0.0)   GO TO 45
      IF(SANGLE(J).GT.ASLOPM)ASLOPM=SANGLE(J)
      IF(ASLOPM.EQ.SANGLE(J))   JM=J
      GO TO 46
   45 IF(SANGLE(J).GT.REPOSE.AND.SANGLE(J).GT.ASLOPP)ASLOPP=SANGLE(J)
      IF(ASLOPP.EQ.SANGLE(J))   JP=J
   46 CONTINUE
      IF(ASLOPM.EQ.-1.0E50.AND.ASLOPP.EQ.0.0)   GO TO 42
      IF(ASLOPM.EQ.-1.0E50)   GO TO 44
      DUM=-BERM/2.
      IF(JM.NE.1)   DUM=DEEP(I,JM-1)
      ALTER=((0.5/SLOPEM*(DEEP(I,JM+1)-DUM))-(Y(I,JM+1)-Y(I,JM)))/
     *   (1.0+((DEEP(I,JM+1)-DEEP(I,JM))/(DEEP(I,JM)-DUM)))
      Y(I,JM+1)=Y(I,JM+1)+ALTER
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      Y(I,JM)=Y(I,JM)-(ALTER*(DEEP(I,JM+1)-DEEP(I,JM))/(DEEP(I,JM)-DUM))
      QYEXP(I,JM+1)=QYEXP(I,JM+1)+DX/DELT*ALTER*(DEEP(I,JM+1)-DEEP(I,JM)
     *   )
      GO TO 43
   44 CONTINUE
      DUM=-BERM/2.
      IF(JP.NE.1)   DUM=DEEP(I,JP-1)
      ALTER=((0.5/SLOPEM*(DEEP(I,JP+1)-DUM))-(Y(I,JP+1)-Y(I,JP)))/
     *   (1.0+((DEEP(I,JP+1)-DEEP(I,JP))/(DEEP(I,JP)-DUM)))
      Y(I,JP+1)=Y(I,JP+1)+ALTER
      Y(I,JP)=Y(I,JP)-(ALTER*(DEEP(I,JP+1)-DEEP(I,JP))/(DEEP(I,JP)-DUM))
      QYEXP(I,JP+1)=QYEXP(I,JP+1)+DX/DELT*ALTER*(DEEP(I,JP+1)-DEEP(I,JP)
     *   )
      GO TO 43
   42 WEQ(I,JMAX+1)=Y(I,JMAX+1)-Y(I,JMAX)
   48 CONTINUE
C*IF WE GET SENT HERE, LOOP 444 WILL CATCH THE CROSSED CONTOURS.
   41 CONTINUE
C*NOW WE CAN COMPUTE QX'S AND QY'S!
      DO 318 I=2,IMAX
C*ALL IMPLIC AND EXPLIC MOVEMENT OF YZERO WILL BE TAKEN CARE OF HERE
      QY(I,1)=-BERM*DX*(Y(I,1)-YOLD(I,1))/DELT
      YZERO(I)=YZERO(I)+(Y(I,1)-YOLD(I,1))
  319 DO 318 J=1,JMAX
      QX(I,J)=RHS1(I,J)-S3(I,J)*YIMP(I,J)+S3(I,J)*YIMP(I-1,J)
  318 QY(I,J+1)=CONST6(I,J+1)*(0.5*(YIMP(I,J)+YOLD(I,J)-YIMP(I,J+1)
     *   -YOLD(I,J+1))+WEQ(I,J+1))
      DO 323 J=1,JMAX
      QX(1,J)=QX(2,J)
  323 QX(IMAX+1,J)=QX(IMAX,J)
C*TOTAL QYS WILL BE COMP FROM IMPLIC AND EXPLIC VALUES.THEN ZERO QYEXP
      DO 39 I=1,IMAX+1
      DO 39 J=1,JMAX+3
       QY(I,J)=QY(I,J)+QYEXP(I,J)
   39 QYEXP(I,J)=0.0
C*THIS CHECK WILL BOMB THINGS OUT IF CONTOURS HAVE CROSSED.
      DO 444 II=1,IMAX
      DO 444 JJ=1,JMAX
C*IF CONTOURS CROSS AT ANY TIME WANT PROGRAM TO STOP!
      IF(Y(II,JJ).LT.Y(II,JJ+1))   GO TO 444
      WRITE(6,103)
      WRITE(6,*/)   NUNIV
      DO 150 J=1,JMAX
  150   WRITE(6,100)   (QX(I,J),I=1,IMAX)
      DO 151 J=1,JMAX
  151   WRITE(6,101)   (QY(I,J),I=1,IMAX)
      DO 152 J=1,JMAX
  152   WRITE(6,100)   (Y(I,J),I=1,IMAX)
  103 FORMAT(2X,'THE CONTOURS HAVE CROSSED AND SOMETHING IS WRONG',/)
      DO 19 J=1,JMAX
   19 WRITE(6,100) (YOLD(I,J),I=1,IMAX)
      GO TO 445
  444 CONTINUE
      WRITE(6,*/)   NUNIV
C*THE FOLLOWING STATEMENT DETERMINES AT WHAT FREQ EVERYTHING IS WRITTEN!
      IF(MOD(NUNIV,10).NE.0)   GO TO 1
C*LET'S WRITE ALL OF IT OUT.
      WRITE(6,926)   NUNIV
  926 FORMAT(2X,'THE TOTAL ELAPSED NUMBER OF TIME-STEPS. NUNIV= ',I5,/)
 800  FORMAT(2X,14(F8.4))
C*     DO 900 I=1,IMAX
C*900  WRITE(6,800)   (THETA(I,J),J=1,JMAX)
C*     DO 903 J=1,JMAX+1
C*903  WRITE(6,801)   DEEP(1,J)
C*     DO 906 I=1,IMAX
C*906  WRITE(6,800)   (H(I,J),J=1,JMAX)
C*    DO 755 J=1,JMAX
C*755 WRITE(6,800) (CONST6(I,J),I=1,IMAX)
 801  FORMAT(2X,14(F8.2))
      WRITE(6,107)
  107 FORMAT(/,2X,'THE LONGSHORE TRANSPORTS,QX, FOLLOW')
      DO 15 J=1,JMAX
   15   WRITE(6,100)   (QX(I,J),I=1,IMAX)
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      WRITE(6,108)
  108 FORMAT(' ',2X,'THE ON OFFSHORE TRANSPORTS  QY  FOLLOW')
      DO 17 J=1,JMAX
   17 WRITE(6,101) (QY(I,J),I=1,IMAX)
      WRITE(6,109)
  109 FORMAT(' ',2X,'THE NEW CONTOUR VALUES  Y  FOLLOW')
      DO 18 J=1,JMAX
   18 WRITE(6,100) (Y(I,J),I=1,IMAX)
  100 FORMAT(2X,11F8.1)
  101 FORMAT(2X,11F8.4)
    1 CONTINUE
      RETURN
      GO TO 446
  445 STOP
  446 CONTINUE
      END
C*****************************************************************
      SUBROUTINE QTRAN
C*THIS SUBROUTINE CALCS THE BREAKER HEIGHT  FOR EACH
C*OF THE I GRID LINES. METHOD: FINDS Y LOCATIONS BEFORE AND AFTER
C*BREAKING HAS OCCURRED BY  REFRAC .  THEN USES SHOALING TO GET THE
C*HBQ SNELL'S LAW IS USED FOR REFRACTION OVER THE SHORT DIST TO BREAKING
C* QY(I,J) IS THE TRANS BETWEEN(I-1,J) AND (I,J) AT THE BLOCKCENT
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/B/ THETA(60,20),QYTOT(60), DIDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/N/USED,JUST,T,CO,CGEN,CGOEN,ANGGEN,DX,BERM,THETAO(10),MMAX
      COMMON/D/SIGMA,G,FLO,JMAX,IMAX,PI,TWOPI,PIO2,HOEN,IJETT(10),SJETTY
      COMMON/G/IBREAK(60),INONBR(20)
      COMMON/E/ RHO,RHOS,POROS,CONST,TKST
      COMMON/P/HBQ(60),DEEPB(60)
      CAPPA=0.78
      DO 1 I=2,IMAX
      DO 2 JJ=1,JMAX
      J=JMAX-JJ+1
      HDUM=(H(I,J)+H(I-1,J))*0.5
      HBDUM=(HB(I,J)+HB(I-1,J))*0.5
C**CAN ONLY USE COND ON ONE SIDE OF STRUCT  CAN'T AVG HERE!
      IF(SJETTY.EQ.0.0)  GO TO 4
      DO 4 M=1,MMAX
      IF(I.NE.IJETT(M)+1)  GO TO 4
      IF(THETAO(M).GE.0.0)  ISIDE=IJETT(M)
      IF(THETAO(M).LT.0.0)  ISIDE=IJETT(M)+1
C****B.C. AT STRUCT TIP ASSUMES QY COMP AS IF NO STRUCT IS PRESENT
      YSEA=0.5*(Y(ISIDE,J)+Y(ISIDE,J+1))
      IF(YSEA.GT.SJETTY)  GO TO 3
      HDUM=H(ISIDE,J)
      HBDUM=HB(ISIDE,J)
      GO TO 3
    4 CONTINUE
    3 IF(HDUM.LT.HBDUM)  GO TO 2
      DEEPB(I)=((0.5*(H(I,J+1)+H(I-1,J+1)))*((0.5*(DEEP(I,J+1)
     *   +DEEP(I-1,J+1))))**0.25)/CAPPA)**0.8
      HBQ(I)=CAPPA*DEEPB(I)
C**HBQ(I) AND DEEPB(I) WILL BE COMPUTED ACCORDING TO THE WAVE DIR
C** AT THE STRUCTURE TIP THETAO
      IF(SJETTY.EQ.0.0)  GO TO 1
      DO 6 M=1,MMAX
      IF(I.NE.IJETT(M)+1)  GO TO 6
C**THE TRANSPORTING WAVES WILL BE COMPUTED USING THE WAVE TO PROP SIDE
      IF(THETAO(M).GT.0.0)  GO TO 11
      DEEPB(I)=(H(IJETT(M)+1,J+1)*DEEP(IJETT(M)+1,J+1)**0.25/CAPPA)**0.8
      IBREAK(I)=IBREAK(IJETT(M)+1)
      GO TO 12
   11 DEEPB(I)=(H(IJETT(M),J+1)*DEEP(IJETT(M),J+1)**0.25/CAPPA)**0.8
      IBREAK(I)=IBREAK(IJETT(M))
   12 HBQ(I)=DEEPB(I)*CAPPA
      GO TO 1
    6 CONTINUE
      GO TO 1
    2 CONTINUE
    1 CONTINUE
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C*IF THE OFFSHORE WAVE HT IS ZERO, NEVER GET TO HERE
C*HOWEVER IF THE H IS SUCH THAT IT WOULD BREAK INSHORE OF Y(I,2)
C*DEEPB(I) WOULD STILL BE ZERO AND DISTR(I,J) WOULD BLOW-UP.
      DO 20 I=1,IMAX
      IF(DEEPB(I).GT.0.0)   GO TO 20
      DEEPB(I)=(H(I,1)*DEEP(I,1)**0.25/CAPPA)**0.8
      HBQ(I)=CAPPA*DEEPB(I)
   20 CONTINUE
      HBQ(1)=HBQ(2)
      HBQ(IMAX+1)=HBQ(IMAX)
      DEEPB(1)=DEEPB(2)
      DEEPB(IMAX+1)=DEEPB(IMAX)
      RETURN
      END
C*******************************************************************
      SUBROUTINE BREAK(IMAX,JMAX)
C*ROUTINE WILL DETERMINE HB AND DEEPB ON THE GRID LINES RATHER
C*  THAN BETWEEN THEM. REQ'D FOR COFF BEYOND SURF ZONE.
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/MP/ RKB(60),HBI(60),DEEPBI(60)
      CAPPA=0.78
      DO 1 I=2,IMAX
      DO 2 JJ=1,JMAX
      J=JMAX-JJ+1
      IF(H(I,J).LT.HB(I,J))   GO TO 2
      DEEPBI(I)=((H(I,J+1)*DEEP(I,J+1)**0.25)/CAPPA)**0.8
      HBI(I)=CAPPA*DEEPBI(I)
C***ONCE THE HEIGHT & DEPTH AT BREAKING ARE FOUND, GO TO NEXT GRID-LINE.
      GO TO 1
  2   CONTINUE
  1   CONTINUE
      DO 20 I=1,IMAX
      IF(DEEPBI(I).GT.0.0)   GO TO 20
      DEEPBI(I)=(H(I,1)*DEEP(I,1)**0.25/CAPPA)**0.8
      HBI(I)=CAPPA*DEEPBI(I)
   20 CONTINUE
      DEEPBI(1)=DEEPBI(2)
      DEEPBI(IMAX+1)=DEEPBI(IMAX)
      HBI(1)=HBI(2)
      HBI(IMAX+1)=HBI(IMAX)
      RETURN
      END
C*******************************************************************
      SUBROUTINE REFRAC(JBEGIN,JEND,NPTS,IBEGIN,IEND,ISTART,M)
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/B/ THETA(60,20),QXTOT(60), OLDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/N USED/JUSE,T,CO,CGEN,CGGEN,ANGGEN,DX,BERM,THETAO(10),MMAX
      COMMON/D/SIGMA,G,ELO,JMAX,IMAX,PI,TWOPI,PIO2,HGEN,IJET(10),SJETTY
      COMMON/G/IBREAK(60),HNONBR(20)
      COMMON/ZZZ/NTIME
      DIMENSION JBEGIN(60),JEND(60)
C***************                            THIS SUBROUTINE WILL DETERMINE H AND
C***************                            THETA AT THE MID PT OF Y VALUES.
C***TAU IS THE FACTOR WHICH RECOUPLES THE REFRACTION EQS.SEE ABBOTT
      TAU=0.25
C*MUST PRESCRIBE THE WAVE ANGLE AT THE OUTERMOSTCONTOUR BOX
C*SNELL'S LAW WILL BE USED TO START THINGS OFF.
C*THETA(I,J) WILL BE AT AREA'S CENTER AND WILL USE Y(I,J) IN NEG Y-DIR
C*WILL INITIALIZE ALL THETA'S USING SNELL'S LAW.
      DO 206 I=IBEGIN,IEND
C*INITIALIZE TWO J-VALUES BEYOND JMAX,IF IN REGION 1.
      IF(JEND(I).EQ.JMAX)   JINIT=2
      IF(JEND(I).NE.JMAX)   JINIT=0
      DO 206 J=JBEGIN(I),JEND(I)+JINIT
C*MUST CORRECT FOR THE CONTOUR ORIENTATION, ALPHAS.
      IF(I.NE.IBEGIN)   GO TO 960
      ALPHAS(I,J)=ATAN((0.5*(Y(I+1,J)+Y(I+1,J+1))-0.5*(Y(I,J)
     *   +Y(I,J+1)))/DX)
      GO TO 962
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  960 IF(I.NE.IEND)   GO TO 961
      ALPHAS(I,J)=ATAN((0.5*(Y(I,J)+Y(I,J+1))-0.5*(Y(I-1,J)
     *  +Y(I-1,J+1)))/DX)
      GO TO 962
  961 ALPHAS(I,J)=ATAN((0.5*(Y(I+1,J)+Y(I+1,J+1))-0.5*
     * (Y(I-1,J)+Y(I-1,J+1)))/(2.*DX))
  962 DALPHA=ANGGEN-ALPHAS(I,J)
      THETA(I,J )=ARSIN((C(I,J )/CGEN)*SIN(DALPHA))
C*MUST GET THETA WRT THE X-AXIS.
      THETA(I,J)=THETA(I,J)+ALPHAS(I,J)
  206 CONTINUE
C*NOW, WE MUST COMP THE BOUN WAVE HTS SO THE HTS CAN BE COMPUTED.
C*WILL USE THE EQ. ******   DEL DOT    (E*CG)=0.0
C*NOW WE WILL CORRECT THE HT FOR SHOALING AND REFRACTION TO THE B.C.
C*WILL ALSO INITIALIZE H'S WITH THESE EQUATIONS FOR ENTIRE ARRAY.
      DO 500 I=IBEGIN,IEND
C*INITIALIZE TWO J-VALUES BEYOND JMAX IF IN REGION 1.
      IF(JEND(I).EQ.JMAX)    JINIT=2
      IF(JEND(I).NE.JMAX)    JINIT=0
      DO 500 J=JBEGIN(I),JEND(I)+JINIT
      H(I,J)=HGEN*SQRT(CGGEN/CG(I,J))*SQRT(COS(ANGGEN)/COS(THETA(I,
     *  J)))
      IF(HB(I,J).LT.H(I,J))    H(I,J)=HB(I,J)
  500 CONTINUE
C*-----------------------------------------------------------------
C*****************************************************************
C*LET'S FILL THE DY ARRAY.
C*DY WILL BE INDEXED AS THE THETA TO WHICH WE ARE GOING.
      DO 209 I=IBEGIN,IEND
      DO 209 J=JBEGIN(I)+1,JEND(I)
      DY(I,J-1)=0.5*(Y(I,J-1)+Y(I,J))-0.5*(Y(I,J)+Y(I,J+1))
  209 CONTINUE
      NITERS=100
      DO 100 NITER=1,NITERS
      SUMANG=0.0
C*DO "60 LOOP" GOES FROM 2 TO IMAX IF ISTART =IBEGIN
C*DO "60 LOOP"  GOES FROM IMAX-1 TO 1 IF ISTART=IEND
      DO 60 II=IBEGIN,IEND
C*MUST HAVE IT SET UP SO THAT  THE KNOWN BOUNDARIES ANGLES AREN'T RECOMP
      IF(ISTART.EQ.IBEGIN)    I=II
      IF(ISTART.EQ.IBEGIN   .AND.  I.EQ.IBEGIN)    GO TO 60
      IF(ISTART.EQ.IEND)    I=IEND-II+IBEGIN
      IF(ISTART.EQ.IEND   .AND.  I.EQ.IEND)    GO TO 60
C*ADX EQUALS ACTUAL DELTA X ACROSS SPACE STEP.
C*ONLY ON BOUNDARIES WHERE FORWARD OR BACKWARD DIFFERENCING.
      IF(I.NE.IBEGIN)    GO TO 6
      ADX=DX
      IP=I+1
      IM=I
      GO TO 12
    6 IF(I.NE.IEND)    GO TO 10
      ADX=DX
      IP=I
      IM=I-1
      GO TO 12
   10 ADX=2.0*DX
      IP=I+1
      IM=I-1
   12   CONTINUE
      DO 40 J=JBEGIN(I),JEND(I)-1
C*WILL GO FROM (JMAX-1) TO 1 BECAUSE THAT'S THE DIR WAVE COMES IN FROM.
      JJ=JEND(I)-1-J+JBEGIN(I)
      OLDANG(I,JJ)=THETA(I,JJ)
C*LOCATE MIDPOINT BETWEEN TWO ADJACENT BLOCK CENTERS
C*BECAUSE THETA'S JJ-VALUE IS THE SAME AS THE FIRST SHOREWARD Y VALUE
C*MUST USE JJ, JJ+1, AND JJ+2 TO COMPUTE YBAR.
      YBAR=0.25*(Y(I,JJ)+2.0*Y(I,JJ+1)+Y(I,JJ+2))
C*LOCATE APPROPRIATE INDICES ON IP AND IM GRID LINES.
      IMINUS=-1
      IPLUS=+1
      CALL LOC(IM,JJ,JOIM,JSIM,YBAR,IMINUS)
      CALL LOC(IP,JJ,JOIP,JSIP,YBAR,IPLUS)
C*NOW USE THE CONSERVATION OF WAVES EQUATION.............
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      PART1C=RK(I,JJ+1)*SIN(THETA(I,JJ+1))
      PART2=-DY(I,JJ)/ADX
C*WILL LINEARLY INTERPOLATE TO DETERMINE RK*COS(THETA) AT I+1 AND I-1.
C*IF NO ADJ SHOREWARD PT EXISTS, PUT IN ZERO FOR TERMS IN GOV. EQ.
      IF(JSIM.NE.0)   GO TO 301
      PART3B=0.0
      GO TO 302
  301 TOPIM=RK(IM,JOIM-1)*COS(THETA(IM,JOIM-1))
      BOTIM=RK(IM,JSIM)*COS(THETA(IM,JSIM))
      TOTALB=0.5*(Y(IM,JOIM)+Y(IM,JOIM-1))-0.5*(Y(IM,JSIM+1)+Y(IM,JSIM))
      DUMB=0.5*(Y(IM,JOIM)+Y(IM,JOIM-1))-YBAR
      PART3B=((TOTALB-DUMB)*(TOPIM-BOTIM)/TOTALB)+BOTIM
  302 IF(JSIP.NE.0)   GO TO 303
      PART3A=0.0
      GO TO 304
  303 TOPIP=RK(IP,JOIP-1)*COS(THETA(IP,JOIP-1))
      BOTIP=RK(IP,JSIP)*COS(THETA(IP,JSIP))
      TOTALA=0.5*(Y(IP,JOIP)+Y(IP,JOIP-1))-0.5*(Y(IP,JSIP+1)+Y(IP,JSIP))
      DUMA=0.5*(Y(IP,JOIP)+Y(IP,JOIP-1))-YBAR
      PART3A=((TOTALA-DUMA)*(TOPIP-BOTIP)/TOTALA)+BOTIP
  304 PART3=PART3A-PART3B
C*NOW MUST FIND RK*SIN(THETA) FOR I+1 AND I-1 AT J+1
      YBARP=0.25*(Y(I,JJ+1)+2.*Y(I,JJ+2)+Y(I,JJ+3))
      CALL LOC(IM,JJ+1,JPOIM,JPSIM,YBARP,IMINUS)
      CALL LOC(IP,JJ+1,JPOIP,JPSIP,YBARP,IPLUS)
      IF(JPSIM.NE.0)   GO TO 305
      PART1B=0.0
      GO TO 306
  305 TOPM=RK(IM,JPOIM-1)*SIN(THETA(IM,JPOIM-1))
      BOTM=RK(IM,JPSIM)*SIN(THETA(IM,JPSIM))
      TOTB=0.5*(Y(IM,JPOIM)+Y(IM,JPOIM-1))-0.5*(Y(IM,JPSIM+1)+
     *   Y(IM,JPSIM))
      DUMPB=0.5*(Y(IM,JPOIM)+Y(IM,JPOIM-1))-YBARP
      PART1B=((TOTB-DUMPB)*(TOPM-BOTM)/TOTB)+BOTM
  306 IF(JPSIP.NE.0)   GO TO 307
      PART1A=0.0
      GO TO 308
  307 TOPP=RK(IP,JPOIP-1)*SIN(THETA(IP,JPOIP-1))
      BOTP=RK(IP,JPSIP)*SIN(THETA(IP,JPSIP))
      TOTA=0.5*(Y(IP,JPOIP)+Y(IP,JPOIP-1))-0.5*(Y(IP,JPSIP+1)+Y(IP,JPSIP
     *   ))
      DUMPA=0.5*(Y(IP,JPOIP)+Y(IP,JPOIP-1))-YBARP
      PART1A=((TOTA-DUMPA)*(TOPP-BOTP)/TOTA)+BOTP
  308 PART1=TAU*PART1B+(1.-2.*TAU)*PART1C+TAU*PART1A
      IF(JPSIM.EQ.0)PART1=(1.-TAU)*PART1C+TAU*PART1A
      IF(JPSIP.EQ.0)PART1=TAU*PART1B+(1.-TAU)*PART1C
      ARG=((PART1+PART2*PART3)/RK(I,JJ))
C*IF THE ROUTINE IS TO BLOWUP,USE SNELLS LAW.
      IF(ABS(ARG).LE.1.0)   GO TO 41
      ARG=(C(I,JJ)/C(I,JJ+1))*SIN(THETA(I,JJ+1))
      IF(ARG.GT.1.0)   ARG=1.0
      THETA(I,JJ)=ARSIN(ARG)
      GO TO 42
   41 THETA(I,JJ)=ARSIN(ARG)
   42 THETA(I,JJ)=0.5*(THETA(I,JJ)+OLDANG(I,JJ))
      SUMANG=SUMANG+(ABS(THETA(I,JJ)-OLDANG(I,JJ)))
  40  CONTINUE
  60  CONTINUE
C*MUST EJECT IF WE HAVE REACHED AN ACCEPTABLE ITERATION ERROR
C*IF THE SUM OF THE ABSOLUTE VALUE OF ANGLE CHANGES DURING AN ITERATION
C*      AVERAGES LESS THAN 0.02 DEGREES PER GRID ITS CLOSE ENOUGH.
      IF(SUMANG.LT.(NPTS*0.0035))   GO TO 215
      IF(NITER.GE.50)   GO TO 215
  100 CONTINUE
      WRITE(6,803)
  215 CONTINUE
C*ITERATION LOOP FOR THE WAVE HEIGHT.
      DO 501 NITER=1,NITERS
      SUMH=0.0
      DO 510  II=IBEGIN,IEND
C*MUST HAVE IT SET UP SO THAT  THE KNOWN BOUNDARIES HTS. AREN'T RECOMP
      IF(ISTART.EQ.IBEGIN)   I=II
      IF(ISTART.EQ.IBEGIN  .AND.  I.EQ.IBEGIN)  GO TO 510
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      IF(ISTART.EQ.IEND)   I=IEND-II+IBEGIN
      IF(ISTART.EQ.IEND  .AND.  I.EQ.IEND)   GO TO 510
C*ADX EQUALS ACTUAL DELTA X ACROSS SPACE STEP.
C*ONLY ON BOUNDARIES WHERE FORWARD OR BACKWARD DIFFERENCING.
      IF(I.NE.IBEGIN)   GO TO 503
      ADX=DX
      IP=I+1
      IM=I
      GO TO 505
 503    IF(I.NE.IEND)   GO TO 504
      ADX=DX
      IP=I
      IM=I-1
      GO TO 505
 504   ADX=2.0*DX
      IP=I+1
      IM=I-1
 505    CONTINUE
      DO 502 J=JBEGIN(I),JEND(I)-1
      JJ=JEND(I)-1-J+JBEGIN(I)
      HOLD(I,JJ)=H(I,JJ)
      YBAR=0.25*(Y(I,JJ)+2.0*Y(I,JJ+1)+Y(I,JJ+2))
      CALL LOC(IM,JJ,JOIM,JSIM,YBAR,IMINUS)
      CALL LOC(IP,JJ,JOIP,JSIP,YBAR,IPLUS)
      PART13=(H(I,JJ+1)**2.)*CG(I,JJ+1)*COS(THETA(I,JJ+1))
      PART2=DY(I,JJ)/ADX
      IF(JSIM.NE.0)   GO TO 311
      PART4B=0.0
      GO TO 312
  311 TOPIMH=(H(IM,JOIM-1)**2.)*CG(IM,JOIM-1)*(SIN(THETA(IM,JOIM-1)))
      BOTIMH=(H(IM,JSIM)**2.)*CG(IM,JSIM)*SIN(THETA(IM,JSIM))
      TOTALB=0.5*(Y(IM,JOIM)+Y(IM,JOIM-1))-0.5*(Y(IM,JSIM+1)+Y(IM,JSIM))
      DUMB=0.5*(Y(IM,JOIM)+Y(IM,JOIM-1))-YBAR
      PART4B=((TOTALB-DUMB)*(TOPIMH-BOTIMH)/TOTALB)+BOTIMH
  312 IF(JSIP.NE.0)   GO TO 313
      PART4A=0.0
      GO TO 314
  313 TOPIPH=(H(IP,JOIP-1)**2.)*CG(IP,JOIP-1)*SIN(THETA(IP,JOIP-1))
      BOTIPH=(H(IP,JSIP)**2 )*CG(IP,JSIP)*SIN(THETA(IP,JSIP))
      TOTALA=0.5*(Y(IP,JOIP)+Y(IP,JOIP-1))-0.5*(Y(IP,JSIP+1)+Y(IP,JSIP))
      DUMA=0.5*(Y(IP,JOIP)+Y(IP,JOIP-1))-YBAR
      PART4A=((TOTALA-DUMA)*(TOPIPH-BOTIPH)/TOTALA)+BOTIPH
  314 PART4=PART4A-PART4B
      YBARP=0.25*(Y(I,JJ+1)+2.*Y(I,JJ+2)+Y(I,JJ+3))
      CALL LOC(IM,JJ+1,JPOIM,JPSIM,YBARP,IMINUS)
      CALL LOC(IP,JJ+1,JPOIP,JPSIP,YBARP,IPLUS)
      IF(JPSIM.NE.0)   GO TO 315
      PART12=0.0
      GO TO 316
  315 TOPMH=(H(IM,JPOIM-1)**2)*CG(IM,JPOIM-1)*COS(THETA(IM,JPOIM-1))
      BOTMH=(H(IM,JPSIM)**2)*CG(IM,JPSIM)*COS(THETA(IM,JPSIM))
      TOTB= 5*(Y(IM,JPOIM)+Y(IM,JPOIM-1))-.5*(Y(IM,JPSIM+1)+Y(IM,JPSIM))
      DUMPB=0.5*(Y(IM,JPOIM)+Y(IM,JPOIM-1))-YBARP
      PART12=((TOTB-DUMPB)*(TOPMH-BOTMH)/TOTB)+BOTMH
  316 IF(JPSIP.NE.0)   GO TO 317
      PART11=0 0
      GO TO 318
  317 TOPPH=(H(IP,JPOIP-1)**2)*CG(IP,JPOIP-1)*COS(THETA(IP,JPOIP-1))
      BOTPH=(H(IP,JPSIP)**2)*CG(IP,JPSIP)*COS(THETA(IP,JPSIP))
      TOTA=.5*(Y(IP,JPOIP)+Y(IP,JPOIP-1))-.5*(Y(IP,JPSIP+1)+Y(IP,JPSIP))
      DUMPA=0.5*(Y(IP,JPOIP)+Y(IP,JPOIP-1))-YBARP
      PART11=((TOTA-DUMPA)*(TOPPH-BOTPH)/TOTA)+BOTPH
  318 PART1H=TAU*PART12+(1.-2.*TAU)*PART13+TAU*PART11
      IF(JPSIM.EQ.0)PART1H=(1.-TAU)*PART13+TAU*PART11
      IF(JPSIP.EQ.0)PART1H=TAU*PART12+(1.-TAU)*PART13
      ARG=((PART1H+PART2*PART4)/(CG(I,JJ)*COS(THETA(I,JJ))))
C*IF THERE IS TO BE AN INVALID SQRT,USE LINEAR SHOALING.
      IF(ARG.GE.0.)   GO TO 44
      ARG=(CG(I,JJ+1)*COS(THETA(I,JJ+1)))/(CG(I,JJ)*COS(THETA(I,JJ)))
      IF(ARG.LT.0.0)   ARG=0.0
      H(I,JJ)=H(I,JJ+1)*SQRT(ARG)
      GO TO 45
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   44 H(I,JJ)=SQRT(ARG)
   45 H(I,JJ)=0.5*(H(I,JJ)+HOLD(I,JJ))
      HNONBR(JJ)=H(I,JJ)
C*IBREAK(I)=JJ, THEREFORE JJ WILL BE LEEWARD SIDE OF GRID AT INIT BREAK
      IF(HB(I,JJ) .LT. H(I,JJ) .AND. HB(I,JJ+1).GE.HNONBR(JJ+1))
     *       IBREAK(I)=JJ
      IF(HB(I,JJ).LT.H(I,JJ))    H(I,JJ)=HB(I,JJ)
      SUMH=SUMH+ABS(H(I,JJ)-HOLD(I,JJ))
  502 CONTINUE
  510 CONTINUE
      IBREAK(IEND)=IBREAK(IEND-1)
      IBREAK(IBEGIN)=IBREAK(IBEGIN+1)
      IF(SUMH.LT.(NPTS*0.01))   GO TO 507
      IF(NITER.GE.50)   GO TO 507
  501 CONTINUE
      WRITE(6,803)
  507 CONTINUE
  802 FORMAT(2X,4(F15.5),////)
  803 FORMAT(2X,"AFTER NITERS ITERATIONS, CONVERGENCE WAS NOT REACHED")
  804 FORMAT(2X,"THE WAVE HT. ROUTINE CONVERGED IN, NITER= ",I5,//)
  805 FORMAT(2X,"THIS IS MY CHECKING WRITE STATEMENT")
  806  FORMAT(2X,"THE WAVE ANGLE ROUTINE CONVERGED IN, NITER= ",I5,//)
      RETURN
      END
C*****************************************************************
      SUBROUTINE DIFF(RHOND,THETAO,ANGLE,AMP)
C****DIFFRACTION ABOUT SEMI INFINITE BREAKWATER (PENNEY-PRICE)
      PI=3.14159265
      ABSS=SIN(0.5*(ANGLE-THETAO))
      ABSP=SIN(0.5*(ANGLE+THETAO))
      ABC=COS(ANGLE-THETAO)
      ABC1=COS(ANGLE+THETAO)
      XX=RHOND*ABC
      XXC=COS(XX)
      XXS=SIN(XX)
      XX1=RHOND*ABC1
      XXC1=COS(XX1)
      XXS1=SIN(XX1)
      AL=SQRT(RHOND/PI)
      SIG=2.0*AL*ABSS
      SIGP=-2.0*AL*ABSP
      CALL FRES(SIG,C,S,FR,FI)
      CALL FRES(SIGP,CP,SP,FRP,FIP)
      SUM1=XXC*FR+XXS*FI+XXC1*FRP+XXS1*FIP
      SUM2=XXC*FI-XXS*FR+XXC1*FIP-XXS1*FRP
      AMP=SQRT(SUM1**2+SUM2**2)
      RETURN
      END
C*****************************************************************
      SUBROUTINE FRES(A,C,S,FR,FI)
C*FRESNEL INTEGRAL SUBROUTINE****AFTER ABROMOWITZ AND STEGUN.
      Z=ABS(A)
      PO2=1.5707963
      FZ=(1.0+0.926*Z)/(2.0+1.792*Z+3.104*Z*Z)
      GZ=1.0/(2.0+4.142*Z+3.492*Z*Z+6.670*Z*Z*Z)
      XX=PO2*Z*Z
      CZ=COS(XX)
      SZ=SIN(XX)
      C=0.5-GZ*CZ+FZ*SZ
      S=0.5-FZ*CZ-GZ*SZ
      IF(A.GT.0.0)   GO TO 50
      C=-C
      S=-S
   50 FR=0.5*(1.0+C+S)
      FI=-0.5*(S-C)
      RETURN
      END
C*****************************************************************
      SUBROUTINE PREDIF
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/B/ THETA(60,20),QXTOT(60), OLDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
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      COMMON/N USED/JUSE,T,CO,CGEN,CGGEN,ANGGEN,DX,BERM,THETAO(10),MMAX
      COMMON/D/SIGMA,G,ELO,JMAX,IMAX,PI,TWOPI,PIO2,HGEN,IJET(10),SJETTY
      COMMON/G/IBREAK(60),HNONBR(20)
      DIMENSION J1(60),J2(60),J1REF(60),J3REF(60)
C*THIS SUB CALCS WHERE DIFFRACTION GOVERNS AND WHERE REFRACT GOVERNS.
C*IT WILL CALL REFRAC FOR OFFSHORE AREA(OFF TIP OF STRUCTURE).
C*THEN IT WILL DO THE SHADOW ZONE USING DIFF(IF THETAO .NE.0.0)
C* IT WILL THEN FINISH THE  OTHERS USING REFRAC AGAIN.
C*LET'S ZERO-OUT THE DIMENSIONED ARRAYS.
      DO 1000 I=1,IMAX+2
      J1(I)=0.0
      J2(I)=0.0
      J1REF(I)=0.0
 1000 J3REF(I)=0.0
C*NOW, LETS FIND C,CG,RK,HB, AND WVNUM.
      DO 202 I=1,IMAX
      DO 202 J=1,JMAX+2
      DEPTH=DEEP(I,J)
      CALL WVNUM(DEPTH,T,DUMK)
      RK(I,J)=DUMK
      C(I,J)=CO*TANH(RK(I,J)*DEEP(I,J))
      EN=0.5*(1.0+((2.*RK(I,J)*DEEP(I,J))/SINH(2.*RK(I,J)*DEEP(I,J))))
      CG(I,J)=EN*C(I,J)
      HB(I,J)=0.78*DEEP(I,J)
 202  CONTINUE
C*WILL ATTRIB AN EQUAL REACH TO EACH SIDE OF EACH M-GROIN.
      DO 200 M=1,MMAX
      IDUML=1
      IF(M.NE.1) IDUML=(IJET(M)+IJET(M-1))/2
      IDUMR=IMAX
      IF(M.NE.MMAX) IDUMR=(IJET(M)+IJET(M+1))/2
      NPTS=0
      DO 1 I=IDUML,IDUMR
      DO 2 J=1,JMAX
      IF(Y(I,J).LT.SJETTY)   GO TO 14
      J1(I)=J
      J2(I)=JMAX
      GO TO 15
   14 CONTINUE
    2 CONTINUE
   15 CONTINUE
C*IF NO STRUCT IS PRESENT(SJETTY=0.0), DO REFRAC THRUOUT GRID SYSTEM
      IF(SJETTY.EQ.0.0)   J1(I)=1
    1 CONTINUE
      DO 16 I=IDUML,IDUMR
C*  'REFRAC' STARTS ON THE NEXT TO LAST J-CONTOUR,NOT THE LAST!
      DO 16 J=J1(I),J2(I)-1
   16 NPTS=NPTS+1
C*WILL NOW DO THE REFRACT FOR THE REGION 1 AREA.
C*ISTART REPRESENTS THE DIRECTION THE SWEEPS WILL BEGIN FROM
C*WILL USE DUMMY IMAX,IJET,IJET+1 IN CALL STTS SO IBEGIN,IEND, AND
C***ISTART  WON'T CHANGE THEM MUST RESET AFTER EACH CALL REFRAC
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
      IF(ANGGEN.GE.0.0) CALL REFRAC(J1,J2,NPTS,IDUMLL,IMAXT,IDUMLL,M)
      IF(ANGGEN.LT.0.0) CALL REFRAC(J1,J2,NPTS,IDUMLL,IMAXT,IMAXT,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
      JDUMN=J1(IJET(M))
      JDUMS=J1(IJET(M)+1)
      XDISTN=(IJET(M)-1.0)*DX+DX/2.
      ELTIP=T*0.5*(C(IJET(M),JDUMN)+C(IJET(M)+1,JDUMS))
C*NOW MUST CHECK THE ANGLE AT THE STRUCTURE'S TIP TO SEE WHERE SHAD ZONE
C*IF NO STRUCT PRESENT(SJETTY=0.0), FUTHER REFRAC/DIFF UNNECESSARY
      IF(SJETTY.EQ.0.0)   GO TO 13
      THETAO(M)=0.5*(THETA(IJET(M),JDUMN)+THETA(IJET(M)+1,JDUMS))
      HINC=0.5*(H(IJET(M),JDUMN)+H(IJET(M)+1,JDUMS))
      IF(THETAO(M))10,11,12
C*THIS SECTION HANDLES REFRAC/DIFF IF THETAO<0.0
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   10 CONTINUE
C*FIRST ALL OF REGION 2 WILL GET REFRACTED.
      NPTS=0
      DO 100 I=IJET(M)+1,IDUMR
      J2(I)=J1(I)
  100 J1(I)=1
      DO 101 I=IJET(M)+1,IDUMR
      DO 101 J=J1(I),J2(I)-1
  101 NPTS=NPTS+1
      IMAXT=IDUMR
      IDUMLL=IDUML
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1,J2,NPTS,IJETP1,IMAXT,IMAXT,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
C*NOW MUST DO REGION 3 OF NEG THETAO CASE-SHADOW ZONE.
      DO 102 I=IDUML,IJET(M)
      J2(I)=J1(I)
  102 J1(I)=1
      DO 103 I=IDUML,IJET(M)
      J1REF(I)=1
      DO 104 J=J1(I),J2(I)+1
      XCOOR=(I-1.0)*DX
      YCOOR=0.5*(Y(I,J)+Y(I,J+1))
      ANGLE=ATAN((XDISTN-XCOOR)/(SJETTY-YCOOR))
      IF(YCOOR.GT.SJETTY)   ANGLE=PI+ANGLE
C*IF MOST SHOREWARD PT OUT OF SHAD ZONE, SO ARE THE OTHERS FOR THAT I.
      IF(ABS(ANGLE).GT.ABS(THETAO(M)))   GO TO 105
      RAD=SQRT((XDISTN-XCOOR)**2+(SJETTY-YCOOR)**2)
      RHOND=RAD*TWOPI/ELTIP
C*DIFFRACTION TREATS THE POS THETAO CASE.
      THE=ABS(THETAO(M))
      CALL DIFF(RHOND,THE,ANGLE,AMP)
      H(I,J)=AMP*HINC
      ANGRAD=-ANGLE
C*WILL NOW REFRACT DIFF WAVES IN THE SHAD ZONE USING SNELL'S.
      CTIP=ELTIP/T
       ALPHAS(I,J)=ATAN((0.5*(Y(I+1,J)+Y(I+1,J+1))-0.5*
     *  (Y(I-1,J)+Y(I-1,J+1)))/(2.*DX))
      IF(I.EQ.IJET(M))ALPHAS(I,J)=ATAN((0.5*(Y(I,J)+Y(I,J+1))-0.5*(Y(I-1
     *    ,J)+Y(I-1,J+1)))/DX)
      DALPHA=ANGRAD-ALPHAS(I,J)
      THETA(I,J)=ARSIN((C(I,J)/CTIP)*SIN(DALPHA))
      THETA(I,J)=THETA(I,J)+ALPHAS(I,J)
C*MUST CHECK TO SEE IF WAVE WOULD HAVE BROKEN.
      IF(HB(I,J).LE.H(I,J).AND.HB(I,J+1).GT.H(I,J+1))IBREAK(I)=J
      IF(HB(I,J).LT.H(I,J))   H(I,J)=HB(I,J)
  104 CONTINUE
      GO TO 103
  105 J1REF(I)=J
  103 CONTINUE
C*NOW MUST DO REFRACTION FOR REGION 4.
      NPTS=0
      DO 106 I=IDUML,IJET(M)
      DO 106 J=J1REF(I),J2(I)-1
  106 NPTS=NPTS+1
      IDUMLL=IDUML
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1REF,J2,NPTS,IDUMLL,IJETT,IDUMLL,M)
      IDUMLL=IDUML
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      GO TO 13
C*THIS HANDLES REFRAC/DIFF IF THETAO IS 0.0.
C*FOR THIS CASE, ONLY THREE REGIONS EXIST
   11 CONTINUE
      NPTS=0
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      DO 120 I=IDUML,IJET(M)
      J2(I)=J1(I)
  120 J1(I)=1
      DO 121 I=IDUML,IJET(M)
      DO 121 J=J1(I),J2(I)-1
  121 NPTS=NPTS+1
      IMAXT=IDUMR
      IDUMLL=IDUML
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1,J2,NPTS,IDUMLL,IJETT,IDUMLL,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
      DO 122 I=IJET(M)+1,IDUMR
      J2(I)=J1(I)
  122 J1(I)=1
      NPTS=0
      DO 123 I=IJET(M)+1,IDUMR
      DO 123 J=J1(I),J2(I)-1
  123 NPTS=NPTS+1
      IMAXT=IDUMR
      IDUMLL=IDUML
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1,J2,NPTS,IJETP1,IMAXT,IMAXT,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
      GO TO 13
C*THIS SECTION HANDLES REFRACT/DIFF IF THETAO>0.0
  12  CONTINUE
C*FIRST, REGION 2- ALL REFRACTION.
      NPTS=0
      DO 110 I=IDUML,IJET(M)
      J2(I)=J1(I)
  110 J1(I)=1
      DO 111 I=IDUML,IJET(M)
      DO 111 J=J1(I),J2(I)-1
  111 NPTS=NPTS+1
      IMAXT=IDUMR
      IDUMLL=IDUML
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1,J2,NPTS,IDUMLL,IJETT,IDUMLL,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
C*NOW WILL DO REGION 3 OF THE POS THETAO CASE.
      DO 112 I=IJET(M)+1,IDUMR
      J2(I)=J1(I)
  112 J1(I)=1
      DO 113 I=IJET(M)+1,IDUMR
      J1REF(I)=1
C*WILL GO ONE PT. BEYOND J2(I) TO MAKE SURE OUTOF DIFF ZONE.
      DO 114 J=J1(I),J2(I)+1
      XCOOR=(I-1.0)*DX
      YCOOR=0.5*(Y(I,J)+Y(I,J+1))
      ANGLE=ATAN((XCOOR-XDISTN)/(SJETTY-YCOOR))
      IF(YCOOR.GT.SJETTY)   ANGLE=PI+ANGLE
C*IF LEAST J-VALUE IS OUT OF SHAD ZONE,SO ARE OTHER J'S (FOR EACH I)
      IF(ANGLE.GT.ABS(THETAO(M)))   GO TO 115
      RAD=SQRT((XCOOR-XDISTN)**2+(SJETTY-YCOOR)**2)
      RHOND=RAD*TWOPI/ELTIP
      THE=THETAO(M)
      CALL DIFF(RHOND,THE,ANGLE,AMP)
      ANGRAD=ANGLE
C*WILL NOW REFRACT DIFFRACTED WAVES IN SHAD ZONE USING SNELL"S
      CTIP=ELTIP/T
      ALPHAS(I,J)=ATAN((0.5*(Y(I+1,J)+Y(I+1,J+1))-0.5*
```

```
     *   (Y(I-1,J)+Y(I-1,J+1)))/(2.*DX))
      IF(I.EQ.IJET(M)+1)ALPHAS(I,J)=ATAN((0.5*(Y(I+1,J)+Y(I+1,J+1))-0.5*
     *    (Y(I,J)+Y(I,J+1))))/DX)
      DALPHA=ANGRAD-ALPHAS(I,J)
      THETA(I,J)=ARSIN((C(I,J)/CTIP)*SIN(DALPHA))
      THETA(I,J)=THETA(I,J)+ALPHAS(I,J)
      H(I,J)=HINC*AMP
C*MUST CHECK TO SEE IF WAVE WOULD HAVE BROKEN
      IF(HB(I,J).LE.H(I,J).AND.HB(I,J+1).GT.H(I,J+1))IBREAK(I)=
      IF(HB(I,J).LT.H(I,J))    H(I,J)=HB(I,J)
  114 CONTINUE
      GO TO 113
  115 J1REF(I)=J
  113 CONTINUE
C*NOW MUST DO REFRAC FOR REGION 4
      NPTS=0
      DO 116 I=IJET(M)+1,IDUMR
      DO 116 J=J1REF(I),J2(I)-1
  116 NPTS=NPTS+1
      IMAXT=IDUMR
      IDUMLL=IDUML
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      CALL REFRAC(J1REF,J2,NPTS,IJETP1,IMAXT,IMAXT,M)
      IMAXT=IDUMR
      IJETT=IJET(M)
      IJETP1=IJET(M)+1
      IDUMLL=IDUML
   13 CONTINUE
  200 CONTINUE
      RETURN
      END
C***************************************************************.......
      SUBROUTINE LOC(IM,JJ,JOIM,JSIM,YBAR,IDUM)
      COMMON/A/ C(60,20),RK(60,20),Y(60,20),DEEP(60,20),ALPHAS(60,20)
      COMMON/AA/YZERO(60)
      COMMON/B/ THETA(60,20),QXTOT(60), OLDANG(60,20), DY(60,20)
      COMMON/C/ H(60,20),CG(60,20),HOLD(60,20),HB(60,20),YB(60)
      COMMON/N USED/JUSE,T,CO,CGEN,CGGEN,ANGGEN,DX,BERM,THETAO(10),MMA
      COMMON/D/SIGMA,G,ELO,JMAX,IMAX,PI,TWOPI,PIO2,HGEN,IJET(10),S,E
C*SUBROUTINE LOC FINDS J-VALUES WHICH ARE GREATER AND LESS THAN YBAR
      JOIM=2
 2    AA=0.5*(Y(IM,JOIM)+Y(IM,JOIM-1))
      IF(AA.GT.YBAR)    GO TO 4
      JOIM=JOIM+1
C*THE FOLLOWING IS REQ'D SO THAT DY/DX>0.5
C*WILL DTERMINE K SIN THETA ON IM-LINE AT A DIST YBAR
C*WILL CALL THIS POINT     JUSE+1
      IF(JOIM.LE.JUSE)    GO TO 2
      JOIM=JUSE+1
      Y(IM,JOIM)=YBAR
C* DEPTH AT THIS POINT WILL BE COMP ASSUMING CONST BEACH SLOPE ON I-IM
      DEL=.5*(Y(IM,JOIM-1)+Y(IM,JOIM-2))-.5*(Y(IM,JOIM-2)+Y(IM,JOIM-3))
      BSLOPE=(DEEP(IM,JOIM-2)-DEEP(IM,JOIM-3))/DEL
      DEEP(IM,JOIM-1)=DEEP(IM,JOIM-2)+BSLOPE*(Y(IM,JOIM)-Y(IM,JOIM-1))
      DEPTH=DEEP(IM,JOIM-1)
      CALL WVNUM(DEPTH,T,DUMK)
      RK(IM,JOIM-1)=DUMK
      C(IM,JOIM-1)=CO*TANH(RK(IM,JOIM-1)*DEEP(IM,JOIM-1))
      EN=0.5*(1.0+((2.0*RK(IM,JOIM-1)*DEEP(IM,JOIM-1))/SINH(
     *    2.*RK(IM,JOIM-1)*DEEP(IM,JOIM-1))))
      CG(IM,JOIM-1)=C(IM,JOIM-1)*EN
C*WILL USE SNELL'S LAW TO  DETERMINE THE WAVE ANGLE HERE
C*ANGLE OF CONTOUR WILL BE ASSUME TO BE THE SAME AS THE JMAX+1 CONTOUR
      IF(IDUM.EQ.1)ALPH=ATAN((Y(IM,JOIM-1)-Y(IM-1,JOIM-1))/DX)
      IF(IDUM.EQ.-1)ALPH=ATAN((Y(IM+1,JOIM-1)-Y(IM,JOIM-1))/DX)
      DALPHA=ANGGEN-ALPH
      THETA(IM,JOIM-1)=ARSIN((C(IM,JOIM-1)/CGEN)*SIN(DALPHA))
      THETA(IM,JOIM-1)=THETA(IM,JOIM-1)+ALPH
 4     JSIM=JMAX-1
 6    AA=0.5*(Y(IM,JSIM)+(Y(IM,JSIM+1)))
      IF(AA.LT.YBAR)    GO TO 8
      JSIM=JSIM-1
```

```
C*IF JSIM=0,THERE IS NO ADJ PT. SUB REFRAC CAN HANDLE IT
      IF(JSIM.EQ.0)   GO TO 8
      GO TO 6
 8    RETURN
      END
C****************************************************************
      SUBROUTINE WVNUM(DEPTH,T,RK)
      G=32.17
      EPS=0.001
      TWOPI=6.283185307
      SIGMA=TWOPI/T
      RK=TWOPI/(T*SQRT(G*DEPTH))
      DO 100 IT=1,20
      ARG=RK*DEPTH
      EK=(G*RK*TANH(ARG))-(SIGMA**2)
      EKPR=G*(ARG*((SECH(ARG))**2)+TANH(ARG))
      RKNEW=RK-EK/EKPR
      IF(ABS(RKNEW-RK).LE.ABS(EPS*RKNEW))   GO TO 120
      RK=RKNEW
 100  CONTINUE
      WRITE(6,1000)   IT,DEPTH,RK
 1000 FORMAT(///,10X,"ITERATION FOR K FAILED TO CONVERGE AFTER"
     * ,3X,I3,"ITERATION",/,"OUTPUT  DEPTH, RK",3X,2F13.5)
      CALL EXIT
 120  RK=RKNEW
      IF(RK.GT.0.0)   GO TO 140
      WRITE(6,1020)   DEPTH,RK
 1020 FORMAT(///,10X," RK IS NEG",/," OUTPUT DEPTH,RK",3X,2F13.5)
      CALL EXIT
 140  RETURN
      END
C****************************************************************
      SUBROUTINE SMOOTH(THETA,IMAX,JMAX,IJET,SJETTY,MMAX,Y)
C*THIS WILL SMOOTH THE WAVE ANGLE FIELD TO ACCT FOR DIFF(ARTIFICIALLY)
      DIMENSION TEMP(60,20),Y(60,20),THETA(60,20),IJET(10)
C*(MMAX+1) IS REQ'D BECAUSE M-GROINS HAVE M+1 REACHES OF SHORELINE
      DO 10 M=1,MMAX+1
      IF(M.NE.1)   GO TO 3
      ILEFT=2
      IRIGHT=IJET(1)
      GO TO 5
    3 IF(M.NE.MMAX+1)   GO TO 4
      ILEFT=IJET(MMAX)+1
      IRIGHT=IMAX-1
      GO TO 5
    4 ILEFT=IJET(M-1)+1
      IRIGHT=IJET(M)
    5 CONTINUE
      DO 1 J=1,JMAX-1
      DO 1 I=ILEFT,IRIGHT
      IF(I.NE.ILEFT.AND.I.NE.IRIGHT)   GO TO 15
C*TO GET HERE, MUST BE ON BOUN OR ADJ TO A STRUCTURE
      IF(I.EQ.2.OR.I.EQ.IMAX-1)   GO TO 15
C*TO GET HERE,ADJ TO A STRUCT AND CAN BE ILEFT OR IRIGHT
      IF(Y(I,J).GE.SJETTY)   GO TO 15
C*IF HERE, WITHIN JETTY AND ADJ TO EITHER SIDE
      IF(I.EQ.ILEFT)TEMP(I,J)=0.5*(THETA(I,J)+THETA(I+1,J))
      IF(I.EQ.IRIGHT)TEMP(I,J)=0.5*(THETA(I,J)+THETA(I-1,J))
      GO TO 1
   15 TEMP(I,J)=0.25*THETA(I-1,J)+0.50*THETA(I,J)+0.25*THETA(I+1,J)
    1 CONTINUE
   10 CONTINUE
      DO 2 J=1,JMAX-1
      DO 2 I=2,IMAX-1
    2 THETA(I,J)=TEMP(I,J)
      RETURN
      END
C****************************************************************
      FUNCTION SECH(A)
      SECH=1.0/COSH(A)
      RETURN
      END
C****HERE IS WHERE THE IMSL ROUTINES MUST GO!
```

APPENDIX C

CONTOURS AND SCHEMATIC ILLUSTRATIONS

This appendix presents tables of the original contours at Oregon Inlet and the final contours for the eight numerical simulations (Tables C-1 to C-9). Also included are schematic illustrations of sediment volumes transported from the nourished region (Figs. C-1 to C-8).

Table C-1. Initial bathymetry for all simulations (prior to any sediment addition).

```
    Increasing I ---->
I=1= 220.000  200.000  200.000  200.000  220.000  220.000  210.000  200.000  220.000  200.000  200.000  170.000  190.000
   180.000  180.000  160.000  160.000  160.000  190.000  190.000  190.000  180.000  180.000  210.000  220.000  230.000     J=1
   220.000  200.000  160.000  160.000  160.000  160.000  170.000  170.000  170.000  180.000  210.000  220.000  220.000
   230.000  250.000  220.000  200.000  200.000  200.000  200.000  200.000  200.000  210.000  200.000
     251.623  231.623  231.623  231.623  251.623  251.623  241.623  231.623  251.623  231.623  231.623  201.623  221.623
   211.623  211.623  191.623  191.623  191.623  221.623  221.623  221.623  211.623  221.623  241.623  251.623  261.623     2
   251.623  231.623  191.623  191.623  191.623  191.623  201.623  201.623  201.623  211.623  241.623  251.623  251.623
   261.623  281.623  251.623  231.623  231.623  23 .623  231.623  231.623  231.623  241.623  231.623
     309.443  289.443  289.443  289.443  309.443  309.443  299.443  289.443  309.443  289.443  289.443  259.443  279.443
   269.443  269.443  249.443  249.443  249.443  279.443  279.443  279.443  269.443  279.443  299.443  309.443  319.443     3
   309.443  289.443  249.443  249.443  249.443  249.443  259.443  259.443  259.443  269.443  299.443  309.443  309.443
   319.443  339.443  309.443  289.443  289.443  289.443  289.443  289.443  289.443  299.443  289.443
     442.028  422.028  422.028  422.028  442.028  442.028  432.028  422.028  442.028  422.028  422.028  392.028  412.028
   402.028  402.028  382.028  382.028  382.028  412.028  412.028  412.028  402.028  412.028  432.028  442.028  452.028     4
   442.028  422.028  382.028  382.028  382.028  382.028  392.028  392.028  392.028  402.028  432.028  442.028  442.028
   452.028  472.028  442.028  422.028  422.028  422.028  422.028  422.028  422.028  432.028  422.028
     684.758  664.758  664.758  664.758  684.758  684.758  674.758  664.758  684.758  664.758  664.758  634.758  654.758
   644.758  644.758  624.758  624.758  624.758  654.758  654.758  654.758  644.758  654.758  674.758  684.758  694.758     5
   684.758  664.758  624.758  624.758  624.758  624.758  634.758  634.758  634.758  644.758  674.759  684.758  684.758
   694.758  714.758  684.758  664.758  664.759  664.758  664.758  664.758  664.758  674.758  664.758
     980.726  960.726  960.726  960.726  980.726  980.726  970.726  960.726  980.726  960.726  960.726  930.726  950.726
   940.726  940.726  920.726  920.726  920.726  950.726  950.726  950.726  940.726  950.726  970.726  980.726  990.726     6
   980.726  960.726  920.726  920.726  920.726  920.726  930.726  930.726  930.726  940.726  970.726  980.726  980.726
   990.726 1010.726  980.726  960.726  960.726  960.726  960.726  960.726  960.726  970.726  960.726
    1270.414 1250.414 1250.414 1250.414 1270.414 1270.414 1260.414 1250.414 1270.414 1250.414 1250.414 1220.414 1240.414
  1230.414 1230.414 1210.414 1210.414 1210.414 1240.414 1240.414 1240.414 1230.414 1240.414 1260.414 1270.414 1280.414     7
  1270.414 1250.414 1210.414 1210.414 1210.414 1210.414 1220.414 1220.414 1220.414 1230.414 1260.414 1270.414 1270.414
  1280.414 1300.414 1270.414 1250.414 1250.414 1250.414 1250.414 1250.414 1250.414 1260.414 1250.414
    1702.228 1682.228 1682.228 1682.228 1702.228 1702.228 1692.228 1682.228 1702.228 1682.228 1682.228 1652.228 1672.228
  1662.228 1662.228 1642.228 1642.228 1642.228 1672.228 1672.228 1672.228 1662.228 1672.228 1692.228 1702.228 1712.228     8
  1702.228 1682.228 1642.228 1642.228 1642.228 1642.228 1652.228 1652.228 1652.228 1662.228 1692.228 1702.228 1702.228
  1712.228 1732.228 1702.228 1682.228 1682.228 1682.228 1682.228 1682.228 1682.228 1692.228 1682.228
```

Table C-2. Final contours, case 2.a.

```
THE NEW CONTOUR VALUES, Y, FOLLOW
   220.000  219.421  218.843  218.266  217.691  217.119  216.550  215.985  215.425  214.871  214.322  213.779  213.244
 212.716  212.197  211.687  211.186  210.694  210.213  209.744  209.265  208.836  208.398  207.969  207.551  207.144
 206.748  206.362  205.987  205.621  205.266  204.922  204.588  204.264  203.950  203.645  203.348  203.060  202.779
 202.506  202.239  201.977  201.721  201.468  201.218  200.972  200.727  200.484  200.241  200.000
   251.623  251.018  250.439  249.873  249.297  248.719  248.161  247.616  247.058  246.491  245.948  245.440  244.937
 244.407  243.855  243.322  242.835  242.375  241.895  241.375  240.839  240.338  239.914  239.561  239.230  238.871
 238.468  238.053  237.664  237.308  236.968  236.619  236.253  235.884  235.535  235.209  234.908  234.651  234.431
 234.201  233.932  233.643  233.364  233.098  232.837  232.584  232.344  232.109  231.868  231.623
   309.443  308.834  308.267  307.716  307.144  306.569  306.023  305.489  304.931  304.362  303.825  303.332  302.838
 302.300  301.732  301.197  300.731  300.295  299.811  299.261  298.708  298.229  297.838  297.504  297.175  296.810
 296.400  295.978  295.589  295.240  294.904  294.542  294.147  293.753  293.398  293.079  292.788  292.546  292.340
 292.114  291.835  291.534  291.249  290.977  290.707  290.447  290.201  289.956  289.703  289.443
   442.028  441.461  440.894  440.328  439.765  439.204  438.646  438.095  437.549  437.008  436.473  435.945  435.424
 434.911  434.407  433.912  433.427  432.950  432.482  432.020  431.564  431.112  430.667  430.231  429.810  429.404
 429.014  428.640  428.276  427.920  427.569  427.223  426.879  426.540  426.206  425.879  425.562  425.259  424.970
 424.696  424.433  424.178  423.925  423.669  423.406  423.136  422.864  422.587  422.307  422.028
   684.758  684.191  683.625  683.064  682.509  681.961  681.420  680.885  680.354  679.827  679.301  678.777  678.254
 677.735  677.220  676.713  676.215  675.730  675.259  674.803  674.361  673.933  673.518  673.116  672.725  672.344
 671.969  671.596  671.221  670.842  670.462  670.081  669.706  669.340  668.990  668.658  668.347  668.057  667.786
 667.5[illegible]0  667.283  667.037  666.786  666.525   66.252  665.966  665.671  665.370  665.065  664.758
   980.726  980.133  979.544  978.962  978.392  977.832  977.284  976.746  976.214  975.687  975.161  974.633  974.104
 973.574  973.043  972.516  971.997  971.489  970.997  970.525  970.076  969.651  969.248  968.865  968.497  968.137
 967.779  967.417  967.047  966.667  966.278  965.867  965.500  965.125  964.769  964.438  964.135  963.859  963.606
 963.379  963.142  962.915  962.681  962.437  962.178  961.906  961.622  961.328  961.028  960.726
  1270.814 1269.804 1269.194 1268.588 1267.984 1267.385 1266.792 1266.204 1265.622 1265.047 1264.479 1263.918 1263.365
1262.820 1262.284 1261.758 1261.243 1260.740 1260.248 1259.770 1259.306 1258.855 1258.418 1257.995 1257.585 1257.187
1256.802 1256.427 1256.063 1255.708 1255.363 1255.029 1254.705 1254.391 1254.089 1253.798 1253.519 1253.250 1252.990
1252.739 1252.495 1252.256 1252.020 1251.788 1251.557 1251.328 1251.099 1250.870 1250.642 1250.414
  1702.228 1701.595 1700.963 1700.333 1699.706 1699.083 1698.464 1697.852 1697.246 1696.647 1696.057 1695.475 1694.904
1694.342 1693.792 1693.253 1692.726 1692.212 1691.711 1691.223 1690.748 1690.287 1689.840 1689.407 1688.988 1688.584
1688.193 1687.816 1687.454 1687.104 1686.768 1686.446 1686.130 1685.838 1685.552 1685.277 1685.014 1684.760 1684.515
1684.279 1684.052 1683.831 1683.617 1683.408 1683.204 1683.004 1682.807 1682.613 1682.420 1682.228
```

Table C-3. Final contours, case 2.b.

```
THE NEW CONTOUR VALUES, Y, FOLLOW
   220.000  219.439  218.880  218.321  217.765  217.210  216.659  216.112  215.568  215.029  214.495  213.967  213.446
 212.931  212.424  211.925  211.434  210.952  210.479  210.016  209.561  209.115  208.680  208.254  207.837  207.430
 207.032  206.643  206.263  205.894  205.533  205.182  204.840  204.508  204.184  203.868  203.561  203.260  202.966
 202.677  202.394  202.116  201.842  201.571  201.304  201.040  200.778  200.518  200.259  200.000
   251.623  251.065  250.508  249.954  249.401  248.850  248.302  247.757  247.216  246.679  246.148  245.621  245.101
 244.588  244.082  243.584  243.095  242.615  242.144  241.683  241.231  240.789  240.355  239.930  239.514  239.107
 238.708  238.317  237.936  237.563  237.200  236.847  236.504  236.170  235.846  235.530  235.222  234.921  234.626
 234.337  234.052  233.771  233.494  233.219  232.948  232.679  232.413  232.149  231.885  231.623
   309.443  308.891  308.341  307.791  307.246  306.702  306.160  305.620  305.084  304.551  304.022  303.498  302.980
 302.469  301.966  301.471  300.985  300.509  300.043  299.586  299.140  298.701  298.271  297.849  297.435  297.028
 296.628  296.234  295.848  295.470  295.102  294.745  294.399  294.063  293.738  293.421  293.114  292.813  292.518
 292.226  291.938  291.653  291.369  291.087  290.807  290.531  290.257  289.985  289.713  289.443
   442.028  441.490  440.954  440.418  439.884  439.350  438.819  438.288  437.759  437.232  436.707  436.187  435.672
 435.163  434.660  434.174  433.694  433.226  432.769  432.323  431.887  431.459  431.038  430.623  430.212  429.803
 429.398  428.997  428.601  428.213  427.835  427.469  427.117  426.779  426.453  426.139  425.834  425.535  425.240
 424.947  424.653  424.359  424.064  423.768  423.474  423.181  422.890  422.601  422.314  422.028
   684.758  684.237  683.717  683.196  682.676  682.156  681.635  681.114  680.591  680.069  679.548  679.029  678.510
 678.011  677.515  677.032  676.561  676.104  675.660  675.228  674.806  674.392  673.982  673.574  673.166  672.756
 672.344  671.933  671.524  671.124  670.735  670.361  670.004  669.665  669.343  669.034  668.735  668.441  668.148
 667.852  667.553  667.248  666.938  666.625  666.311  665.997  665.685  665.375  665.066  664.758
   980.7[illegible]  980.191  979.657  979.122  978.587  978.050  977.513  976.975  976.436  975.899  975.363  974.833  974.309
 973.795  973.292  972.803  972.328  971.868  971.422  970.988  970.564  970.146  969.734  969.322  968.911  968.499
 968.087  967.678  967.275  966.882  966.502  966.140  965.795  965.468  965.157  964.859  964.571  964.287  964.003
 963.718  963.428  963.133  962.835  962.533  962.230  961.927  961.624  961.324  961.024  960.726
   1270.414 1269.827 1269.239 1268.653 1268.069 1267.487 1266.907 1266.332 1265.760 1265.194 1264.634 1264.081 1263.536
 1263.000 1262.470 1261.959 1261.456 1260.964 1260.485 1260.018 1259.562 1259.117 1258.683 1258.259 1257.844 1257.440
 1257.045 1256.661 1256.286 1255.927 1255.578 1255.241 1254.917 1254.605 1254.304 1254.013 1253.731 1253.455 1253.186
 1252.921 1252.660 1252.401 1252.146 1251.893 1251.642 1251.393 1251.147 1250.901 1250.658 1250.414
   1702.228 1701.600 1700.973 1700.348 1699.726 1699.108 1698.494 1697.886 1697.284 1696.689 1696.103 1695.525 1694.957
 1694.399 1693.851 1693.315 1692.791 1692.279 1691.780 1691.293 1690.820 1690.361 1689.915 1689.483 1689.065 1688.660
 1688.270 1687.893 1687.530 1687.180 1686.844 1686.520 1686.209 1685.910 1685.623 1685.346 1685.080 1684.823 1684.575
 1684.336 1684.104 1683.878 1683.659 1683.445 1683.236 1683.030 1682.827 1682.626 1682.427 1682.228
```

Table C-4. Final contours, case 2.cl.

```
THE NEW CONTOUR VALUES, X, FOLLOW
   220.000  222.852  225.655  228.459  231.234  233.970  236.655  239.278  241.828  244.294  246.665  248.929  251.075
 253.[illegible]  254.964  256.685  258.239  259.618  260.810  261.805  262.594  263.168  263.520  263.643  263.533  263.189
 262.[illegible]  261.791  260.740  259.459  257.953  256.229  254.295  252.160  249.834  247.327  244.651  241.817  238.837
 235.725  232.492  229.151  225.714  222.193  218.602  214.951  211.253  207.519  203.764  200.000
   251.623  254.652  257.687  260.718  263.712  266.652  269.545  272.386  275.145  277.797  280.346  282.804  285.152
 287.389  289.369  291.215  292.907  294.439  295.772  296.865  297.705  298.308  298.694  298.868  298.818  298.515
 297.935  297.079  295.972  294.634  293.068  291.265  289.216  286.935  284.449  281.778  278.928  275.925  272.796
 269.535  266.119  262.565  258.909  255.172  251.353  247.456  243.493  239.492  235.520  231.623
   309.443  313.514  317.59[illegible]  321.665  325.686  329.646  333.542  337.363  341.075  344.657  348.113  351.445  354.628
 357.618  360.390  362.951  365.314  367.461  369.345  370.917  372.171  373.126  373.798  374.180  374.249  373.972
 373.330  372.334  371.011  369.385  367.455  365.207  362.637  359.769  356.641  353.273  349.677  345.882  341.915
 337.766  333.464  328.873  324.214  319.450  314.581  309.623  304.600  299.541  294.483  289.443
   442.028  447.543  453.038  458.495  463.895  469.219  474.449  479.566  484.553  489.390  494.060  498.542  502.816
 506.862  510.658  514.183  517.414  520.328  522.901  525.111  526.937  528.362  529.371  529.958  530.118  529.849
 529.149  528.019  526.458  524.470  522.061  519.242  516.025  512.428  508.470  504.176  499.571  494.685  489.545
 484.171  478.590  472.817  466.868  460.760  454.511  448.146  441.690  435.170  428.608  422.028
   684.75[illegible]  693.500  702.203  710.828  719.341  727.712  735.913  743.920  751.708  759.253  766.528  773.507  780.161
 786.463  792.381  797.887  802.950  807.539  811.626  815.180  818.172  820.578  822.373  823.541  824.065  823.934
 823.139  821.676  819.544  816.751  813.310  809.242  804.571  799.327  793.544  787.256  780.500  773.310  765.723
 757.770  749.480  740.882  732.003  722.872  713.521  703.986  694.304  684.514  674.654  664.758
   980.726  997.388 1014.001 1030.517 1046.888 1063.067 1079.004 1094.650 1109.953 1124.857 1139.303 1153.231 1166.576
 1179.271 1191.249 1202.442 1212.781 1222.200 1230.635 1238.024 1244.309 1249.436 1253.359 1256.035 1257.430 1257.518
 1256.285 1253.726 1249.649 1244.675 1238.238 1230.584 1221.770 1211.859 1200.921 1189.030 1176.260 1162.684 1148.372
 1133.393 1117.812 1101.693 1085.097 1068.088 1050.725 1033.071 1015.185  997.127  978.954  960.726
   1270.414 1275.696 1280.964 1286.207 1291.410 1296.559 1301.640 1306.637 1311.533 1316.308 1320.943 1325.417 1329.706
 1333.786 1337.633 1341.221 1344.522 1347.511 1350.162 1352.451 1354.354 1355.850 1356.922 1357.553 1357.733 1357.453
 1356.712 1355.509 1353.852 1351.750 1349.218 1346.273 1342.938 1339.236 1335.192 1330.832 1326.184 1321.275 1316.130
 1310.775 1305.234 1299.529 1293.683 1287.717 1281.649 1275.498 1269.283 1263.020 1256.725 1250.414
   1702.228 1701.697 1701.166 1700.636 1700.109 1699.584 1699.062 1698.544 1698.030 1697.521 1697.017 1696.519 1696.026
 1695.541 1695.061 1694.589 1694.125 1693.664 1693.213 1692.769 1692.331 1691.901 1691.478 1691.062 1690.652 1690.250
 1689.854 1689.464 1689.082 1688.705 1688.335 1687.972 1687.614 1687.263 1686.917 1686.577 1686.243 1685.913 1685.589
 1685.268 1684.952 1684.640 1684.331 1684.025 1683.722 1683.420 1683.121 1682.822 1682.525 1682.228
```

Table C-5. Final contours, case 2.c2.

```
THE NEW CONTOUR VALUES, Y, FOLLOW
  220.000  222.574  225.142  227.698  230.232  232.735  235.198  237.610  239.962  242.243  244.443  246.550  248.554
 250.442  252.202  253.822  255.290  256.593  257.720  258.660  259.402  259.937  260.256  260.355  260.227  259.869
 259.280  258.461  257.414  256.145  254.659  252.965  251.073  248.994  246.739  244.321  241.751  239.043  236.209
 233.260  230.209  227.068  223.848  220.560  217.215  213.824  210.398  206.946  203.477  200.000
  251.623  254.387  257.181  259.990  262.787  265.552  268.275  270.949  273.563  276.104  278.562  280.926  283.187
 285.327  287.332  289.187  290.877  292.388  293.708  294.821  295.716  296.382  296.811  296.995  296.924  296.589
 295.985  295.117  293.992  292.618  291.000  289.148  287.075  284.796  282.327  279.682  276.875  273.917  270.824
 267.614  264.301  260.896  257.407  253.846  250.224  246.555  242.849  239.119  235.374  231.623
  309.443  312.369  315.475  318.666  321.824  324.931  327.987  330.992  333.933  336.779  339.523  342.195  344.810
 347.324  349.679  351.847  353.825  355.606  357.172  358.504  359.591  360.427  361.022  361.390  361.498  361.247
 360.580  359.556  358.265  356.739  354.956  352.905  350.596  348.053  345.302  342.360  339.231  335.913  332.433
 328.855  325.224  321.524  317.726  313.833  309.864  305.835  301.762  297.660  293.549  289.443
  442.028  445.538  449.186  452.859  456.485  460.091  463.669  467.192  470.631  473.936  477.076  480.140  483.249
 486.354  489.283  491.944  494.342  496.498  498.422  500.102  501.506  502.601  503.403  504.010  504.467  504.530
 503.870  502.55[illegible]  500.893  499.043  496.955  494.589  491.946  489.038  485.882  482.495  478.884  475.030  470.948
 466.773  462.654  458.548  454.299  449.861  445.303  4[illegible].687  436.037  431.362  426.686  422.028
  684.758  693.580  702.347  711.058  719.730  728.360  736.919  745.379  753.715  761.903  769.943  780.355  795.635
 813.218  830.484  847.336  863.718  879.584  894.889  909.591  923.639  936.984  949.588  961.432  972.431  977.454
 971.427  959.366  946.416  932.695  918.235  903.074  887.251  870.809  853.788  836.240  818.222  799.793  781.024
 764.526  752.893  743.635  734.202  724.578  714.804  704.920  694.953  684.921  674.849  664.758
  980.726  996.141 1011.526 1026.850 1042.081 1057.184 1072.120 1086.848 1101.319 1115.484 1129.283 1146.126 1169.417
1195. 11 1221.161 1245.989 1270.014 1293.157  315.340 1329.771 1340.535 1350.556 1359.797 1368.228 1375.828 1379.114
1374.609 1365.793 1356.149 1345.701 1334.479 1322.518 1305.427 1281.833 1257.273 1231.821 1205.553 1178.548 1150.882
1126.103 1107.752 1092.429 1076.727 1060.706 1044.422 1027.926 1011.265  994.484  977.624  960.726
 1270.414 1273.892 1277.367 1280.834 1284.289 1287.728 1291.146 1294.535 1297.888 1301.196 1304.445 1307.622 1310.710
1313.688 1316.535 1319.224 1321.730 1324.022 1326.073 1334.572 1345.336 1355.357 1364.598 1373.029 1380.629 1383.915
1379.410 1370.594 1360.950 1350.502 1339.280 1327.319 1319.094 1316.071 1312.810 1309.337 1305.680 1301.864 1297.913
1293.850 1289.694 1285.465 1281.177 1276.844 1272.478 1268.089 1263.682 1259.265 1254.841 1250.414
 1702.228 1701.684 1701.140 1700.596 1700.054 1699.513 1698.974 1698.438 1697.906 1697.377 1696.854 1696.336 1695.825
1695.321 1694.824 1694.336 1693.856 1693.385 1692.923 1692.469 1692.023 1691.586 1691.156 1690.734 1690.319 1689.912
1689.513 1689.122 1688.738 1688.364 1687.997 1687.640 1687.291 1686.950 1686.617 1686.292 1685.974 1685.662 1685.356
1685.055 1684.759 1684.467 1684.178 1683.893 1683.611 1683.331 1683.053 1682.777 1682.502 1682.228
```

Table C-6. Final contours, case 2.c3.

```
 THE NEW CONTOUR VALUES, Y, FOLLOW
   220.000  221.845  223.683  225.509  227.313  229.089  230.828  232.523  234.167  235.753  237.272  238.716  240.077
 241.348  242.519  243.583  244.530  245.353  246.044  246.595  246.999  247.250  247.344  247.277  247.047  246.654
 246.101  245.388  244.519  243.498  242.328  241.012  239.554  237.957  236.227  234.369  232.393  230.308  228.125
 225.855  223.507  221.089  218.609  216.071  213.480  210.843  208.168  205.462  202.736  200.000
   251.623  253.498  255.392  257.293  259.177  261.030  262.846  264.621  266.350  268.025  269.639  271.186  272.660
 274.050  275.345  276.533  277.605  278.550  279.355  280.006  280.498  280.816  280.956  280.915  280.693  280.287
 279.700  278.935  278.004  276.914  275.671  274.278  272.735  271.046  269.215  267.249  265.161  262.964  260.672
 258.303  255.866  253.369  250.814  248.203  245.537  242.819  240.057  237.263  234.447  231.623
   309.443  311.420  313.526  315.697  317.843  319.940  321.993  324.009  325.987  327.907  329.763  331.568  333.333
 335.036  336.684  338.135  339.505  340.744  341.834  342.746  343.458  343.962  344.255  344.343  344.224  343.875
 343.258  342.373  341.264  339.974  338.519  336.896  335.104  333.146  331.029  328.772  326.388  323.885  321.286
 318.625  315.921  313.167  310.354  307.487  304.568  301.600  298.590  295.551  292.498  289.443
   442.028  445.418  448.880  452.348  455.784  459.206  462.618  466.027  469.418  472.725  475.932  479.175  482.583
 486.080  489.479  492.681  495.692  498.535  501.184  503.567  505.609  507.274  508.581  509.611  510.362  510.567
 509.876  508.291  506.130  503.646  500.898  497.879  494.580  491.005  487.184  483.176  479.013  474.675  470.194
 465.732  461.411  457.184  452.802  448.382  443.943  439.518  435.114  430.730  426.368  422.028
   684.758  696.120  706.490  716.309  726.334  736.809  747.685  758.888  770.505  782.855  796.988  813.373  831.732
 852.453  874.788  897.290  919.475  941.203  962.353  982.669 1001.785 1019.385 1035.244 1048.852 1058.086 1061.185
1057.956 1048.151 1033.429 1016.170  997.242  977.029  955.720  933.448  910.379  886.753  862.903  839.272  816.819
 796.596  779.055  764.002  750.342  737.126  724.257  711.804  699.726  687.912  676.278  664.758
   980.726  998.702 1015.781 1032.198 1048.818 1066.111 1083.912 1101.890 1120.103 1139.021 1159.881 1183.592 1210.260
1239.961 1271.244 1302.237 1329.025 1346.090 1362.660 1378.627 1393.615 1407.198 1419.140 1429.286 1436.598 1439.352
1436.553 1428.600 1417.146 1403.576 1388.502 1372.372 1355.413 1337.697 1319.213 1289.330 1256.251 1223.058 1190.661
1161.109 1136.338 1115.348 1095.497 1075.419 1055.283 1035.610 1016.477  997.673  979.099  960.726
  1270.414 1273.743 1276.935 1279.784 1282.576 1285.699 1289.080 1292.438 1295.774 1299.326 1303.099 1306.958 1311.005
1315.456 1320.326 1325.418 1333.826 1350.891 1367.461 1383.426 1398.416 1411.999 1423.941 1434.087 1441.399 1444.153
1441.354 1433.401 1421.947 1408.377 1393.303 1377.173 1360.214 1342.498 1324.014 1315.513 1309.511 1303.873 1298.513
1293.316 1288.477 1284.093 1279.828 1275.396 1270.916 1266.612 1262.487 1258.424 1254.393 1250.414
  1702.228 1701.647 1701.066 1700.481 1699.896 1699.317 1698.746 1698.179 1697.615 1697.057 1696.506 1695.961 1695.423
1694.894 1694.377 1693.869 1693.368 1692.875 1692.393 1691.923 1691.467 1691.019 1690.579 1690.149 1689.733 1689.329
1688.934 1688.549 1688.173 1687.806 1687.449 1687.104 1686.769 1686.446 1686.131 1685.825 1685.527 1685.238 1684.958
1684.684 1684.418 1684.161 1683.910 1683.660 1683.413 1683.170 1682.932 1682.696 1682.461 1682.228
```

Table C-7. Final contours, case 2.c4.

```
THE NEW CONTOUR VALUES, Y, FOLLOW
  220.000  221.849  223.694  225.532  227.357  229.164  230.947  232.699  234.414  236.083  237.699  239.25[illegible]  240.734
242.133  243.440  244.642  245.728  246.688  247.511  248.186  248.704  249.057  249.239  249.244  249.067  248.708
248.163  247.434  246.522  245.433  244.170  242.740  241.153  239.418  237.545  235.545  233.430  231.211  228.899
226.504  224.036  221.504  218.918  216.286  213.617  210.920  208.203  205.473  202.737  200.000
  251.623  253.492  255.355  257.214  259.064  260.901  262.716  264.502  266.251  267.956  269.609  271.205  272.731
274.176  275.526  276.771  277.898  278.896  279.753  280.459  281.004  281.381  281.583  281.605  281.437  281.075
280.520  279.777  278.847  277.734  276.443  274.982  273.359  271.585  269.671  267.629  265.471  263.208  260.853
258.420  255.916  253.349  250.728  248.061  245.356  242.622  239.864  237.096  234.344  231.623
  309.443  311.824  314.207  316.594  318.982  321.367  323.741  326.099  328.431  330.730  332.986  335.188  337.320
339.366  341.307  343.125  344.802  346.317  347.652  348.789  349.712  350.405  350.858  351.058  350.994  350.662
350.062  349.197  348.071  346.688  345.061  343.202  341.126  338.853  336.401  333.790  331.038  328.166  325.192
322.133  319.003  315.814  312.579  309.309  306.014  302.702  299.380  296.058  292.744  289.443
  442.028  445.709  449.391  453.072  456.753  460.429  464.098  467.751  471.380  474.973  478.514  481.986  485.366
488.632  491.756  494.710  497.464  499.986  502.246  504.214  505.862  507.163  508.095  508.641  508.786  508.522
507.847  506.762  505.276  503.402  501.157  498.564  495.646  492.432  488.951  485.234  481.309  477.208  472.958
468.585  464.11   459.566  454.960  450.309  445.628  440.926  436.211  431.487  426.758  422.0[illegible]
  684.758  699.616  714.513  729.483  744.557  759.753  775.078  790.525  806.069  821.667  837.259  855.367  878.494
903.921  928.912  953.321  976.995  999.774 1021.498 1042.010 1061.164 1078.824 1094.873 1109.216 1121.782 1127.320
1120.607 1106.864 1091.341 1074.105 1055.249 1034.889 1013.156  990.193  966.154  941.196  915.473  889.135  862.324
837.770  818.190  801.075  783.899  766.720  749.579  732.498  715.486  698.536  681.634  664.758
  980.726 1000.366 1020.022 1039.705 1059.422 1079.168 1098.927 1118.668 1138.343 1157.890 1177.226 1199.724 1228.739
1260.672 1291.908 1322.296 1351.684 1373.324 1389.434 1404.547 1418.545 1431.322 1442.782 1452.850 1461.468 1465.130
1460.350 1450.611 1439.418 1426.823 1412.903 1397.747 1381.461 1364.160 1339.492 1308.613 1276.690 1243.878 1210.323
1179.637 1155.419 1134.309 1112.934 1091.375 1069.696 1047.945 1026.156 1004.350  982.538  960.726
  1270.414 1275.657 1280.936 1286.285 1291.736 1297.314 1303.040 1308.927 1314.976 1321.182 1327.523 1333.970 1340.476
1346.987 1353.432 1359.733 1365.801 1378.125 1394.235 1409.348 1423.346 1436.123 1447.583 1457.651 1466.269 1469.931
1465.151 1455.412 1444.219 1431.624 1417.704 1402.548 1386.262 1368.961 1357.242 1350.198 1342.912 1335.469 1327.947
1320.413 1312.922 1305.514 1298.220 1291.055 1284.025 1277.123 1270.336 1263.641 1257.011 1250.414
  1702.228 1701.648 1701.068 1700.491 1699.917 1699.347 1698.782 1698.223 1697.671 1697.127 1696.591 1696.063 1695.545
1695.036 1694.536 1694.047 1693.567 1693.096 1692.636 1692.185 1691.743 1691.310 1690.886 1690.471 1690.064 1689.665
1689.275 1688.892 1688.517 1688.151 1687.792 1687.441 1687.097 1686.762 1686.433 1686.113 1685.800 1685.494 1685.195
1684.902 1684.616 1684.335 1684.060 1683.789 1683.523 1683.260 1682.999 1682.741 1682.484 1682.228
```

Table C-8. Final contours, case 3 (17 weeks plus sediment addition).

```
  220.000  219.071  218.148  217.236  216.342  215.470  214.625  213.812  213.033  212.291  211.589  210.926  210.303
 209.719  209.172  208.660  208.180  207.729  207.305  206.903  206.522  206.158  205.809  205.473  205.147  204.829
 204.516  204.207  203.901  203.596  203.292  202.992  202.695  202.406  202.125  201.857  201.603  201.367  201.149
 200.950  200.773  200.616  200.480  200.363  200.265  200.185  200.121  200.071  200.033  200.000
  251.623  250.708  249.802  248.909  248.035  247.187  246.370  245.589  244.848  244.150  243.495  242.884  242.317
 241.792  241.307  240.857  240.440  240.049  239.682  239.333  238.998  238.673  238.355  238.039  237.724  237.408
 237.089  236.767  236.437  236.101  235.758  235.412  235.066  234.722  234.384  234.056  233.741  233.443  233.165
 232.909  232.676  232.467  232.282  232.119  231.978  231.860  231.766  231.696  231.651  231.623
  309.443  308.558  307.680  306.817  305.980  305.175  304.411  303.694  303.027  302.414  301.855  301.349  300.896
 300.492  300.131  299.808  299.517  299.249  298.999  298.757  298.516  298.268  298.007  297.730  297.434  297.118
 296.781  296.423  296.042  295.640  295.220  294.784  294.336  293.883  293.429  292.979  292.541  292.120  291.723
 291.353  291.013  290.703  290.422  290.167  289.936  289.735  289.578  289.478  289.440  289.443
  442.028  441.318  440.621  439.951  439.321  438.745  438.232  437.789  437.421  437.129  436.913  436.770  436.696
 436.683  436.722  436.798  436.899  437.013  437.127  437.225  437.289  437.302  437.252  437.136  436.952  436.699
 436.373  435.968  435.482  434.916  434.278  433.576  432.822  432.026  431.198  430.354  429.508  428.679  427.882
 427.128  426.422  425.764  425.148  424.568  424.020  423.508  423.043  422.643  422.312  422.028
  684.758  684.608  684.472  684.364  684.297  684.283  684.331  684.448  684.639  684.907  685.250  685.665  686.145
 686.682  687.264  687.876  688.502  689.122  689.717  690.266  690.747  691.140  691.424  691.581  691.597  691.459
 691.160  690.696  690.068  689.284  688.352  687.289  686.110  684.837  683.490  682.091  680.661  679.221  677.788
 676.378  675.004  673.675  672.398  671.175  670.006  668.887  667.813  666.774  665.760  664.758
  980.726  991.845 1002.983 1014.160 1025.389 1036.678 1048.028 1059.430 1070.864 1082.300 1095.693 1108.458 1129.998
 1154.760 1179.183 1203.166 1226.603 1249.386 1271.402 1292.545 1312.709 1331.801 1349.737 1366.447 1381.880 1389.056
 1381.020 1364.718 1347.118 1328.266 1308.222 1287.065 1264.885 1241.782 1217.863 1193.239 1168.019 1142.310 1116.214
 1093.293 1077.103 1064.242 1051.298 1038.315 1025.327 1012.356  999.412  986.497  973.605  960.726
  1270.414 1262.593 1294.812 1307.106 1319.506 1332.036 1344.706 1357.516 1370.450 1383.476 1396.543 1413.058 1436.496
 1463.017 1489.303 1515.137 1540.383 1564.897 1588.535 1611.156 1632.624 1652.816 1671.623 1683.044 1690.753 1694.217
 1689.958 1681.451 1668.760 1648.974 1627.783 1605.291 1581.617 1556.892 1531.254 1504.845 1477.803 1450.263 1422.348
 1397.644 1379.718 1365.183 1350.630 1336.109 1321.650 1307.270 1292.970 1278.740 1264.563 1250.414
  1702.228 1701.147 1700.070 1699.001 1697.943 1696.900 1695.676 1694.874 1693.897 1692.948 1692.029 1691.143 1690.292
 1689.478 1688.742 1687.966 1687.269 1686.613 1685.998 1685.424 1684.891 1684.399 1683.946 1687.080 1691.555 1693.498
 1690.821 1685.609 1682.010 1681.807 1681.635 1681.491 1681.376 1681.287 1681.223 1681.183 1681.164 1681.166 1681.187
 1681.226 1681.280 1681.349 1681.431 1681.523 1681.626 1681.737 1681.854 1681.976 1682.101 1682.228
  2180.943 2179.761 2178.582 2177.412 2176.254 2175.111 2173.988 2172.888 2171.815 2170.771 2169.760 2168.784 2167.840
 2166.989 2166.094 2165.282 2164.516 2163.797 2163.125 2162.501 2161.925 2161.398 2160.919 2160.487 2160.102 2159.763
 2159.468 2159.217 2159.008 2158.840 2158.710 2158.617 2158.559 2158.533 2158.539 2158.573 2158.634 2158.719 2158.828
 2158.956 2159.103 2159.267 2159.445 2159.636 2159.838 2160.048 2160.266 2160.488 2160.715 2160.943
```

Table C-9. Final contours, case 4.

```
THE NEW CONTOUR VALUES, Y, FOLLOW
   220.000  221.328  222.652  223.967  225.270  226.555  227.817  229.050  230.248  231.406  232.517  233.575  234.573
  235.504  236.361  237.136  237.823  238.415  238.904  239.284  239.549  239.693  239.711  239.601  239.358  238.982
  238.471  237.826  237.049  236.143  235.110  233.957  232.688  231.309  229.827  228.249  226.582  224.833  223.011
  221.121  219.172  217.169  215.120  213.032  210.910  208.760  206.588  204.401  202.203  200.000
   288.041  289.564  291.081  292.592  294.092  295.576  297.036  298.467  299.863  301.216  302.521  303.769  304.954
  306.068  307.101  308.046  308.893  309.634  310.260  310.763  311.134  311.368  311.456  311.395  311.181  310.812
  310.287  309.606  308.771  307.786  306.656  305.388  303.988  302.464  300.825  299.079  297.235  295.301  293.288
  291.204  289.056  286.853  284.601  282.308  279.980  277.623  275.242  272.844  270.440  268.041
   442.028  444.472  446.915  449.354  451.787  454.207  456.608  458.984  461.324  463.622  465.865  468.045  470.148
  472.161  474.070  475.857  477.506  478.999  480.317  481.442  482.356  483.045  483.493  483.688  483.621  483.288
  482.685  481.815  480.684  479.299  477.678  475.822  473.760  471.505  469.075  466.490  463.766  460.922  457.975
  454.938  451.825  448.650  445.421  442.140  438.841  435.507  432.152  428.784  425.408  422.028
   573.553  577.760  581.968  586.160  590.395  594.611  598.826  603.032  607.222  611.386  615.512  619.581  623.571
  627.458  631.210  634.791  638.163  641.285  644.113  646.605  648.719  650.415  651.659  652.423  652.683  652.427
  651.650  650.356  648.560  646.285  643.558  640.417  636.901  633.053  628.914  624.530  619.940  615.184  610.296
  605.307  600.244  595.126  589.971  584.790  579.595  574.391  569.182  563.972  558.763  553.553
   646.571  654.696  662.837  671.010  679.232  687.518  695.882  704.336  712.884  721.522  730.234  738.991  747.787
  756.443  765.001  773.328  781.319  788.860  795.828  802.100  807.554  812.074  815.559  817.921  819.097  819.046
  817.756  815.240  811.554  806.753  800.933  794.199  786.669  778.466  769.715  760.537  751.045  741.343  731.518
  721.647  711.788  701.983  692.261  682.634  673.197  663.674  654.322  645.034  635.791  626.571
   724.021  737.477  750.945  764.449  777.975  791.504  805.222  818.958  832.772  846.656  860.583  874.511  888.376
  902.091  915.548  928.618  941.154  952.996  963.970  973.899  982.609  989.931  995.712  999.821 1002.157 1002.653
 1001.281  998.054  993.025  986.283  977.948  968.165  957.098  944.919  931.805  917.927  903.452  888.531  873.297
  857.866  842.330  826.762  811.213  795.714  780.284  764.926  749.634  734.397  719.199  704.021
   805.662  819.050  832.453  845.888  859.370  872.914  886.531  900.226  913.994  927.815  941.649  955.434  969.080
  982.470  995.482 1007.951 1019.720 1030.626 1040.517 1049.255 1056.723 1062.839 1067.595 1070.703 1072.390 1072.551
 1071.178 1068.274 1063.852 1057.940 1050.582 1041.846 1031.821 1020.622 1008.385  995.262  981.416  967.006  952.185
  937.092  921.841  906.525  891.269  875.936  860.735  845.608  830.551  815.553  800.597  785.662
   980.720  986.961  993.178  999.355 1005.475 1011.517 1017.462 1023.292 1028.988 1034.533 1039.910 1045.105 1050.103
 1054.890 1059.450 1063.766 1067.816 1071.574 1075.006 1078.064 1080.703 1082.866 1084.498 1085.546 1085.969 1085.740
 1084.850 1083.310 1081.149 1078.411 1075.147 1071.415 1067.267 1062.752 1057.911 1052.776 1047.371 1041.718 1035.832
 1029.728 1023.423 1016.933 1010.276 1003.472  996.541  989.505  982.383  975.199  967.973  960.726
  1270.414 1270.105 1269.796 1269.486 1269.176 1268.866 1268.556 1268.245 1267.933 1267.620 1267.306 1266.990 1266.672
 1266.353 1266.031 1265.705 1265.376 1265.043 1264.704 1264.358 1264.005 1263.644 1263.274 1262.895 1262.505 1262.105
 1261.695 1261.275 1260.845 1260.407 1259.960 1259.506 1259.045 1258.576 1258.102 1257.621 1257.134 1256.642 1256.144
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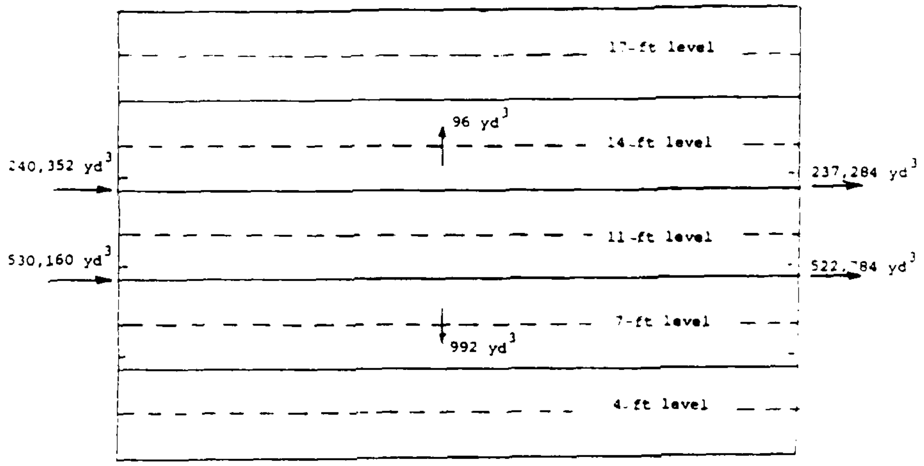


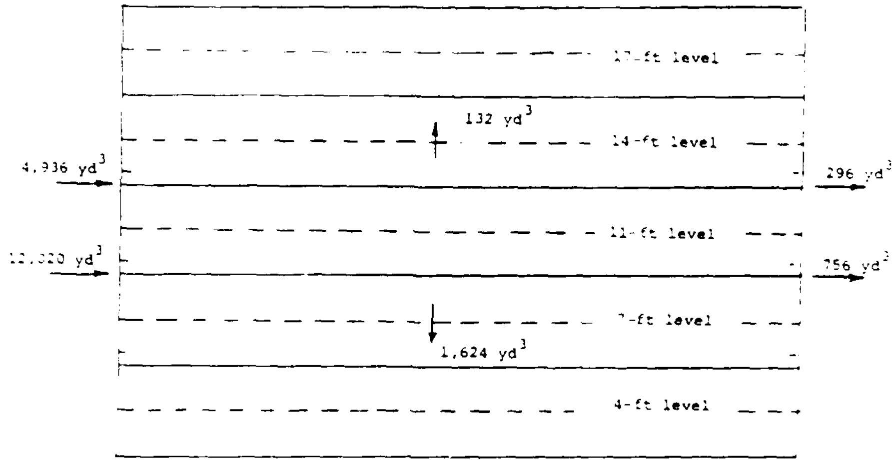
Case 2.a.

Period Considered: Twelve months, January through December, using [illegible] WIS wave hindcasts

Sediment Budget Summary:

Amount of sediment added:	None
Amount of sediment transported shoreward from nourished region:	992 yd^3
Amount of sediment transported seaward from nourished region:	96 yd^3
Net amount of sediment transported alongshore from nourished region:	-10,444 yd^3
Total amount of sediment transported from nourished region:	-9,356 yd^3

Figure C-1. Schematic illustration of sediment volumes transported from region, case 2.a.



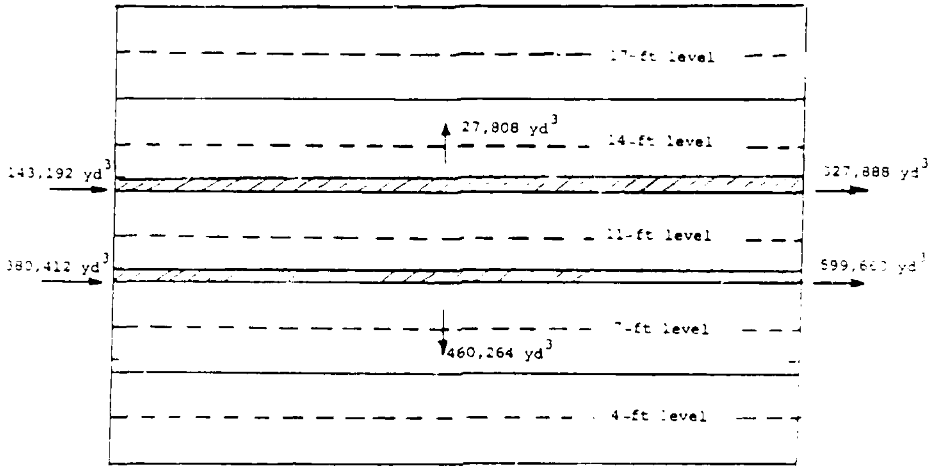
Case No. 2b.

Period considered: Twelve months, January through December, using 197[illegible] WIS wave hindcasts, but wave angle always set equal to [illegible]

Sediment Budget Summary:

Amount of sediment added:	None
Amount of sediment transported shoreward from nourished region:	1,624 yd^3
Amount of sediment transported seaward from nourished region:	132 yd^3
Net amount of sediment transported alongshore from nourished region:	−15,904 yd^3
Total amount of sediment transported from nourished region:	−14,148 yd^3

Figure C-2. Schematic illustration of sediment volumes transported from region, case 2.b.



Case 2.c1:

Period simulated: twelve months, [illegible] through December, using [illegible] WIS wave hindcasts.

Sediment Budget Summary:

Amount of sediment added: [illegible] yd^3 on 7- and 11-ft contours

Amount of sediment transported shoreward from nourished region:	460,264 yd^3 (21.[illegible])
Amount of sediment transported seaward from nourished region:	27,808 yd^3 (1.[illegible])
Net amount of sediment transported alongshore from nourished region:	403,944 yd^3 (27.[illegible])
Total amount of sediment transported from nourished region:	892,016 yd^3 (61.4 [illegible])

Figure C-3. Schematic illustration of sediment volumes transported from nourished region, case 2.c1.

AD-A130 197 A NUMERICAL MODEL TO SIMULATE SEDIMENT TRANSPORT IN THE VICINITY OF COAST..(U) COASTAL AND OFFSHORE ENGINEERING AND RESEARCH INC NEWARK DE M PERLIN ET AL. MAY 83

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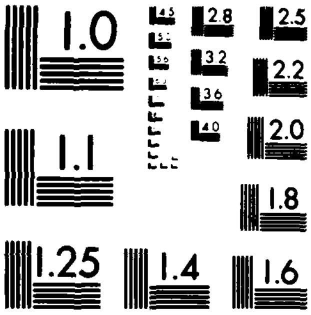
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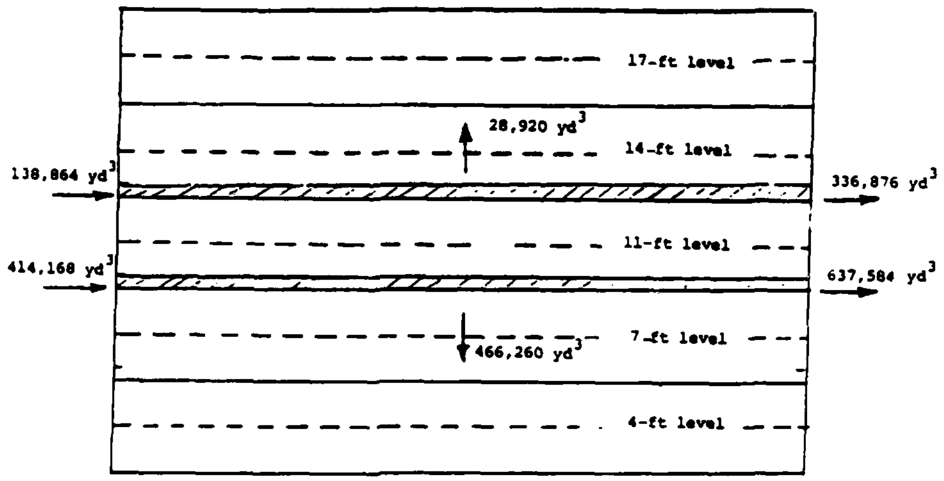


Case 2.c2.

Period considered: Twelve months, April through March, using 1975 WIS wave hindcasts.

Sediment Budget Summary:

Amount of sediment added: 1,452, 000 yd^3 (on 7- and 11-ft contours)

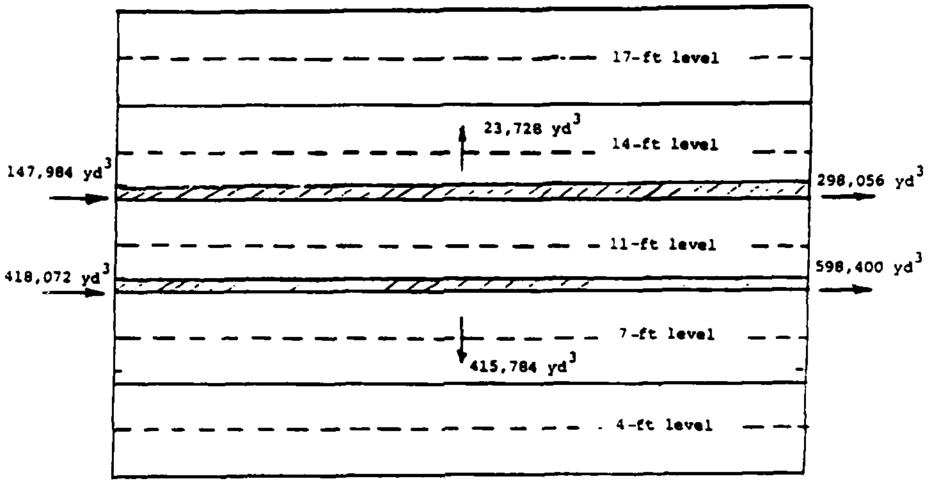
Amount of sediment transported shoreward from nourished region: **466,260** yd^3 (32.1pct)

Amount of sediment transported seaward from nourished region: **28,920** yd^3 (2.0 pct)

Net amount of sediment transported alongshore from nourished region: **421,428** yd^3 (29.0pct)

Total amount of sediment transported from nourished region: **916,608** yd^3 (63.1pct)

Figure C-4. Schematic illustration of sediment volumes transported from nourished region, case 2.c2.



Case 2.c3.

Period considered: Twelve months, July through June, using 1975 WIS wave hindcasts.

Sediment Budget Summary

Amount of sediment added: 1,452,000 yd^3 (on 7- and 11-ft contour)

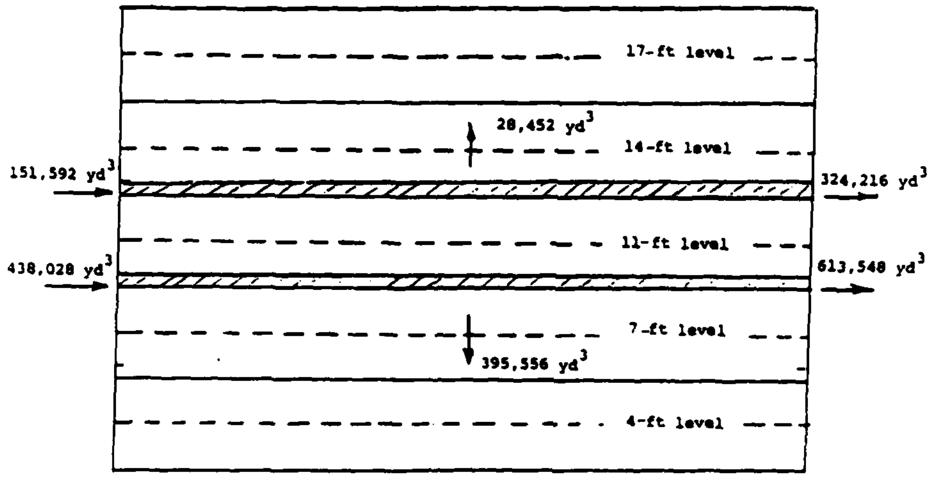
Amount of sediment transported shoreward from nourished region: **415,784 yd^3** (28.6 pct)

Amount of sediment transported seaward from nourished region: **23,728 yd^3** (1.6 pct)

Net amount of sediment transported alongshore from nourished region **330,400 yd^3** (22.8 pct)

Total amount of sediment transported from nourished region: **769,912 yd^3** (53.0 pct)

Figure C-5. Schematic illustration of sediment volumes transported from nourished region, case 2.c3.



Case 2.c4.

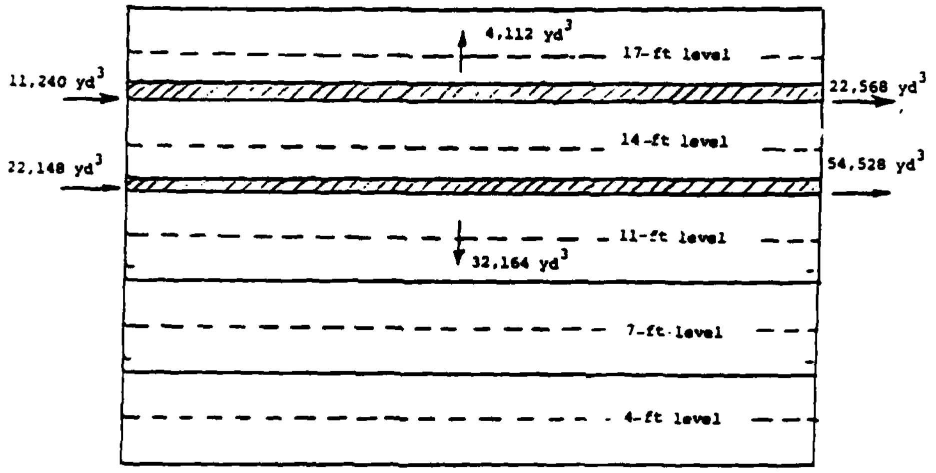
Period considered: Twelve months, October through September, using 1975 WIS wave hindcasts.

Sediment Budget Summary:

Amount of sediment added: 1,452,000 yd^3 (on 7- and 11-ft contours).

Amount of sediment transported shoreward form nourished region:	395,556 yd^3	(27.2 pct)
Amount of sediment transported seaward from nourished region:	28,452 yd^3	(2.0 pct)
Net amount of sediment transported alongshore from nourished region:	348,144 yd^3	(24.0 pct)
Total amount of sediment transported from nourished region:	772,152 yd^3	(53.2 pct)

Figure C-6. Schematic illustration of sediment volumes transported from nourished region, case 2.c4.



Case 3.

Period considered: Four months, January through April, using 1975 WIS wave hindcasts.

Sediment Budget Summary:

Amount of sediment added 363,000 yd^3 (on 11- and 14-ft contours).

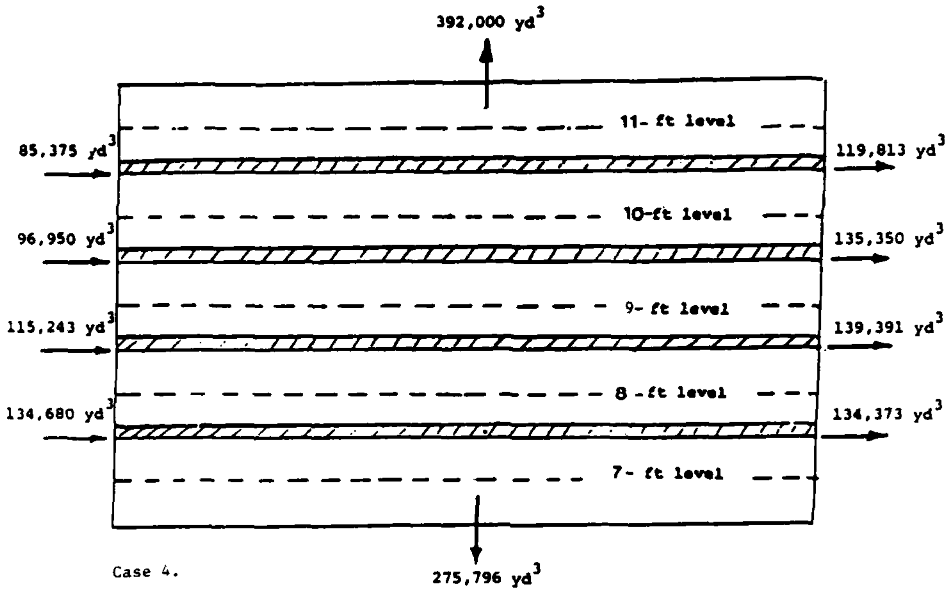
Amount of sediment transported shoreward from nourished region: **32,164 yd^3** (8.9pct)

Amount of sediment transported seaward from nourished region: **4,112 yd^3** (1.1pct)

Net amount of sediment transported alongshore from nourished region: **43,708 yd^3** (12.0pct)

Total amount of sediment transported from nourished region: **79,984 yd^3** (22.0pct)

Figure C-7. Schematic illustration of sediment volumes transported from nourished region, case 3.



Period considered: Twelve months, January through December, using 1975 WIS wave hindcasts.

Sediment Budget Summary:

Amount of sediment added: 1,452,000 yd^3 (on 7-, 8-, 9-, and 10-ft contours).

Amount of sediment transported shoreward from nourished region: **275,796 yd^3 (19.0 pct)**

Amount of sediment transported seaward from nourished region: **392,000 yd^3 (27.0 pct)**

Net amount of sediment transported alongshore from nourished region: **96,679 yd^3 (6.7 pct)**

Total amount of sediment transported from nourished region: **764,475 yd^3 (52.6 pct)**

Figure C-8. Schematic illustration of sediment volumes transported from nourished region, case 4.

APPENDIX D

METHODOLOGY AND PROGRAM LISTING OF COMPUTER PROGRAM WHICH CONVERTS BATHYMETRIC DATA INTO MONOTONICALLY DECREASING DEPTH CONTOURS

In order to simulate prototype shorelines (and in this case to help verify the numerical model via Channel Islands Harbor data), the (x, y, z) data points must be transformed into a form suitable for use in the model (i.e., bars can not be present). First, the bathymetric data have to be put into a form with fixed longshore and offshore spacings (i.e., Δx and Δy equal constants). This can be accomplished using one of the many available canned programs which do the interpolation. The problem is then one of finding the most suitable value of the constant, A, in the equation $h = Ay^{2/3}$. However, as is usually the case, the exact location of the shoreline (h = 0) is unknown. In addition, one requires the added constraint is required that the volumes of sediment (or conversely, the water above the profiles) balance. The problem is solved using LaGrange Multipliers and the Newton Raphson technique for non linear equations.

The equation to be minimized is

$$F(A, ydel_1, ydel_2, \ldots ydel_{IMAX}) = \sum_{i=1}^{IMAX} \sum_{j=1}^{IMAX} (h_{meas_{i,j}} - h_{pred_{i,j}})^2 \qquad \text{(D-1)}$$

where A is the scale parameter in the equilibrium beach profile, $ydel_i$ are the locations of the shoreline for the IMAX profiles, h_{meas} is the interpolated depth from the survey, and h_{pred} is the depth predicted by the equation

$$h_{pred_{i,j}} = A(y_{i,j} - ydel_i)^{2/3} \qquad \text{(D-2)}$$

The constraint equation is as follows

$$g(A, ydel_1, \ldots ydel_{IMAX}) = V_{pred} = \sum_{i=1}^{IMAX} \Delta x \left\{ \int_{ydel_i}^{y_f} A(y - ydel_i)^{2/3}\, dy \right\}$$

$$= \sum_{i=1}^{IMAX} \frac{3}{5} \Delta x\, A(y_f - ydel_i)^{5/3} = V_{meas} \qquad \text{(D-3)}$$

where V_{pred} is the predicted volume of water above the profile to the reference datum, V_{meas} is the measured volume computed from the survey, Δx is the longshore distance between onshore-offshore profiles, and y_f is the distance offshore to the last point on each of the measured profiles (it was a constant after the interpolation routine was used).

LaGrange Multipliers procedure says to form the quantify F* as

$$\cdot \quad F^* = F - \lambda g \qquad \text{(D-4)}$$

take the total differential of equation (D-4)

$$dF^* = dF - \lambda\, dg = \left(\frac{dF}{dA}\, dA + \frac{dF}{d(ydel_1)}\, d(ydel_1) + \ldots \frac{dF}{d(ydel_{IMAX}})d(ydel_{IMAX}) \right)$$

$$- \lambda \left(\frac{dg}{dA}\, dA + \frac{dg}{d(ydel_1)}\, d(ydel_1) + \ldots \frac{dg}{d(ydel_{IMAX}}\, d(ydel_{IMAX}) \right) \qquad \text{(D-5)}$$

Rearranging

$$0 = dF^* = \left(\frac{dF}{dA} - \lambda \frac{dg}{dA} \right) dA + \left(\frac{dF}{d(ydel_1)} - \lambda \frac{dg}{d(ydel_1)} \right) \quad d(ydel_1) + \ldots \qquad \text{(D-6)}$$

It is clear that the terms in brackets in equation (D-6) must individually equal zero, however this leaves (IMAX + 2) unknown (udel i = to IMAX, A, and λ) and only (IMAX = 1) Equations. The (IMAX + 2)th equation is taken as equation (D-3). The following system of equation then results:

$$0 = \frac{dF}{dA} - \lambda \frac{dg}{dA} \quad = \sum_{i=1}^{IMAX} \sum_{j=1}^{JMAX} [-2(h_{meas_{i,j}} - A(y_{i,j} - ydel_i)^{2/3})(y_{i,j} - ydel_i)^{2/3}]$$

$$- \lambda \sum_{i=1}^{IMAX} \frac{3}{5}\, \Delta x\, (y_f - ydel_i)^{5/3} \qquad \text{(D-7-1)}$$

$$0 = \frac{dF}{d(ydel_1)} - \lambda \frac{dg}{d(ydel_1)} = \sum_{j=1}^{JMAX} [2(h_{meas_{1,j}} - A(y_{1,j} - ydel_1)^{2/3})$$

$$*(2/3\ A(y_{1,j} - ydel_1)^{-1/3} \qquad + \lambda\ \Delta x\ A\ (y_f - ydel_1)^{2/3}$$

(D-7-2)

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$$0 = \frac{dF}{d(ydel_{IMAX})} - \lambda \frac{dg}{d(ydel_{IMAX})} = \sum_{j=1}^{JMAX} \Big[2(h\ meas_{IMAX,j} - A(y_{IMAX,j} - ydel_{IMAX})^{2/3})$$

$$*(2/3\ A(y_{IMAXj} - ydel_{IMAX})^{-1/3}] \qquad + \lambda\ \Delta x\ A\ (y_f - ydel_{IMAX})^{2/3}$$

(D-7-(IMAX+1))

$$V_{meas} = \sum_{i=1}^{IMAX} (3/5\ \Delta x\ A(y_f - ydel_I)^{5/3})$$

(D-7-(IMAX+2))

Because Equations (D-7) is a system of nonlinear equations, it can not be written in matrix form as a [D] [x] = [E] system of equations (the brackets denote matrices). To solve the equations, a Newton-Raphson Iteration technique for nonlinear equations was used. This is done by differentiating each of the (IMAX + 2) equations with respect to each of the unknowns, the resulting equations are then linear in terms of Δa, $\Delta ydel_1$, . . . $\Delta ydel_{IMAX}$, $\Delta\lambda$. The resulting matrix is inverted to obtain the Δ(unknown) and the quantities are added to the original estimates to produce a better estimate. This iterative procedure is continued until the changes become acceptably small. The solution converged rapidly. Generally, the first row of the matrix to be inverted is (a_{kl} represents the kth row and the lth column of the matrix).

$$a_{11} = \sum_{i=1}^{IMAX} \sum_{j=1}^{JMAX} 2(y_{i,j} - ydel_i)^{4/3}$$

$$a_{1,2} = \sum_{j=1}^{JMAX} \frac{4}{3} (y_{1,j} - ydel_1)^{-1/3} (h_{meas_{i,j}} - 2A(y_{1,j} - ydel_1)^{2/3})$$

$$a_{1,IMAX+1} = \sum_{j=1}^{JMAX} \frac{4}{3} (y_{IMAX,j} - ydel_{IMAX})^{-1/3} (h\ meas_{IMAX,j}$$

$$- 2A(y_{IMAX,j} - ydel_{IMAX})^{2/3})$$

$$a_{1,IMAX+2} = \sum_{i=1}^{IMAX} [\frac{3}{5}\ \Delta x\ (y_f - ydel_I)^{5/3}] \qquad (D\text{-}8)$$

The second row of the matrix is as follows:

$$a_{2,1} = \sum_{j=1}^{JMAX} [\frac{4}{3} h\ meas_{1,j} (y_{1,j} - ydel_1)^{-1/3} - \frac{8}{3} A(y_{1,j}-ydel_1)^{1/3}]$$

$$+ \lambda \Delta x\ (y_f - ydel_1)^{2/3}$$

$$a_{2,2} = \sum_{j=1}^{JMAX} [\frac{4}{9} A\ h\ meas_{i,j}\ (y_{1,j} - ydel_1)^{-4/3} + \frac{4}{9} A^2\ (y_{1,j} - ydel_1)^{-2/3}]$$

$$- \lambda\ (2/3)\ \Delta x\ A\ (y_f - ydel_1)^{-1/3}$$

$$a_{2,3} = 0$$

$$\vdots$$

$$a_{2,IMAX+1} = 0$$

$$a_{2,IMAX+2} = \Delta x\ A\ (y_f - ydel_1)^{2/3} \qquad (D\text{-}9)$$

The third row is simply these elements repeated except that the ones on the right-hand side of the first and last elements are changed to twos, and the $a_{3,3}$ element is similar to the $a_{2,2}$ except the ones on the right hand side become twos. The remaining column elements (i.e., those when the k = 1) are zeroes. This process is continued to fill the array, except for the last row.

The (IMAX+2)th row is as follows:

$$a_{IMAX+2,1} = \sum_{i=1}^{IMAX} \frac{3}{5}\Delta x \; (y_f - ydel_i)^{5/3}$$

$$a_{IMAX+2,2} = -\Delta x \; A \; (y_f - ydel_1)^{2/3}$$

$$\vdots$$

$$a_{IMAX+2,\; IMAX+1} = -\Delta x \; A \; (Y_f - ydel_{IMAX}) \; 2/3$$

$$a_{IMAX+2,\; IMAX+2} = 0 \qquad (D\text{-}10)$$

The E matrix in the [D] [x] = [E] system of equations is

$$E_1 = - \sum_{i=1}^{IMAX} \sum_{j=1}^{JMAX} - 2(h\;meas_{i,j} - A(y_{i,j} - ydel_i)^{2/3})(y_{i,j} - ydel_i)^{2/3}$$

$$- \lambda \sum_{i=1}^{IMAX} (\tfrac{3}{5}) \; \Delta x \; (y_f - ydel_i)^{5/3}]$$

$$E_2 = - [\sum_{j=1}^{JMAX} \; 2(h\;meas_{i,j} - A(y_{1,j} - ydel_1)^{2/3})((\tfrac{2}{3}) \; A \; (y_{1,j} - ydel_1)^{-1/3})$$

$$+ \lambda \; (\Delta x \; A \; (y_f - ydel_1)^{2/3})]$$

$$E_{IMAX+1} = - [\sum_{j=1}^{JMAX} \; 2(h\;meas_{IMAX,j} - A(y_{IMAX,j} - ydel_{IMAX})^{2/3})$$

$$*((\tfrac{2}{3}) \; A \; (y_{1,j} - ydel_1)^{-1/3}) \qquad + \lambda \; (\Delta x \; A \; (y_f - ydel_1)^{2/3})]$$

$$E_{IMAX+2} = - [\sum_{i=1}^{IMAX} \; ((\tfrac{3}{5}) \; \Delta x \; A(y_f - ydel_I)^{5/3}) - V_{meas}] \qquad (D\text{-}11)$$

The [D] [x] = [E] system of equations was then solved, as explained previously, by solving the x column vector (which represents the changes in the unknowns, ΔA, $\Delta ydel_1$ ... $\Delta ydel_{IMAX}$, $\Delta\lambda$), adding these changes to the respective variables and iterating until a final solution is obtained.

The computer program which did these calculations for the Channel Island Harbor simulation follows. A user-supplied matrix inversion routine is required (Line 37,200).

```
 $RESET FREE
C****************PROGRAM   CIH/BVALUE1
FILE  5(KIND=PACK,TITLE="CIH42076A",FILETYPE=7)
FILE  6(KIND=REMOTE)
C*THIS PROGRAM USES THE INTERPOLATED PROFILES OF CIH.
C*IT FINDS THE LOCATION OF THE SHORELINE, YDEL AND THE BEST
C*FIT LEAST SQUARES  "B" VALUE  FOR H=BY**2/3
C*USES LAGRANGE MULTIPLIERS TO CONSTRAIN THE VOLUMES(SO THEY ARE EQUAL)
C*THEN IT USES NEWTON-RAPHSON ITER FOR NON-LIN EQS
      DIMENSION X(40)
      DIMENSION WKAREA(600),AMATRX(23,23),BMATRX(23,1)
      DIMENSION Y(40,20),Z(40,20),YDEL(40),JBEGIN(40),YDELI(40)
      DIMENSION DYTWO(40,20),DYONE(40,20),DYMTWO(40,20),DYMONE(40,20)
      DIMENSION DYMFOR(40,20),DYFOR(40,20),YDONE(40,20),YDMTWO(40,20)
      DIMENSION YDMONE(40,20),YETWO(40),YEONE(40),YEMONE(40)
      DIMENSION YEMTWO(40),YEMFOR(40),YEFIVE(40)
      EXPON=2./3.
      THIRD=0.3333333333333333
C*FIRST READ IN THE PROFILES FROM DISKPACK.
      DO 1 I=1,34
      DO 1 J=1,15
    1 READ(5,100)   X(I),Y(I,J),Z(I,J)
  100 FORMAT(14X,F6.0,F5.0,F5.0)
C*NOW WE MUST GET A FIRST APPROX FOR YDEL
C*WE WILL USE LINEAR INTERPOLATION TO DETERMINE IT.
      IBEGIN=1
      IMAX=21
      JMAX=15
C*CHANGE PROFILE TO SPAN 1 TO IMAX(IF ALREADY DONE,WON'T HARM THINGS)
      ITEMP1=1
      ITEMP2=IMAX-IBEGIN+1
      K=-1
      DO 777 I=1,ITEMP2
      K=K+1
      DO 777 J=1,JMAX
      Y(I,J)=Y(IBEGIN+K,J)
  777 Z(I,J)=Z(IBEGIN+K,J)
      IMAX=ITEMP2
      DX=100.00
      DO 2 I=1,IMAX
      DO 3 J=1,JMAX
      IF(Z(I,J).GE.0.0)   GO TO 3
C*FIRST NEG POINT ON THE PROFILE IS SEAWARD OF Z=0.0
C* WE MUST ALSO REMEMBER THIS LOCATION.
C*IF Z(I,1)<0.,CHOOSE ARBITRARY PT, ROUTINE ITERATES TO SOLN.
      ZDUM=1.0
      IF(J.NE.1)   ZDUM=Z(I,J-1)
      YDUM=Y(I,J)-50.0
      IF(J.NE.1)   YDUM=Y(I,J-1)
      DELY=ZDUM/((ZDUM-Z(I,J))/(Y(I,J)-YDUM))
      YDEL(I)=YDUM+DELY
      JBEGIN(I)=J
      GO TO 2
    3 CONTINUE
    2 CONTINUE
C*THE VALUES FOR Z ARE NEG ON FILE, MUST NOW MAKE POS.
C*THE Z VALUES ARE ALSO *10.
      DO 35 I=1,IMAX
      DO 35 J=JBEGIN(I),JMAX
   35 Z(I,J)=-Z(I,J)/10.0
C*MUST INITIALIZE "B" SO WILL MAKE A FIRST GUESS.
C*MUST ALSO GUESS LAMBDA  (XLAMB)
      B=0.30
      XLAMB=-2.0
      DO 10 ITER=1,100
C*LET'S  CALCULATE THE VOL OF WATER ABOVE THE PROFILE,VMEAS.
C*ITS OUR CONSTRAINT,BUT SINCE YDEL IS NOT KNOWN,A PRIORI,IT WILL CHANGE
      VMEAS=0.0
      DO 200 I=1,IMAX
      DO 200 J=JBEGIN(I),JMAX
      IF(J.NE.JBEGIN(I))   GO TO 201
```

```
      VMEAS=VMEAS+DX*Z(I,J)*(0.5*(Y(I,J)+Y(I,J+1))-YDEL(I))
      GO TO 200
  201 IF(J.EQ.JMAX)   GO TO 202
      VMEAS=VMEAS+DX*0.5*(Y(I,J+1)-Y(I,J-1))*Z(I,J)
      GO TO 200
  202 VMEAS=VMEAS+DX*Z(I,J)*(Y(I,J)-0.5*(Y(I,J)+Y(I,J-1)))
  200 CONTINUE
C*PRIOR TO EQS,COMPUTE AND STORE SEVERAL VALUES WE NEED OVER AND OVER
C*BECAUSE COMPUTER CAN'T RAISE A NEG VALUE TO AN EXPONENT
C*MUST PRESERVE THE SIGN.
      DO  400 II=1,IMAX
      DO  401 JJ=JBEGIN(II),JMAX
      ARG1=Y(II,JJ)-YDEL(II)
      DYSIGN=SIGN(1.,ARG1)
      DY=ABS(Y(II,JJ)-YDEL(II))
      DYTWO(II,JJ)=DY**EXPON
       DYONE(II,JJ)=DYSIGN*DY**THIRD
      DYMTWO(II,JJ)=DY**(-EXPON)
      DYMONE(II,JJ)=DYSIGN*DY**(-THIRD)
      DYMFOR(II,JJ)=DY**(-2.*EXPON)
      DYFOR(II,JJ)=DY**(2.*EXPON)
  401 CONTINUE
      ARG2=1400.-YDEL(II)
      DSIGN=SIGN(1.,ARG2)
      DYE=ABS(ARG2)
      YETWO(II)=DYE**EXPON
      YEONE(II)=DSIGN*DYE**THIRD
      YEMONE(II)=DSIGN*DYE**(-THIRD)
      YEMTWO(II)=DYE**(-EXPON)
      YEMFOR(II)=DYE**(-2.*EXPON)
      YEFIVE(II)=DSIGN*DYE**(5.*THIRD)
  400 CONTINUE
C*LET'S INPUT THE FIRST ROW OF THE MATRIX, A
      SUM1B=0.0
      DO 300 II=1,IMAX
      DO 300 JJ=JBEGIN(II),JMAX
  300 SUM1B=SUM1B+2.*DYFOR(II,JJ)
      AMATRX(1,1)=SUM1B
      SUMLAM=0.0
      DO 305 K=1,IMAX
      SUM1K=0.0
      DO 306  JJ=JBEGIN(K),JMAX
  306 SUM1K=SUM1K+2.*EXPON*DYMONE(K,JJ)*(Z(K,JJ)-2.*B*
     *   DYTWO(K,JJ))
      SUMLAM=SUMLAM-0.6*DX*YEFIVE(K)
  305 AMATRX(1,K+1)=SUM1K
      AMATRX(1,IMAX+2)=SUMLAM
C*NOW THE MIDDLE ROWS OF THE AMATRX.
      DO 410  LROW=2,IMAX+1
      SUM2B=0.0
      II=LROW-1
      DO 415 JJ=JBEGIN(II),JMAX
  415 SUM2B=SUM2B+2.*EXPON*Z(II,JJ)*DYMONE(II,JJ)-4.*EXPON*
     *   B*DYONE(II,JJ)
      AMATRX(LROW,1)=SUM2B+XLAMB*DX*YETWO(II)
      DO 430 II=1,IMAX
      SUM2Y=0.0
      DO 425  JJ=JBEGIN(II),JMAX
  425 SUM2Y=SUM2Y+2.*EXPON*THIRD*B*Z(II,JJ)*DYMFOR(II,JJ)+THIRD*EXPON
     *   *2.*B*B*DYMTWO(II,JJ)
      IF((II+1).EQ.LROW)   GO TO 431
      AMATRX(LROW,II+1)=0.0
      GO TO 430
  431 AMATRX(LROW,II+1)=SUM2Y-XLAMB*EXPON*DX*B*YEMONE(II)
  430 CONTINUE
  410 AMATRX(LROW,IMAX+2)=DX*B*YETWO(LROW-1)
C*NOW THE LAST ROW OF THE MATRIX A
      SUMFB=0.0
      DO 450   II=1,IMAX
  450 SUMFB=SUMFB+0.6*DX*YEFIVE(II)
      AMATRX(IMAX+2,1)=SUMFB
```

```
      DO 453   II=1,IMAX
  453 AMATRX(IMAX+2,II+1.)=-DX*B*YETWO(II)
      AMATRX(IMAX+2,IMAX+2)=0.0
C*NOW MUST INPUT THE BMATRX.
      SUMF1A=0.0
      SUMF1B=0.0
      DO 455   II=1,IMAX
      SUMF1B=SUMF1B+XLAMB*0.6*DX*YEFIVE(II)
      DO 455    JJ=JBEGIN(II),JMAX
  455 SUMF1A=SUMF1A-2.*(Z(II,JJ)-B*DYTWO(II,JJ))*DYTWO(II,JJ)
      BMATRX(1,1)=-(SUMF1A-SUMF1B)
      DO 460   II=1,IMAX
      SUMFII=0.0
      DO 462    JJ=JBEGIN(II),JMAX
  462 SUMFII=SUMFII+2.*(Z(II,JJ)-B*DYTWO(II,JJ))*EXPON*B*DYMONE(II,JJ)
      SUMFII=SUMFII+XLAMB*DX*B*YETWO(II)
  460 BMATRX(II+1,1)=-SUMFII
      SUMV=0.0
      DO 465   II=1,IMAX
  465 SUMV=SUMV+0.6*DX*B*YEFIVE(II)
      BMATRX(IMAX+2,1)=-(SUMV-VMEAS)
C*NEXT LET'S CALL THE MATRIX INVERSION ROUTINE VIA IMSL
      CALL LEQT2F(AMATRX,1,IMAX+2,23,BMATRX,3,WKAREA,IER)
C*THE SOLN IS RETURNED IN THE VECTOR BMATRX
C*FINALLY, WE MUST UPDATE THE X VECTOR IN AX=B.
      B=B+BMATRX(1,1)
      XLAMB=XLAMB+BMATRX(IMAX+2,1)
      DO 470   II=1,IMAX
  470 YDEL(II)=YDEL(II)+BMATRX(II+1,1)
C*CHECK THE CRITERION FOR COMPLETION
      SUMVEC=0.0
      DO 475    II=1,IMAX
  475 SUMVEC=SUMVEC+ABS(BMATRX(II,1))
      IF(SUMVEC.LT.(0.1*(IMAX+2)))   GO TO 11
      WRITE(6,*/) B,ITER,(I,YDEL(I),I=1,IMAX),XLAMB
   10 CONTINUE
   11 CONTINUE
C*LET'S WRITE IT ALL OUT.
      WRITE(6,*/)   ITER,B,(I,YDEL(I),I=1,IMAX)
      STOP
      END
```

APPENDIX E

USER DOCUMENTATION AND INPUT AND OUTPUT FOR PROGRAM VERIFICATION

The computer program presented in Appendix B was run on a Burroughs B-7700 computer. The B7000/B6000 series FORTRAN language was designed so several existing programs written in FORTRAN would be compatible with minimal changes. It was designed to be compatible with Fortram IV, H level and to contain ANSI X3.9-1966 Standard FORTRAN as a subset.

Line 37,200 of the coding (see App. B) requires a subroutine from the IMSL subroutine package, LEQT1B and its associated subroutines. If the user's computing center has access to this package of subroutine programs they need only bind them to the program (note: copyright laws prohibited the inclusion of the IMSL coding). If not, a substitute subroutine must be user supplied. It must facilitate the solution of a banded storage mode matrix.

The program input will be described here using a card deck set-up, however, the use of diskpack or magnetic tape input follows directly. Lines 3100, 4100, 5500, 5900, 6800, 7500, and 12,900 are read statements. The cards used for the simulation presented in this appendix are shown in Figure E-1. The first card contains the value of WDEPTH, the depth of water (in meters) to which the input wave conditions are to be transformed (a partial list of variables used in the program is presented beginning on page A-8 of Appendix A). The format statements are obviously in the program coding.

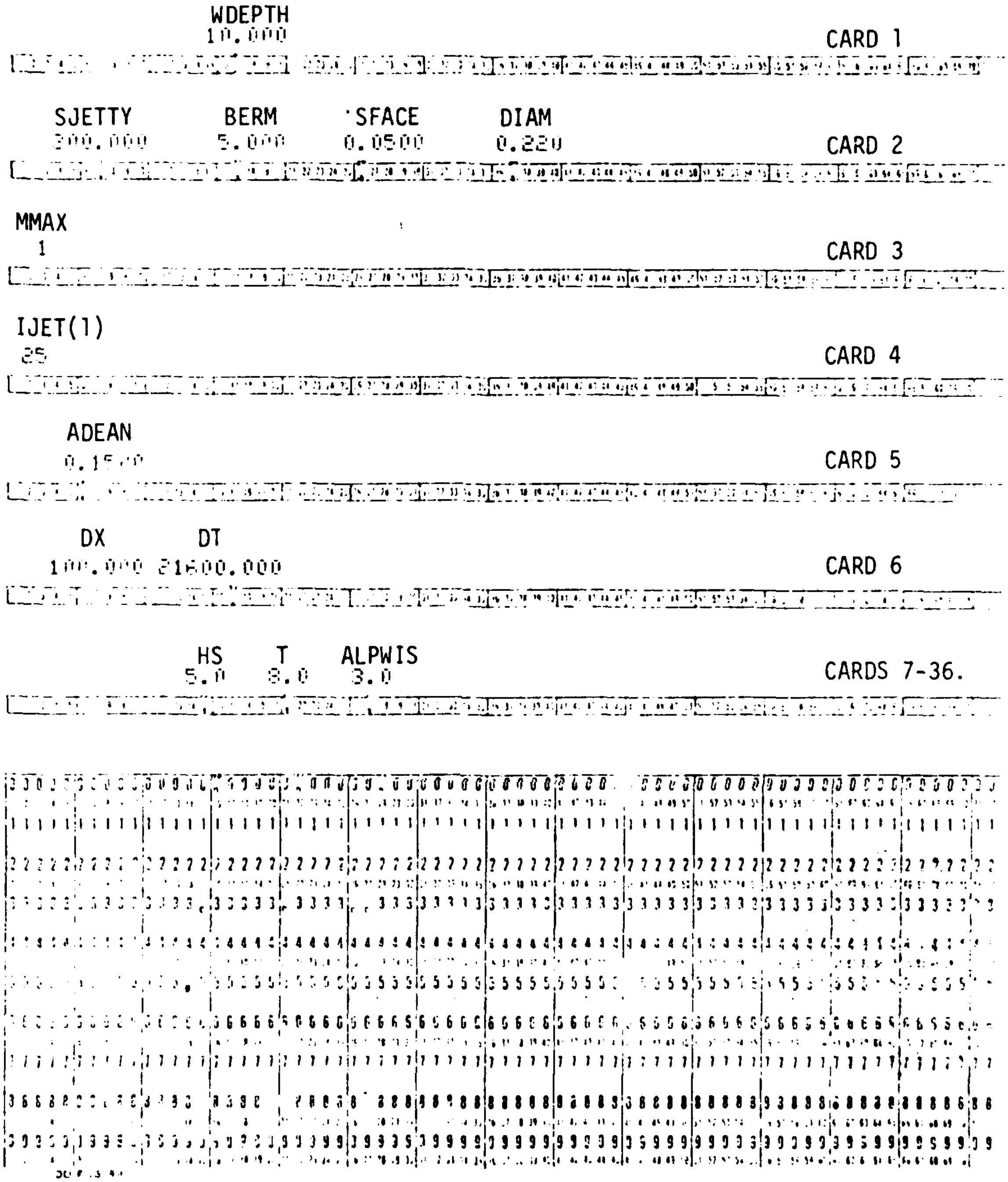
The second data input card is read by line 4100 where the variables SJETTY, BERM, SFACE, and DIAM are required (length of the structure, berm height, shore face slope, and sediment diameter, respectively).

Lines 5500 reads MMAX, the number of structures to be simulated (as set-up here, a maximum of 10 structures can be modeled, however, appropriate changes in array dimensions would allow additions (structures). Line 5900, which is in a "DO" loop reads the lesser I grid value adjacent to where the structure is desired. The number of structures, MMAX, determines the number of data cards required here; 3 structures require 3 cards with the 3 I grid locations (note, the present configuration of the refraction and diffraction subroutines requires evenly spaced structures, however this can be altered if necessary).

The parameter ADEAN, which represents the value of A in the equilibrium profile used is the next value input (line 6800). As mentioned previously, whenever possible a site-specific value should be used. The space-step and time-step (DX and DELT in the coding) are input next (line 7500).

The last input values are the wave data, HS, T, ALPWIS read by line 12,900. This statement is in a loop made by the unconditional GO TO statement (line 16,400) and the read statement. There is an action specifier included in the read statement to transfer the program to statement 1000, thereby stopping execution of the program once all the wave climate data have been used. The number of data cards required for this read statement is dictated by the length of the simulation and the time-step used.

The input file and output for program verification follow.



WDEPTH
10.000
CARD 1

SJETTY BERM SFACE DIAM
300.000 5.000 0.0500 0.220
CARD 2

MMAX
1
CARD 3

IJET(1)
25
CARD 4

ADEAN
0.1500
CARD 5

DX DT
100.000 21600.000
CARD 6

HS T ALPWIS
5.0 8.0 3.0
CARDS 7-36.

Figure E-1. Card deck input for program verification.

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INPUT: FILE DUM

```
100               10.000
200   300.000      5.000    0.0500     0.220
300  1
400 25
500     0.1500
600     100.00   21600.00
700               5.0   8.0   3.0
800               5.0   8.0   3.0
900               5.0   8.0   3.0
1000               5.0   8.0   3.0
1100               5.0   8.0   3.0
1200               5.0   8.0   3.0
1300               5.0   8.0   3.0
1400               5.0   8.0   3.0
1500               5.0   8.0   3.0
1600               5.0   8.0   3.0
1700               5.0   8.0   3.0
1800               5.0   8.0   3.0
1900               5.0   8.0   3.0
2000               5.0   8.0   3.0
2100               5.0   8.0   3.0
2200               5.0   8.0   3.0
2300               5.0   8.0   3.0
2400               5.0   8.0   3.0
2500               5.0   8.0   3.0
2600               5.0   8.0   3.0
2700               5.0   8.0   3.0
2800               5.0   8.0   3.0
2900               5.0   8.0   3.0
3000               5.0   8.0   3.0
3100               5.0   8.0   3.0
3200               5.0   8.0   3.0
3300               5.0   8.0   3.0
3400               5.0   8.0   3.0
3500               5.0   8.0   3.0
3600               5.0   8.0   3.0
```

```
****************************************************
 TO WHAT DEPTH ARE THE WAVES TO BE TRANSFORMED
 THE DEPTH (IN FT) WAVES TRANSFORMED TO  WDEPTH=      32 808
****************************************************
 ITS TIME FOR SJETTY, BERM, SFACE, AND DIAM

 THE LENGTH OF THE STRUCTURE, SJETTY=     300 000
 THE HEIGHT OF THE BERM  BERM=        5 000
 THE SLOPE OF THE BEACH FACE, SFACE=       0 0500
 THE SEDIMENT DIAMETER, DIAM=       0 220
****************************************************
 THE NUMBER     1 GROIN IS LOCATED AT GRID   25
****************************************************
 NOW ENTER THE VALUE OF ADEAN
 THE VALUE OF ADEAN=      0 1500  IN THE EQ  H=AY**2/3
****************************************************
 READ IN THE SPACE STEP,TIMESTEP

 THE VALUE OF THE LONGSHORE SPACE STEP, DX=     100 000
 THE TIME STEP IN SECONDS, DELT=  21600 000
****************************************************
 THE BOUNDARY Y VALUES, I=1,IMAX ARE AS FOLLOWS

     0 00    31 62    68 01   137 71   252 98   464 76   760 73 1050 41 1656 50 2674 85
     0 00    31 62    68 01   137 71   252 98   464 76   760 73 1050 41 1656 50 2674 85
****************************************************

 THE DEPTHS BETWEEN CONTOURS ARE AS FOLLOWS

     1 00     2 00     3 00     5 00     7 00    11 00    14 00    17 00    25 00    32 81
****************************************************

NUNIV=1.

NUNIV=2.

NUNIV=3.

NUNIV=4.

NUNIV=5.

NUNIV=6.

NUNIV=7.

NUNIV=8.

NUNIV=9.

NUNIV=10.
 THE TOTAL ELAPSED NUMBER OF TIME-STEPS, NUNIV=     10


 THE LONGSHORE TRANSPORTS QX, FOLLOW
     0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
   0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
   0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
   0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
```

```
THE NEW CONTOUR VALUES, Y, FOLLOW
      0 000   -0 000    0 000    0 000    0 000   -0 000   -0 000   -0 000   -0 000    0 000    0 000    0 000    0 000
    0 001    0 002    0 003    0 005    0 009    0 016    0 029    0 057    0 092    0 150    0 219    0.273    0 273
   -0 220   -0 150    0 092    0 053    0 029   -0 016   -0 009   -0 005   -0 003   -0 001   -0 001   -0 000   -0 000
    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
     31 623   31 623   31 623   31 623   31 623   31 623   31 623   31 623   31 624   31 625   31 627   31 630   31 636
   31 644   31 657   31 677   31 708   31 758   31 835   31 958   32 148   32 434   32 842   33 393   33 886   29 357
   29 850   30 402   30 811   31 098   31 288   31 411   31.489   31 538   31 569   31 589   31 602   31 610   31 615
   31 619   31 620   31 622   31 622   31 623   31 623   31 623   31 623   31 623   31 623   31 623
     68 041   68 041   68 042   68 043   68 044   68 045   68 048   68 052   68 059   68 069   68 084   68 107   68 140
   68 187   68 257   68 356   68 497   68 697   68 978   69 369   69 907   70 638   71 595   72 851   73 990   62 084
   63 226   64 485   65 443   66 176   66 715   67 107   67 388   67 588   67 729   67 828   67 897   67 944   67 977
   67 999   68 014   68 024   68 031   68 035   68 038   68 039   68 040   68 041   68 041   68 041
    137 706  137 711  137 717  137 724  137 734  137 747  137 767  137 794  137 833  137 886  137 960  138 061  138 196
  138 377  138 612  138 924  139 322  139 829  140 469  141 266  142 250  143 453  144 892  146 676  148 273  127 129
  128 728  130 514  131 956  133 161  134 146  134 945  135 585  136 093  136 491  136 801  137 038  137 218  137 354
  137 454  137 528  137 581  137 619  137 647  137 666  137 679  137 689  137 695  137 701  137 706
    252 982  253 009  253 036  253 065  253 097  253 133  253 174  253 222  253.277  253.341  253 414  253 498  253 594
  253 702  253 822  253 954  254 098  254 252  254 413  254 576  254 737  254 887  255 017  255 114  255 168  250 794
  250 848  250 945  251 075  251 225  251 386  251 550  251 712  251 866  252 010  252 143  252 264  252 372  252 467
  252 552  252 625  252 689  252 744  252 791  252 832  252 868  252 900  252.928  252 955  252 982
    464 758  464 758  464 758  464 759  464 759  464 759  464 759  464 760  464 760  464 760  464 761  464 761  464 762
  464 763  464 763  464 764  464 764  464 765  464 765  464 765  464 765  464.764  464.763  464 762  464 760  464 757
  464 754  464 752  464 751  464 750  464 750  464 750  464 751  464 751  464 752  464 753  464 754  464 754  464 755
  464 756  464 756  464 757  464 757  464 757  464 757  464 758  464 758  464 758  464.758  464 758
    760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726
  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726
  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726
  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726  760 726
   1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414
 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414
 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414
 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414 1050 414

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NUNIV=12

NUNIV=13.

NUNIV=14.

NUNIV=15.

NUNIV=16.

NUNIV=17.

NUNIV=18.

NUNIV=19.

NUNIV=20.
 THE TOTAL ELAPSED NUMBER OF TIME STEPS  NUNIV=     20


 THE LONGSHORE TRANSPORTS, QS, FOLLOW
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    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
```

```
    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002
 0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 000
 0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002
 0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002    0 002
    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011
 0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 010    0 010    0 009    0 007    0 008    0 000
 0 008    0 007    0 009    0 010    0 010    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011
 0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011    0 011
    0 129    0 129    0 129    0 129    0 129    0 128    0 126    0 128    0 128    0 127    0 126    0 125    0 124
 0 122    0 119    0 116    0 112    0 107    0 101    0 093    0 084    0 073    0 062    0 043    0 054    0 000
 0 054    0 043    0 062    0 074    0 084    0 093    0 101    0 107    0 112    0 116    0 119    0 122    0 124
 0 125    0 126    0 127    0 128    0 128    0 128    0 128    0 129    0 129    0 129    0 129
    0 360    0 360    0 360    0 360    0 359    0 359    0 358    0 358    0 357    0 356    0 355    0 353    0 352
 0 351    0 349    0 338    0 346    0 335    0 334    0 344    0 344    0 345    0 347    0 351    0 356    0 363
 0 356    0 351    0 337    0 335    0 334    0 334    0 344    0 345    0 346    0 348    0 349    0 351    0 352
 0 351    0 355    0 356    0 357    0 358    0 358    0 359    0 360    0 360    0 360    0 360
    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379
 0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379
 0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379
 0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379    0 379
    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001
 0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001
 0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001
 0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001    0 001
    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
 0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
 0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000
 0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000    0 000

THE ON OFFSHORE TRANSPORTS  QY. FOLLOW
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000    0 0000   -0 0000
 0 0000    0 0000    0 0000    0 0001    0 0001   -0 0001   -0 0002    0 0004   -0 0006   -0 0008   -0 0010   -0 0011    0 0011
 0 0010    0 0008    0 0006    0 0004    0 0002    0 0001    0 0001    0 0001    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0001    0 0001    0 0002    0 0003    0 0004   -0 0007   -0 0010   -0 0015   -0 0019    0 0019
 0 0015    0 0010    0 0007    0 0004    0 0003    0 0002    0 0001    0 0001    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000    0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000   -0 0001
 0 0001    0 0002    0 0002    0 0003    0 0005    0 0007    0 0009    0 0013    0 0017   -0 0023   -0 0030    0 0036    0 0036
 0 0030    0 0023    0 0017    0 0013    0 0009    0 0007    0 0005    0 0003    0 0002    0 0002    0 0001    0 0001    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0001   -0 0001   -0 0001    0 0002    0 0003    0 0001
 0 0005    0 0006    0 0009    0 0011    0 0014    0 0018    0 0022    0 0026   -0 0031   -0 0036   -0 0041   -0 0046    0 0046
 0 0041    0 0036    0 0031    0 0026    0 0022    0 0018    0 0014    0 0011    0 0009    0 0006    0 0005    0 0004    0 0003
 0 0002    0 0001    0 0001    0 0001    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
    0 0000    0 0000    0 0000    0 0001    0 0001    0 0001    0 0001    0 0001   -0 0002   -0 0002    0 0002    0 0002   -0 0001
 0 0001    0 0000    0 0002    0 0004    0 0008    0 0012    0 0019    0 0027    0 0037    0 0050    0 0067    0 0082    0 0082
 0 0067    0 0050    0 0037    0 0027    0 0018    0 0012   -0 0008    0 0004   -0 0002   -0 0000    0 0001    0 0001    0 0002
 0 0002    0 0002    0 0002    0 0001    0 0001    0 0001    0 0001    0 0001    0 0000    0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001
 0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0000   -0 0000    0 0000   -0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000    0 0000    0 0000   -0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000    0 0000    0 0000    0 0000   -0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000    0 0000    0 0000
    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
 0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
```

```
THE NEW CONTOUR VALUES, Y, FOLLOW
    0.000    0.000    0.001    0.001    0.002    0.003    0.004    0.006    0.008    0.011    0.015    0.021    0.029
  0.040    0.056    0.078    0.109    0.152    0.212    0.294    0.401    0.534    0.685    0.831    0.928   -0.929
  0.832    0.686    0.535    0.401    0.294   -0.212   -0.151   -0.108   -0.078   -0.056   -0.040   -0.029   -0.021
 -0.015   -0.011   -0.008   -0.005    0.004    0.003   -0.002   -0.001   -0.001   -0.000    0.000
   31.621   31.626   31.631   31.636   31.643   31.650   31.661   31.674   31.692   31.714   31.744   31.784   31.835
  31.902   31.988   32.101   32.248   32.438   32.683   32.997   33.396   33.893   34.491   35.191   35.766   27.475
  28.051   28.751   29.351   29.849   30.248   30.563   30.809   30.999   31.146   31.258   31.345   31.412   31.463
  31.502   31.532   31.555   31.572   31.585   31.595   31.603   31.610   31.615   31.619   31.623
   68.011   68.058   68.076   68.096   68.119   68.147   68.182   68.226   68.281   68.351   68.439   68.550   68.690
  68.864   69.083   69.356   69.695   70.115   70.634   71.274   72.057   73.013   74.157   75.548   76.758   59.315
  60.527   61.920   63.066   64.024   61.809   65.449   65.969   66.390   66.729   67.002   67.221   67.395   67.535
  67.645   67.733   67.803   67.858   67.902   67.936   67.964   67.988   68.007   68.025   68.041
  137.706  137.763  137.819  137.878  137.945  138.022  138.114  138.225  138.358  138.519  138.713  138.947  139.229
 139.568  139.973  140.457  141.032  141.714  142.521  143.471  144.586  145.895  147.408  149.231  150.840  124.560
 126.171  127.997  129.512  130.822  131.939  132.891  133.698  134.381  134.957  135.441  135.847  136.185  136.468
 136.702  136.896  137.057  137.190  137.300  137.392  137.469  137.536  137.594  137.650  137.706
  252.982  253.067  253.150  253.234  253.321  253.411  253.507  253.608  253.716  253.831  253.952  254.081  254.217
 254.359  254.505  254.655  254.805  254.951  255.090  255.215  255.319  255.393  255.427  255.404  255.349  250.614
 250.559  250.535  250.569  250.643  250.747  250.872  251.011  251.158  251.308  251.458  251.605  251.747  251.883
 252.012  252.134  252.249  252.357  252.458  252.554  252.645  252.731  252.815  252.898  252.982
  464.758  464.759  464.760  464.761  464.763  464.764  464.765  464.766  464.767  464.768  464.770  464.771  464.772
 464.773  464.774  464.775  464.775  464.775  464.775  464.774  464.773  464.771  464.769  464.766  464.761  464.756
 464.752  464.748  464.744  464.742  464.741  464.740  464.739  464.740  464.740  464.741  464.742  464.744  464.745
 464.746  464.748  464.749  464.750  464.752  464.753  464.754  464.755  464.756  464.757  464.758
  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726
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  1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414
 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414 1050.414
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NUNIV=23.

NUNIV=24.

NUNIV=25.

NUNIV=26.

NUNIV=27.

NUNIV=28.

NUNIV=29.

NUNIV=30.
 THE TOTAL ELAPSED NUMBER OF TIME STEPS, NUNIV=     30


 THE LONGSHORE TRANSPORTS, Q, FOLLOW
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  0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000
  0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000
  0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000    0.000
```

```
     0 002     0 002     0 002     0 002     0 002     0 002     0 002       002     0 002     0 002     0 002     0 002     0 002
   0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 000
   0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0.002     0 002     0 002     0 002
   0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0 002     0.002     0.002
     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011
   0 011     0 011     0 010     0 010     0 010     0 010     0 009     0 009     0 008     0 008     0 006     0 008     0 000
   0 008     0 006     0 008     0 008     0 009     0 009     0 010     0 010     0 010     0 010     0 011     0 011     0 011
   0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011     0 011
     0 125     0 125     0 125     0 125     0 124     0 124     0 123     0 122     0 121     0 120     0 118     0 116     0 114
   0 111     0 108     0 104     0 100     0 095     0 089     0 082     0 074     0 065     0 055     0 038     0 051     0 000
   0 051     0 038     0 055     0 065     0 074     0 082     0 089     0 095     0 100     0.104     0 108     0 111     0 114
   0 116     0 118     0 120     0 121     0 122     0 123     0 124     0 124     0 125     0 125     0 125
     0 349     0 349     0 348     0 348     0 348     0 348     0 347     0 347     0 346     0 346     0 345     0 345     0 345
   0 345     0 345     0 345     0 346     0 347     0 349     0 352     0 355     0 360     0 366     0 374     0 377     0 398
   0 377     0 374     0 366     0 360     0 355     0 352     0 349     0 347     0 346     0 345     0 345     0 345     0 345
   0 345     0 345     0 346     0 346     0 347     0 347     0 348     0 348     0 348     0 348     0 349
     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0.379     0.379     0.379     0 379     0.379
   0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 380     0 380     0 380     0 380
   0 380     0 380     0 380     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379
   0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0 379     0.379     0 379
     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0.001     0 001     0.001
   0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001
   0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0.001     0 001     0.001     0 001
   0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001     0 001
     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0.000     0.000     0.000     0 000
   0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0.000     0 000     0 000     0 000
   0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0.000     0 000     0.000     0 000     0 000
   0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000     0 000

THE ON OFFSHORE TRANSPORTS  QY, FOLLOW
     0 0000    0 0000    0 0000    0 0000    0 0001   -0 0001   -0 0001   -0 0001   -0 0001   -0.0002   -0 0002   -0 0003   -0 0003
   0 0004    0 0005    0 0006    0 0008    0 0009   -0 0011   -0 0013   -0 0015   -0 0017   -0 0018   -0 0019   -0 0020    0 0020
   0 0019    0 0018    0 0017    0 0015    0 0013    0 0011    0 0009    0 0008    0 0006    0.0005    0.0004    0.0003    0 0003
   0 0002    0 0002    0 0001    0 0001    0 0001    0 0001    0 0001    0 0000    0 0000    0.0000    0.0000
     0 0000    0 0000    0 0000    0 0000    0 0001   -0 0001   -0 0001   -0 0001   -0 0002   -0.0002   -0.0003   -0.0003   -0.0004
   0 0005    0 0006    0 0007    0 0009    0 0011   -0 0013   -0 0016   -0 0019   -0 0022   -0.0027   -0.0032   -0.0037    0.0037
   0 0033    0 0027    0 0022    0 0019    0 0016    0 0013    0 0011    0 0009    0 0007    0.0006    0.0005    0 0004    0 0003
   0 0003    0 0002    0 0002    0 0001    0 0001    0 0001    0 0001    0 0000    0 0000    0.0000    0.0000
     0 0000    0 0000    0 0001    0 0001    0 0001   -0 0002   -0 0002   -0 0003   -0 0003   -0 0004   -0 0005   -0 0006   -0 0007
   0 0008    0 0010    0 0012   -0 0014    0 0016   -0 0019   -0 0022   -0 0026   -0 0030   -0.0035   -0.0042   -0.0048    0.0048
   0 0042    0 0035    0 0030    0 0026    0 0022    0 0019    0 0016    0 0014    0 0012    0.0010    0.0008    0.0007    0.0006
   0 0005    0 0004    0 0003    0 0003    0 0002    0 0002    0 0001    0 0001    0 0001    0.0000   -0 0000
     0 0000    0 0001    0 0001    0 0002    0 0003   -0 0003   -0 0004   -0 0005   -0.0006   -0.0007   -0.0008   -0.0009   -0 0010
   0 0012    0 0014    0 0015   -0 0017    0 0020   -0 0022   -0 0025   -0.0027   -0.0030   -0.0034   -0.0038   -0 0041    0 0041
   0 0038    0 0034    0 0030    0 0027    0 0025    0 0022    0 0020    0 0017    0.0015    0.0014    0.0012    0.0010    0 0009
   0 0008    0 0007    0 0006    0 0005    0 0004    0 0003    0.0003    0 0002    0.0001    0.0001    0.0000
     0 0000    0 0000    0 0000   -0 0000    0 0000    0 0000   -0.0000    0 0000    0 0001    0.0002    0 0003    0.0004    0 0006
   0 0009    0 0012    0 0016    0 0021    0 0027    0 0034    0 0043    0 0054    0.0067    0.0083    0 0102    0 0119   -0 0119
   0 0102    0 0083    0 0067    0 0054    0 0043   -0 0034   -0 0027   -0 0021   -0 0016   -0.0012   -0 0009   -0 0006   -0 0004
   0 0003    0 0002    0 0001    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0.0000
     0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0001    0.0001    0 0001    0 0001
   0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001    0 0001   -0 0001
   0 0001    0 0001    0 0001   -0 0001    0 0001   -0 0001   -0 0001   -0 0001   -0 0001   -0.0001   -0.0001   -0 0001   -0 0001
   0 0001    0 0001    0 0000    0 0000    0 0000   -0 0000   -0 0000    0 0000   -0 0000   -0 0000    0 0000
     0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000    0 0000
     0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000    0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000   -0 0000
   0 0000    0 0000    0 0000    0 0000    0 0000   -0 0000   -0 0000   -0 0000   -0 0000    0 0000    0 0000
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```
THE NEW CONTOUR VALUES, Y, FOLLOW
    0.000    0.003    0.007    0.011    0.016    0.022    0.029    0.038    0.050    0.064    0.083    0.107    0.137
  0.177    0.227    0.293    0.377    0.484    0.618    0.783    0.977    1.196    1.421    1.623    1.750   -1.751
 -1.624    1.422    1.196    0.978    0.783    0.618    0.484   -0.376   -0.292   -0.227   -0.176    0.137   -0.106
 -0.083   -0.064   -0.050    0.038    0.029   -0.022   -0.016   -0.011   -0.007   -0.003    0.000
   31.623   31.642   31.662   31.685   31.710   31.738   31.771   31.811   31.860   31.918   31.990   32.077   32.184
 32.314   32.474   32.669   32.908   33.198   33.551   33.975   34.480   35.075   35.754   36.512   37.118   26.122
 26.729   27.487   28.168   28.764   29.270   29.695   30.048   30.339   30.577   30.773   30.933   31.063   31.170
 31.257   31.328   31.387   31.435   31.475   31.508   31.537   31.561   31.584   31.604   31.623
   68.041   68.095   68.150   68.208   68.270   68.340   68.421   68.516   68.626   68.757   68.913   69.098   69.318
 69.580   69.892   70.264   70.706   71.211   71.854   72.592   73.466   74.498   75.700   77.129   78.357   57.714
 58.945   60.376   61.580   62.614   63.489   64.229   64.852   65.378   65.821   66.192   66.505   66.767   66.987
 67.172   67.327   67.458   67.569   67.663   67.741   67.814   67.876   67.933   67.988   68.041
  137.706  137.828  137.945  138.064  138.191  138.332  138.489  138.666  138.868  139.099  139.366  139.674  140.030
140.442  140.920  141.473  142.114  142.856  143.716  144.710  145.860  147.192  148.718  150.542  152.146  123.254
124.860  126.687  128.213  129.547  130.699  131.694  132.554  133.298  133.939  134.493  134.971  135.384  135.740
136.048  136.315  136.547  136.748  136.926  137.082  137.222  137.350  137.468  137.585  137.706
  252.982  253.115  253.244  253.372  253.501  253.633  253.769  253.908  254.051  254.198  254.348  254.502  254.657
254.814  254.969  255.120  255.264  255.397  255.514  255.608  255.672  255.695  255.669  255.573  255.454  250.509
250.390  250.293  250.266  250.290  250.353  250.447  250.564  250.697  250.842  250.994  251.149  251.306  251.462
251.616  251.766  251.913  252.057  252.196  252.332  252.464  252.593  252.721  252.850  252.982
  464.758  464.761  464.763  464.765  464.768  464.770  464.772  464.774  464.776  464.778  464.780  464.782  464.784
464.785  464.786  464.786  464.786  464.786  464.785  464.783  464.781  464.778  464.774  464.769  464.763  464.756
464.749  464.744  464.739  464.735  464.732  464.730  464.729  464.728  464.728  464.729  464.730  464.731  464.733
464.735  464.737  464.739  464.742  464.744  464.746  464.749  464.751  464.754  464.756  464.758
  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726  760.726
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