

A Parallel Multi-Verse Optimizer for Application in Multilevel Image Segmentation

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This work was supported by the Natural Science Foundation of China under Grant 61872085.

ABSTRACT Multi-version optimizer (MVO) inspired by the multi-verse theory is a new optimization algorithm for challenging multiple parameter optimization problems in the real world. In this paper, a novel parallel multi-verse optimizer (PMVO) with the communication strategy is proposed. The parallel mechanism is implemented to randomly divide the initial solutions into several groups, and share the information of different groups after each fixed iteration. This can significantly promote the cooperation individual of MVO algorithm, and reduce the deficiencies that the original MVO is premature convergence, search stagnation and easily trap into local optimal search space. To confirm the performance of the proposed scheme, the PMVO algorithm was compared with the other well-known optimization algorithms, such as gray wolf optimizer (GWO), particle swarm optimization (PSO), multi-version optimizer (MVO), and parallel particle swarm optimization (PPSO) under CEC2013 test suite. The experimental results prove that the PMVO is superior to the other compared algorithms. In addition, PMVO is also applied to solve complex multilevel image segmentation problems based on minimum cross entropy thresholding. The application results appear that the proposed PMVO algorithm can achieve higher quality image segmentation compared to other similar algorithms.

INDEX TERMS Meta-heuristic optimization, parallel multi-verse optimizer, multilevel image segmentation, minimum cross entropy thresholding.

I. INTRODUCTION

In recent decades, meta-heuristic optimization techniques have attracted extensive research interest and have been successfully applied in various fields of the engineering community [1], [2]. A lot of optimizers could be inspired by the behavior of animals, social events and physical phenomena. These algorithms usually start by randomly initializing a set of solutions in the search space, and then the generated solutions try to obtain the best solution by moving, combining, and evolving during the iteration process. For different optimization algorithms, they usually have different moving, combining, and evolutionary strategies. The performance of a newly proposed optimization algorithm will be evaluated by conducting various experiments on different test suites. Currently, many popular optimization algorithms have been

proposed to solve optimization problems in real life. For example, the well-known genetic algorithm (GA) [3], particle swarm optimization (PSO) [4], [36], cat swarm optimization (CSO) [6], parallel particle swarm optimization (PPSO) [5], multi-verse optimizer (MVO) [7], grey wolf optimizer (GWO) [8], artificial bee colony optimization (ABC) [9], quasi-affine transformation evolution algorithm (QUATR) [10] and other recently proposed effective algorithms in the literature [27]–[31].

Different optimization algorithms have different evolutionary mechanisms, but they all include the two common features of exploration and exploitation in the optimization process. The purpose of exploration is to identify as wide a range of promising areas as possible in the search space. However, exploitation refers to the ability to perform local search and convergence around the obtained promising area. Due to the unknown nature of the search space and the randomness of the meta-heuristic algorithm, how to find an

The associate editor coordinating the review of this manuscript and approving it for publication was Jihad Aljaam¹.

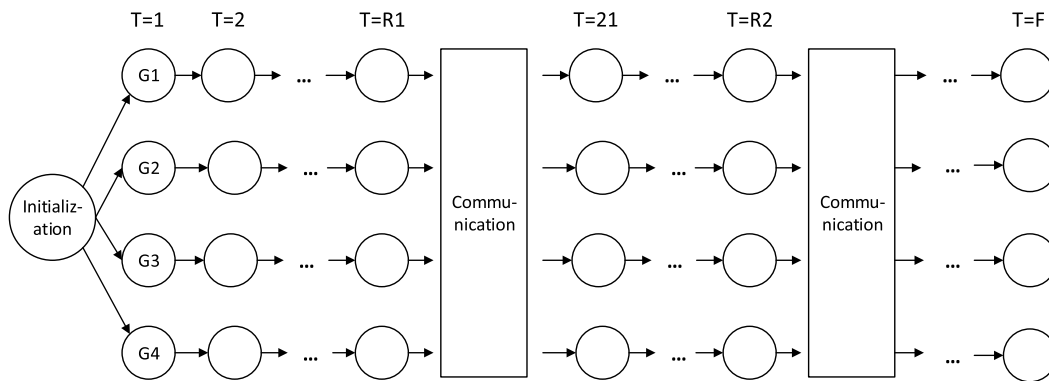


FIGURE 1. The main framework of PMVO.

appropriate balance between exploration and exploitation is a major problem for an optimization algorithm. Therefore, when designing an optimization algorithm, the main challenge is how to transition from exploration to development at an appropriate time.

A multi-verse optimizer (MVO) algorithm was proposed in [7], which was derived from three main concepts in physics: white holes, black holes, and wormholes. White and black holes were used for exploration and wormholes were used for development. In order to emphasize the exploitation and improve the accuracy of local search during the optimization process, the MVO algorithm also named two important coefficients: the wormhole existence probability (WEP) and the traveling distance rate (TDP).

The MVO has a lot of advantages, such as simplicity, robustness, few control parameters and outputting powerful performance. The MVO has been demonstrated to be a competitive algorithm in the literature [7], [12], [13]. However, when dealing with some complex optimization situations, the MVO algorithm also has some inherent shortcomings. For example, it sometimes produces results that converge prematurely, stagnate the search, and easily fall into a local optimal search space. Motivated by the parallel evolution mechanism [5], [26], we adopt the parallel mechanism with a new communication strategy based on the MVO and proposed a novel algorithm called parallel multi-verse optimizer (PMVO) to overcome the deficiencies of the original MVO. In PMVO, the initialized solution is randomly divided into several groups, and then evolved separately based on the MVO algorithm. After each fixed iteration, the best solution is selected from each group to achieve information flow between different groups. This helps to increase the diversity of the population. We have performed a large number of simulation experiments under the CEC2013 test suite. Compared with MVO, the proposed PMVO algorithm can produce more competitive results.

Image segmentation technology is a basic problem in the field of pattern recognition and computer vision. It currently has a very wide range of applications, such as surveillance, object tracking, medical imaging, character recognition, and so on. The threshold-based scheme is very effective for image

segmentation. This method is based on information such as the pixel histogram of the image to select a few appropriate thresholds and divide all pixels into different regions. For example, the bi-level thresholding problem is to choose an appropriate threshold to divide all pixels into two classes: object and background, which is easy to implement. However, multilevel thresholding is more popular to solve the challenge tasks such as multilevel image segmentation, mixed-typed document analysis and so on. In this paper, we have put more emphasis on the proposed MVO algorithm to solve the multilevel image segmentation problem. [13].

As more and more researchers begin to focus on entropy-based on threshold for segmentation. Cross entropy [16], Tsallis entropy [14], Renyi entropy [15], etc. have been proposed and widely used in image research. Since the minimum cross-entropy threshold (MCET) [17]–[20] has the advantage of being able to deal with multilevel threshold constraints very well and obtain accurate threshold values, we apply the proposed PMVO algorithm to optimize the MCET function in order to obtain the thresholds to segment the color image. The segmented image quality is ultimately assessed in three well-known metrics: peak signal noise ratio (PSNR), structural similarity index (SSIM), and feature similarity index (FSIM) [21]–[25]. The results show that the proposed PMVO algorithm can obtain higher quality segmented images than those compared algorithms.

The main contributions of this paper are as follows.

- A new optimizer is proposed namely PMVO, which reduces the deficiencies of the original MVO.
- A parallel mechanism with the new communication strategy is implemented to reserve population diversity.
- The proposed algorithm is compared with the well-known MVO, PSO, PPSO and GWO algorithms under the CEC2013 test suite. The results indicate that the PMVO algorithm is superior to other algorithms.
- The application results also demonstrate that the proposed PMVO algorithm is better than the other algorithms in solving multilevel image segmentation.

The rest of this paper deals with the following. In section 2, we briefly review the original MVO algorithm and the multilevel image segmentation problem. Section 3 introduces

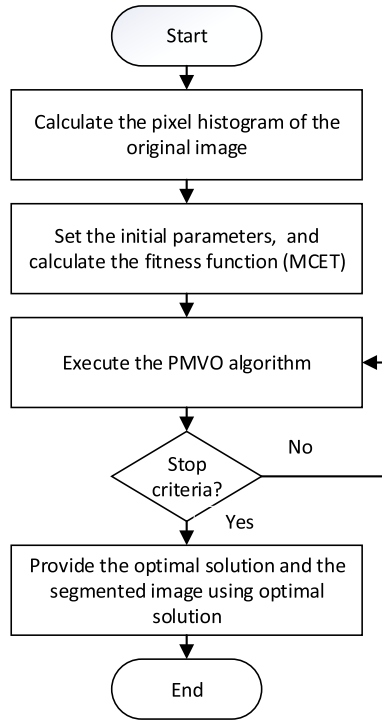


FIGURE 2. The flowchart by using PMVO to solve the multilevel image segmentation.

the proposed PMVO algorithm and its application in the multilevel image segmentation problem. In section 4, the experimental results under the CEC2013 test suite and multilevel image segmentation problem are described. Eventually, the newly proposed PMVO algorithm is summarized in section 5.

II. RELATED WORKS

A. CANONICAL MVO ALGORITHM

Reference [7] presented a novel and promising optimization algorithm called MVO. Inspired by the multi-universe theory in physics, Mirjalili et al. introduced the concepts of white holes, black holes, and wormholes into the algorithm. In the search space, white and black holes are responsible for exploration, and wormholes are responsible for development. At the same time, some new concepts are applied, for example, a universe corresponds to a candidate solution, an individual in the universe corresponds to a variable of the solution, and the inflation rate corresponds to the fitness value.

The MVO algorithm conforms to the following rules.

1. The larger the value of the inflation rate, the more likely a white hole will appear, and the less likely a black hole will appear.
2. A universe with a large inflation rate tends to transmit objects, and a universe with a low inflation rate tends to receive objects through black and white tunnels.
3. All objects in the universe will randomly move around the best universe through the wormhole, regardless of the numerical value of the inflation rate

TABLE 1. The pseudo code of PMVO.

Initialization:

Set the universe space V , initialize the N universes and randomly divide evenly them into G groups (G is equal to 4), Each universe contains D (dimension) objects, $f(U_i)$ represents the fitness function of the U_i universe, current generation $T=1$, the maximal number of generation F , R is the generation to trigger the communication strategy.

Iteration:

```

1: while  $T < F$  do
2:   for  $gth = 1$  to  $G$  do
3:     Calculate the inflation rate of each university in the  $G(gth)$  ( $G(gth)$  denotes the  $gth$  group), and sort them to obtain the best university so far.
4:     for  $i = 1$  to  $N/G$  do
5:       Choosing the white hole to change each  $y_{ij}$  in the roulette mechanism by using Eq. (2)
6:        $y_{ij}$  is mutated based on the best universe via the wormhole by using Eq. (3)
7:     end for
8:   end for
9:   if  $T = R$ 
10:    Communication strategy: apply Eq. (13) to update  $y_{ij}$ 
11:   end if
12:    $T = T + 1$ 
13: end while
  
```

Output:

The global best universe U_{gbest} , and the best inflation rate value $f(U_{gbest})$

In each iteration of the MVO algorithm, a roulette wheel mechanism is used to select a white hole from all universes according to the inflation rate. The purpose is to promote object exchange in different universes and enhance exploration capability.

MVO scheme assumes that

$$U = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{bmatrix}, \quad (1)$$

where U is a matrix of all universes, p is the number of individuals in a universe and n is the total number of initialized universes.

$$y_{ij} = \begin{cases} y_{hj} & r1 < NI(U_i) \\ y_{ij} & r1 \geq NI(U_i) \end{cases}, \quad (2)$$

where y_{ij} denotes the j th object of i th universe, y_{hj} represents the j th object of the selected h th universe according to the roulette wheel mechanism, the i th universe is indicated as U_i , the normalized inflation rate of i th universe is represented as $NI(U_i)$, and $r1$ is a random number between 0 and 1.

Wormhole tunnels have been established between each universe and the best universe currently available to increase the local variations of each universe and its inflation rate.

TABLE 2. Fidelity parameters evaluate the efficiency of segmented image results.

	Parameters	Formulas	Remarks
1.	<i>MSE</i> and <i>PSNR</i>	$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N [I(i, j) - I'(i, j)]^2$ $PSNR = 20 \log_{10} \frac{255}{\sqrt{MSE}}$	<i>MSE</i> measures the deviation between actual and expected values. <i>PSNR</i> represents a ratio between <i>MSE</i> and the maximum pixel value.
2.	<i>SSIM</i>	$SSIM(I, I') = \frac{(2\mu_I \mu_{I'} + c_1)(2\sigma_{II'} + c_2)}{(\mu_I^2 + \mu_{I'}^2 + c_1)(\sigma_I^2 + \sigma_{I'}^2 + c_2)}$	<i>SSIM</i> is a similarity parameter of two different image structures.
3.	<i>FSIM</i>	$FSIM = \frac{\sum_{i=1}^N S_L(i) PC_{max}(i)}{\sum_{i=1}^N PC_{max}(i)}$	<i>FSIM</i> is a similarity parameter of two different image features.

The detailed mechanism is formulated as follows.

$$y_{ij} \begin{cases} Y_j + TDR \times ((ub_j - lb_j) \times r4 + lb_j) & r3 < 0.5 \\ Y_j - TDR \times ((ub_j - lb_j) \times r4 + lb_j) & r3 \geq 0.5 \end{cases} \begin{matrix} r2 < WEP \\ r2 \geq WEP \end{matrix} \quad (3)$$

where Y_j is the j th object of best universe obtained at present, *TDR* is an acronym for traveling distance rate, *WEP* is an acronym for wormhole existence probability, the upper boundary of j th object is represented by ub_j , and its lower boundary is represented by lb_j , $r2$, $r3$, and $r4$ are three random numbers ranging from 0 to 1.

The *TDR* is an important factor that helps to teleport objects through the wormholes around the best universe currently available. The *TDR* increases with the number of iterations to achieve more explicit exploitation.

$$TDR = 1 - \frac{T^{1w}}{F^{1w}}, \quad (4)$$

where the current iteration is represented by T and the maximum iteration is represented by F . w describes the local search capability during the optimization process. As the number of iterations increases, a high w value can achieve more accurate local search capability. This paper sets w to 6.

The *WEP* represents the existence probability of wormhole and is defined to increase linearly during the optimization process. Therefore, the MVO algorithm emphasizes exploitation over the iterations.

$$WEP = Wmin + T \times \left(\frac{Wmax - Wmin}{F} \right), \quad (5)$$

where *Wmin* denotes the minimum and *Wmax* denotes the maximum of the *WEP*. In this paper, *Wmin* is set to 0.2, *Wmax* is set to 1.

In the MVO algorithm, the general steps are summarized as follows. Firstly, initialize the parameters and randomly generate some universes as candidate solutions. Then in the optimization process, the universe with a high inflation rate teleports objects to the universe with a low inflation rate through white and black hole tunnels. At the same time, all universes have a chance to move towards the best universe via

TABLE 3. Parameters settings.

Algorithm	Parameters settings
PMVO	$G = 4, R = 20, 40, \dots, 2000, w = 6, Wmin = 0.2, Wmax = 1$
MVO	$w = 6, Wmin = 0.2, Wmax = 1$
PSO	$Vmin = 10, Vmax = 10, w = 0.9 \text{ to } 0.4, c1, c2 = 1.49455$
PPSO	$G = 4, R = 20, 40, \dots, 2000, Vmin = 10, Vmax = 10, w = 0.9 \text{ to } 0.4, c1, c2 = 1.49455$
GWO	$a = 2 \text{ to } 0$

the wormhole. Finally, the termination condition is satisfied, and the optimal universe and its inflation rate are output.

B. MULTILEVEL IMAGE SEGMENTATION PROBLEM

The thresholding technique is fundamental and important for image segmentation. Multilevel thresholding focuses on determining boundaries to divide the image into multiple regions. For example, determining n thresholds divides all pixels of the original image into $(n + 1)$ classes. The n thresholds are denoted by t_1, t_2, \dots, t_n . *class*₁ belongs to the region $\{0, \dots, t_1\}$, *class*₂ belongs to the region $\{t_1, \dots, t_2\}$, \dots , *class* _{$n+1$} belongs to $\{t_n, \dots, B\}$. Therefore, the optimal n thresholds $\{t_1^*, \dots, t_n^*\}$ can be obtained as follows.

$$\{t_1^*, \dots, t_n^*\} = \operatorname{argmin} \{f(t_1, \dots, t_n)\}$$

$$\text{Subject to } 0 < t_1 < t_2 < \dots < t_n < B, \quad (6)$$

where $f(t_1, \dots, t_n)$ represents the objective function.

Reference [35] presented the concept of cross entropy. Cross entropy is used to measure the theoretical information distance of two probability distributions. Let two probabilistic distributions $P = \{p_1, p_2, \dots, p_n\}$, $Q = \{q_1, q_2, \dots, q_n\}$. The cross entropy can be formulated as

$$D(P, Q) = \sum_{i=1}^n p_i \log \frac{p_i}{q_i} \quad (7)$$

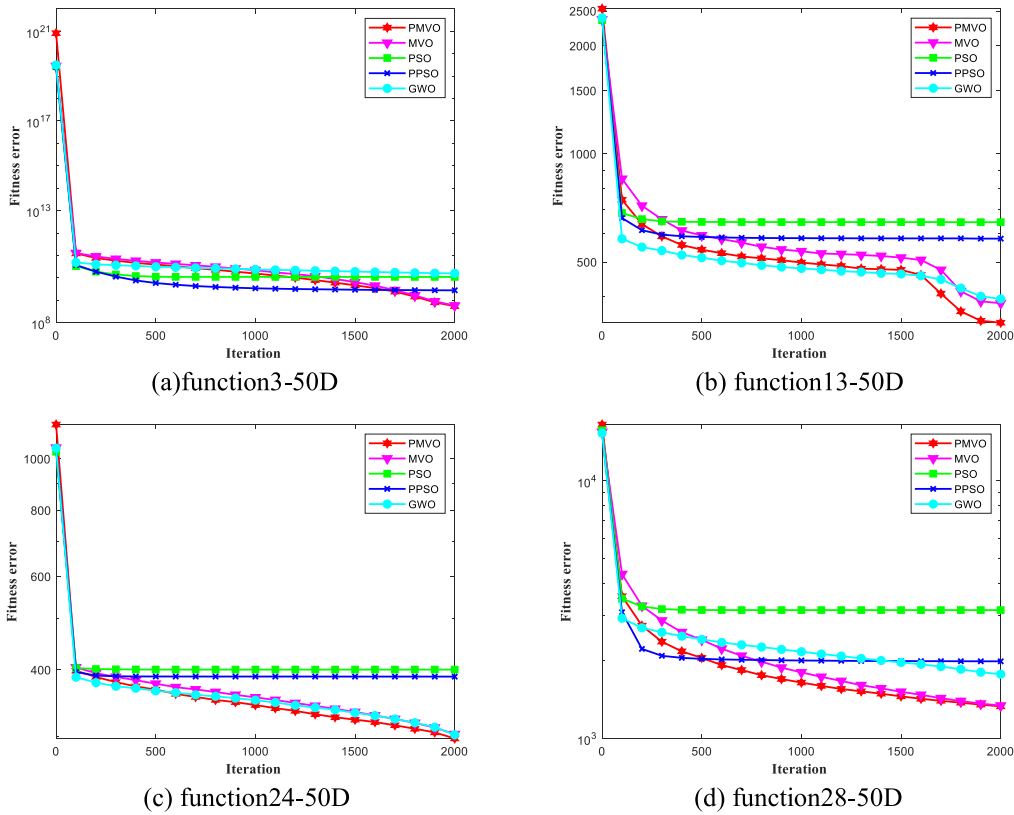


FIGURE 3. Comparison of the best fitness errors for functions f3, f13, f24, f28 with 50D optimization.

In [17], Li et al. first applied the minimum cross entropy threshold (MCET) method to segment images. Let I be an original image with histogram $z(i) = 1, 2, \dots, B$, and B is the number of gray levels. The bi-level segmented image as I_s can be constructed by

$$I_s(x, y) = \begin{cases} u(1, t), & I(x, y) < t \\ u(t, B+1), & I(x, y) \geq t, \end{cases} \quad (8)$$

where t is the threshold value to segment the original image,

$$u(a, b) = \sum_{i=a}^{b-1} iz(i) / \sum_{i=a}^{b-1} z(i).$$

For an image that needs to be segmented, the cross entropy is given by the formula:

$$D(t) = \sum_{i=1}^{t-1} iz(i) \log\left(\frac{i}{u(1, t)}\right) + \sum_{i=t}^B iz(i) \log\left(\frac{i}{u(t, B+1)}\right) \quad (9)$$

Eq. (9) is modified as

$$D(t) = \sum_{i=1}^B iz(i) \log(i) - \sum_{i=1}^{t-1} iz(i) \log(u(1, t)) - \sum_{i=t}^L iz(i) \log(u(t, B+1)) \quad (10)$$

To extend the n thresholds case to achieve multilevel image segmentation. Eq. (10) can be extended as follows.

$$D(t_1, \dots, t_n) = \sum_{i=1}^B iz(i) \log(i) - \sum_{i=1}^{t_1-1} iz(i) \log(u(1, t_1)) - \sum_{i=t_1}^{t_2-1} iz(i) \log(u(t_1, t_2)) - \dots - \sum_{i=t_n}^B iz(i) \log(u(t_n, B+1)) \quad (11)$$

Since the first phase in Eq. (11) is constant for an original image. When determining the optimal threshold values, we can add $t_0 = 1, t_{n+1} = B+1$, and then the objective function obtained by MCET is redefined as follows.

$$f(t_1, \dots, t_n) = - \sum_{k=0}^n \sum_{i=t_k}^{t_{k+1}-1} iz(i) \log(u(t_k, t_{k+1})) \quad (12)$$

III. PMVO AND ITS APPLICATION IN MULTILEVEL IMAGE SEGMENTATION PROBLEM

A. PARALLEL MULTI-VERSE OPTIMIZER (PMVO)

The PMVO will be depicted in this subsection. The original MVO faces some problems such as it may lose population diversity early in search process and stagnate

TABLE 4. Performance for PMVO, MVO and PSO under CEC2013.

50D	MVO			PSO			PMVO		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
1	1.21E-01	1.92E-01	4.11E-02	1.01E+01	1.41E+02	2.79E+02	1.41E-01	2.34E-01	4.18E-02
2	3.10E+06	8.08E+06	2.86E+06	3.79E+06	1.09E+07	5.89E+06	3.67E+06	9.00E+06	2.23E+06
3	1.13E+08	5.85E+08	3.49E+08	3.11E+09	1.10E+10	7.26E+09	3.78E+07	5.48E+08	4.16E+08
4	1.93E+02	4.12E+02	1.41E+02	5.71E+03	1.23E+04	3.54E+03	1.87E+02	4.67E+02	1.57E+02
5	1.25E-01	1.91E-01	2.88E-02	3.20E+01	1.24E+02	1.20E+02	1.46E-01	2.08E-01	2.70E-02
6	4.15E+01	4.76E+01	8.36E+00	5.04E+01	1.11E+02	3.90E+01	4.35E+01	5.27E+01	1.69E+01
7	4.27E+01	8.70E+01	2.48E+01	8.40E+01	1.34E+02	3.25E+01	2.95E+01	8.47E+01	2.65E+01
8	2.11E+01	2.12E+01	3.48E-02	2.10E+01	2.11E+01	4.61E-02	2.11E+01	2.12E+01	4.00E-02
9	2.72E+01	3.49E+01	4.16E+00	5.36E+01	6.18E+01	4.79E+00	2.15E+01	3.09E+01	3.91E+00
10	1.14E+00	1.58E+00	2.91E-01	1.67E+01	8.71E+01	5.93E+01	1.36E+00	2.30E+00	5.58E-01
11	1.07E+02	1.97E+02	4.20E+01	3.52E+02	4.86E+02	6.84E+01	1.17E+02	1.87E+02	4.34E+01
12	1.36E+02	2.07E+02	4.13E+01	3.19E+02	5.13E+02	8.45E+01	1.04E+02	1.90E+02	3.86E+01
13	2.62E+02	3.84E+02	6.44E+01	4.91E+02	6.46E+02	7.75E+01	1.45E+02	3.39E+02	5.72E+01
14	4.26E+03	6.51E+03	7.88E+02	5.82E+03	7.83E+03	9.38E+02	4.80E+03	6.65E+03	7.55E+02
15	4.64E+03	6.43E+03	8.48E+02	7.04E+03	8.82E+03	9.96E+02	4.44E+03	6.56E+03	1.01E+03
16	5.35E-01	1.07E+00	3.60E-01	1.01E+00	1.99E+00	5.19E-01	2.52E-01	1.02E+00	4.96E-01
17	2.46E+02	3.61E+02	6.76E+01	3.75E+02	5.61E+02	9.56E+01	2.23E+02	3.25E+02	5.55E+01
18	2.59E+02	3.40E+02	4.79E+01	2.83E+02	5.55E+02	1.02E+02	2.55E+02	3.29E+02	3.93E+01
19	1.27E+01	1.90E+01	3.69E+00	3.14E+01	6.86E+01	9.62E+01	1.00E+01	1.71E+01	3.17E+00
20	2.03E+01	2.27E+01	1.43E+00	2.14E+01	2.36E+01	7.22E-01	1.94E+01	2.22E+01	1.40E+00
21	2.07E+02	8.91E+02	3.73E+02	2.88E+02	9.35E+02	2.33E+02	2.08E+02	8.10E+02	3.90E+02
22	4.82E+03	8.01E+03	1.53E+03	8.51E+03	1.09E+04	1.14E+03	5.81E+03	7.69E+03	1.02E+03
23	4.86E+03	7.25E+03	1.12E+03	9.40E+03	1.18E+04	1.22E+03	4.65E+03	7.47E+03	1.59E+03
24	2.77E+02	3.03E+02	1.26E+01	3.66E+02	4.01E+02	1.86E+01	2.69E+02	2.97E+02	1.24E+01
25	3.04E+02	3.33E+02	1.33E+01	4.00E+02	4.55E+02	3.15E+01	2.99E+02	3.31E+02	1.36E+01
26	2.01E+02	3.75E+02	5.26E+01	2.00E+02	4.48E+02	6.33E+01	2.01E+02	3.79E+02	3.85E+01
27	9.16E+02	1.27E+03	1.20E+02	1.82E+03	2.13E+03	1.44E+02	9.19E+02	1.22E+03	1.20E+02
28	4.02E+02	1.34E+03	1.47E+03	4.94E+02	3.15E+03	2.36E+03	4.02E+02	1.33E+03	1.45E+03
win	15	17	14	26	27	24	-	-	-
lose	10	10	13	1	1	4	-	-	-
draw	3	1	1	1	0	0	-	-	-

the search. In order to avoid above drawbacks, the parallel multi-verse optimizer is designed based on the original MVO. In the parallel method, all the initialized universes are divided into G groups. Each universe of the G groups evolves independently by the MVO algorithm during the iterations. A new communication strategy is presented in this paper. Different universes will exchange objects between the G groups after each fixed iteration, which products the advantage of inter-group cooperation. The new communication strategy adopts a stochastic mechanism. For instance, the value of G groups is set to 4, which is described in detail as follows.

$$y_{ij} = y_{ij} + (y_* - y_{ij}) \times r5 \quad (13)$$

$$y_* = \begin{cases} y_b^1, & r6 \leq 0.25 \\ (y_b^1 + y_b^2) / 2, & 0.25 < r6 \leq 0.5 \\ (y_b^1 + y_b^2 + y_b^3) / 3, & 0.5 < r6 \leq 0.75 \\ (y_b^1 + y_b^2 + y_b^3 + y_b^4) / 4, & 0.75 < r6 \leq 1 \end{cases} \quad (14)$$

where y_* denotes the combined value between different groups, y_b^g represents the best universe in the g th group, $r5$ and $r6$ are two random numbers in $[0, 1]$.

FIGURE 1 gives the main framework of the propose PMVO. The current iteration is represented as T , the maximal number of pre-defined iteration is represented as F , and R_i is a fixed iteration set to communicate between

TABLE 5. Performance for PMVO, PPSO and GWO under CEC2013.

50D	PPSO			GWO			PMVO		
	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
1	5.76E-02	1.46E+00	2.15E+00	9.23E+02	2.79E+03	1.02E+03	1.41E-01	2.34E-01	4.18E-02
2	9.85E+05	3.41E+06	1.19E+06	1.20E+07	4.74E+07	1.73E+07	3.67E+06	9.00E+06	2.23E+06
3	4.58E+08	2.78E+09	1.83E+09	6.64E+09	1.59E+10	6.17E+09	3.78E+07	5.48E+08	4.16E+08
4	3.11E+03	8.05E+03	3.06E+03	2.43E+04	4.48E+04	8.48E+03	1.87E+02	4.67E+02	1.57E+02
5	1.02E+00	4.05E+01	2.33E+01	1.46E+02	9.15E+02	4.02E+02	1.46E-01	2.08E-01	2.70E-02
6	4.39E+01	8.76E+01	3.18E+01	1.47E+02	2.54E+02	7.88E+01	4.35E+01	5.27E+01	1.69E+01
7	6.38E+01	1.04E+02	1.81E+01	3.55E+01	6.69E+01	1.62E+01	2.95E+01	8.47E+01	2.65E+01
8	2.09E+01	2.11E+01	7.66E-02	2.11E+01	2.12E+01	3.58E-02	2.11E+01	2.12E+01	4.00E-02
9	4.94E+01	5.92E+01	4.74E+00	3.17E+01	3.89E+01	3.68E+00	2.15E+01	3.09E+01	3.91E+00
10	2.32E+00	1.34E+01	8.40E+00	2.08E+02	6.69E+02	1.99E+02	1.36E+00	2.30E+00	5.58E-01
11	2.91E+02	4.11E+02	7.05E+01	1.36E+02	2.24E+02	4.08E+01	1.17E+02	1.87E+02	4.34E+01
12	2.43E+02	4.16E+02	6.14E+01	1.54E+02	2.76E+02	9.67E+01	1.04E+02	1.90E+02	3.86E+01
13	4.56E+02	5.81E+02	7.97E+01	2.22E+02	3.95E+02	7.94E+01	1.45E+02	3.39E+02	5.72E+01
14	5.52E+03	7.32E+03	9.14E+02	4.11E+03	6.02E+03	1.82E+03	4.80E+03	6.65E+03	7.55E+02
15	6.08E+03	8.15E+03	1.01E+03	4.42E+03	8.32E+03	3.16E+03	4.44E+03	6.56E+03	1.01E+03
16	6.45E-01	1.62E+00	4.70E-01	2.95E+00	3.62E+00	2.52E-01	2.52E-01	1.02E+00	4.96E-01
17	2.98E+02	4.22E+02	6.88E+01	2.09E+02	3.40E+02	6.94E+01	2.23E+02	3.25E+02	5.55E+01
18	2.73E+02	3.96E+02	5.64E+01	3.51E+02	5.31E+02	5.20E+01	2.55E+02	3.29E+02	3.93E+01
19	1.76E+01	3.21E+01	1.02E+01	2.25E+01	5.98E+02	8.41E+02	1.00E+01	1.71E+01	3.17E+00
20	2.14E+01	2.37E+01	8.07E-01	1.94E+01	2.11E+01	7.71E-01	1.94E+01	2.22E+01	1.40E+00
21	2.16E+02	9.25E+02	2.01E+02	8.06E+02	1.99E+03	6.66E+02	2.08E+02	8.10E+02	3.90E+02
22	6.82E+03	1.00E+04	1.39E+03	4.78E+03	7.35E+03	2.21E+03	5.81E+03	7.69E+03	1.02E+03
23	7.79E+03	1.08E+04	1.24E+03	4.59E+03	8.82E+03	2.92E+03	4.65E+03	7.47E+03	1.59E+03
24	3.47E+02	3.89E+02	1.70E+01	2.75E+02	3.02E+02	1.32E+01	2.69E+02	2.97E+02	1.24E+01
25	3.89E+02	4.39E+02	2.55E+01	3.19E+02	3.43E+02	1.27E+01	2.99E+02	3.31E+02	1.36E+01
26	2.00E+02	3.46E+02	1.29E+02	3.68E+02	4.00E+02	1.21E+01	2.01E+02	3.79E+02	3.85E+01
27	1.75E+03	2.04E+03	1.40E+02	1.01E+03	1.28E+03	1.10E+02	9.19E+02	1.22E+03	1.20E+02
28	4.04E+02	1.99E+03	2.17E+03	6.28E+02	1.78E+03	1.17E+03	4.02E+02	1.33E+03	1.45E+03
win	23	25	21	21	23	18	-	-	-
lose	3	3	6	5	4	10	-	-	-
draw	2	0	1	2	1	0	-	-	-

different groups. TABLE 1 is the pseudo code of the proposed PMVO algorithm.

B. PMVO FOR MULTILEVEL IMAGE SEGMENTATION PROBLEM

The MVO algorithm has been proven to produce good results on image segmentation. Our proposed PMVO algorithm reduces the performance deficiency of the original MVO and can achieve better segmentation results. Image segmentation divides the pixels in the original image into several meaningful regions. Determining the threshold is a key step in image segmentation using the threshold method. The threshold is

the boundary of the divided area. Appropriate thresholds can produce good segmentation results. The minimum cross entropy thresholding method can provide accurate and fast segmentation results for an image. Therefore, the MCET is chosen as the objective function for an image, and then the PMVO algorithm is used to solve the objective function to obtain the appropriate thresholds. Finally, they are used as the boundaries of the segmented areas to obtain the segmented image. The flowchart of applying the PMVO algorithm to solve the multilevel image segmentation problem is shown in FIGURE 2.

A given color image contains information of three channels (red, green, blue). We calculate a histogram of the pixel



FIGURE 4. Test color image (Img1-Img8).

values of each color band. Corresponding thresholds obtained through the process of FIGURE 2 are used to segment each color band separately. The final segmented color image is generated by concatenating the segmentation results of each color band together. Three metrics are used to evaluate the results of image segmentation. They are peak signal-to-noise ratio (PSNR) [21]–[23], structural similarity index (SSIM) [25], and feature similarity index (FSIM) [24]. TABLE 2 is a brief summary of the three parameters.

IV. EXPERIMENTAL ANALYSIS

In this section, we confirm the performance of the newly proposed PMVO, which has been experimentally tested on CEC2013 and multi-layer image segmentation problems.

A. EXPERIMENTAL RESULTS FOR GLOBAL OPTIMIZATION

In this subsection, the test suite CEC2013 is used to evaluate the performance of the proposed PMVO algorithm for real-parameter optimization. The CEC2013 includes 28 benchmark functions, in which f1-f5 are five unimodal functions, f6-f20 are fifteen multi-modal functions and f21-f28 are composition functions. All of these benchmarks have been moved to the same global minimum for testing. More detailed descriptions of CEC2013 can be found in the literature [33], [34].

MVO, PSO, PPSO (strategy 1), and GWO are used to compare with the proposed PMVO algorithm. In order to achieve a fair competition, we tested 51 times for each optimization algorithm. The dimensions of all benchmark functions were set to 50, the number of iterations was 2000, the number of initial solutions was 100, and the initial solutions range were -100 to 100. TABLE 3 presents the different parameters remaining in each algorithm. TABLE 4 - 5 show the performance of each algorithm in the experiments on the best, the mean and standard deviation of the function error $f = f_i - f_i^*$. A small value means a corresponding excellent optimization result. FIGURE 3 shows the simulation results

of five optimization algorithms on the benchmark functions f3, f13, f24, and f28.

According to TABLE 4 and TABLE 5, from the optimization accuracy of the CEC2013 test suite, the proposed PMVO algorithm is superior to the other four compared algorithms. From Table 4, comparing with the MVO algorithm, the proposed PMVO achieves 15 better performances, 10 worse performances, 3 similar performances in 28 benchmarks from the “best” perspective of view. It achieves 17 better performances, 10 worse performances and 1 similar performance from “mean” perspective of view. It also achieves 14 better performance, 13 worse performances and 1 similar performance from “standard deviation” perspective of view. Comparing with PSO algorithm, the proposed PMVO achieves 26 better performances, 1 worse performance, 1 similar performance in 28 benchmarks from “best” perspective of view. It achieves 27 better performances, 1 worse performance and 0 similar performance from “mean” perspective of view. It also achieves 24 better performance, 4 worse performances and 0 similar performance from “standard deviation” perspective of view. The convergence curves of best values for these algorithms are plotted in FIGURE 3. It can be seen from the results that the performance of the proposed PMVO algorithm is superior to the other competed algorithms on the functions f3, f13, f24, f28.

From Table 5, for the “best” value, compared with the PPSO algorithm, the proposed PMVO algorithm obtains 23 better performances, 3 worse performances, and 2 similar performances. For the “mean” value, it obtains 25 better performances, 3 worse performances, and 0 similar performance. For the “standard deviation” value, it obtains 21 better performances, 6 worse performances, and 1 similar performance. Comparing with the GWO algorithm, the proposed PMVO algorithm obtains 21 better performances, 5 worse performances, and 2 similar performances. For the “mean” value, it obtains 23 better performances, 4 worse performances, and 1 similar performance. For the “standard deviation” value, it obtains 18 better performances, 10 worse performances,

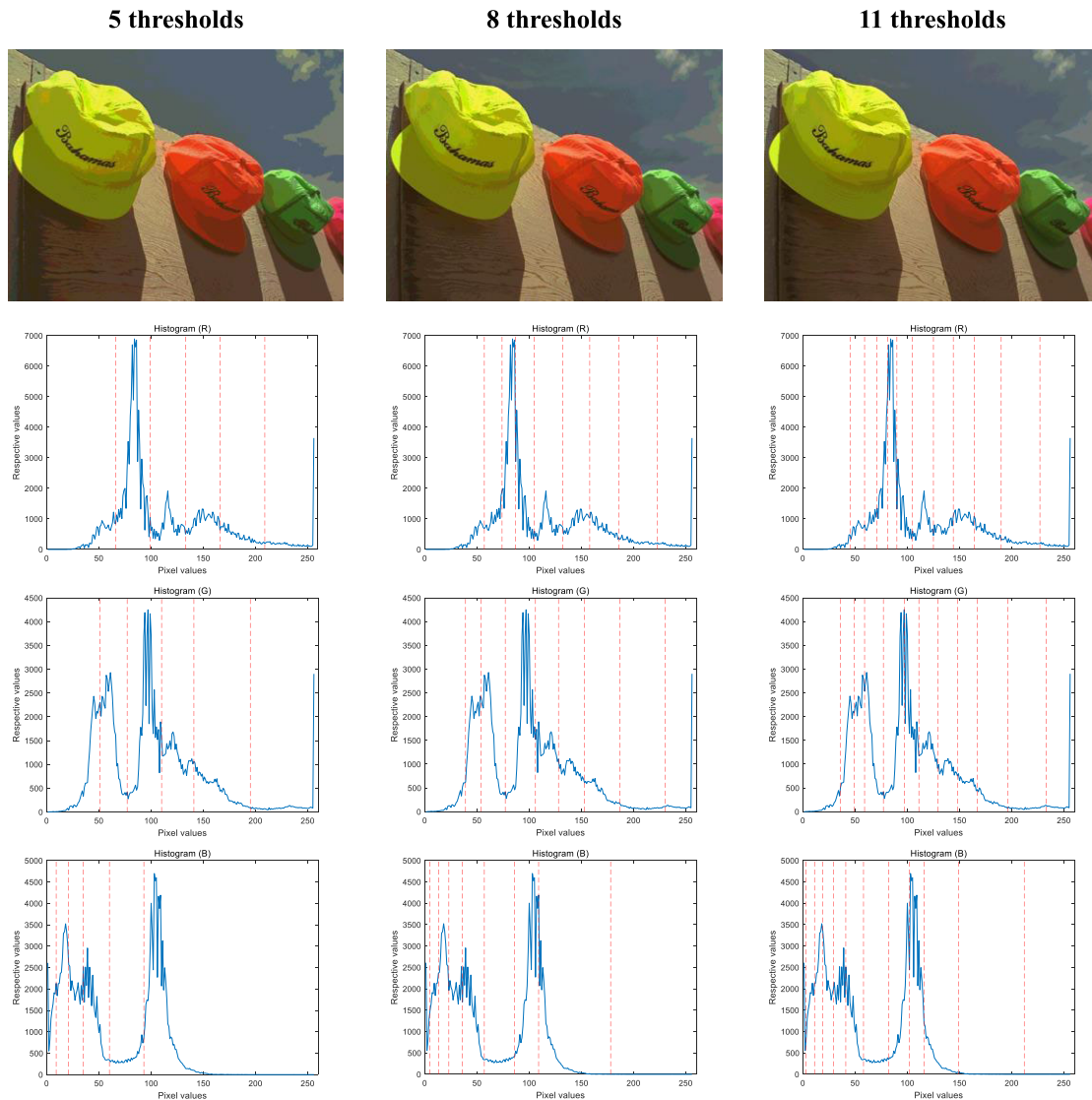


FIGURE 5. The segmented results of *Img5* by applying the PMVO algorithm and their histogram plots of each band (RGB) with 5, 8, 11 threshold values.

and 0 similar performance. Overall, the proposed PMVO algorithm can perform better than the contrasted MVO, PSO, PPSO and GWO algorithms under the CEC2013 test suite.

B. EXPERIMENTAL RESULTS FOR MULTILEVEL IMAGE SEGMENTATION PROBLEM

In this subsection, the minimum cross-entropy threshold is used as a fitness function to deal with multi-level threshold segmentation of color images. The eight original images are shown in FIGURE 4. Each color image contains three bands (RGB) with multimodal properties. They are used to evaluate the performance of the proposed PMVO application on image segmentation problems. In order to have a fair competition to avoid any stochastic discrepancy, each color image is run 51 times using each algorithm respectively, the iteration is set to 100, and the number of initialized solutions is 100.

The rest parameter settings in each algorithm are also the same as TABLE 3.

In order to visually show the image segmentation effect achieved by the proposed PMVO algorithm, the segmented information of *Img5* is presented in FIGURE 5, in which the thresholds are set to 5, 8, and 11 for three channels (RGB). According to the pixel histogram information of each band, the MCET function is optimized to obtain the thresholds for multilevel image segmentation. Red dash lines at the histogram of each band of *Img5* indicate the obtained threshold values. FIGURE 5 shows the obtained segmentation boundary positions, which sequentially divide the pixel values into different regions.

In addition, set the threshold number of three channels (RGB) to 5, 8, and 11. Apply the proposed PMVO and contrast MVO, PSO, PPSO, GWO algorithms to segment

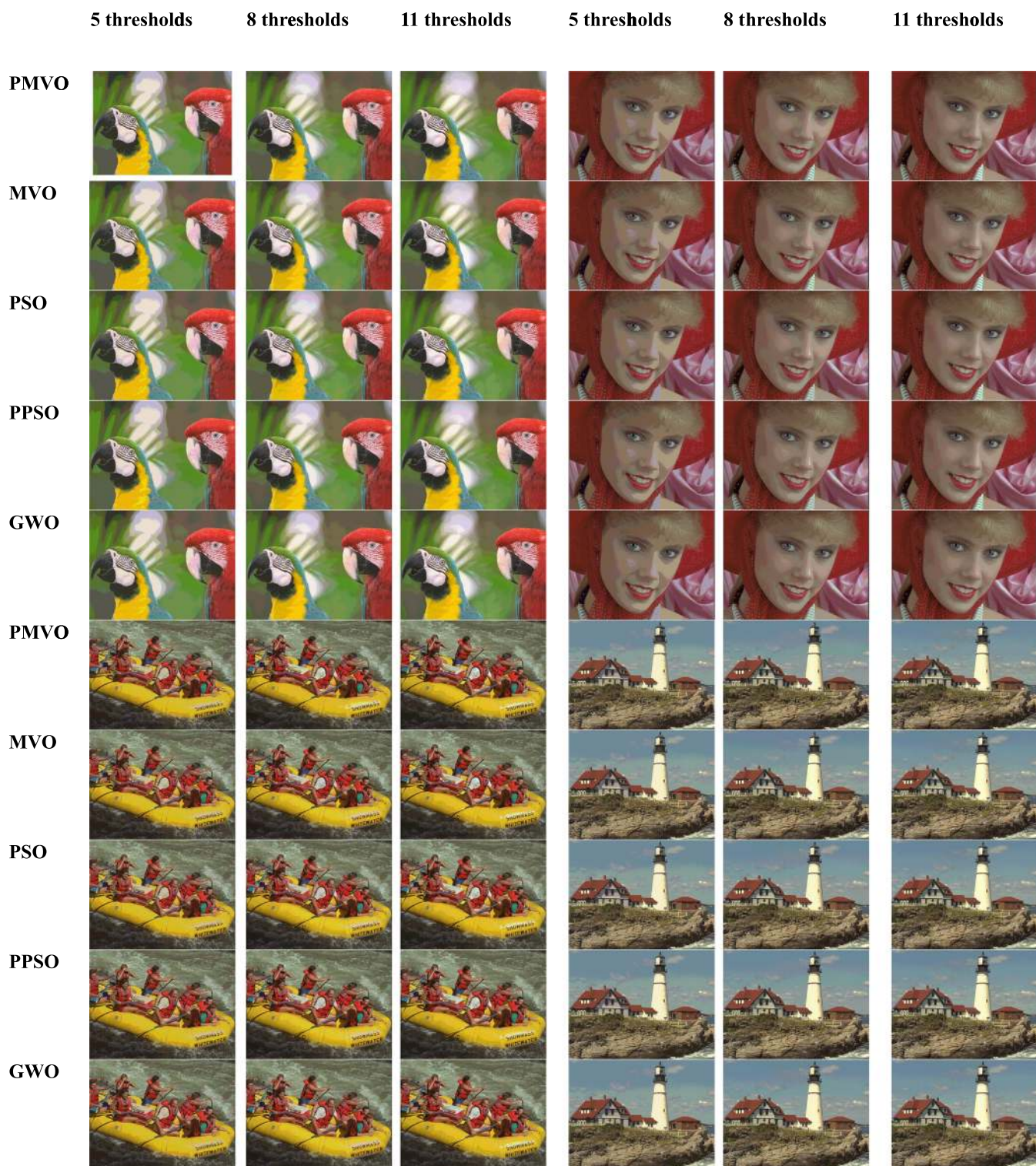


FIGURE 6. Segmented images for 5, 8, 11 thresholding values (Img1, Img4, Img6, Img7).

each channel of the image respectively, and then concatenate the segmented results to form the final segmented image. Several segmented images (Img1, Img4, Img6, Img7) after using different optimization algorithms are presented in FIGURE 6. From the results of segmentation, we also find

that the quality of the segmented image will improve with the increase in the number of thresholds.

Since each color image contains various information features and the optimization algorithm is random, this will lead to some changes in the experimental results obtained.

TABLE 6. Comparison of PSNR, SSIM and FSIM computed by different algorithms.

	PMVO	MVO	PSO	PPSO	GWO		PMVO	MVO	PSO	PPSO	GWO
	PSNR						PSNR				
Img1	27.7274	27.7199	27.7141	27.2663	27.2663	Img5	29.6954	29.6873	29.6873	29.6873	29.6873
	30.6858	30.9967	30.6901	30.7035	30.7011		32.6991	32.4827	32.4533	32.2906	32.5644
	33.4599	33.0558	33.1826	32.9758	33.1563		34.6938	34.9113	35.2136	34.9044	35.1311
Img2	29.0463	29.0463	29.0463	29.0463	29.0463	Img6	27.5589	27.5321	27.5470	27.5470	27.5435
	32.1381	32.2276	32.4814	32.4769	32.3486		30.8249	30.7159	30.6805	30.6875	30.7457
	35.5308	35.3965	35.0102	34.7354	34.1941		33.3312	33.0073	33.1884	33.1925	32.9527
Img3	28.4154	28.4154	28.4154	28.4154	28.4154	Img7	28.9318	28.9318	28.9318	28.9318	28.9374
	31.8368	31.4503	31.6986	31.7971	31.7461		32.2885	32.1075	31.7156	31.8798	31.8416
	33.8523	33.6884	34.2001	34.1342	34.4203		34.5372	34.6341	34.3909	34.4589	34.5840
Img4	29.1437	29.1437	29.1437	29.1437	29.1437	Img8	26.8469	26.8469	26.8469	26.8469	26.8520
	32.3280	31.9904	32.2766	32.0305	32.2498		30.1058	29.8766	30.0492	30.0492	30.0803
	34.2932	34.3055	34.8105	34.8220	34.8143		33.9510	33.6632	32.5908	32.5580	33.0472
	SSIM						SSIM				
Img1	0.8638	0.8618	0.8620	0.8614	0.8614	Img5	0.9104	0.9106	0.9106	0.9106	0.9106
	0.9131	0.9134	0.9137	0.9138	0.9138		0.9397	0.9383	0.9385	0.9389	0.9405
	0.9457	0.9435	0.9456	0.9444	0.9443		0.9568	0.9564	0.9617	0.9608	0.9601
Img2	0.9335	0.9335	0.9335	0.9335	0.9335	Img6	0.9228	0.9227	0.9228	0.9228	0.9228
	0.9595	0.9615	0.9627	0.9625	0.9619		0.9631	0.9601	0.9598	0.9597	0.9601
	0.9792	0.9754	0.9764	0.9762	0.9754		0.9760	0.9738	0.9767	0.9768	0.9747
Img3	0.8985	0.8985	0.8985	0.8985	0.8985	Img7	0.9352	0.9352	0.9352	0.9352	0.9352
	0.9467	0.9456	0.9449	0.9456	0.9454		0.9589	0.9631	0.9605	0.9576	0.9568
	0.9627	0.9650	0.9695	0.9695	0.960		0.9774	0.9775	0.9766	0.9764	0.9777
Img4	0.8827	0.8827	0.8827	0.8827	0.8827	Img8	0.9528	0.9528	0.9528	0.9528	0.9528
	0.9363	0.9302	0.9358	0.9309	0.9359		0.9785	0.9755	0.9766	0.9766	0.9767
	0.9528	0.9526	0.9616	0.9612	0.9613		0.9875	0.9867	0.9766	0.9870	0.9875
	FSIM						FSIM				
Img1	0.9291	0.9217	0.9210	0.9147	0.9147	Img5	0.9511	0.9511	0.9511	0.9511	0.9511
	0.9466	0.9508	0.9464	0.9466	0.9466		0.9767	0.9691	0.9690	0.9687	0.9707
	0.9654	0.9654	0.9645	0.9643	0.9645		0.9772	0.9794	0.9775	0.9787	0.9812
Img2	0.9765	0.9765	0.9765	0.9765	0.9765	Img6	0.9736	0.9736	0.9735	0.9735	0.9735
	0.9885	0.9880	0.9884	0.9883	0.9885		0.9842	0.9855	0.9851	0.9850	0.9852
	0.9928	0.9935	0.9938	0.9937	0.9929		0.9918	0.9911	0.9909	0.9908	0.9910
Img3	0.9432	0.9432	0.9432	0.9432	0.9432	Img7	0.9727	0.9727	0.9727	0.9727	0.9727
	0.9752	0.9713	0.9750	0.9729	0.9716		0.9870	0.9875	0.9857	0.9871	0.9871
	0.9790	0.9725	0.9815	0.9820	0.9802		0.9922	0.9919	0.9920	0.9921	0.9919
Img4	0.9479	0.9479	0.9479	0.9479	0.9479	Img8	0.9715	0.9715	0.9715	0.9715	0.9715
	0.9688	0.9598	0.9646	0.9633	0.9684		0.9835	0.9832	0.9825	0.9825	0.9827
	0.9799	0.9773	0.9796	0.9791	0.9799		0.9914	0.9904	0.9907	0.9907	0.9914

For instance, the threshold values generated by using an optimization algorithm are inappropriate, which may not obtain the best segmentation result. We used PSNR, SSIM, and

FSIM parameters to comprehensively evaluate the quality of segmented images obtained by applying different optimization algorithms. PSNR is a ratio parameter between the

maximum pixel value and MSE, SSIM represents the structural information similarity between two images, and FSIM represents the similarity of feature information between two images. For these three parameters, a large value indicates that the segmented image is of high quality, and the results obtained are similar to the original image. TABLE 6 lists all experimental data, and the best values are shown in bold. We can observe that as the number of thresholds increases, the values of the three evaluation parameters increase accordingly, which means that the quality of the segmented image will be better. For the eight color images used for testing, the segmentation image quality obtained by each algorithm is similar when the threshold is set to 5. For 8, 11 thresholds, the proposed PMVO algorithm will generally produce higher quality segmentation results compared to the other algorithms. In general, the proposed scheme can effectively and feasibly solve the multilevel image segmentation problem.

V. CONCLUSION

In this paper, a new optimization algorithm named PMVO is presented. The parallel mechanism with a communication strategy is significantly used to achieve the cooperation individual of optimization algorithms. In PMVO, the initialized universe is randomly divided into several groups, and each group of universes evolves with the number of iterations through the original MVO algorithm. At the same time, a new communication strategy is presented. After reaching a pre-set fixed iteration, the universes between different groups will share information, speeding up the flow of information between groups to increase the diversity of the population. Firstly, we tested the proposed scheme under the CEC2013 test suite, and the experimental results confirmed that the proposed PMVO is superior to the MVO, PSO, PPSO, and GWO algorithms compared. In addition, we also applied the proposed PMVO to solve the problem of multilevel image segmentation. The obtained segmented images have been evaluated by three parameters: PSNR, SSIM, and FSIM. The results prove that the proposed scheme is effective and feasible, and it is more competitive than the compared algorithms.

In the next work, we will further modify the communication strategy and evolution scheme to enhance the information exchange between populations. This will improve the performance of the optimization algorithm. We will also use the proposed scheme to deal with more challenging problems in reality.

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