A Particle Swarm Optimization for Economic Dispatch With Nonsmooth Cost Functions

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***Abstract*—This paper presents a new approach to economic dispatch (ED) problems with nonsmooth cost functions using a particle swarm optimization (PSO) technique. The practical ED problems have nonsmooth cost functions with equality and inequality constraints that make the problem of finding the global optimum difficult using any mathematical approaches. In this paper, a modified PSO (MPSO) mechanism is suggested to deal with the equality and inequality constraints in the ED problems. A constraint treatment mechanism is devised in such a way that the dynamic process inherent in the conventional PSO is preserved. Moreover, a dynamic search-space reduction strategy is devised to accelerate the optimization process. To show its efficiency and effectiveness, the proposed MPSO is applied to test ED problems, one with smooth cost functions and others with nonsmooth cost functions considering valve-point effects and multi-fuel problems. The results of the MPSO are compared with the results of conventional numerical methods, Tabu search method, evolutionary programming approaches, genetic algorithm, and modified Hopfield neural network approaches.**

***Index Terms*—Constrained optimization, economic dispatch (ED), nonsmooth optimization, particle swarm optimization (PSO).**

I. Introduction

MOST of power system optimization problems including economic dispatch (ED) have complex and nonlinear characteristics with heavy equality and inequality constraints. Recently, as an alternative to the conventional mathematical approaches, the heuristic optimization techniques such as genetic algorithms, Tabu search, simulated annealing, and recently-introduced particle swarm optimization (PSO) are considered as realistic and powerful solution schemes to obtain the global or quasiglobal optimums in power system optimization problems [1].

Recently, Eberhart and Kennedy suggested a particle swarm optimization (PSO) based on the analogy of swarm of bird and school of fish [2]. The PSO mimics the behavior of individuals in a swarm to maximize the survival of the species. In PSO, each individual makes his decision using his own experience together with other individuals' experiences [3]. The algorithm, which is based on a metaphor of social interaction, searches a space by adjusting the trajectories of moving points in a multidimensional space. The individual particles are drawn stochastically toward the position of present velocity of each individual, their own previous best performance, and the best previous performance of their neighbors [4]. The main advantages of the PSO algorithm are summarized as: simple concept, easy implementation, robustness to control parameters, and computational efficiency when compared with mathematical algorithm and other heuristic optimization techniques.

Recently, PSO have been successfully applied to various fields of power system optimization such as power system stabilizer design [5], reactive power and voltage control [3], and dynamic security border identification [6]. The original PSO mechanism is directly applicable to the problems with continuous domain and without any constraints. Therefore, it is necessary to revise the original PSO to reflect the equality/inequality constraints of the variables in the process of modifying each individual's search. Yoshida *et al.* [3] suggested a modified PSO to control reactive power and voltage considering voltage security assessment. Since the problem was a mixed-integer nonlinear optimization problem with inequality constraints, they applied the classical penalty method to reflect the constraint-violating variables. Abido [5] developed a revised PSO for determining the optimal values of parameters for power system stabilizers. In the study, the velocity of each parameter is limited to a certain value to reflect the inequality constraint problem in the dynamic process.

The practical ED problems with valve-point and multi-fuel effects are represented as a nonsmooth optimization problem with equality and inequality constraints, and this makes the problem of finding the global optimum difficult. To solve this problem, many salient methods have been proposed such as a mathematical approach [7], dynamic programming [8], evolutionary programming [9], [15], [16], Tabu search [14], neural network approaches [10], [11], and genetic algorithm [12].

In this paper, an alternative approach is proposed to the nonsmooth ED problem using a modified PSO (MPSO), which focuses on the treatment of the equality and inequality constraints when modifying each individual's search. The equality constraint (i.e., the supply/demand balance) is easily satisfied by specifying a variable (i.e., a generator output) at random in each iteration as a slag generator whose value is determined by the difference between the total system demand and the total generation excluding the slag generator. However, the inequality constraints in the next position of an individual produced by the PSO algorithm can violate the inequality constraints. In this case, the position of any individual violating the constraints is set to its minimum or maximum position

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depending on the velocity evaluated. Additionally, to accelerate the convergence speed, a dynamic search-space reduction strategy is devised based on the distance between the best position of the group and the inequality boundaries.

II. FORMULATION OF ED PROBLEM

A. ED Problem With Smooth Cost Functions

The ED problem is to find the optimal combination of power generations that minimizes the total generation cost while satisfying an equality constraint and inequality constraints. The most simplified cost function of each generator can be represented as a quadratic function as given in (2) whose solution can be obtained by the conventional mathematical methods [8]:

$$C = \sum_{j \in J} F_j(P_j) \tag{1}$$

$$F_j(P_j) = a_j + b_j P_j + c_j P_j^2 \tag{2}$$

where

C	total generation cost;
F_j	cost function of generator j;
a_j, b_j, c_j	cost coefficients of generator j;
P_j	electrical output of generator j;
J	set for all generators.

While minimizing the total generation cost, the total generation should be equal to the total system demand plus the transmission network loss. However, the network loss is not considered in this paper for simplicity. This gives the equality constraint

$$\sum_{j \in J} P_j = D \tag{3}$$

where D is the total system demand.

The generation output of each unit should be between its minimum and maximum limits. That is, the following inequality constraint for each generator should be satisfied

$$P_{j\min} \le P_j \le P_{j\max} \tag{4}$$

where $P_{j\min}, P_{j\max}$ is the minimum, maximum output of generator j.

B. ED Problem With Nonsmooth Cost Functions

In reality, the objective function of an ED problem has nondifferentiable points according to valve-point effects and change of fuels; therefore, the objective function should be composed of a set of nonsmooth cost functions. In this paper, two cases of nonsmooth cost functions are considered. One is the case with the valve-point loading problem where the objective function is generally described as the superposition of sinusoidal functions and quadratic functions. The other is the case with the multiple fuel problem where the objective function is expressed as the piecewise quadratic cost functions. In both cases, the problems have multiple minima, therefore, the task of finding the global solution still remains to be tackled [7], [14]–[16].

1) Nonsmooth Cost Functions With Valve-Point Effects: The generator with multi-valve steam turbines has very different input-output curve compared with the smooth cost function. Typically, the valve point results in, as each steam valve starts to

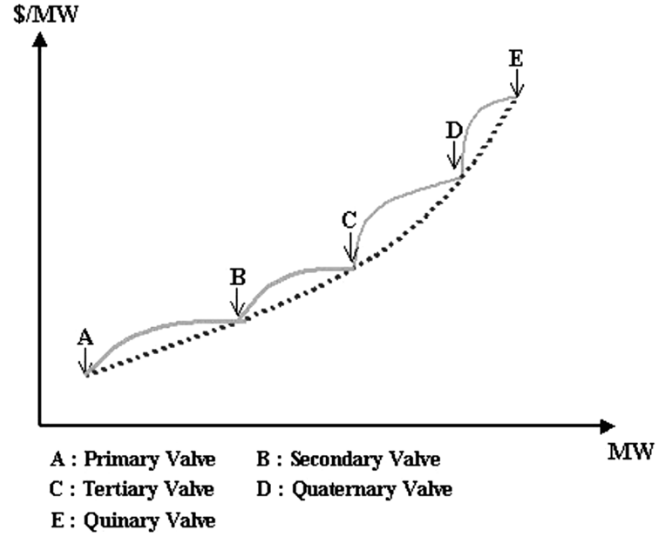

Fig. 1. Example of cost function with 5 valves.

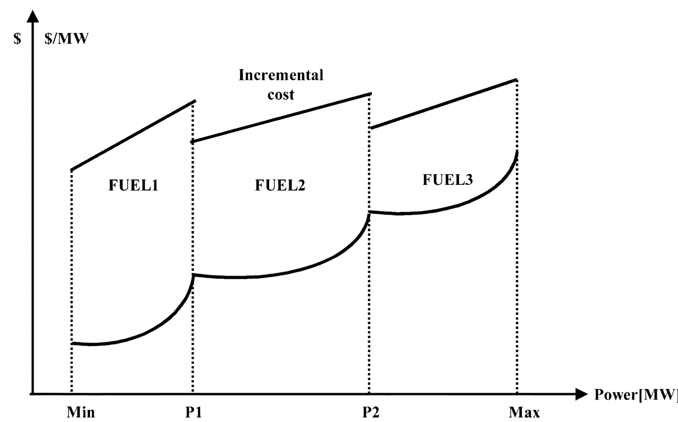

Fig. 2. Piecewise quadratic and incremental cost functions of a generator.

open, the ripples like in Fig. 1, [12], [15], [16]. To take account for the valve-point effects, sinusoidal functions are added to the quadratic cost functions as follows:

$$F_j(P_j) = a_j + b_j P_j + c_j P_j^2 + |e_j \times \sin(f_j \times (P_{j\min} - P_j))| \tag{5}$$

where e_j and f_j are the coefficients of generator j reflecting valve-point effects.

2) Nonsmooth Cost Functions With Multiple Fuels: Generally, a piecewise quadratic function is used to represent the input-output curve of a generator with multiple fuels [7]. The piecewise quadratic function is described as (6) and the cost and the incremental cost functions are illustrated in Fig. 2

$$F_j(P_j) = \begin{cases} a_{j1} + b_{j1} P_j + c_{i1} P_j^2 & \text{if } P_{j\min} \le P_j \le P_{j1} \\ a_{j2} + b_{j2} P_j + c_{j2} P_j^2 & \text{if } P_{j1} \le P_j \le P_{j2} \\ \vdots & \vdots \\ a_{jn} + b_{jn} P_j + c_{jn} P_j^2 & \text{if } P_{jn-1} \le P_j \le P_{j\max} \end{cases} \tag{6}$$

where a_{jp}, b_{jp}, c_{jp} are the cost coefficients of generator j for the pth power level.

III. Implementation of PSO for ED Problems

A. Overview of the PSO

Kennedy and Eberhart [2] developed a PSO algorithm based on the behavior of individuals (i.e., particles or agents) of a swarm. Its roots are in zoologist's modeling of the movement of individuals (e.g., fishes, birds, or insects) within a group. It has been noticed that members within a group seem to share information among them, a fact that leads to increased efficiency of the group [13]. The PSO algorithm searches in parallel using a group of individuals similar to other AI-based heuristic optimization techniques [17]. An individual in a swarm approaches to the optimum or a quasioptimum through its present velocity, previous experience, and the experience of its neighbors.

In a physical n-dimensional search space, the position and velocity of individual i are represented as the vectors $X_i = (x_{i1}, \ldots, x_{\text{in}})$, and $V_i = (v_{i1}, \ldots v_{\text{in}})$, respectively, in the PSO algorithm. Let $\text{Pbest}_i = (x_{i1}^{\text{Pbest}}, \ldots, x_{\text{in}}^{\text{Pbest}})$, and $\text{Gbest}_i = (x_{i1}^{\text{Gbest}}, \ldots, x_{\text{in}}^{\text{Gbest}})$, respectively, be the best position of individual i and its neighbors' best position so far. Using the information, the updated velocity of individual i is modified under the following equation in the PSO algorithm:

$$V_i^{k+1} = \omega V_i^k + c_1 \text{rand}_1 \times \left(\text{Pbest}_i^k - X_i^k\right) + c_2 \text{rand}_2 \times \left(\text{Gbest}^k - X_i^k\right) \quad (7)$$

where

V_i^k	velocity of individual i at iteration k;
ω	weight parameter;
c_1, c_2	weight factors;
$\text{rand}_1, \text{rand}_2$	random numbers between 0 and 1;
X_i^k	position of individual i at iteration k;
Pbest_i^k	best position of individual i until iteration k;
Gbest^k	best position of the group until iteration k.

Each individual moves from the current position to the next one by the modified velocity in (7) using the following equation:

$$X_i^{k+1} = X_i^k + V_i^{k+1}. \quad (8)$$

The search mechanism of the PSO using the modified velocity and position of individual i based on (7) and (8) is illustrated in Fig. 3.

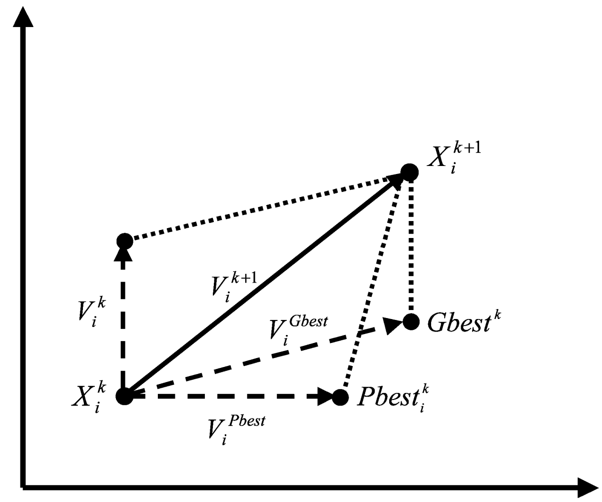

Fig. 3. The search mechanism of the particle swarm optimization.

B. Modified PSO for ED Problems

In this section, a new approach to implement the PSO algorithm will be described in solving the ED problems. Especially, a suggestion will be given on how to deal with the equality and inequality constraints of the ED problems when modifying each individual's search point in the PSO algorithm. Additionally, to accelerate the convergence speed, the dynamic search-space reduction strategy is devised. The process of the modified PSO algorithm can be summarized as follows:

Step 1) Initialization of a group at random while satisfying constraints.
Step 2) Velocity and position updates while satisfying constraints.
Step 3) Update of *Pbest* and *Gbest*.
Step 4) Activation of space reduction strategy.
Step 5) Go to Step 2 until satisfying stopping criteria.

In the subsequent sections, the detailed implementation strategies of the MPSO are described.

1) Initialization and Structure of Individuals: In the initialization process, a set of individuals is created at random. In this paper, the structure of an individual for ED problem is composed of a set of elements (i.e., generation outputs). Therefore, individual i's position at iteration 0 can be represented as the vector of $X_i^0 = (P_{i1}^0, \ldots, P_{\text{in}}^0)$ where n is the number of generators. The velocity of individual i (i.e., $V_i^0 = (v_{i1}^0, \ldots v_{\text{in}}^0)$) corresponds to the generation update quantity covering all generators. The elements of position and velocity have the same dimension, i.e., MW in this case. Note that it is very important to create a group of individuals satisfying the equality constraint (3) and inequality constraints (4). That is, summation of all elements of individual i (i.e., $\sum_{j=1}^{n} P_{ij}^0$) should be equal to the total system demand D and the created element j of individual i at random (i.e., P_{ij}^0) should be located within its boundary. Although we can create element j of individual i at random satisfying the inequality constraint by mapping [0, 1] into $[P_{j\min}, P_{j\max}]$, it is necessary to develop a new strategy to handle the equality constraint. To do this, the following procedure is suggested for any individual in a group:

Step 1) Set $j = 1$.
Step 2) Select an element (i.e., generator) of an individual at random.
Step 3) Create the value of the element (i.e., generation output) at random satisfying its inequality constraint.
Step 4) If $j = n - 1$ then go to Step 5; otherwise $j = j + 1$ and go to Step 2.
Step 5) The value of the last element of an individual is determined by subtracting $\sum_{j=1}^{n-1} P_{ij}^0$ from the total system demand D. If the value is in the range of its operating region then go to Step 6; otherwise go to Step 1.
Step 6) Stop the initialization process.

After creating the initial position of each individual, the velocity of each individual is also created at random. The following strategy is used in creating the initial velocity:

$$(P_{j\min} - \varepsilon) - P_{ij}^0 \leq v_{ij}^0 \leq (P_{j\max} + \varepsilon) - P_{ij}^0 \quad (9)$$

where ε is a small positive real number. The velocity of element j of individual i is generated at random within the boundary. The developed initialization scheme always guarantees to produce individuals satisfying the constraints while maintaining the concept of the PSO algorithm. The initial Pbest_i of individual i is set as the initial position of individual i and the initial *Gbest* is determined as the position of an individual with minimum payoff of (1).

2) Velocity Update: To modify the position of each individual, it is necessary to calculate the velocity of each individual in the next stage, which is obtained from (7). In this velocity updating process, the values of parameters such as ω, c_1, and c_2 should be determined in advance. In this paper, the weighting function is defined as follows [1], [3]:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{\text{Iter}_{\max}} \times \text{Iter} \quad (10)$$

where

$\omega_{\max}, \omega_{\min}$ initial, final weights;
$\text{Iter}_{\max}$ maximum iteration number;
Iter current iteration number.

3) Position Modification Considering Constraints: The position of each individual is modified by (8). The resulting position of an individual is not always guaranteed to satisfy the inequality constraints due to over/under velocity. If any element of an individual violates its inequality constraint due to over/under speed then the position of the individual is fixed to its maximum/minimum operating point. Therefore, this can be formulated as follows:

$$P_{ij}^{k+1} = \begin{cases} P_{ij}^k + v_{ij}^{k+1} & \text{if } P_{ij,\min} \leq P_{ij}^k + v_{ij}^{k+1} \leq P_{ij,\max} \\ P_{ij,\min} & \text{if } P_{ij}^k + v_{ij}^{k+1} < P_{ij,\min} \\ P_{ij,\max} & \text{if } P_{ij}^k + v_{ij}^{k+1} > P_{ij,\max} \end{cases} \quad (11)$$

Fig. 4 illustrates how the position of element j of individual i is adjusted to its maximum when the over-velocity situation occurs.

Although the aforementioned method always produces the position of each individual satisfying the inequality constraints (4), the problem of equality constraint still remains to be resolved. Therefore, it is necessary to develop a new strategy such that the summation of all elements in an individual (i.e., $\sum_{j=1}^{n} P_{ij}^k$) is equal to the total system demand. To resolve the equality constraint problem without intervening the dynamic process inherent in the PSO algorithm, we propose the following heuristic procedures:

Step 1) Set $j = 1$. Let the present iteration be k.
Step 2) Select an element (i.e., generator) of individual i at random and store in an index array $A(n)$.
Step 3) Modify the value of element j using (7), (8), and (11).
Step 4) If $j = n - 1$ then go to Step 5, otherwise $j = j + 1$ and go to Step 2.

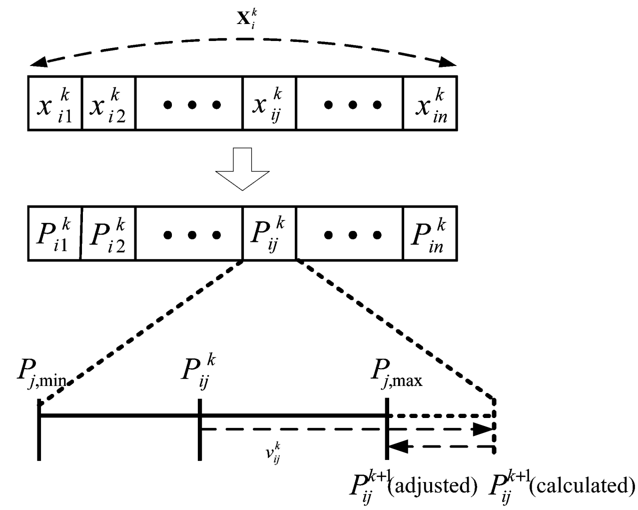

Fig. 4. Adjustment strategy for an individual's position within boundary.

Step 5) The value of the last element of individual i is determined by subtracting $\sum_{j=1}^{n-1} P_{ij}^k$ from D. If the value is not within its boundary then adjust the value using (11) and go to Step 6, otherwise go to Step 8.
Step 6) Set $l = 1$.
Step 7) Readjust the value of element l in the index array $A(n)$ to the value satisfying equality condition (i.e., $D - \sum_{\substack{j=1 \\ j \neq l}}^{n} P_{ij}^k$). If the value is within its boundary then go to Step 8; otherwise, change the value of element l using (11). Set $l = l + 1$, and go to Step 7. If $l = n + 1$, go to Step 1.
Step 8) Stop the modification procedure.

4) Update of Pbest and Gbest: The *Pbest* of each individual at iteration $k + 1$ is updated as follows:

$$\begin{aligned} \text{Pbest}_i^{k+1} &= X_i^{k+1} \quad \text{if } \text{TC}_i^{k+1} < \text{TC}_i^k \\ \text{Pbest}_i^{k+1} &= \text{Pbest}_i^k \quad \text{if } \text{TC}_i^{k+1} \geq \text{TC}_i^k \end{aligned} \quad (12)$$

where

TC_i the object function evaluated at the position of individual i.

Additionally, *Gbest* at iteration $k + 1$ is set as the best evaluated position among Pbest_i^{k+1}.

5) Space Reduction Strategy: To accelerate the convergence speed to the solutions, the MPSO has introduced the search-space reduction strategy. This strategy is activated in the case when the performance is not increased during a prespecified iteration period. In this case, the search space is dynamically adjusted (i.e., reduced) based on the "distance" between the *Gbest* and the minimum and maximum output of generator j. To determine the adjusted minimum/maximum output of generator j at iteration k, the distance is multiplied by the predetermined step-size Δ and subtracted (added) from the maximum (minimum) output at iteration k as described in (13)

$$\begin{aligned} P_{j\max}^{k+1} &= P_{j\max}^k - \left(P_{j\max}^k - \text{Gbest}_j^k\right) \times \Delta \\ P_{j\min}^{k+1} &= P_{j\min}^k + \left(P_{j\min}^k - \text{Gbest}_j^k\right) \times \Delta. \end{aligned} \quad (13)$$

Fig. 5 illustrates how the search space of each generator is dynamically reduced when activated.

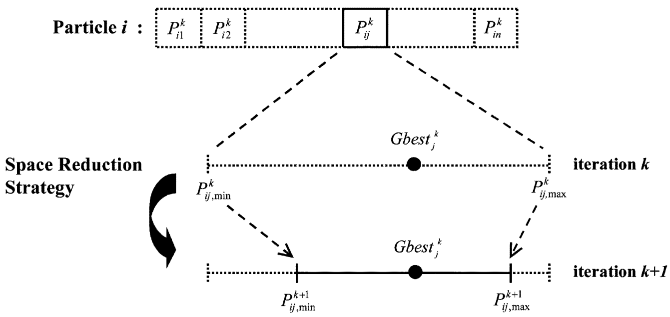

Fig. 5. Schematic of the dynamic space reduction strategy.

6) Stopping Criteria: The MPSO is terminated if the iteration approaches to the predefined maximum iteration.

IV. Case Studies

To assess the efficiency of the proposed MPSO, it has been applied to ED problems where the objective functions can be either smooth or nonsmooth. The results obtained from the MPSO are compared with those of other methods: the numerical lambda-iteration method (NM) [8], the hierarchical numerical method (HM) [7], the improved evolutionary programming (IEP) [9], the genetic algorithm (GA) [12], the Tabu search (TS) [14], the evolutionary programming (EP) [15], [16], the modified Hopfield neural network (MHNN) [10], and the Adaptive Hopfield neural network (AHNN) [11].

A. ED Problems With Nonsmooth Cost Functions Considering Valve-Point Effects

1) Test Systems: The MPSO is applied to two ED problems, one with 3 generators and another with 40 generators, where valve-point effects are considered for both problems. The input data for 3-generator system are given in [14] and those for 40-generators in [16]. Here, the total demand for the 3-generator and 40-generator systems are set as 850 MW and 10 500 MW, respectively. It was reported in [14] that the global optimum solution found for the 3-generator system is 8234.07 [$], while the global solution for the 40-generator system is not discovered yet. The best local solution reported until now is 122 624.35 [$] [16].

2) Parameter Determination Strategy: There exist several parameters to be determined for the implementation of the MPSO such as $\omega_{\max}, \omega_{\min}, c_1, c_2$ in (7) and (10) as well as Δ in (13). In this paper these parameters have been determined through the experiments for the 3-generator system. To avoid the problem of the curse of dimensionality, the procedures and strategies are determined as follows [17]: 1) The values of c_1 and c_2 have the same value, which implies the same weights are given between *Pbest* and *Gbest* in the evolution processes. 2) The values of $\omega_{\max}$ are varied from 1.0 to 0.5 and $\omega_{\min}$ from 0.5 to 0.1. 3) The values of Δ are also varied from 0.01 to 0.8 with increments of 0.01 under the assumption that the parameters of $\omega_{\max}, \omega_{\min}, c_1, c_2$ are tuned in the processes 1) and 2).

In Table I, the effects of parameters are illustrated, where 100 random trials are performed for each parameter set.

TABLE I
Effects of Parameters in MPSO Performance

Case	$\omega_{\max}$	$\omega_{\min}$	c_1, c_2	Minimum Cost	No. of Hits to Global
1	1.0	0.5	1	8234.07	56
2	0.9	0.4	1	8234.07	27
3	0.8	0.3	1	8234.07	47
4	0.7	0.2	1	8234.07	29
5	0.6	0.1	1	8234.07	11
6	0.5	0.1	1	8234.07	23
7	1.0	0.5	2	8234.07	58
8	0.9	0.4	2	8234.07	49
9	0.8	0.3	2	8234.07	44
10	0.7	0.2	2	8234.07	37
11	0.6	0.1	2	8234.07	30
12	0.5	0.1	2	8234.07	38

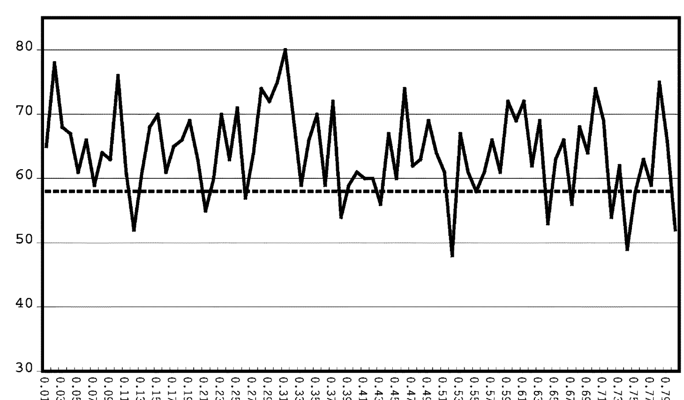

Fig. 6. Number of hits to the global solution in terms of Δ values.

Among 12 sets of parameters in Table I, Case 7 shows the best performance in terms of the number of hits to the global solution, which was obtained 58 times among 100 random trials. Therefore, the parameter values for Case 7 are selected for subsequent studies considering valve-point effects.

Next, the value of step-size Δ is determined by varying it from 0.01 to 0.8 with 0.01 increments. Fig. 6 illustrates the number of hits to the global in terms of Δ values, where the dotted horizontal line corresponds to the number of hits to the global when the space-reduction technique is not used (i.e., Δ is zero). In most cases, the results have been improved (i.e., 69 times among 80 cases) while the exceptional cases were limited to 11 times (i.e., when $\Delta = 0.12, 0.21, 0.26, 0.38, 0.43, 0.52, 0.64, 0.67, 0.72, 0.74,$ and 0.8). From these experiments, the best step-size is determined as 0.31 where the global solution is obtained 80 times among 100 random trials.

3) Numerical Results: The obtained results for the 3-generator system using the predetermined parameters are given in Table II and the results are compared with those from GA [12], IEP [9], and EP [15]. It shows that the MPSO has succeeded in finding the global solution presented in [14] with a high probability (i.e., 80 times among 100 trials) always satisfying the equality and inequality constraints.

Fig. 7 illustrates the convergence characteristics of the proposed MPSO for different number of particles and Fig. 8 shows the impact of initialization randomly created. These provide a

TABLE II
COMPARISON OF SIMULATION RESULTS OF EACH METHOD CONSIDERING VALVE-POINT EFFECTS (3-GENERATOR SYSTEM)

Unit	GA	IEP (pop=20)	EP	MPSO (par=20)
1	300.00	300.23	300.26	300.27
2	400.00	400.00	400.00	400.00
3	150.00	149.77	149.74	149.73
TP	850.00	850.00	850.00	850.00
TC	8237.60	8234.09	8234.07	8234.07

* pop : population size, par : number of particles, TP : total power [MW], TC : total generation cost [$].

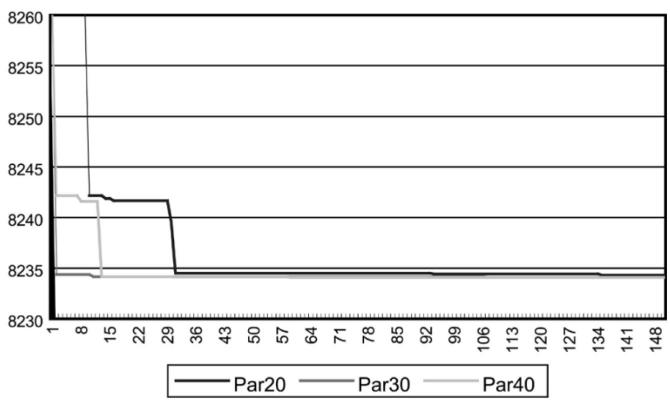

Fig. 7. Convergence characteristics of the MPSO for different number of particles when considering valve-point effects.

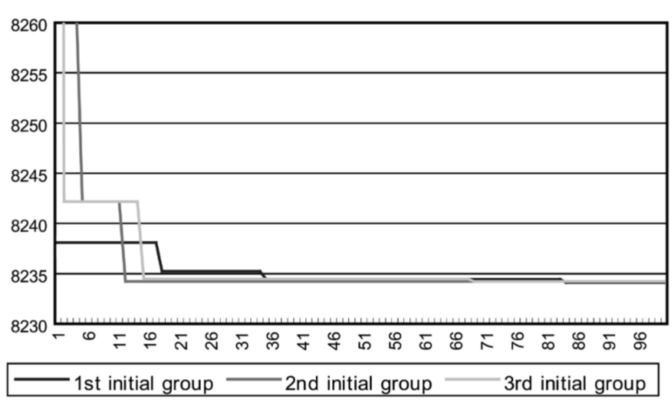

Fig. 8. Convergence characteristics of the MPSO for different initial group at random with 20 particles when considering valve-point effects.

robustness of the MPSO regarding to the population size and initial group.

The MPSO with the same parameter values of 3-generator system has been applied to the system with 40 generators. In Table III, the obtained best result is compared with results from other methods in [16] such as classical EP (CEP), fast EP (FEP), modified FEP (MFEP), and improved FEP (IFEP). Although the obtained best solution among 100 trials is not guaranteed to be the global solution, the MPSO has shown the superiority to the existing methods as one can see in Table III. The generation outputs and the corresponding costs of the best solution are provided in Table IV.

TABLE III
COMPARISON OF SIMULATION RESULTS OF EACH METHOD CONSIDERING VALVE-POINT EFFECTS (40-GENERATOR SYSTEM)

	CEP	FEP	MFEP	IFEP	MPSO
Minimum Cost	123,488.29	122,679.71	122,647.57	122,624.35	122,252.265

TABLE IV
GENERATION OUTPUT OF EACH GENERATOR AND THE CORRESPONDING COST IN 40-GENERATOR SYSTEM

Unit	$P_{i,min}$	$P_{i,max}$	Generation	Cost
1	36	114	114.000	978.156
2	36	114	114.000	978.156
3	60	120	120.000	1544.653
4	80	190	182.222	2195.814
5	47	97	97.000	853.178
6	68	140	140.000	1596.464
7	110	300	300.000	3216.424
8	135	300	299.021	3035.919
9	135	300	300.000	3071.990
10	130	300	130.000	2502.065
11	94	375	94.000	1893.305
12	94	375	94.000	1908.167
13	125	500	125.000	2541.681
14	125	500	304.485	5149.599
15	125	500	394.607	6444.856
16	125	500	305.323	5190.458
17	220	500	490.272	5318.085
18	220	500	500.000	5517.134
19	242	550	511.404	5543.630
20	242	550	512.174	5560.299
21	254	550	550.000	5575.329
22	254	550	523.655	5078.899
23	254	550	534.661	5283.585
24	254	550	550.000	5558.049
25	254	550	525.057	5311.518
26	254	550	549.155	5771.718
27	10	150	10.000	1140.524
28	10	150	10.000	1140.524
29	10	150	10.000	1140.524
30	47	97	97.000	843.178
31	60	190	190.000	1643.991
32	60	190	190.000	1643.991
33	60	190	190.000	1643.991
34	90	200	200.000	2101.017
35	90	200	200.000	2043.727
36	90	200	200.000	2043.727
37	25	110	110.000	1220.166
38	25	110	110.000	1220.166
39	25	110	110.000	1220.166
40	242	550	512.964	5577.439
Total Generation & Total Cost			10,500.000	122,252.265

To compare the results between MPSO and various methods in [16] in a statistical manner, the relative frequency of convergence is provided for each range of cost among 100 trials in Table V. From Table V, one can observe the robustness and superiority to the existing heuristic methods.

4) ED Problem With Smooth Cost Functions: The MPSO with the same parameters determined is applied to an ED problem with 3 generators and the quadratic cost functions. The input data of the system are given in [8] where the system

TABLE V
COMPARISON OF METHODS ON RELATIVE FREQUENCY OF CONVERGENCE IN THE RANGES OF COST

Methods	Range of Cost [k$]									
	127.0 - 126.5	126.5 - 126.0	126.0 - 125.5	125.5 - 125.0	125.0 - 124.5	124.5 - 124.0	124.0 - 123.5	123.5 - 123.0	123.0 - 122.5	122.5 - 120.0
CEP[16]	10	4	-	16	22	42	4	2	-	-
FEP[16]	6	-	4	2	10	20	26	24	6	-
MFEP[16]	-	-	-	-	-	14	26	50	10	-
IFEP[16]	-	-	2	-	4	4	18	50	22	-
MPSO	-	-	-	-	-	-	-	-	53	47

TABLE VI
COMPARISON OF SIMULATION RESULTS OF EACH METHOD

Unit	NM	MHNN	IEP (pop=10)	MPSO (par=10)
1	393.170	393.800	393.170	393.170
2	334.604	333.100	334.603	334.604
3	122.226	122.300	122.227	122.226
TP	850.000	849.200	850.000	850.000
TC	8194.35612	8187.00000	8194.35614	8194.35612

demand considered is 850 MW. Table VI shows the comparison of the results from MPSO, NM [8], IEP [9], and MHNN [10].

As seen in Table VI, the MPSO has provided the global solution with a very high probability, the same result of the lambda-iteration method, exactly satisfying the equality and inequality constraints.

To test the robustness of the MPSO for the smooth cost functions, we have changed the demand from 300 MW to 1200 MW with 50 MW increments and we have compared the results with those of NM [8]. As we can see in Table VII, the MPSO has successfully provided the same results of NM with a very high probability in every case. Among 1900 simulations reflecting demand and initial group changes, 1866 simulations have succeeded in finding the global solution (i.e., the probability to obtain the global optimum is about 98%).

B. ED Problem With Nonsmooth Cost Functions Considering Multiple Fuels

The developed MPSO has also been applied to the ED problem with 10 generators where the multiple-fuel effects are considered. In this case, the objective function is represented as the piecewise quadratic cost function. The input data and related constraints of the test system are given in [7], [9], and [11]. In this case, the total system demand is varied from 2400 MW to 2700 MW with 100 MW increments.

For these problems, the same parameter determination strategy is adopted as the case of valve-point loading problems. The resulting values of parameters are $\omega_{\max} = 0.5, \omega_{\min} = 0.1, c_1 = c_2 = 2$, and $\Delta = 0.05$. The best results from the MPSO are compared with those of HM [7], IEP [9], MHNN [10] and AHNN [11] and given in Tables VIII–XI. In this case, the global solution is not known,

TABLE VII
COMPARISON OF SIMULATION RESULTS BETWEEN MPSO AND NM

Demand	Minimum Cost (PSO)	Minimum Cost (NM)	No. of Hits to the Global Solution
300	3387.095	3387.095	100
350	3803.711	3803.711	91
400	4226.192	4226.192	96
450	4652.427	4652.427	100
500	5082.330	5082.330	100
550	5515.901	5515.901	100
600	5953.141	5953.141	100
650	6394.048	6394.048	100
700	6838.623	6838.623	100
750	7286.866	7286.866	100
800	7738.777	7738.777	100
850	8194.356	8194.356	100
900	8653.603	8653.603	99
950	9116.518	9116.518	98
1000	9583.102	9583.102	100
1050	10053.679	10053.679	100
1100	10529.921	10529.921	97
1150	11012.061	11012.061	85
1200	11500.520	11500.520	100

TABLE VIII
COMPARISON OF OPTIMIZATION METHODS (DEMAND = 2400 [MW])

S	U	HM		MHNN		AHNN		IEP(pop=30)		MPSO (par=30)	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	1	193.2	1	192.7	1	189.1	1	190.9	1	189.7
	2	1	204.1	1	203.8	1	202.0	1	202.3	1	202.3
	3	1	259.1	1	259.1	1	254.0	1	253.9	1	253.9
	4	3	234.3	2	195.1	3	233.0	3	233.9	3	233.0
2	5	1	249.0	1	248.7	1	241.7	1	243.8	1	241.8
	6	1	195.5	3	234.2	1	233.0	3	235.0	3	233.0
	7	1	260.1	1	260.3	1	254.1	1	253.2	1	253.3
3	8	3	234.3	3	234.2	3	232.9	3	232.8	3	233.0
	9	1	325.3	1	324.7	1	320.0	1	317.2	1	320.4
	10	1	246.3	1	246.8	1	240.3	1	237.0	1	239.4
TP		2401.2		2399.8		2400.0		2400.0		2400.0	
TC		488.500		487.87		481.700		481.779		481.723	

TABLE IX
COMPARISON OF OPTIMIZATION METHODS (DEMAND = 2500 [MW])

S	U	HM		MHNN		AHNN		IEP(pop=30)		MPSO (par=30)	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	206.6	2	206.1	2	206.0	2	203.1	2	206.5
	2	1	206.5	1	206.3	1	206.3	1	207.2	1	206.5
	3	1	265.9	1	265.7	1	265.7	1	266.9	1	265.7
	4	3	236.0	3	235.7	3	235.9	3	234.6	3	236.0
2	5	1	258.2	1	258.2	1	257.9	1	259.9	1	258.0
	6	3	236.0	3	235.9	3	235.9	1	236.8	3	236.0
	7	1	269.0	1	269.1	1	269.6	1	270.8	1	268.9
3	8	3	236.0	3	235.9	3	235.9	3	234.4	3	235.9
	9	1	331.6	1	331.2	1	331.4	1	331.4	1	331.5
	10	1	255.2	1	255.7	1	255.4	1	254.9	1	255.1
TP		2501.1		2499.8		2500.0		2500.0		2500.0	
TC		526.700		526.13		526.230		526.304		526.239	

or it may be impossible to find the global solution with the numerical approach for piecewise quadratic cost functions.

As seen in Tables VIII–XI, the MPSO has always provided better solutions than HM [7] (except for 2600 MW case), IEP [9], and MHNN [10]. Furthermore, it has provided solutions

TABLE X
COMPARISON OF OPTIMIZATION METHODS (DEMAND = 2600 [MW])

S	U	HM		MHNN		AHNN		IEP(pop=30)		MPSO (par=30)	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	216.4	2	215.3	2	215.8	2	213.0	2	216.5
	2	1	210.9	1	210.6	1	210.7	1	211.3	1	210.9
	3	1	278.5	1	278.9	1	279.1	1	283.1	1	278.5
	4	3	239.1	3	238.9	3	239.1	3	239.2	3	239.1
2	5	1	275.4	1	275.7	1	276.3	1	279.3	1	275.5
	6	3	239.1	3	239.1	3	239.1	1	239.5	3	239.1
	7	1	285.6	1	286.2	1	286.0	1	283.1	1	285.7
3	8	3	239.1	3	239.1	3	239.1	3	239.2	3	239.1
	9	1	343.3	1	343.5	1	342.8	1	340.5	1	343.5
	10	1	271.9	1	272.6	1	271.9	1	271.9	1	272.0
TP			2600.0		2599.8		2600.0		2600.0		2600.0
TC			574.030		574.26		574.370		574.473		574.381

TABLE XI
COMPARISON OF OPTIMIZATION METHODS (DEMAND = 2700 [MW])

S	U	HM		MHNN		AHNN		IEP(pop=30)		MPSO (par=30)	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	2	218.4	2	224.5	2	225.7	2	219.5	2	218.3
	2	1	211.8	1	215.0	1	215.2	1	211.4	1	211.7
	3	1	281.0	3	291.8	1	291.8	1	279.7	1	280.7
	4	3	239.7	3	242.2	3	242.3	3	240.3	3	239.6
2	5	1	279.0	1	293.3	1	293.7	1	276.5	1	278.5
	6	3	239.7	3	242.2	3	242.3	1	239.9	3	239.6
	7	1	289.0	1	303.1	1	302.8	1	289.0	1	288.6
3	8	3	239.7	3	242.2	3	242.3	3	241.3	3	239.6
	9	3	429.2	1	355.7	1	355.1	3	425.1	3	428.5
	10	1	275.2	1	289.5	1	288.8	1	277.2	1	274.9
TP			2702.2		2699.7		2700.0		2700.0		2700.0
TC			625.180		626.12		626.240		623.851		623.809

satisfying the equality and inequality constraints while HM [7] and MHNN [10] do not satisfy the equality constraint.

When compared with AHNN [11], the MPSO has provided better solution for the demand of 2700 MW. Note that the fuel types and dispatch levels from the MPSO is quite different from those of other approaches. Although the AHNN has provided better solutions than the MPSO in the other cases, the generation configurations are quite similar between the AHNN and MPSO.

V. CONCLUSION

This paper presents a new approach to nonsmooth ED problems based on the PSO algorithm. A position adjustment strategy is incorporated in the PSO framework in order to provide the solutions satisfying the inequality constraints. The equality constraint in the ED problem is resolved by reducing the degree of freedom by one at random. The strategies for handling constraints are devised while preserving the dynamic process of the PSO algorithm. Additionally, the dynamic search-space reduction strategy is applied to accelerate the convergence speed.

The MPSO has provided the global solution satisfying the constraints with a very high probability for the ED problems with smooth cost functions. For the ED problems with nonsmooth cost functions due to the valve-point effects, the MPSO has also provided the global solution with a high probability for 3-generator system and provided a set of quasioptimums for 40-generator system which are better than other heuristic methods. In the case of nonsmooth function problem due to multi-fuel effects, the MPSO has shown superiority to the conventional numerical method, the conventional Hopfield neural network, and the evolutionary programming approach, while providing very similar results with the modified Hopfield neural networks.

REFERENCES

[1] K. Y. Lee and M. A. El-Sharkawi, Eds., *Modern Heuristic Optimization Techniques with Applications to Power Systems*: IEEE Power Engineering Society (02TP160), 2002.

[2] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE Int. Conf. Neural Networks (ICNN'95)*, vol. IV, Perth, Australia, 1995, pp. 1942–1948.

[3] H. Yoshida, K. Kawata, Y. Fukuyama, S. Takayama, and Y. Nakanishi, "A particle swarm optimization for reactive power and voltage control considering voltage security assessment," *IEEE Trans. Power Syst.*, vol. 15, pp. 1232–1239, Nov. 2000.

[4] M. Clerc and J. Kennedy, "The particle swarm-explosion, stability, and convergence in a multidimensional complex space," *IEEE Trans. Evol. Comput.*, vol. 6, no. 1, pp. 58–73, Feb. 2002.

[5] M. A. Abido, "Optimal design of power-system stabilizers using particle swarm optimization," *IEEE Trans. Energy Conv.*, vol. 17, no. 3, pp. 406–413, Sept. 2002.

[6] I. N. Kassabalidis, M. A. El-Sharkawi, R. J. Marks, L. S. Moulin, and A. P. A. da Silva, "Dynamic security border identification using enhanced particle swarm optimization," *IEEE Trans. Power Syst.*, vol. 17, pp. 723–729, Aug. 2002.

[7] C. E. Lin and G. L. Viviani, "Hierarchical economic dispatch for piecewise quadratic cost functions," *IEEE Trans. Power App. Syst.*, vol. PAS-103, no. 6, pp. 1170–1175, June 1984.

[8] A. J. Wood and B. F. Wollenbergy, *Power Generation, Operation, and Control*. New York: Wiley, 1984.

[9] Y. M. Park, J. R. Won, and J. B. Park, "A new approach to economic load dispatch based on improved evolutionary programming," *Eng. Intell. Syst. Elect. Eng. Commun.*, vol. 6, no. 2, pp. 103–110, June 1998.

[10] J. H. Park, Y. S. Kim, I. K. Eom, and K. Y. Lee, "Economic load dispatch for piecewise quadratic cost function using Hopfield neural network," *IEEE Trans. Power Syst.*, vol. 8, pp. 1030–1038, Aug. 1993.

[11] K. Y. Lee, A. Sode-Yome, and J. H. Park, "Adaptive Hopfield neural network for economic load dispatch," *IEEE Trans. Power Syst.*, vol. 13, pp. 519–526, May 1998.

[12] D. C. Walters and G. B. Sheble, "Genetic algorithm solution of economic dispatch with the valve point loading," *IEEE Trans. Power Systems*, vol. 8, pp. 1325–1332, Aug. 1993.

[13] G. Ciuprina, D. Ioan, and I. Munteanu, "Use of intelligent-particle swarm optimization in electromagnetics," *IEEE Trans. Magn.*, vol. 38, pp. 1037–1040, Mar. 2002.

[14] W. M. Lin, F. S. Cheng, and M. T. Tsay, "An improved Tabu search for economic dispatch with multiple minima," *IEEE Trans. Power Syst.*, vol. 17, pp. 108–112, Feb. 2002.

[15] H. T. Yang, P. C. Yang, and C. L. Huang, "Evolutionary programming based economic dispatch for units with nonsmooth fuel cost functions," *IEEE Trans. Power Syst.*, vol. 11, no. 1, pp. 112–118, Feb. 1996.

[16] N. Sinha, R. Chakrabarti, and P. K. Chattopadhyay, "Evolutionary programming techniques for economic load dispatch," *IEEE Trans. Evol. Comput.*, vol. 7, pp. 83–94, Feb. 2003.

[17] J. Kennedy and R. C. Eberhart, *Swarm Intelligence*. San Francisco, CA: Morgan Kaufmann, 2001.

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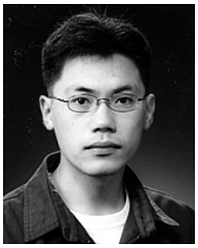

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