Mon. Not. R. astr. Soc. (1989) **241**, 375–392

A period–luminosity–colour relation for Mira variables

M. W. Feast, I. S. Glass, P. A. Whitelock and
R. M. Catchpole *South African Astronomical Observatory, PO Box 9, Observatory 7935, South Africa*

Accepted 1989 April 28. Received 1989 April 28; in original form 1989 March 21

Summary. Time-averaged *J, H, K* and m_{bol} magnitudes are given for 29 O-rich and 20 C-rich Mira variables in the Large Magellanic Cloud. PL and PC relations are derived. The PL relations for O-Miras and C-Miras are very similar at *K*. For the O-Miras the residuals from the PL and PC relations are correlated showing the existence of PLC relations. Such relations are derived and compared with the theoretical pulsation equation. Data are also given for six variables with periods greater than 420 d. These are brighter by 0.7 mag than expected from extrapolations of the PL or PLC relations. Using Miras in galactic globular clusters to estimate the cluster distances (from the PLC relation) gives a mean horizontal branch luminosity of $+0.71 \pm 0.09$ (internal standard error) at [Fe/H] $= -0.58$, in good agreement with that derived in other ways. For the C-Miras, a significant colour term is estimated for the PLC relation at *J*. There is, however, little evidence for a significant colour term at *K* and the evidence for one in m_{bol} is marginal.

1 Introduction

That the Mira variables show a period–(visual) luminosity relation seems to have first been suggested by Gerasimovic (1928) on the basis of statistical parallaxes. Later work (e.g. Gyllenberg 1929, 1930; Wilson & Merrill 1942; Osvalds & Risley 1961; Clayton & Feast 1969) confirmed and extended this relation although the scatter in visual absolute magnitudes at a given period always seemed to be quite large (probably ~ 0.5 mag). The results were extended to the near-infrared and to M_{bol} by Robertson & Feast (1981) on the basis of extensive *JHKL* (1.2, 1.6, 2.2 and 3.4 μm) observations of nearby Miras made by Catchpole *et al.* (1979). Glass & Lloyd Evans (1981) and Glass & Feast (1982a) demonstrated that a rather precise period–luminosity (PL) relation ($\sigma \sim 0.25$ mag) existed at *JHK* and m_{bol} on the basis of observations of 11 Miras in the LMC. These latter relations were further refined by Feast (1984) by including data from Wood, Bessell & Paltoglou (1985) making available a total of 43 LMC Miras. The carbon-rich and oxygen-rich Miras were found to follow the same PL relation at *K*. A study of Miras in galactic globular clusters (Menzies & Whitelock 1985) suggested that similar PL relations applied to these objects as well. Infrared observations by Glass *et al.* (1987) of 73 LMC Miras showed that well-defined PL relations can be obtained with a scatter of $\sigma \sim 0.15$ mag. The data of Glass & Feast (1982b) show a PL relation at K for Miras in the Galactic Bulge (Feast 1986) and van den Bergh (1984) finds a PL relation in m_{pg} for bulge Miras.

Essentially all of the above LMC work (except an earlier report of the present programme, Glass *et al.* 1987) depended on only a few (often only one) infrared observations per star. Thus a considerable part of the observed scatter in the derived PL relations could be observational. The primary purpose of the present investigation was to obtain reasonably good phase coverage in *JHK* for a limited number of LMC Miras with the intention of investigating the intrinsic width of the PL relations and the possible existence of period–luminosity–colour (PLC) relations. PLC relations are expected, from the pulsation equation $P\sqrt{\rho}=Q$, for a homologous set of variable stars occupying an instability strip of finite width in the HR diagram and for which the mass is a function of M_{bol}, colour and/or period, as was pointed out (for Cepheids) by Sandage (1958). A subset of the observations analysed here was previously used in a discussion by Reid, Glass & Catchpole (1988).

2 Observations

A total of 82 long-period variables from the lists of Glass & Lloyd Evans (1981); Wood, Bessell & Paltoglou (1985); Glass & Reid (1985); Reid *et al.* (1988) and Lloyd Evans (private communication) were selected for observation. The work was carried out using the Mk III infrared photometer attached to the 1.9-m telescope at SAAO Sutherland. The filters are the same as those described in Glass (1973) and used, in particular, during the work of Glass & Lloyd Evans (1981) and Glass & Feast (1982a). The results were reduced using standards from Carter (1989) transformed to the natural system of the Mk III photometer and are given on that system. It should be noted that, because of their unusual energy distribution, it has not yet been possible to derive completely satisfactory transformations for Mira variables between the infrared *JHKL* photometric systems in use at various observatories and telescopes. In particular, work is still in progress to define the relation between the system used in the present paper and the Carter system which is the natural system of the Mk II photometer usually attached to the SAAO 0.75-m telescope. Of the 82 stars on the initial list, three were abandoned early on, two due to crowding and one due to apparent lack of variation.

A total of 884 *J*, *H* and *K* sets, including those of Glass & Feast (1982a), were obtained for the 79 remaining objects. In some cases, L measures were also obtained. Absorption corrections $A_J=0.06$, $A_H=0.03$, $A_K=0.02$, $A_L=0.01$ were adopted corresponding to a mean value of $E_{B-V}=0.074$ as derived for Cepheids in the LMC by Caldwell & Coulson (1985). Values of the bolometric magnitude (m_{bol}) were calculated for each dereddened set of $JHK(L)$ values using blackbody curves as interpolation devices and adopting the following values of $\log f_\nu$ (when f_ν is the flux per unit frequency for an 0th magnitude star), -22.83 at J, -23.01 at H, -23.21 at K and -23.55 at L ($\mathrm{W\,m^{-2}\,Hz^{-1}}$) (Wilson *et al.* 1972).

All the measures for each star were plotted against phases determined from the periods given in the discovery papers. In a number of cases the scatter of the points suggested that the periods might be improved. Revised periods for these stars were then derived using a Fourier fitting programme kindly made available by Dr L. A. Balona. Many of the stars with revised periods are from the list of Glass & Lloyd Evans (1981). For these stars the infrared observations now span 10 yr, much longer than the plate series from which the original periods were derived. Eight individual observations were rejected. In most cases examination of the original data suggested that inadequate weather conditions or crowding had interfered with the observations. (Crowding effects occur due to extraneous stars in the observing aperture or in either of the two reference positions, usually ± 30 arcsec away from the star in declination.)

Some estimate of the internal consistency of the photometry can be obtained from the observations of GRV 0515-6451 which was accidentally observed twice on seven nights due to confusion in its nomenclature. The rms differences are 0.05, 0.02 and 0.03 for *J*, *H* and *K*, respectively. The errors in *L* for this sample should not be regarded as lower than about 0.05.

The light curves for each star have been classified subjectively into the categories 'good, fair, poor' according to phase coverage and scatter about a mean curve. 'Good' curves show little scatter and have reasonably well-spaced phase coverage, especially around maxima and minima. 'Fair' curves show some scatter or have less adequate coverage but still give a fair picture of an undoubtedly periodic variable. 'Poor' curves have been so classified when no periodicity is discernable, severe crowding has interfered or the phase coverage is exceptionally poor, especially for the latter if the shape of a complete curve would have to be very unusual. Eighteen curves were considered 'good', 37 'fair' and 24 'poor'.

'Poor' quality can be introduced by observations being contaminated by extraneous nearby objects, as already mentioned, and this cause is, in fact, suspected in several cases. Apparently, poor curves can also be caused by the mis-classification of the star as a Mira or erroneous periods. (The photographic classification programmes have often relied on observations over quite short periods of time.) The stars with light curves classified as poor are not included in the following discussions. After eliminating the 'poor' stars, 55 remain. The individual observations will be published elsewhere (Glass *et al.* 1989), the mean results for these stars are listed in Table 1.

Table 1. Mean results. Designation – W = Wood *et al.* (1985); Co-ordinate designation or GR = Glass & Reid (1985) or Reid *et al.* (1988); other designations Glass & Lloyd Evans (1981). Adopted period – where the period is a revision of a previously published value it is marked by an asterisk. Light curve quality (Q) (1 = good, 2 = fair). Number of *JHK* sets (N). Spectral type (if available) or tentative classification as oxygen-rich (M) or carbon-rich (C), derived from the position of the variable in the *J–H, H–K* diagram (*cf.* Feast *et al.* 1982). J_1, H_1, K_1 the average of the observed maximum and minimum magnitudes in *JHK*. m_{b3} Intensity Means derived from Fourier fits to the bolometric light curves.

Designation	Period (Days)	Q	N	Type	J_1	H_1	K_1	m_{b3}
C38	128*	1	16	(M)	13.14	12.40	12.12	14.87
C11	202*	2	16	(M)	12.75	11.87	11.51	14.52
C20	210*	2	16	(M)	12.94	12.04	11.54	14.56
R120	217*	1	20	M	12.56	11.64	11.38	14.27
R141	255*	1	19	(M)	12.19	11.30	10.99	13.87
R110	261*	2	13	(M)	12.70	11.81	11.29	14.23
C7	326*	2	15	C	12.64	11.39	10.69	13.99
R153	370*	2	12	C	13.16	11.61	10.52	14.03
R105	420*	1	15	(M)	11.64	10.67	10.29	13.40
W132	155	2	12	M	12.72	11.88	11.67	14.48
W151	172	2	14	M	12.90	12.02	11.74	14.67
W148	185	2	13	(M)	13.04	12.15	11.82	14.74
W158	185	2	9	(M)	12.96	12.00	11.77	14.75
W19	189	2	10	M	12.66	11.76	11.51	14.33
W77	217	2	10	S	12.44	11.54	11.25	14.20
W94	220	2	7	M	12.50	11.58	11.28	14.23
W74	227	2	10	M	12.80	11.82	11.49	14.53
W1	233	1	14	M	12.86	11.93	11.48	14.55

Table 1 – *continued.*

Designation	Period (Days)	Q	N	Type	J_1	H_1	K_1	m_{b3}
W140	244	2	8	(M)	12.44	11.42	11.19	14.17
W48	279	2	8	(M)	12.18	11.28	10.99	13.97
W46	286	2	5	C	12.69	11.55	11.00	14.20
W126	323	2	11	K?	12.28	11.34	10.89	13.91
W103	351	2	9	C	13.70	12.03	10.78	14.14
W30	400	2	9	C	12.34	11.10	10.48	13.73
0517-6551	117	1	10	(M)	13.38	12.56	12.25	15.14
0512-6559	141	1	16	M	13.37	12.52	12.13	15.05
0530-6437	157	1	14	C	13.12	12.42	12.08	14.71
0526-6754	160	1	10	M	12.86	11.98	11.79	14.64
0528-6531	195	1	12	M	12.52	11.65	11.48	14.31
GR13	202	1	11	(M)	12.74	11.86	11.59	14.50
0507-6639	208*	1	13	M	12.78	11.90	11.57	14.51
0515-6617	211*	2	11	C	13.26	12.07	11.16	14.56
0528-6520	231*	2	9	C	12.71	11.66	11.08	14.29
0520-6528	234*	2	11	C	12.52	11.60	11.28	14.17
0519-6454	242	2	10	(C)	12.85	11.70	11.09	14.33
0533-6807	247	1	12	M	12.57	11.76	11.38	14.28
0529-6759	274	2	10	(C)	12.70	11.56	10.91	14.25
0515-6451	284	1	25	(C)	12.96	11.70	10.81	14.21
0537-6607	284	1	10	(M)	12.32	11.42	11.02	13.99
0514-6605	305	2	11	(C)	12.76	11.46	10.64	14.02
0505-6657	311	2	13	(M)	11.98	11.10	10.67	13.73
0534-6531	312	1	11	(C)	13.64	12.18	10.98	14.28
0524-6543	312	1	12	(M)	11.94	11.08	10.71	13.71
0529-6739	319	2	11	(C)	12.98	11.62	10.60	13.91
0541-6631	328	2	10	(C)	13.22	11.94	10.50	14.09
0515-6438	365*	2	11	(C)	13.12	11.91	10.90	14.27
W220	286	2	6	C	12.76	11.50	10.83	14.12
0502-6711	308*	2	12	(C)	12.64	11.38	10.53	13.92
0537-6740	418*	2	11	(C)	12.58	11.28	10.47	13.90
Variables with $P>420$ d.								
C2	573		10		11.32	10.15	9.46	12.86
0515-6510	438		9		11.29	10.42	9.98	12.93
0503-6620	597		10		10.64	9.74	9.23	12.36
0515-6608	555		10		10.60	9.70	9.23	12.24
0523-6644	649*		10		10.03	9.18	8.76	11.87
GR17	780		11		10.30	9.30	8.75	11.87

The bolometric magnitudes have always been derived using observations dereddened as indicated in Section 2. Other magnitudes are followed by the subscript zero if they have been dereddened. The general symbol m is used to mean any magnitude, i.e. J, H, K or $m_{\rm bol}$.

A variety of types of means was investigated. Menzies & Whitelock (1985) concluded that the average of the maximum and minimum magnitudes was close to the Intensity Mean Fourier fits. This is also clear from our work, since for the 49 stars with $P<420$ d with which we shall be primarily concerned in the following, the mean differences are

$K_1 - K_3 = +0.012 \pm 0.009$ ($\sigma = 0.064$) (K_3 = Intensity Means from Fourier Fits),

$m_{bl} - m_{b3} = +0.030 \pm 0.010$ ($\sigma = 0.069$) (m_{bl} = average of maximum and minimum bolometric magnitudes).

Also

$K_2 - K_3 = -0.043 \pm 0.008$ ($\sigma = 0.060$), $m_{b2} - m_{b3} = -0.037 \pm 0.008$ ($\sigma = 0.055$),

where K_2 (and m_{b2}) are the magnitudes of the intensity average of maximum and minimum magnitudes at K and m_{bol}, respectively and

$K_4 - K_3 = +0.018 \pm 0.004$ ($\sigma = 0.024$),

where K_4 is the Magnitude Mean Fourier fit to the light curve at K.

The data of Table 1 were subdivided in a number of different ways in carrying out the analysis. The principal two groupings are the oxygen-rich or probable oxygen-rich objects [types M, S, K or (M)] and the carbon-rich or probable carbon-rich objects [C or (C)]. In general, results will be given only for these two groups (called for simplicity, O-Miras and C-Miras). Restricting the analysis to only spectroscopically proven members of these groups does not yield significantly different results. The main analysis is restricted to the 49 stars for which $P < 420$ d. The stars with periods longer than this are discussed in Section 7.

3 Period–luminosity (PL) relations

Least-squares solutions of the form

$$m_0 = \rho \log P + \delta, \tag{1}$$

were made and some of them are listed in Table 2. σ is the standard deviation in m_0. Here, and throughout, the coefficients of the various equations are those appropriate for magnitudes (and colours) after dereddening according to the prescription in Section 2. The equations can be converted to relations in absolute magnitude by adopting a distance modulus for the LMC (in

Table 2. Period–luminosity relations (equation 1).

Group	Mag	N	ρ ± s.e.	δ ± s.e.	σ	Solution
O-Miras	$(J_1)_0$	29	−2.92 ± .28	19.37 ± .32	0.19	1
C-Miras	$(J_1)_0$	20	−0.19 ± .83	13.33 ± 2.05	0.37	2
O-Miras	$(H_1)_0$	29	−3.14 ± .27	19.04 ± .64	0.18	3
C-Miras	$(H_1)_0$	20	−1.72 ± .63	15.90 ± 1.56	0.28	4
O-Miras	$(K_1)_0$	29	−3.47 ± .19	19.48 ± .45	0.13	5
C-Miras	$(K_1)_0$	20	−3.30 ± .40	18.98 ± .98	0.18	6
All	$(K_1)_0$	49	−3.57 ± .16	19.70 ± .39	0.15	7
O-Miras	m_{b3}	29	−3.00 ± .24	21.35 ± .57	0.16	8
C-Miras	m_{b3}	20	−1.86 ± .30	18.73 ± .74	0.13	9
All	m_{b3}	49	−2.34 ± .19	19.86 ± .45	0.17	10

the present paper we use 18.47 for the true modulus, *cf.* Feast & Walker 1987; Feast 1988a). The main results of the PL solutions are as follows:

(i) The O-Miras show a rather good PL relation in $(K_1)_0$ (solution 5, $\sigma=0.13$, *cf.* Fig. 1). These stars show PL relations in $(J_1)_0$, $(H_1)_0$ and m_{b3} also (*cf.* Fig. 2) but with somewhat larger scatter.

(ii) The C-Miras also show a PL relation at $(K_1)_0$ (solution 6) but with more scatter ($\sigma=0.18$) than for the O-Miras. As Fig. 1 shows, the C- and O-Miras fit a single PL relation rather well in $(K_1)_0$ (solution 7). If the shortest period C-Mira were omitted the rest would define a line of somewhat shallower slope than the O-Miras. The C-Miras show a good PL relation in m_{b3} ($\sigma=0.13$) but this lies below that of the O-Miras over most of the period range occupied by C-Miras (Fig. 2). As previously noted (Glass *et al.* 1987), it is quite possible that flux removed by the high molecular opacities of C star atmospheres in the J and H bands is

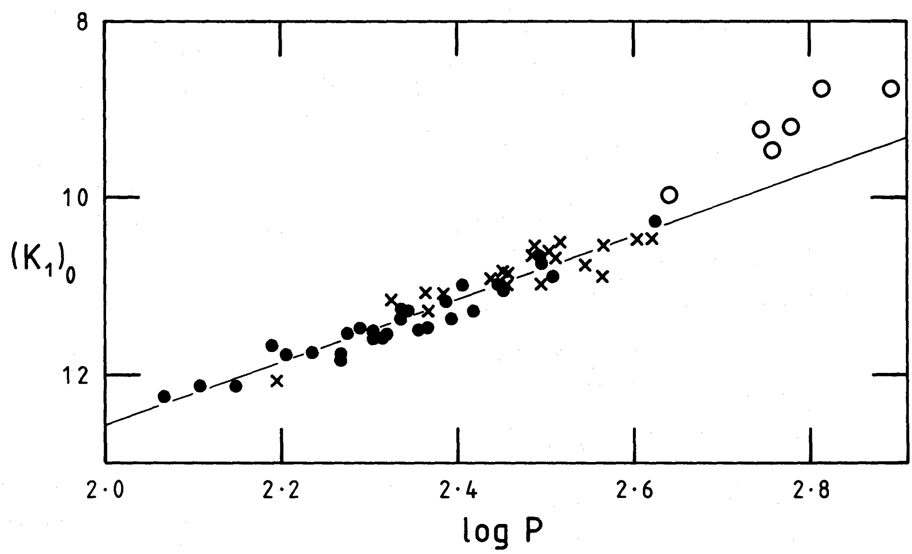

Figure 1. Period–luminosity relation at K: filled circles = O-Miras ($P<420$ d); open circles = O-Miras ($P>420$ d); crosses = C-Miras. The line is the least-squares fit to the O and C-Miras with $P<420$ d.

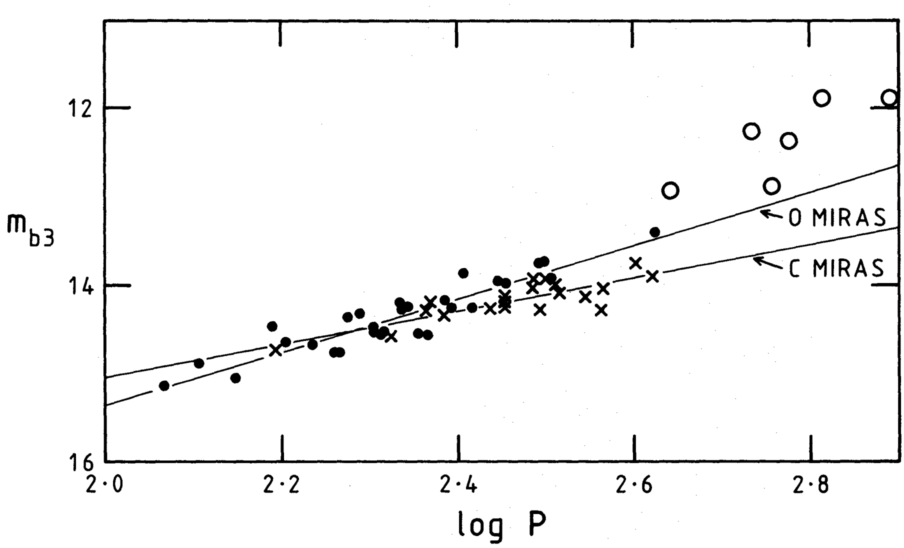

Figure 2. Period–luminosity relations in m_{b3}. Symbols as in Fig. 1. The lines are least-squares fits to the O-Miras ($P<420$ d) and to the C-Miras.

re-radiated at much longer wavelengths or that this opacity otherwise distorts the values of $m_{\rm bol}$ obtained by the present method. Thus it is at present uncertain whether the apparent displacement between the C- and O-Miras in a period–$m_{\rm bol}$ diagram is real or not. A mean relation for all stars in m_{b3} is given (solution 10) but this is probably of limited value.

(iii) The PL relations for C-Miras in $(J_1)_0$ and $(H_1)_0$ (solutions 2 and 4) have a lower slope and a much larger scatter than the corresponding relation in $(K_1)_0$. In fact the scatter in $(J_1)_0$ is no smaller than that about the straight mean of the 20 $(J_1)_0$ values ($\sigma = 0.36$). The PL relation in $(J_1)_0$ is therefore not useful and that in $(H_1)_0$ is of limited usefulness only. In addition such PL relations as exist at J and H are distinctly different from those for O-Miras so there seems little point in giving a mean relation for all the stars.

(iv) Whilst the PL relation in $m_{\rm bol}$ is the most significant for discussions of stellar pulsation and evolution, the relation at K is likely to be the most practical one when Miras are being used as distance indicators. This is due to the common, narrow, PL relation for O- and C-Miras and the relative insensitivity of K to interstellar reddening.

4 Period–colour relations

Throughout this paper we adopt $(J_1 - K_1)$ as the most suitable colour with which to work. Least-squares solutions of the form

$$(J_1 - K_1)_0 = \tau \log P + \phi, \tag{2}$$

with σ the standard deviation in $(J_1 - K_1)_0$, were made and two of these are listed in Table 3. There is a significant relation for O-rich Miras (solution 11, Table 2). Fig. 3 shows a plot of $(J_1 - K_1)_0$ against log P for the O-Miras. The scatter about the mean relation has $\sigma = 0.082$, a significant part of which is likely to be observational. Also plotted in Fig. 3 are the results for (O-rich) Miras in galactic globular clusters (Menzies & Whitelock 1985). These results have

Table 3. Period–colour relations (equation 2).

Group	N	τ ± s.e.	φ ± s.e.	σ	Solution
O-Miras	29	0.56 ± .12	-.12 ± .29	0.08	11
C-Miras	20	2.32 ± 1.07	-3.77 ± 2.64	0.48	12

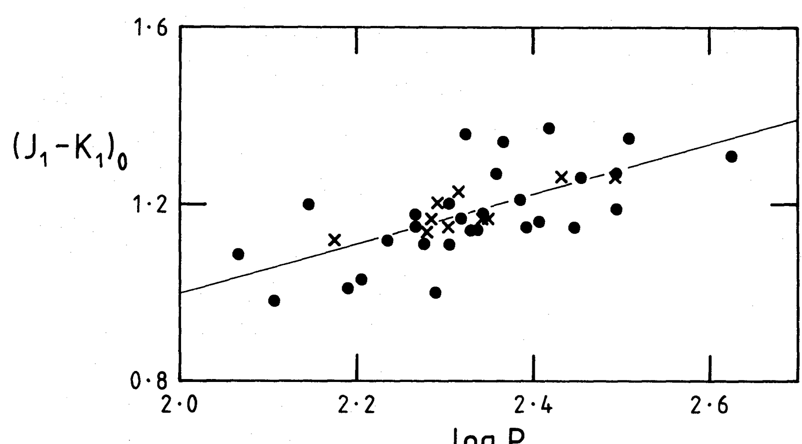

Figure 3. Period–colour relation for O-Miras: filled circles = LMC Miras; crosses = Miras in galactic globular clusters. The line is a least-squares fit to the LMC O-Miras.

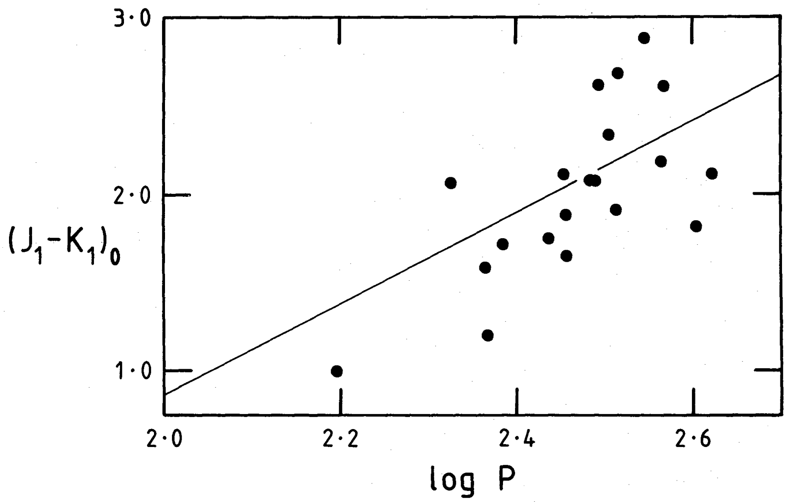

Figure 4. Period–colour relation for C-Miras. The line is a least squares fit. Note the large scatter.

been placed on the same system as the LMC observations (see Appendix). The globular cluster Miras follow the LMC relation remarkably well, a mean deviation (observed minus calculated) of only $+0.020 \pm 0.009$ mag with a scatter (per star) of $\sigma = 0.035$. It is known that there is at least a rough period–metallicity relation for O-Miras in galactic globular clusters (e.g. Lloyd Evans & Menzies 1973; Feast 1981; Lloyd Evans 1983). Thus either the period–$(J_1-K_1)_0$ relation for O-Miras is independent of abundance or else, in the mean, the LMC Miras follow the same period–metallicity relation as those in galactic globular clusters.

For the C-Miras there is a much larger spread in colour at a given period (Fig. 4) and the existence of a PC relation is therefore much less firmly established. The line shows the least-squares fit.

5 Period–luminosity–colour relations (O-Miras)

That the data for the O-Miras are better represented by a PLC relation than by a simple PL relation can be directly demonstrated by plotting residuals from a PL relation against those from the corresponding PC relation. Such a plot for the residuals in m_{b3} (solution 8) and those in $(J_1-K_1)_0$ (solution 11) is shown in Fig. 5. The two regression lines are drawn. There is an evident correlation of the two sets of residuals. Similar plots can be made for magnitudes at J, H and K. [Fig. 6 shows the $(J_1)_0$, $(J_1-K_1)_0$ residuals, solutions 1 and 11.]

Solutions were then made for equations of the form

$$m_0 = \alpha \log P + \beta (J_1-K_1)_0 + \gamma, \tag{3}$$

with σ = the standard deviation of a fit in m_0 (per star).

The results are shown in Table 4. In each case three solutions are tabulated: (a) a least-squares solution with all the errors in m_0; (b) a least-squares solution with all the errors in $(J_1-K_1)_0$; (c) a maximum likelihood solution with equal errors in m_0 and $(J_1-K_1)_0$ and very small errors in $\log P$. A maximum likelihood-type of solution undoubtedly gives the best (unbiased) estimates of the coefficients α, β and γ, provided the relative errors of the various observational quantities have been properly assessed. Caldwell & Coulson (1986) showed for LMC Cepheids that, with comparable errors in the magnitudes and colours and with the PL width considerably greater than the PC width (as it is both for Cepheids and Miras), then a

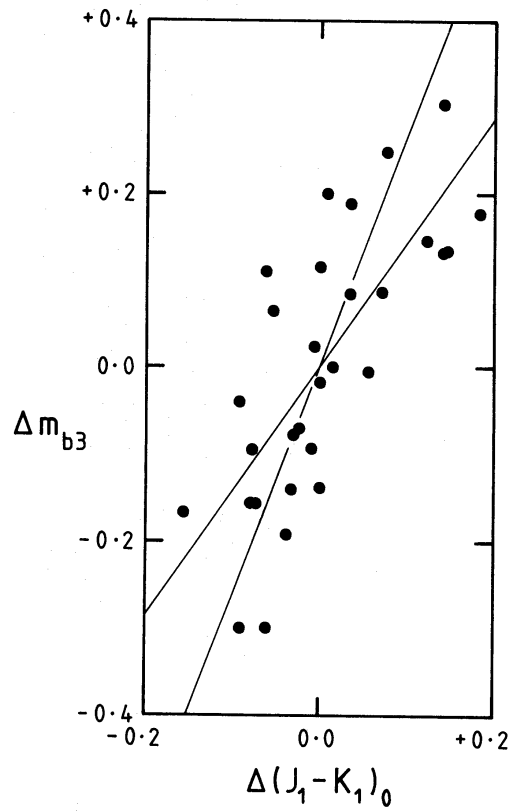

Figure 5. Residuals from the period–luminosity relation in m_{b3} for O-Miras plotted against residuals from the period–colour relation. The two regression lines are shown.

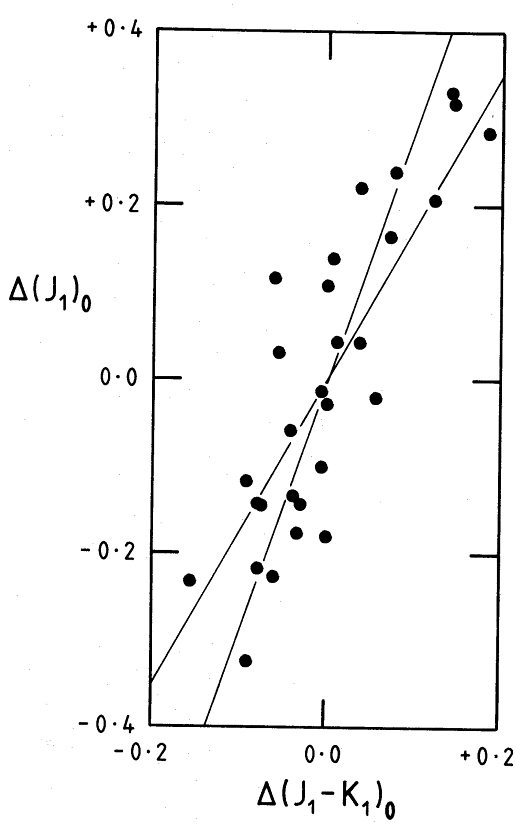

Figure 6. Residuals from the period–luminosity relation in $(J_1)_0$ for O-Miras plotted against residuals from the period–colour relation. The two regression lines are shown.

Table 4. Period–luminosity relations (equation 3) for O-Miras (29 stars).

Mag	Method	α ± s.e.	β ± s.e.	γ ± s.e.	σ	Solution
$(J_1)_0$	a	-3.98 ± .21	1.91 ± .24	19.58 ± .37	0.10	13
	b	-4.42 ± .27	2.72 ± .35	19.70 ± .45	0.12	14
	c	-4.34	2.58	19.66	0.11	15
$(H_1)_0$	a	-4.11 ± .22	1.74 ± .26	19.24 ± .40	0.11	16
	b	-4.67 ± .30	2.76 ± .41	19.36 ± .25	0.14	17
	c	-4.57	2.57	19.34	0.13	18
$(K_1)_0$	a	-3.98 ± .21	0.91 ± .24	19.58 ± .37	0.10	19
	b	-4.91 ± .48	2.60 ± .70	19.78 ± .21	0.17	20
	c	-4.58	2.00	19.71	0.13	21
m_{b3}	a	-3.80 ± .22	1.43 ± .26	21.52 ± .39	0.11	22
	b	-4.47 ± .42	2.64 ± .48	21.65 ± .33	0.15	23
	c	-4.32	2.37	21.63	0.13	24

a = least squares with all error in magnitude; b = least squares with all error in colour; c = maximum likelihood solution.

maximum likelihood PLC solution gave coefficients rather close to those obtained from a least-squares solution with all the error in colour. A similar result for the Miras is shown in Table 4.

Caldwell & Coulson demonstrate that introducing the uncertainties in the reddening estimates of individual Cepheids as well as the range in individual distance moduli due to the finite thickness of the LMC in the line-of-sight and also the tilt of the LMC, can result in maximum likelihood solutions closer to the least-squares solution with all the errors in m_0. Allowance for these effects in the case of the Miras will obviously move the maximum likelihood solution nearer to the least-squares solution in m_0. However, the effect is unlikely to be large since interstellar absorptions are small in the infrared and, whilst an old system such as the Miras is expected to be thicker in the line-of-sight (which is essentially perpendicular to the plane of the LMC) than are younger subsystems, the estimated scaleheight (from kinematics, Bessell, Freeman & Wood 1986) is only 0.3 kpc. It is now well established that young objects in the LMC belong to a flattened system inclined to the plane of the sky. If the Mira subsystem were similarly inclined it would have only a small effect on the solutions. For instance, correcting the Mira residuals in solutions 19 and 21 for such a tilt on the model of Caldwell & Coulson (1986) makes no significant difference to the derived σ. In the following we adopt values of the coefficients from the maximum likelihood solutions. Values obtained from the two types of least-squares solutions are useful in showing the upper and lower limits on these coefficients which result from extreme assumptions regarding the relative uncertainties in magnitudes and colours.

As in the case of the Cepheids it is necessary to decide whether the colour term, β, is intrinsic or due to differential interstellar reddening. A normal reddening law leads, on the second hypothesis to the following expected values of β at J, H and K, using $(J-K)$ as the colour: $\beta_J = 1.56$, $\beta_H = 0.89$, $\beta_K = 0.56$. These values are distinctly different from the maximum likelihood values of β listed in Table 4 [Method (c)]. The 'interstellar' values are in fact smaller than the lower limits to β which are obtained with the unlikely assumption that all error is in the magnitudes m_0 [Method (a) in Table 4]. We conclude that the colour term β is intrinsic. In principle the colour term could be due to differing amounts of circumstellar reddening. Table 5

Table 5. Effective values of β for a colour term produced by circumstellar extinction for various extinction laws.

	$Q\propto\lambda^{-1}$	$Q\propto\lambda^{-1.3}$	$Q\propto\lambda^{-2}$	Dirty Silicates	R For
β_J	2.2	1.8	1.4	2.0	2.0
β_H	1.6	1.3	0.8	1.4	1.5
β_K	1.2	0.8	0.4	1.0	1.0

lists calculated effective values of β_J, β_H and β_K (defined as above) for various extinction laws in the circumstellar shells. Extinction cross-sections (Q) varying with wavelength according to powers between -1 and -2 have frequently been used in modelling circumstellar shells. The data for dirty silicates are from the grain model of Jones & Merrill (1976) with $Q_{\rm abs}$ as tabulated by Le Bertre *et al.* (1984). The data for R For use the empirical determinations for the C-Mira R For by Feast *et al.* (1984). Whilst all these values, except those for $Q\propto\lambda^{-2}$, are at least marginally consistent with the set of solutions (a) of Table 4 (least squares with all errors in m_0), none are consistent with the maximum likelihood solutions. These results strongly suggest, therefore, that the reddening term is not due to circumstellar reddening and thus leave a photospheric temperature effect (as expected from the pulsation equation) as the most likely origin of the term.

The pulsation equation may be written

$$M_{\rm bol}=-3.333\log P-1.667\log M-10\log T_{\rm e}+3.333\log Q+42.37, \tag{4}$$

($M_{\rm bol}$ = absolute bolometric magnitude, M = mass, $T_{\rm e}$ = effective temperature, Q = pulsation constant.)

Two different empirical $(J\text{–}K)_0$–log $T_{\rm e}$ relationships for O-Miras have been used in recent times. One (*cf.* Bessell *et al.* 1983) assumes that a calibration based on the measured angular diameters of ordinary M giants (Ridgway *et al.* 1980) can be applied to O-Miras. The other (Glass & Feast 1982a) uses temperatures derived from blackbody fits to infrared photometry of O-Miras. The justification for these latter temperatures is that they are in agreement with the small number of (infrared) angular diameter measurements of Miras themselves. Whilst these two calibrations have distinctly different zero points, they have very similar slopes. At the mean $(J_1\text{–}K_1)_0$ colour of the LMC Miras the calibration of Bessell *et al.* gives $d(\log T_{\rm e})/d(J\text{–}K)_0=-0.209$. On the other hand the relation between $(J_1\text{–}K_1)_0$ and log $T_{\rm e}$ derived from the O-Miras of Table 1 by the blackbody method is closely linear over most of its range and has $d(\log T_{\rm e})/d(J_1\text{–}K_1)_0=-0.216$.

Thus, apart from a constant, the temperature term in equation (4), $-10\log T_{\rm e}$, can be written 2.09 $(J_1\text{–}K_1)_0$ or 2.16 $(J_1\text{–}K_1)_0$. Both these values are the same (within the errors) as the empirical term in the PLC equation for which solution (24) gives 2.37 $(J_1\text{–}K_1)_0$. This implies that to a good approximation we can write log M as a function of log P only.

Putting

$$\log M=A\log P+B, \tag{5}$$

we find $A=0.59$ with an uncertainty of about 0.15 from the maximum likelihood solution (solution 24).

It is inferred from studies of the galactic kinematics of Miras (e.g. Feast 1963) that their initial mass increases with the period. The present discussion shows that M, the pulsation mass i.e. the present mass, also increases with period. The range of pulsation masses involved is quite modest. Thus the mass of a 400-d Mira is 1.5 times that of a 200-d Mira. Or, if we adopt

$M \sim 0.64$ at 200 d as appropriate to an AGB member of 47 Tuc which has Miras of this period, we find $M = 0.96$ at 400 d.

6 Period–luminosity–colour relations (C-Miras)

Plotting the residuals from equation (1) (PL relation) for C-Miras against those from equation (2) (PC relation) shows the existence of a strong correlation in the case of the $(J_1)_0$ magnitudes and $(J_1-K_1)_0$ colours (see Fig. 7). Solutions for a PLC relation (equation 3) were carried out as described in Section 5 for O-Miras and the results are listed in Table 6. The evidence for a colour term for the relation in $(J_1)_0$ shown by Fig. 7 is reflected in the much lower σ for the PLC relation in $(J_1)_0$ (Table 6, solution 27) ($\sigma = 0.17$) compared with the corresponding PL solution (Table 2, solution 2) ($\sigma = 0.37$). Plots of the PL residuals in m_{b3} against those in

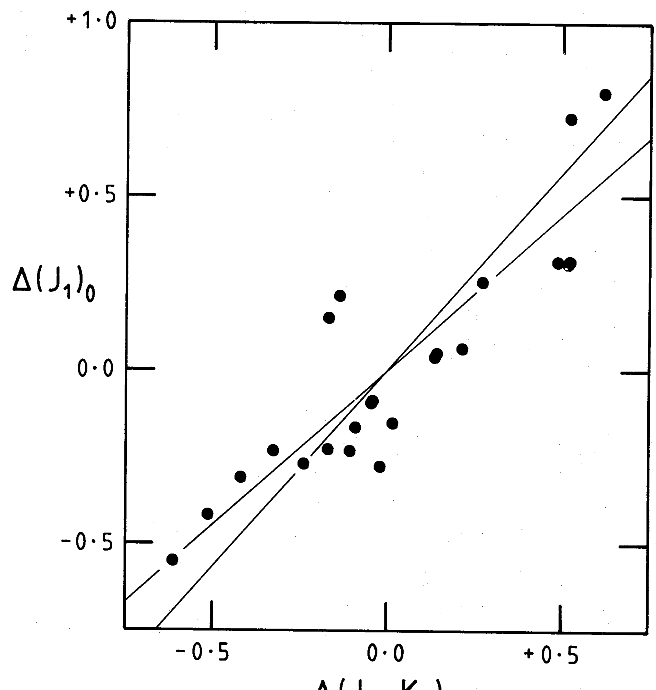

Figure 7. Residuals from the period–luminosity relation in $(J_1)_0$ for C-Miras plotted against residuals from the period–colour relation. The two regression lines are shown.

$(J_1-K_1)_0$ show some sign of a correlation but the need for a colour term remains marginal. In the case of $(K_1)_0$ there seems little evidence for a colour term within the scatter of the present observations, though formal PLC solutions are given in Table 6.

Although one might suspect that at least the reddest of the LMC C-Miras would be affected by circumstellar reddening the value of β_J given in Table 6, solution (27) ($= +1.0$) is less than that expected if the whole colour term were due to circumstellar dust with properties similar to that of the various examples listed in Table 5. The temperature calibration of the C-Miras in terms of colour is quite uncertain and a comparison with the theoretical pulsation equation, similar to that for the O-Miras in Section 5, has not been made.

7 Variables with $P > 420$ d

The six variables with $P > 420$ d in Table 1 were omitted in the above analyses. Five of these have $J-H$, $H-K$ colours suggesting that they are likely to be O-Miras, the status of the sixth star

Table 6. Period–luminosity–colour relations (equation 3) for C-Miras (20 stars).

Mag	Method	α ± s.e.	β ± s.e.	γ ± s.e.	σ	Solution
$(J_1)_0$	a	−2.97 ± .54	+0.89 ± .11	18.38 ± 1.17	0.18	25
	b	−3.74 ± .64	+1.14 ± .15	19.79 ± 1.36	0.20	26
	c	−3.34	+1.01	19.05	0.17	27
$(H_1)_0$	a	−3.46 ± .59	+0.56 ± .13	19.06 ± 1.31	0.20	28
	b	−4.99 ± .96	+1.05 ± .24	21.84 ± 1.94	0.27	29
	c	−3.89	+0.70	19.85	0.19	30
$(K_1)_0$	a	−2.97 ± .53	−0.11 ± .11	18.38 ± 1.17	0.18	31
	b	+3.57 ± 7.7	−2.21 ± 2.4	6.31 ± 14	0.81	32
	c	−2.88	−0.14	18.22	0.17	33
m_{b3}	a	+2.36 ± .37	+0.16 ± .08	19.65 ± .81	0.12	34
	b	−4.38 ± 1.5	+0.81 ± .40	23.33 ± 3.8	0.27	35
	c	−2.42	+0.18	19.75	0.12	36

a, b, c as in Table 4.

(C2) is less certain. At $(K_1)_0$ and m_{b3} these stars are more luminous than predicted by linear extrapolations of the PL and PLC relations for O-Miras (~ 0.7 mag more luminous than the PL or PLC predictions). Further work on a larger sample of variables of these long periods is evidently desirable. The present results suggest that we are detecting a non-linearity in the PLC relation. The theoretical relation (equation 4) is converted to the observational relation (equation 3) by assuming a linear relation between $(J_1-K_1)_0$ and log T_e (T_e = effective temperature), a linear relation between log M (M = mass) and M_{bol}, log T_e and/or log P, and a constant value of Q (the pulsation 'constant'). It is possible that these long-period objects are relatively massive (perhaps $\sim 2M_\odot$) and may be evolving into very long-period Miras (OH/IR

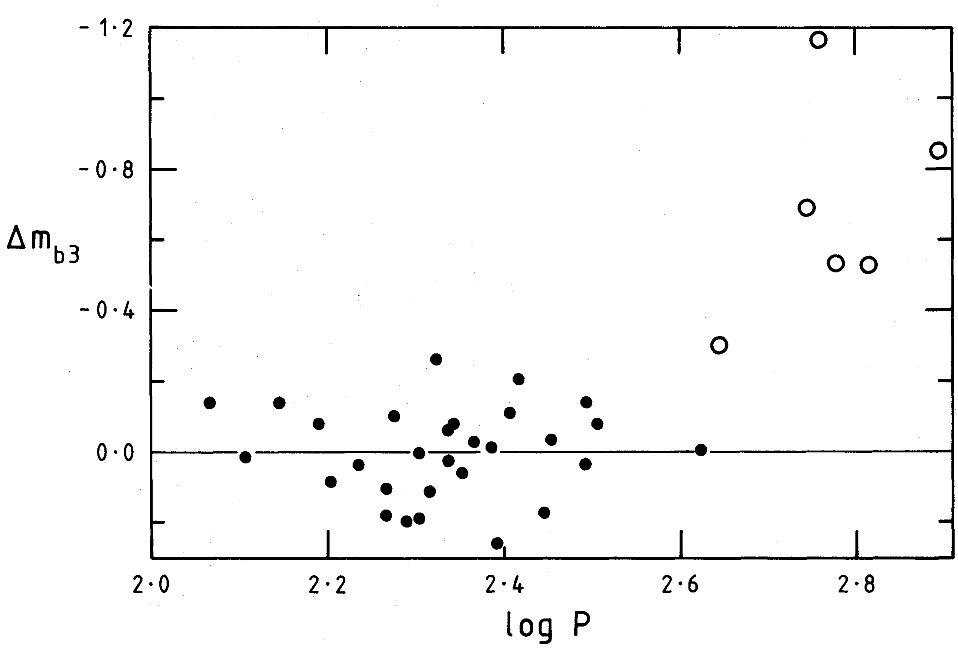

Figure 8. Residuals from the period–luminosity–colour relation in m_{b3} for O-Miras (solution 24, Table 4): filled circles = variables, with $P<420$ d, from which solution 24 was derived; open circles = variables with $P>420$ d.

sources) with periods in the range 1000–2000 d or into carbon stars. For stars of such masses it may be that some of these linearities break down. If the whole observed effect were due to a variation of Q, this would have to decrease by a factor of ~ 1.4 at the masses of these long-period objects. Fig. 8 shows that there is no evidence for a correlation between PLC residual and period for O-Miras with $P < 420$ d so that the effect becomes significant relatively suddenly.

8 The PLC relation and the distances of galactic globular clusters

The Mira PL and PLC relations are important tools for distance scale studies. In this paper we have based the calibration of these relations on a distance modulus of 18.47 (± 0.15) for the LMC (*cf.* Section 3). This modulus is based primarily on the Cepheid scale. Since O-Miras are present in some of the more metal-rich galactic globular clusters it is possible to use these to calibrate Mira zero points if the globular cluster distances are known. This method of calibration was used by various workers in the past including Menzies & Whitelock (1985) who used it together with their infrared photometry of globular cluster Miras. Most of the distances of the relevant globular clusters depend on adopted absolute magnitudes for RR Lyrae variables, $\langle M_v(\mathrm{RR})\rangle$, or (the equivalent) horizontal branch absolute magnitude M_v(HB). In the present section we derive the RR Lyrae/HB absolute magnitudes which result when the globular cluster distances are derived from the Miras they contain using our adopted zero points. We then compare these absolute magnitudes with other estimates.

The results given in Table 7(a) were derived using $(J_1 - K_1)$ and K_1 values from Menzies & Whitelock (1985) converted to the present photometric system (see Appendix A) and the values of E_{B-V} tabulated by them. We adopt $A_v = 3.05 E_{B-V}$. Values of [Fe/H] and the apparent magnitudes of the HB were taken from the recent compilation of Armandroff (1989). The [Fe/H] values are a revision of the Zinn scale. Distance moduli were obtained from the cluster-Miras using the PLC relation (maximum likelihood solution = solution 21) in $(K_1)_0$ and $(J_1 - K_1)_0$. Distance moduli were also derived using the PL relation at $(K_1)_0$ (solution 5, Table 2) and the resulting values of M_v(HB) (the absolute V magnitude of the HB corrected for reddening) are also listed. Table 7(b) contains results for three Magellanic Cloud clusters for which there is accurate RR Lyrae BVI photometry (Walker 1985; Walker & Mack 1988a, b). Distance moduli of 18.47 and 18.78 were adopted for the LMC and SMC, respectively (*cf.* Feast & Walker 1987; Feast 1988). Table 7(c) contains the values of M_v(HB) for four galactic globular clusters whose distances are derived from main-sequence fits to subdwarfs (Richer & Fahlman 1987).

The absolute visual magnitudes M_v(HB) of Table 7 are shown plotted against [Fe/H] in Fig. 9. In the case of the Mira determinations the PLC values were used. The value for NGC 5139 (ω Cen) is plotted at the adopted mean [Fe/H] for this cluster in which a range of [Fe/H] values exists. The value of M_v(HB) we derive for this cluster is particularly interesting since our value ($+0.28$) is bright compared to the 'conventional' mean value of $+0.60$. However, it agrees well with a recent estimate by Dickens (1988) [$M_v(\mathrm{HB}) = +0.25$] which is based on main-sequence fitting to ω Cen photometry. Fig. 9 also shows the relation

$$M_v(\mathrm{HB}) = \langle M_v(\mathrm{RR})\rangle = 0.92 + 0.2[\mathrm{Fe/H}], \quad (6)$$

derived recently from synthetic HB models (Lee, Demarque & Zinn 1987). There seems to be some observational support for this relation from Baade-Wesselink studies (Fernley, private communication; *cf.* Feast 1988). Also shown is the line

$$\langle M_v(\mathrm{RR})\rangle = 0.95 + 0.35[\mathrm{Fe/H}]. \quad (7)$$

Table 7. (a) Galactic globular clusters with Mira distances.

NGC	True Mod (PLC)	M_V(HB) (PLC)	M_V(HB) (PL)	[Fe/H]
104	13.22	+0.66	+0.62	-0.71
5139	13.90	+0.28	+0.21	(-1.59)
5927	14.91	+0.51	+0.57	-0.31
6352	15.98	+0.87	+0.83	-0.54
6553	13.55	+1.02	+0.97	-0.29
6637	14.68	+1.00	+0.92	-0.59
6712	14.51	+0.41	+0.43	-1.01
6838	13.09	+0.53	+0.48	-0.58
Mean (5139 Omitted)		+0.71 ± .09	+0.63 ± .12	-0.58
Predicted from Equation 6		+0.80		
Predicted from Equation 7		+0.75		

(b) Magellanic Cloud clusters with RR Lyrae photometry.

NGC	<M_V(RR)>	[Fe/H]
2210 (LMC)	+0.53	-1.9
1786 (LMC)	+0.35	-2.3
121 (SMC)	+0.68	-1.4

(c) Galactic globular clusters with distances from main-sequence fitting (Richer & Fahlman 1987).

Cluster	Mv(HB)	[Fe/H]
M15	0.44	-2.15
M13	0.56	-1.40
M5	0.81	-1.13
M4	0.83	-0.93

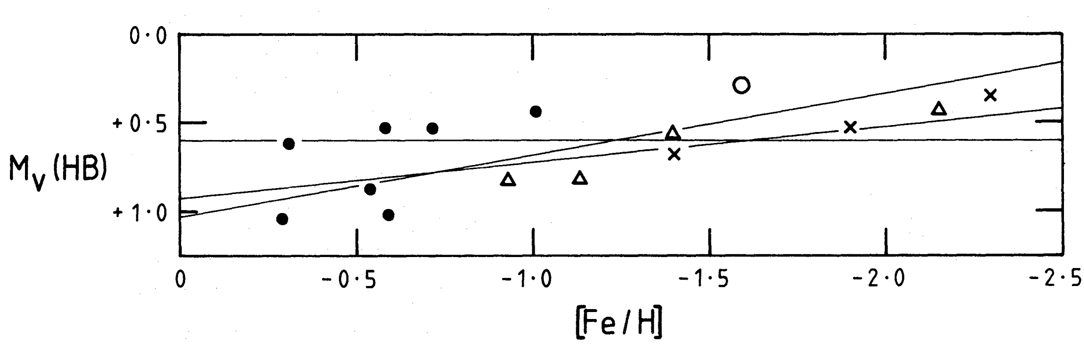

Figure 9. Absolute visual magnitudes of RR Lyrae variables (or horizontal branches) plotted against metallicity. The different symbols indicate the different ways M_v(HB) was derived: filled circles = from Miras in galactic globular clusters; open circle = from the Mira in ω Cen; crosses = from RR Lyraes in Magellanic Cloud clusters; triangles = from galactic globular clusters with distances from main-sequence fits. See text for details.

This has the slope originally proposed by Sandage (1982) to explain the Oosterhoff effect and has a zero point derived from statistical parallaxes (*cf.* Feast 1987). The constant 'conventional' value of $+0.6$ is also shown in Fig. 9.

Fig. 9 suggests that there is a dependence of $\langle M_v(\mathrm{RR})\rangle$ on metallicity though it does not allow one to distinguish between equations (6) and (7). The figure also shows that within the current uncertainties (~ 0.15 mag in the LMC modulus) there is adequate consistency between the globular cluster and Cepheid distance scales and (since this is demonstrated using Miras) that there are no significant luminosity differences (at a given period) between Miras in globular clusters and those in the LMC. It was shown earlier (Feast 1988) that the same difference in moduli between the LMC and SMC is obtained using either Cepheids or Miras.

9 Scatter in the PLC relations

The PLC relations of Table 4 can reproduce the magnitudes of the LMC O-Miras with an uncertainty of ~0.13. It is important to know how much, if any, of this scatter is intrinsic. As already mentioned (Section 2) a long series of infrared observations is now available for the LMC Miras listed by Glass & Lloyd Evans (1981). For the nine Miras in this group which appear in Table 1 the mean value without regard to sign of log $(P_{\mathrm{original}}/P_{\mathrm{revised}})$ is 0.010 corresponding to a standard error of this quantity of 0.012. Propagated through the PLC relation, this will produce a standard error in the predicted magnitudes of ~0.05. The mean magnitudes of Table 1 derived from Fourier fits to the observations have internal standard errors in the range 0.01 to 0.05 mag (with three higher values: 0.06, 0.08 and 0.12). The true external values will be higher, though by how much is not certain. These errors, which obviously affect both the magnitudes and the colours, together with the uncertainty resulting from errors in the periods can probably account for the observed scatter in the PLC relations. Thus if the standard errors in mean magnitudes and colours are ~0.05 mag each and the standard error in the period is 0.012, then the expected scatter in the PLC relation in m_{bol} and $(J_1-K_1)_0$ for O-Miras (maximum likelihood, solution 24) is 0.14, close to the value, 0.13, actually found. The depth of the Mira subsystem in the LMC and differential interstellar extinction will also contribute to the scatter though these effects are likely to be small. Within the current uncertainties there are no grounds for assuming any significant intrinsic width to the PLC relations for Miras.

10 Possible PL relations at shorter wavelengths

As mentioned in Section 1, PL relations for O-Miras exist at V(or B or m_{pg}) and these will no doubt continue to be of use in some circumstances.

It would be useful to obtain a PL relation in the I band since this is a useful band in which to carry out photographic surveys for Miras. For the 21 LMC Miras with photographic $\langle I\rangle$ values and (single) periods in table 2 of Reid *et al.* (1988) which are classified as M or (M) as defined previously we find

$$\langle I\rangle = -1.23\,(\pm 0.76)\log P + 17.12\,(\pm 1.78),$$

with a standard deviation $\sigma = 0.42$. At least some of this scatter is probably due to photometric errors and it would be interesting to see whether this rather high value of σ could be reduced by photometry of greater accuracy.

11 Conclusions

In this paper we have shown that the Miras, and in particular the O-Miras, conform to the predictions of pulsation theory by obeying PLC relations. Distance moduli with an uncertainty of only ~0.13 mag may be derived using these relations. The present results show rather clearly that Mira variables are 'well-behaved' pulsating stars occupying an instability strip of finite width in the HR diagram. It was shown earlier (Whitelock *et al.* 1987) that the mass of circumstellar dust round O-Miras depends on both the period and the bolometric amplitude, the scatter in the relevant relation being within the observational errors. This result, together with the present work, suggests that the Miras are a physically well-defined class of object which should be amenable to theoretical treatment in models of AGB evolution.

Acknowledgments

We are indebted to Dr C. A. Laney, Dr J. A. R. Caldwell, Dr L. A. Balona, Dr T. Lloyd Evans, Miss Y. Thomas, Mrs C. Strydom and Mrs C. Black for help in various ways.

References

Armandroff, T. E., 1989. *Astr. J.*, **97**, 375.
Bessell, M. S., Wood, P. R. & Lloyd Evans, T., 1983. *Mon. Not. R. astr. Soc.*, **202**, 59.
Bessell, M. S., Freeman, K. C. & Wood, P. R., 1986. *Astrophys. J.*, **310**, 710.
Caldwell, J. A. R. & Coulson, I. M., 1985. *Mon. Not. R. astr. Soc.*, **212**, 879.
Caldwell, J. A. R. & Coulson, I. M., 1986. *Mon. Not. R. astr. Soc.*, **218**, 223.
Carter, B. S., 1989. *Mon. Not. R. astr. Soc.*, in press.
Catchpole, R. M., Robertson, B. S. C., Lloyd, Evans, T. H. H., Feast, M. W., Glass, I. S. & Carter, B. S., 1979. *S. Afr. astr. Obs. Circ.*, **4**, 61.
Clayton, M. L. & Feast, M. W., 1969. *Mon. Not. R. astr. Soc.*, **146**, 411.
Dickens, R. J., 1989. *IAU Colloq. No. 111*, in press.
Feast, M. W., 1963. *Mon. Not. R. astr. Soc.*, **125**, 367.
Feast, M. W. 1981. In: *Physical Processes in Red Giants*, p. 193, ed. Iben, I. & Renzini, A., Reidel, Dordrecht.
Feast, M. W., 1984. *Mon. Not. R. astr. Soc.*, **211**, 51P.
Feast, M. W., 1986. In: *Light on Dark Matter*, p. 339, ed. Israel, F. P., Reidel, Dordrecht.
Feast, M. W., 1987. In: *The Galaxy*, p. 1, ed. Gilmore, G. & Carswell, B., Reidel, Dordrecht.
Feast, M. W., 1988. In: *The Extragalactic Distance Scale, ASP Conf. Series*, Vol. 4, p. 9, eds van den Bergh, S. & Pritchet, C. J., *Astr. Soc. Pacif.*, San Francisco.
Feast, M. W. & Walker, A. R., 1987. *Ann. Rev. Astr. Astrophys.*, **25**, 345.
Feast, M. W., Whitelock, P. A., Catchpole, R. M., Roberts, G. & Overbeek, M. D., 1984. *Mon. Not. R. astr. Soc.*, **211**, 331.
Feast, M. W., Robertson, B. S. C., Catchpole, R. M., Lloyd Evans, T., Glass, I. S. & Carter, B. S., 1982. *Mon. Not. R. astr. Soc.*, **201**, 439.
Gerasimovic, B. P., 1928. *Proc. natn. Acad. Sci. U.S.A.*, **14**, 963 (Harv. Reprint 54).
Glass, I. S., 1973. *Mon. Not. R. astr. Soc.*, **164**, 155.
Glass, I. S. & Feast, M. W., 1982a. *Mon. Not. R. astr. Soc.*, **199**, 245.
Glass, I. S. & Feast, M. W., 1982b. *Mon. Not. R. astr. Soc.*, **198**, 199.
Glass, I. S. & Lloyd Evans, T., 1981. *Nature*, **291**, 303.
Glass, I. S. & Reid, N., 1985. *Mon. Not. R. astr. Soc.*, **214**, 405.
Glass, I. S., Catchpole, R. M., Feast, M. W., Whitelock, P. A. & Reid, I. N., 1987. In: *Late Stages of Stellar Evolution*, p. 51, eds Kwok, S. & Pottasch, S. R., Reidel, Dordrecht.
Gyllenberg, W., 1929. *Lund Medd Ser. II*, No. 53.
Gyllenberg, W., 1930. *Lund Medd Ser. II*, No. 54.
Jones, T. W. & Merrill, K. M., 1976. *Astrophys. J.*, **209**, 509.
Le Bertre, T., Epchtein, N., Gispert, R., Nguyen-Q-Rieu & Truong-Bach, 1984. *Astr. Astrophys.*, **132**, 75.
Lee, W. Y., Demarque, P. & Zinn, R., 1987. In: *The Second Conference on Faint Blue Stars, IAU Colloq. No. 95*, p. 137, eds Davis Philip, A. G., Hayes, D. S. & Liebert, J. W., Davis Press.

Lloyd Evans, T., 1983. *Mon. Not. R. astr. Soc.,* **204,** 961.
Lloyd Evans, T. & Menzies, J. W., 1973. In: *Variable Stars in Globular Clusters and in Related Systems,* p. 151, ed. Fernie, J. D., Reidel, Dordrecht.
Menzies, J. W. & Whitelock, P. A., 1985. *Mon. Not. R. astr. Soc.,* **212,** 783.
Osvalds, V. & Risley, A. M., 1961. *Publs Leander McCormick,* **11,** 147.
Reid, N., Glass, I. S. & Catchpole, R. M., 1988. *Mon. Not. R. astr. Soc.,* **232,** 53.
Richer, H. B. & Fahlman, G. G., 1987. *Astrophys. J.,* **316,** 189.
Ridgway, S. T., Joyce, R. R., White, N. M. & Wing, R. F., 1980. *Astrophys. J.,* **235,** 126.
Robertson, B. S. C. & Feast, M. W., 1981. *Mon. Not. R. astr. Soc.,* **196,** 111.
Sandage, A., 1958. *Astrophys. J.,* **127,** 513.
Sandage, A., 1982. *Astrophys. J.,* **252,** 553.
van den Bergh, S., 1984. *Astrophys. Space Sci.,* **102,** 295.
Walker, A. R., 1985. *Mon. Not. R. astr. Soc.,* **212,** 343.
Walker, A. R. & Mack, P., 1988a. *Astr. J.,* **96,** 1362.
Walker, A. R. & Mack, P., 1988b. *Astr. J.,* **96,** 872.
Whitelock, P. A., Pottasch, S. R. & Feast, M. W., 1987. In: *Late Stages of Stellar Evolution,* p. 269, ed. Kwok, S. & Pottasch, S. R., Reidel, Dordrecht.
Wilson, R. E. & Merrill, P. W., 1942. *Astrophys. J.,* **95,** 248.
Wilson, W. J., Schwartz, P. R., Neugebauer, G., Harvey, P. M. & Becklin, E. E., 1972. *Astrophys. J.,* **177,** 523.
Wood, P. R., Bessell, M. S. & Paltoglou, G., 1985. *Astrophys. J.,* **290,** 477.

Appendix A

The corrections specified in the table below must be added to the photometry of Menzies & Whitelock (1985)★ in order to correct it to the photometric system used in this paper. The corrections are the result of the high accuracy of standard star photometry now available.

Cluster NGC	ΔJ	ΔH	ΔK	ΔL
		(Mag)		
5139	-0.044	-0.043	-0.030	+0.013
5927	-0.040	-0.073	-0.056	-0.025
6352, 6388	-0.028	-0.051	-0.012	-0.035
6356, Terzan 5	-0.028	-0.049	-0.022	-0.067
6553, 6637, 6712, 6723	-0.018	-0.046	-0.020	0.000
6838	-0.023	-0.071	-0.016	-
104, 362	+0.006	-0.008	+0.046	+0.047
288	+0.023	-0.016	+0.025	+0.025
1904	-0.002	-0.032	-0.009	-0.022

★There was a misprint in Appendix B of Menzies & Whitelock (1985). The lines for HR 6378 and HR 0180 should have read as follows:

Standard	J	H	K	L	Cluster
HR 6378	2.31	2.30	2.27	2.31	N6356, Terzan 5
HR 0180	2.95	2.46	2.34	2.26	N288