



A parametric study of wind wave spectrum representations for shallow water

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Abstract

Different wind wave spectrum representations for shallow water are compared from a parametric point of view, including formulations with both constant (TMA and FRF spectra) and variable (Wallops-SW and GLERL spectra) frequency exponents. The results of the research clearly show the necessity of at least four parameters in order to achieve an adequate representation of the full range of the spectrum in non-saturated or decaying seas propagating in shallow water. Furthermore, when the modelling of the high frequency range is not of special interest, the total number of independent parameters could be reduced to only three without considerable loss of accuracy. Comparisons of theoretical shallow water spectral forms with spectra calculated from actual field measurements do not show any definite tendency in performance, being possible to obtain good results with any of the studied spectra. So, the selection of which spectrum to use in a particular application is still a quite subjective decision, being of importance the availability of input parameters needed for each model.

Introduction

Interest in obtaining a proper representation of the wind wave spectrum in shallow water has increased during the last decades due to the economic and environmental incidence of physical processes taking place in coastal regions. Detailed information on the shape of shallow water spectra is generally needed in many engineering applications, e.g.: stability and oscillation of floating bodies; resonance in harbours and estuaries; surf zone dynamics and longshore sediment transport; determination of transfer functions for structural response of coastal structures.

A number of functional forms have been proposed to represent the wind wave spectrum in deep water. From all of them, the Pierson-Moskowitz

spectrum (Pierson and Moskowitz²⁵) has become a standard for the representation of fully developed seas. For active growing seas and, even, for decaying seas, the JONSWAP spectrum (Hasselmann *et al.*¹⁰) and the Wallops spectrum (Huang *et al.*¹³) can be outlined as the most successful formulations in wave modelling and engineering design. Giménez and Sánchez-Carratalá⁷ compare several of these deep water spectra, determining the minimum number of independent parameters needed for the representation of both fully and partially developed seas.

On the other hand, different functional forms have been developed to represent the spectrum of wind generated waves in water of arbitrary depth. Several experimental studies have been carried out during the last years in order to test the accuracy of the available models from actual field measurements. However, it has not been possible yet to definitely establish the applicability and limitations of every formulation.

This paper presents an objective comparison of four wind wave spectrum representations for shallow water, recognised nowadays by researchers and engineers as the most reliable ones, namely: the TMA, the FRF, the Wallops-SW and the GLERL spectra. The comparison is accomplished by using a new parametric approach recently proposed by Sánchez-Carratalá²⁷ for the study of spectral shapes.

Spectral formulations with constant frequency exponents (CFE)

The TMA spectrum

Based on dimensional arguments, Phillips²³ suggests the existence of an equilibrium range due to wave breaking, deducing the following asymptote in the deep water directional wave number spectrum:

$$S_{K1}(\vec{k}) = \beta_{K1}(\theta) k^{-4} \quad (1)$$

where \vec{k} is the wave number vector; k is the wave number; and $\beta_{K1}(\theta)$ is a dimensionless function specifying the directional distribution of wave components.

Kitaigorodskii *et al.*¹⁷ show that the wave number spectrum maintains similarity in water of arbitrary depth. Using this assumption and applying the linear dispersion relationship, Equation (1) can be transformed into the corresponding shallow water one-dimensional frequency formulation:

$$S_{K1}(f; \alpha_{K1}, d) = \alpha_{K1} g^2 (2\pi)^{-4} f^{-5} \Phi_{K1}(f; d) \quad (2)$$

where f is the frequency; d is the water depth; g is the gravitational acceleration; α_{K1} is a dimensionless coefficient; and $\Phi_{K1}(f; d)$ is the transformation factor, a dimensionless function considering both the spectral

transformation from deep to shallow water and the change of variable in the spectrum. The transformation factor for an f^{-5} equilibrium range is given by:

$$\Phi_{K1}(f;d) = \frac{\left[k^{-3} \frac{\partial k}{\partial f} \right]_{(2\pi f)^2 = gk \tanh(kd)}}{\left[k^{-3} \frac{\partial k}{\partial f} \right]_{(2\pi f)^2 = gk}} \quad (3)$$

Let us define the dimensionless angular frequency as $\omega_d = 2\pi f(d/g)^{0.5}$. In deep water (frequencies with $\omega_d > 1.8$), the spectrum in Equation (2) takes the form of the well-known depth-independent Phillips' f^{-5} equilibrium range, while in very shallow water (frequencies with $\omega_d < 0.25$) it turns into a depth-dependent f^{-3} power law.

Assuming that the transformation factor can be applied to the entire range of frequencies, Bouws *et al.*² propose an extension of the self-similar JONSWAP spectral shape to shallow water depth, commonly referred to as TMA spectrum, after the three data sets used to confirm the validity of their assumptions:

$$S_T(f; \alpha_{K1}, f_p, \gamma, \sigma_a, \sigma_b, d) = S_{K1}(f; \alpha_{K1}, d) \Phi_{PM}(f; f_p) \Phi_J(f; f_p, \gamma, \sigma_a, \sigma_b) \quad (4)$$

with

$$\Phi_{PM}(f; f_p) = e^{-\frac{5}{4} \left(\frac{f}{f_p} \right)^{-4}} \quad (5)$$

$$\Phi_J(f; f_p, \gamma, \sigma_a, \sigma_b) = \gamma \exp \left[-\frac{(f - f_p)^2}{2\sigma^2 f_p^2} \right] \quad (6)$$

where $\Phi_{PM}(f; f_p)$ is the exponential high-pass filter used by Pierson and Moskowitz²⁵ to model the wind wave spectrum in the low frequency range (front face); $\Phi_J(f; f_p, \gamma, \sigma_a, \sigma_b)$ is the peak enhancement factor proposed by Hasselmann *et al.*¹⁰ to fit the narrowly peaked spectra observed in active growing seas; f_p is the peak frequency; γ is the peak enhancement parameter, defined as the ratio of the maximum spectral density in the JONSWAP spectrum to the maximum in the corresponding Pierson-Moskowitz spectrum with the same values of α_{K1} and f_p ; and σ is the width parameter that specifies the side width around the spectral peak with two different values: σ_a for $f \leq f_p$; and σ_b for $f > f_p$.

From Equation (4), it is evident that the TMA spectrum has 5+1 independent parameters, i.e.: 2 scale parameters (α_{K1}, f_p); 3 shape parameters ($\gamma, \sigma_a, \sigma_b$); and 1 transformation parameter (d). Furthermore, it is apparent that



five of them (α_{K1} , f_p , γ , σ_a , σ_b) play the same roll as those in the JONSWAP spectrum.

The FRF spectrum

Considerable empirical evidence supporting an equilibrium range proportional to f^{-4} , as originally suggested by Toba^{28,29}, has been presented in the last two decades (Kahma¹⁵, Forristall⁶, Donelan *et al.*⁵, Battjes *et al.*¹, Liu²¹, Young³¹). This high frequency tail formulation is also supported on theoretical grounds (Kitaigorodskii¹⁶, Phillips²⁴). Resio²⁶ suggests that the equilibrium range of a wind wave spectrum should be depth-independent in terms of the spectral action density rather than in terms of the spectral energy density, implying an f^{-4} power law for the deep water wave spectrum and an f^{-2} for the very shallow water case.

Based on dimensional arguments, Kitaigorodskii¹⁶ suggests the existence of an equilibrium range due to non-linear interactions amongst spectral components, deducing the following asymptote in the deep water directional wave number spectrum:

$$S_{K2}(\vec{k}; U_{10}) = \beta_{K2}(\theta) g^{-0.5} U_{10} k^{-3.5} \quad (7)$$

where $\beta_{K2}(\theta)$ is a dimensionless function specifying the directional distribution of wave components; and U_{10} is the wind speed at 10 m above the MSL.

Due to similarity principles, Equation (7) can be transformed into the corresponding shallow water one-dimensional frequency formulation:

$$S_{K2}(f; \alpha_{K2}, U_{10}, d) = \alpha_{K2} g U_{10} (2\pi)^{-3} f^{-4} \Phi_{K2}(f; d) \quad (8)$$

where α_{K2} is a dimensionless coefficient; and $\Phi_{K2}(f; d)$ is the transformation factor for an f^{-4} equilibrium range. From here, derivation of the FRF spectrum carried out by Miller and Vincent²² parallels that of the TMA spectrum, giving rise to the following formulation:

$$S_F(f; \alpha_{K2}, U_{10}, f_p, \gamma, \sigma_a, \sigma_b, d) = S_{K2}(f; \alpha_{K2}, U_{10}, d) \Phi_{PM}(f; f_p) \Phi_J(f; f_p, \gamma, \sigma_a, \sigma_b) \quad (9)$$

From Equation (9), it is evident that the FRF spectrum has, in its raw form, 6+1 independent parameters, i.e.: 3 scale parameters (α_{K2} , U_{10} , f_p); 3 shape parameters (γ , σ_a , σ_b); and 1 transformation parameter (d). Furthermore, it is apparent that five of them (α_{K2} , f_p , γ , σ_a , σ_b) play the same roll as those in the modified JONSWAP spectrum proposed by Donelan *et al.*⁵.

Reparameterizations of CFE spectral formulations

Several reparameterizations of the JONSWAP spectral form have been proposed (Houmb and Overvik¹¹, Chakrabarti⁴, Isherwood¹⁴, Young³¹), providing different empirical relationships for estimating all the spectral parameters in terms of some characteristic wave height and wave period (e.g., significant wave height H_s ; and peak period T_p). Relationships given by Young³¹ also suggest that an additional free parameter could be necessary to determine whether a sea is locally wind generated and fetch limited. Goda⁸ rewrites the JONSWAP spectrum in terms of H_s , f_p and γ . All of the aforementioned reparameterizations seem to indicate the possibility of representing a deep water wind wave spectrum with 2 or, at most, 3 independent parameters.

Taking into account that the TMA spectral form is nothing but a depth-dependent extension of the deep water JONSWAP spectral form, it may be argued that similar reparameterizations could be developed for the TMA spectrum, e.g. based on the relationships given by Bouws *et al.*³. Consequently, the number of independent parameters in the TMA spectrum could be reduced to only 2+1 (e.g., α_{K1} , f_p , d) or, at most, 3+1 (e.g., α_{K1} , f_p , γ , d), when considering the depth dependency of the shallow water spectrum.

With respect to the FRF spectrum, Miller and Vincent²² recommend that α_{K2} , σ_a and σ_b be set equal to their mean observed values, thus reducing to 3+1 (e.g., U_{10} , f_p , γ , d) the number of independent parameters needed for the modelling of a shallow water wind wave spectrum. Moreover, if α_{K2} , σ_a and σ_b are held constant, an empirical relationship between the peak enhancement parameter γ and the significant slope of the wave field ζ ($\zeta = H_s/4L_p$, with L_p the wave length corresponding to the peak frequency) can be obtained that simplifies the practical application of the FRF spectrum in such a way that only 2+1 (e.g., U_{10} , f_p , d) independent parameters would be necessary.

According to the previous argumentations, it could be concluded that the mean characteristics of the wind wave spectrum in shallow water can be reasonably represented by an CFE spectral formulation with only 3 or, at most, 4 independent parameters.

Spectral formulations with variable frequency exponents (VFE)

The Wallops-SW spectrum

Based on theoretical studies and laboratory data, Huang *et al.*¹³ propose a different formulation for the deep water wave spectrum that only depends on internal sea state parameters. The so-called Wallops spectrum is the first to use a variable frequency exponent in the equilibrium range, providing to the spectral form both variable bandwidth and variable spectral peakedness:



$$S_w(f) = \alpha_w g^2 (2\pi)^{-4} f_p^{m-5} f^{-m} e^{-\frac{m}{4} \left(\frac{f}{f_p}\right)^4} \quad (10)$$

Huang *et al.*¹² extend the Wallops spectrum to finite depth water (Wallops-SW spectrum), distinguishing two different cases depending on the nondimensional depth $\bar{d} = k_p d$, with k_p the wave number corresponding to the peak frequency. For $0.75 \leq \bar{d} < 3$ they use Stokes' wave theory that leads to:

$$\alpha_w = \frac{(2\pi \zeta)^2 m^{(m-1)/4} \tanh^2(k_p d)}{4^{(m-5)/4} \Gamma\left(\frac{m-1}{4}\right)} \quad (11)$$

$$m = \left| \frac{\log \left\{ \sqrt{2} \zeta \left[1 + \frac{3}{2 \sinh^2(k_p d)} \right] \coth(k_p d) \right\}}{\log \sqrt{2}} \right| \quad (12)$$

and for $\bar{d} < 0.75$ they apply solitary wave theory to obtain:

$$\alpha_w = \frac{(2\pi \zeta)^2 m^{(m-1)/4} (c_p^2 k_p / g)^2}{4^{(m-5)/4} \Gamma\left(\frac{m-1}{4}\right)} \quad (13)$$

$$m = \left| \frac{\log \left(\cosh \frac{\pi}{\sqrt{7.5} U_R} \right)}{\log \sqrt{2}} \right| \quad (14)$$

where Γ is the Gamma function; $c_p = 2\pi f_p / k_p$ is the phase celerity corresponding to the peak frequency; and $U_R = 2\pi \zeta / (k_p d)^3$ is the Ursell number. Liu²⁰ finds a better agreement with experimental data if solitary wave theory rather than Stokes' wave theory is also used in the interval $0.75 \leq \bar{d} < 1.5$.

From Equations (10)-(14), it is evident that the Wallops-SW spectrum has 2+1 independent parameters, e.g.: H_s , f_p and d .

The GLERL spectrum

Liu¹⁸ presents a generalized spectral form, commonly named as GLERL spectrum, that can be expressed as follows:

$$S_L(f) = \alpha_L H_s^2 f_p^{m-1} f^{-m} e^{-\frac{m}{n} \left(\frac{f}{f_p}\right)^n} \quad (15)$$

with

$$\alpha_L = \frac{0,06238 m^{(m-1)/n}}{n^{(m-n-1)/n}} \frac{1}{\Gamma\left(\frac{m-1}{n}\right)} \quad (16)$$

where n is a dimensionless frequency exponent.

There is no explicit depth dependency in this purely empirical spectrum. However, as shown in Liu¹⁹, it applies equally well in shallow water and in deep water.

From Equations (15) and (16), it is evident that the GLERL spectrum has 4 independent parameters, e.g.: H_s , f_p , m and n .

Reparameterizations of VFE spectral formulations

Goda⁹ presents a modified Wallops spectrum that depends on 3 independent parameters (H_s , f_p and m), so that it should be considered a new spectrum rather than a reparameterization.

With respect to the GLERL spectrum, several reparameterizations could be performed without reducing the number of free parameters. However, if a fixed value is adopted for the frequency exponent in the exponential high-pass filter (e.g., $n=4$ as done in many other spectra), it would be possible to obtain a modified GLERL spectrum with only 3 independent parameters, closely related with the modified Wallops spectrum mentioned above.

According to the previous argumentations, it could be concluded that the mean characteristics of the wind wave spectrum in shallow water can be reasonably represented by an VFE spectral formulation with only 3 or, at most, 4 independent parameters.

Comparison of CFE and VFE spectral formulations from field measurements

Vincent and Resio³⁰ present an eigenvector analysis for the parameterization of ocean wave spectra, showing that as few as five free parameters are required to reproduce 85-90% of the total energy in the spectrum. Considering that many of the spectra analysed by Vincent and Resio³⁰ have two or more peaks, it can be followed that for unimodal spectra not more than 3 or 4 independent parameters should be necessary.

Miller and Vincent²² point out that spectral analysis of field measurements seems to indicate that, for most practical purposes, it is generally possible to



fit adequately any well-behaved observation with any of the studied CFE spectra, if the statistical variability of the spectral estimates is considered.

According to Liu²⁰, the choice of which model to use depends upon the availability of input data, and is still at present a subjective decision. Unexpectedly, spectra specifically developed for depth effects (like TMA and Wallops-SW) tend to be less effective than spectra without depth effects explicitly included in their formulation (like JONSWAP and GLERL). Moreover, while the TMA, FRF and GLERL spectra attempt to fit the full range of the spectrum, the Wallops-SW spectrum tends to fit only the portion of the spectrum in the vicinity of the spectral peak, as should be expected from its theoretical derivation. Liu²⁰ also indicates that the spectral shape is basically unaffected by depth. This gives support to the existence of a similarity law in shallow water, allowing us to hypothesize that a transformation factor like those of the TMA or the FRF spectra could be used to extend any deep water spectrum to water of arbitrary depth.

Conclusions

Different representations of the wind wave spectrum in shallow water have been compared from a parametric point of view, using the data and analyses of several field measurements previously published. The results of the research clearly show the necessity of at least four parameters in order to achieve an adequate representation of the full range of the spectrum in non-saturated or decaying seas propagating in shallow water. Furthermore, when the modelling of the high frequency range is not of special interest, the total number of independent parameters could be reduced to only three without considerable loss of accuracy.

Comparisons of theoretical shallow water spectral forms with spectra calculated from actual field measurements do not show any definite tendency in performance, being possible to obtain good results with any of the studied spectra. So, the selection of which spectrum to use in a particular application is still a quite subjective decision, being of importance the availability of input parameters needed for each model. In other words, none of the studied CFE and VFE spectral formulations can be pointed out as clearly better than the others. However, some studies seem to show a certain superiority of those spectral forms not having the depth as an explicit parameter.



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