SANDIA REPORT
SAND2001-2903
Unlimited Release
Printed December 2001

A p-Adic Metric for Particle Mass Scale Organization with Genetic Divisors

Yang Dai, Alexey B. Borisov, Keith Boyer, and Charles K. Rhodes

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation,
a Lockheed Martin Company, for the United States Department of
Energy under Contract DE-AC04-94AL85000.

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865)576-8401
Facsimile: (865)576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: http://www.doe.gov/bridge

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd
Springfield, VA 22161

Telephone: (800)553-6847
Facsimile: (703)605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: http://www.ntis.gov/ordering.htm

SAND2001-2903
Unlimited Release
Printed December 2001

A p-Adic Metric for Particle Mass Scale Organization with Genetic Divisors

Yang Dai[†‡], *Alexey B. Borisov*[‡], *Keith Boyer*[‡], *and Charles K. Rhodes*[‡]

[†]Department of Mathematical and Computing Sciences,
Tokyo Institute of Technology, Tokyo, *JAPAN*

[‡]Department of Physics, University of Illinois at Chicago,
Chicago, IL 60607-7059, *USA*

ABSTRACT

The concept of genetic divisors can be given a quantitative measure with a non-Archimedean p-adic metric that is both computationally convenient and physically motivated. For two particles possessing distinct mass parameters x and y, the metric distance D(x, y) is expressed on the field of rational numbers $\mathbf{Q}$ as the inverse of the greatest common divisor [gcd (x, y)]. As a measure of genetic similarity, this metric can be applied to (1) the mass numbers of particle states and (2) the corresponding subgroup orders of these systems. The use of the Bézout identity in the form of a congruence for the expression of the gcd (x, y) corresponding to the ν_e and ν_μ neutrinos (a) connects the genetic divisor concept to the cosmic seesaw congruence, (b) provides support for the δ-conjecture concerning the subgroup structure of particle states, and (c) quantitatively strengthens the interlocking relationships joining the values of the prospectively derived (i) electron neutrino (ν_e) mass (0.808 meV), (ii) muon neutrino (ν_μ) mass (27.68 meV), and (iii) unified strong-electroweak coupling constant ($\alpha^{*-1} = 34.26$).

I. Introduction

Arithmetic conditions relating particle masses can be defined on the basis of (A) the supersymmetric conservation of congruence and (B) the observed characteristics of particle reactions and stabilities [1]. Stated in the form of common divisors of the particle mass parameters, these relations can be interpreted as expressions of genetic elements that represent particle characteristics [2]. In order to illustrate this concept, it has been shown that the pion triplet ($\pi^{\pm}$, π^{o}) can be associated with the existence of a greatest common divisor $d_{o\pm}$ in a way that can account for both the highly similar physical properties of these particles and the observed $\pi^{\pm}/\pi^{o}$ mass splitting. Classification of the respective physical states is achieved by association of the common divisors (genes) with corresponding residue class designations [3] in a finite field $\mathbb{F}_{P_\alpha}$. Further, the existence of the finite field $\mathbb{F}_{P_\alpha}$ and the corresponding group of units $\mathbb{F}^*_{P_\alpha}$ leads immediately to the definition of a new physical entity, the inverse state [3], a concept that allows supersymmetry to be directly expressed in terms of hierarchical relationships between odd and even order subgroups of $\mathbb{F}^*_{P_\alpha}$. The invention of the inverse state further enables (1) the formulation of the cosmic seesaw congruence, the statement that fuses the concepts of mass and space [3], (2) the creation of a new definition of particle elementarity [4], and (3) the computation of the Higgs mass [3].

The structure established by the use of these mathematical procedures transforms the theoretical organization of the particle mass scale into a cryptographic analysis [5,6]. It has been shown [4] that the law of quadratic reciprocity [7] arranges the group structure of physical particle states by powerfully constraining the subgroup relationships in $\mathbb{F}^*_{P_\alpha}$. A profound consequence of the subgroup ordering is the <u>doubling</u> of the genetic function [2] of the divisors and their classification with the residue classes of $\mathbb{F}_{P_\alpha}$. On the basis of the pattern found, the genetic divisor interpretation can be applied to both (α) the designation of subgroup orders and (β) the specification of the masses and intrinsic attributes of individual particle systems [2]. It was generally concluded [4] that the constraint on the group relationships expressed jointly by quadratic reciprocity and the seesaw congruence creates a universal <u>optimized</u> structure that plays a fundamental regulatory role in a large array of complex phenomena.

II. Genetic Divisor Metric

The genetic divisor concept can be placed on a firm quantitative footing with the development of an appropriate metric. It will be shown below that this metric finds its natural voice through expression in the field of p-adic numbers Ω, the non-Archimedean analogue [8-11] of the algebraically closed field $\mathbb{C}$.

For any rational number $x \in \mathbb{Q}$ and prime p, consider the map $|x|_p$ of the form

$$|x|_p = \left\{ \begin{array}{r} \dfrac{1}{p^{\mathrm{ord}_p x}},\ \text{if } x \neq 0; \\ 0,\ \text{if } x = 0. \end{array} \right\} . \tag{1}$$

It can be shown [8] that $|x|_p$ is a norm on $\mathbb{Q}$ and that the statement

$$\prod_p |x|_p = 1 \tag{2}$$

holds. It is understood that Eq.(2) includes $|x|_\infty$, an Archimedean norm which we will take as the customary absolute value.

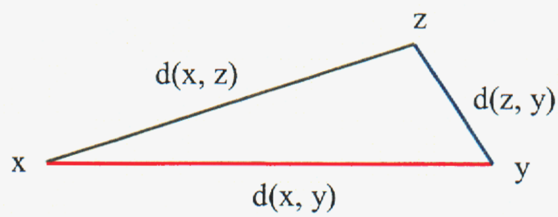

Fig.(1): Triangle formed by three points x, y, z.

Consider three points (x, y, z) as shown in Fig.(1). From the definition of a metric, we must have

$$d(x, y) \leq d(x, z) + d(z, y), \qquad (3)$$

the "triangle inequality." With no loss of generality, we can set z = 0. Following the development of Koblitz [8] for the construction of a metric on a field $\mathbb{F}$ with a non-Archimedean norm, we write

$$\| x-y \| \leq \max (\| x \|, \| y \|), \qquad (4)$$

assume

$$\| x \| < \| y \|, \qquad (5)$$

and obtain

$$\| x-y \| \leq \| y \| . \qquad (6)$$

However,

$$\| y \| = \| x -(x-y) \| \leq \max (\| x \|, \| x-y \|), \qquad (7)$$

and with Eq.(5) above, the relation

$$\| y \| \leq \| x-y \| \qquad (8)$$

must hold. We conclude that

$$\| y \| = \| x-y \| . \qquad (9)$$

Hence, if x and y are unequal, the larger must have a length equal to the magnitude of the third side. In this sense, the non-Archimedean norm always produces an isosceles triangle. Accordingly, in relation to Fig.(1), we have

$$|z|_p = |0|_p = 0, \tag{10}$$

$$d(x, z) = d(x, 0) = |x - 0|_p = |x|_p, \tag{11}$$

$$d(z, y) = d(0, y) = |-y|_p = |y|_p, \tag{12}$$

and

$$d(x, y) = d(z, y) = |y|_p . \tag{13}$$

Consider two unequal (|| x || < || y ||) mass numbers x and y such that

$$x = d_{xy}x_o \tag{14}$$

and

$$y = d_{xy}y_o \tag{15}$$

with

$$d_{xy} = \gcd(x, y), \tag{16}$$

a quantity given by the composition of primes

$$d_{xy} = \prod_{p}^{n} p_i^{\alpha_i} . \tag{17}$$

With the norm given by Eq.(1), we can then define the metric "distance" D(x, y) between x and y as

$$D(x, y) = \prod_{i=1}^{n} |x - y|_{p_i} = \prod_{i=1}^{n} |y|_{p_i} \tag{18}$$

with the elementary result that

$$D(x, y) = \frac{1}{\gcd(x, y)} = \frac{1}{d_{xy}} . \tag{19}$$

Hence, on the rational number field $\mathbb{Q}$, the metric distance between a pair of unequal mass parameters x and y is given by the inverse of the greatest common divisor. In terms of the concept of genetic divisors [2], this means that similar particles (x, y), which naturally possess a large gcd(x, y), perforce have a small value for D(x, y). This outcome has an obvious physical motivation; namely, particles with closely related physical properties and a large corresponding genetic similarity enjoy a small metric separation.

This genetic interpretation is reinforced by Bézout's identity [12] which states that the greatest common divisor of two integers can be expressed as a linear combination of those integers. It follows that we can equivalently express Eq.(16) as

$$d_{xy} = \gcd(x,y) = rx + sy \tag{20}$$

for suitable integer values of r and s. Accordingly, the (r,s) doublet functions as an independent index for the specific association of the d_{xy} genetic element (gene) with the (x,y) pair. It is of practical significance that, for a given unfactored pair (x,y), the quantities d_{xy}, r, and s can be efficiently determined with the Euclidean algorithm [12].

III. Bézout Congruence and the Cosmic Seesaw

The Bézout identity given by Eq.(20) can be expressed as a congruence on the finite field of $\mathbb{F}^*_{P_\alpha}$ used in the determination of the Higgs mass [3], a formulation that yields the statement

$$rx + sy \equiv d_{xy}(\text{mod } P_\alpha). \tag{21}$$

A previous study [4] of the group structure of particle states showed that the respective mass numbers B_{ν_e} and B_{ν_μ} of the electron and muon neutrinos correspond to the important special case $d_{xy} = 2$ in Eq.(21). Specifically, in the previously established notation [4], with the identification $x = B_{\nu_e} = [g_\alpha]_{P_\alpha}$ and $y = B_{\nu_\mu} = [g_\beta]^{-1}_{P_\alpha}$, we have

$$d_{xy} = \gcd(x, y) = \gcd([g_\alpha]_{P_\alpha}, [g_\beta]^{-1}_{P_\alpha}) = d_{\nu_e\nu_\mu} = 2 \tag{22}$$

and

$$r[g_\alpha]_{P_\alpha} + s[g_\beta]^{-1}_{P_\alpha} \equiv 2(\text{mod } P_\alpha) \tag{23}$$

from Eq.(21). By inspection we immediately obtain the elementary solution of Eq.(23) as

$$1 + 1 \equiv 2(\text{mod } P_\alpha)\,, \tag{24}$$

with

$$r = [g_\alpha]^{-1}_{P_\alpha} \tag{25}$$

and

$$s = [g_\beta]_{P_\alpha}\,, \tag{26}$$

the inverses corresponding to the known $([g_\alpha]_{P_\alpha}, [g_\beta]^{-1}_{P_\alpha})$ pair. Hence, the final relation reads

$$[g_\alpha]^{-1}_{P_\alpha}[g_\alpha]_{P_\alpha} + [g_\beta]_{P_\alpha}[g_\beta]^{-1}_{P_\alpha} \equiv 2(\text{mod } P_\alpha)\,, \tag{27}$$

an additive statement connecting the electron and muon neutrino masses, the masses of the corresponding inverse and supersymmetric states, and the residue class $[2]_{P_\alpha}$, the latter quantity reflecting the common genetic divisor content of the mass numbers of the ν_e and ν_μ neutrinos.

Three important results flow directly from Eq.(27). First, the expression relates the four states illustrated in Fig. (2) that have been specified in an earlier study [3] as the fundamental organizers of the particle mass scale. Second, Eq.(27) only holds for the special case $d_{xy} = 2$ in Eq.(21); it is otherwise invalid. This point is of considerable significance, since the value of Eq.(22), namely, $d_{xy} = 2$, is uniquely selected from alternative possibilities by the demand for agreement of the neutrino masses [3,4,13] with the measured magnitude of the fine-structure constant α, a physical parameter that is experimentally very accurately known ($\Delta\alpha/\alpha \sim 10^{-8}$) [14]. Third, the four mass states shown in Fig. (2) obey two parallel seesaw congruences [3], specifically, the multiplicative relations

$$[g_\alpha]_{P_\alpha}[g_\beta]_{P_\alpha}^{-1} \equiv B_{Higgs}^2 \pmod{P_\alpha} \tag{28}$$

and

$$[g_\beta]_{P_\alpha}[g_\alpha]_{P_\alpha}^{-1} \equiv B_{Higgs}^2 \pmod{P_\alpha}\,. \tag{29}$$

Therefore, the supersymmetric Higgs doublet, the organizers of the mass scale, and the subgroup orders corresponding to them [4], represent themselves with two combined theoretical patterns. One is the multiplicative statement of the cosmic seesaw [3]; the other is the independent additive relationship based on the Bézout identity.

MASS SCALE GENERATORS

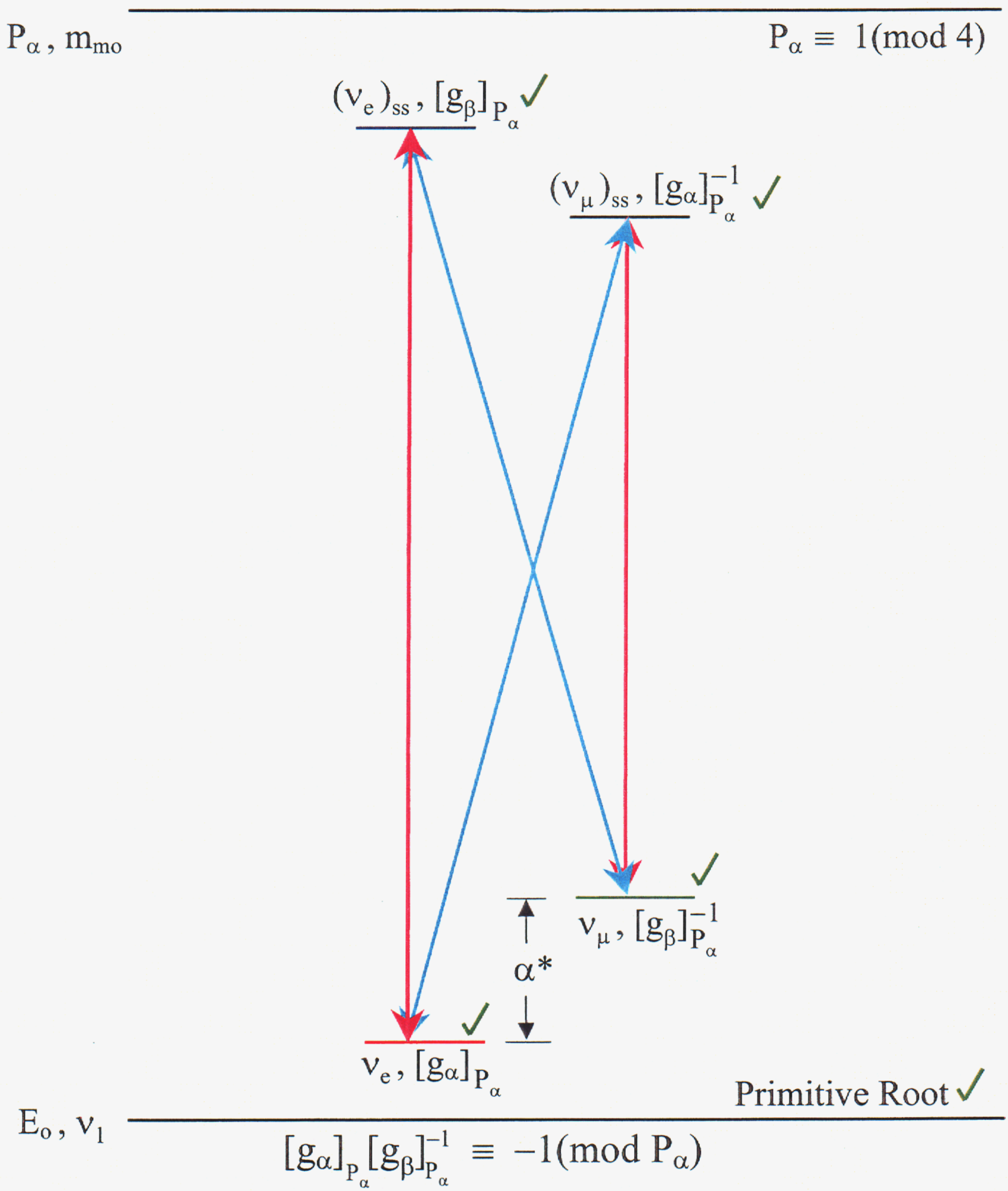

Fig.(2): Illustration of the primitive root states that organize the mass scale, construct an eight-dimensional index space, and specify the unified strong-electroweak coupling constant α^*. From Eq.(22), $d_{\nu_e \nu_\mu} = 2$. The magnitude of P_α represents the monopole mass m_{mo}, E_o denotes the energy unit, and ν a boson neutrino with a mass equal to E . See references [1–4] for further discussion.

IV. Conclusions

The concept of genetic divisors can be equipped with a quantitative measure of genetic similarity through the construction of a suitable metric. Expressed with p-adic numbers, this metric is both computationally convenient and physically motivated. In this language, for two particles with corresponding distinct mass numbers x and y, the metric distance D(x, y) expressed on the field of rational numbers $\mathbb{Q}$ is given by the inverse of the greatest common divisor [gcd(x, y)]. This resulting non-Archimedean genetic metric can be applied to both (1) the mass numbers of particle states and (2) the corresponding subgroup orders of these systems. The use of the Bézout identity in the form of a congruence to express the greatest common divisor ($d_{\nu_e \nu_\mu} = 2$) corresponding to the mass numbers of the electron (ν_e) and muon (ν_μ) neutrinos (a) connects the genetic divisor concept to the cosmic seesaw congruence, (b) provides support for the δ-conjecture concerning the subgroup structure of particle states, and (c) quantitatively strengthens the interlocking relationships joining the values of the prospectively derived (i) electron neutrino mass (0.808 meV), (ii) muon neutrino mass (27.68 meV), and (iii) unified strong-electroweak coupling constant ($\alpha^{*-1} = 34.26$).

Acknowledgements

One of the authors (CKR) thankfully acknowledges valuable conversations with J. W. Longworth. Support for this research was partially provided under contracts with the Army Research Office (DAAG55-97-1-0310), the Department of Energy at the Sandia National Laboratories (DE-AC04-94AL85000 and BF3611), and a Grant-in Aid of the Ministry of Education, Science, Sport, and Culture of Japan.

Correspondence should be addressed to C.K.R. (e-mail: rhodes@uic.edu).

References

1. Y. Dai, A. B. Borisov, J. W. Longworth, K. Boyer, and C. K. Rhodes, "Gamma-Ray Bursts and the Particle Mass Scale," *Proceedings of the International Conference on Electromagnetics in Advanced Applications,* edited by R. Graglia (Politecnico di Torino, Torino, 1999) p. 3.
2. Y. Dai, A. B. Borisov, K. Boyer, and C. K. Rhodes, "Determination of Supersymmetric Particle Masses and Attributes with Genetic Divisors," Sandia National Laboratories, *Report SAND2001-1608,* June 2001.
3. Y. Dai, A. B. Borisov, K. Boyer, and C. K. Rhodes, "Computation with Inverse States in a Finite Field $\mathbb{F}_{p_\alpha}$: The Muon Neutrino Mass, the Unified Strong-Electroweak Coupling Constant, and the Higgs Mass," Sandia National Laboratories, *Report SAND2000-2043,* August 2000.
4. Y. Dai, A. B. Borisov, J. W. Longworth, K. Boyer, and C. K. Rhodes, "Quadratic Reciprocity and the Group Orders of Particle States," Sandia National Laboratories, *Report SAND2001-1534,* June 2001.
5. Neal Koblitz, *A Course in Number Theory and Cryptography* (Springer-Verlag, Berlin/New York, 1987).
6. Neal Koblitz, *Algebraic Aspects of Cryptography* (Springer-Verlag, Berlin, 1998).
7. F. Lemmermeyer, *Reciprocity Laws* (Springer-Verlag, Berlin/New York, 2000).
8. Neal Koblitz, *p-adic Numbers, p-adic Analysis, and Zeta-Functions,* Second edition (Springer-Verlag, Berlin/New York, 1984).
9. George Bachman, *Introduction to p-Adic Numbers and Valuation Theory*, (Academic Press, New York/London, 1964).
10. Kurt Mahler, *p-Adic Numbers and their Functions* (Cambridge University Press, Cambridge, 1981).
11. Fernando Q. Gouvêa, *p-Adic Numbers* (Springer-Verlag, Berlin/Heidelberg, 1993).
12. L. N. Childs, *A Concrete Introduction to Higher Algebra* (Springer-Verlag, New York/Berlin, 1995).
13. Y. Dai, A. B. Borisov, and C. K. Rhodes, "Nature, Number, and Universality," manuscript in preparation.
14. Peter J. Mohr and Barry N. Taylor. "CODATA Recommended Values of the Fundamental Physical Constants: 1998," *J. Phys. Chem. Ref. Data* **28,** 1713 1999).

DISTRIBUTION

8 Department of Physics (m/c 273)
University of Illinois at Chicago
ATTN: Alex B. Borisov
Keith Boyer
Charles K. Rhodes
Chicago, IL 60607-7059

1 MS-0101 Paul Robinson, 00001

1 MS-0188 Chuck Meyers, 1030

1 MS-1153 Nancy Ries, 15331

1 MS-1188 Roy Hamil, 15334

1 MS-1188 John Wagner, 15334

1 MS-9018 Central Technical Files, 8945-1

2 MS-0899 Technical Library, 9616

1 MS-0612 Review and Approval Desk, 9612 for DOE/OSTI

1 Perry Gray
Los Alamos National Laboratory
P. O. Box 1663 B-287
Los Alamos, New Mexico 87545