A PERIODIC REVIEW INVENTORY MODEL WITH STOCK DEPENDENT DEMAND, PERMISSIBLE DELAY IN PAYMENT AND PRICE DISCOUNT ON BACKORDERS

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Abstract: In this paper we study a periodic review inventory model with stock dependent demand. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. Permissible delay in payments allowed to the inventory manager is also taken into account. Numerical examples are cited to illustrate the model.

Keywords: Periodic review model; stock dependent demand; shortage; price discount on backorder; delay in payment.

MSC: 90B05.

1. INTRODUCTION

In traditional inventory models, it is generally assumed that the demand rate is independent of factors like stock availability, price of items, etc. However, in actual practice, it is observed that demand for certain items is greatly influenced by the stock level. For example, an increase in shelf space for an item is seen to induce more consumers to buy it owing to its visibility, popularity or variety. Conversely, low stocks of certain goods might raise the perception that they are not fresh. Levin et al. (1972) pointed out that large piles of consumer goods displayed in a supermarket attract the customer to buy more. Silver and Peterson (1985) noted that sales at the retail level tend to be proportional to the stock displayed. Baker and Urban (1988) established an EOQ model for a power-form inventory-level-dependent demand pattern. Padmanabhan and

Vrat (1990) developed a multi-item inventory model for deteriorating items with stock-dependent demand under resource constraints. Datta and Pal (1990) presented an inventory model in which the demand rate is dependent on the instantaneous inventory level until a given inventory level is achieved after which, the demand rate becomes constant. Urban and Baker (1997) deliberated the EOQ model in which the demand is a multivariate function of price, time, and level of inventory. Giri and Chaudhuri (1998) expanded the EOQ model to allow for a nonlinear holding cost. Roy and Maiti (1998) developed multi-item inventory models of deteriorating items with stock-dependent demand in a fuzzy environment. Datta and Paul (2001) analyzed a multi-period EOQ model with stock-dependent, and price-sensitive demand rate. Kar et al. (2001) proposed an inventory model for deteriorating items sold from two shops, under single management dealing with limitations on investment and total floorspace area. Other papers related to this area are Gerchak and Wang (1994), Padmanabhan and Vrat (1995), Ray et al. (1998), Hwang and Hahn (2000), Chang (2004), Panda (2010), Chang and Feng (2010), Roy and Chaudhuri (2012), Yadav et al. (2012), among others.

In inventory models with shortages, the general assumption is that the unmet demand is either completely lost or completely backlogged. However, it is quite possible that while some customers leave, others are willing to wait till fulfillment of their demand. In some situations, the inventory manager may offer a discount on backorders and/or reduction in waiting time to tempt customers to wait. Ouyang et al. (1999) considered reduction in lead time and ordering cost in a continuous review model with partial backordering. Chuang et al. (2004) discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Uthayakumar and Parvati (2008) considered a model with only first two moments of the lead time demand known, and obtained the optimum backorder price discount and order quantity in that situation. See also Chung and Huang (1998), Trevino et al. (1993), Kim et al. (1992).

In many real-life situations, the supplier allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period but beyond it, the manager has to pay an interest to the supplier. During the permitted time period, the manager is free to sell his goods, accumulate revenue and earn interest. Hence, it is profitable to the manager to delay his payment till the last day of the settlement period. Goyal (1985) first developed the EOQ model under conditions of permissible delay in payment. Chand and Ward (1987) analyzed Goyal's problem under assumptions of the classical economic order quantity model, obtaining different results. Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) extended Goyal's model to the case of deteriorating items. Jamal et al. (1997) and Chang and Dye (2001) extended Aggarwal and Jaggi's model to allow shortages. Shinn et al. (1996) investigated the problem of price and lot size determination under permissible delay in payment and quantity discount on freight cost. Liao et al. (2000) considered an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible, but no shortages are allowed. Ouyang et al. (2005) developed an inventory model for deteriorating items with partial backlogging under permissible delay in payment. Pal and Ghosh (2007a) considered deterministic inventory models allowing shortage for deteriorating items under stock dependent demand, when delay in payment is allowed. Pal and Ghosh (2006, 2007b) studied quantity dependent settlement period in deterministic inventory models. Ghosh (2007) discussed stochastic inventory model for deteriorating items with permissible delay in payment. Das et al. (2011) developed a

deterministic EOQ inventory model with time dependent demand under permissible delay in payment and the cost parameters are taken as hybrid numbers

In this paper, we consider a periodic review inventory model with stock dependent demand. The supplier allows the inventory manager a fixed time interval to settle his dues and the manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal order quantity and backorder price discount determined. In Section 4, numerical examples are cited to illustrate the policy and to analyze the sensitivity of the model with respect to the cost parameters. Concluding remarks are given in Section 5.

2. NOTATIONS AND ASSUMPTIONS

To develop the model, we use the following notations and assumptions.

Notations

(a) Given variables

K = ordering cost per order

P = purchase cost per unit

h = holding cost per unit per unit time

s_1 = backorder cost per unit backordered per unit time

s_2 = cost of a lost sale

π_0 = marginal profit per unit

I_e = interest that can be earned per unit time

I_r = interest payable per unit time beyond the permissible delay period ($I_r > I_e$)

M = permissible delay in settling the accounts

b_0 = upper bound on backorder ratio, $0 \leq b_0 \leq 1$.

(b) Decision variables

b = fraction of the demand during stock-out period which is allowed or accepted to be backlogged

π = price discount on unit backorder offered

T = length of a replenishment cycle

T_1 = time taken for stock on hand to be exhausted, $0 < T_1 < T$

S = maximum stock height in a replenishment cycle.

Further, let

$I(t)$ = inventory level at time point t, $0 \leq t \leq T$.

Assumptions

1. The model considers only one item in inventory.

2. Replenishment of inventory occurs instantaneously on ordering, that is, lead time is zero.

3. Shortages are allowed, and a fraction b of unmet demands during stock-out is backlogged.

4. Demand rate $R(t)$ at time t is

$$R(t) = \alpha + \beta I(t) \quad \text{for } 0 < t < T_1$$
$$\quad\quad\, = \alpha \quad\quad\quad\quad \text{for } T_1 < t < T$$

    where α = fixed demand per unit time, $\alpha > 0$ and β = fraction of total inventory demanded per unit time under the influence of stock on hand, $0 < \beta < 1$.

5. During the stock-out period, the backorder fraction b is directly proportional to the price discount π offered by the inventory manager. Thus,

$$b = \frac{b_0}{\pi_0} \pi \text{ , where } \quad 0 \leq \pi \leq \pi_0 \text{ .}$$

3. MODEL FORMULATION

The planning period is divided into reorder intervals, each of length T units. Orders are placed at time points 0, T, $2T$, $3T$, …, the order quantity being just sufficient to bring the stock height to a certain maximum level S. Assuming that at the beginning of the first reorder interval the stock level is zero just before ordering, the order quantity in this interval is equal to S.

Depletion of inventory occurs due to demand during the period $(0, T_1)$, $T_1 < T$, and in the interval (T_1, T) shortage occurs, of which a fraction b is backlogged. Hence, the variation in inventory level with respect to time is given by

$$\frac{d}{dt} I(t) = -\alpha - \beta I(t), \text{ if } 0 \leq t \leq T_1$$
$$\quad\quad\; = -b\alpha, \quad\quad\; \text{if } T_1 < t \leq T.$$

Since $I(T_1) = 0$, we get

$$I(t) = \frac{\alpha}{\beta}(e^{\beta(T_1 - t)} - 1), \quad \text{if} \ \ 0 \le t \le T_1$$

$$= b\alpha(T_1 - t), \qquad \text{if} \ \ T_1 < t \le T.$$

Hence, $S = \dfrac{\alpha}{\beta}(e^{\beta T_1} - 1).$

Then,

$H(T_1, T, b) = $ inventory carried during a cycle

$$= \int_0^{T_1} I(t)\,dt$$

$$= \frac{\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\}$$

$S(T_1, T, b) = $ number of backorders during a cycle

$$= \int_{T_1}^{T} I(t)\,dt$$

$$= b\alpha(T - T_1)^2 / 2$$

$$= E(T_1, T, b) = \text{number of lost sales during a cycle}$$

$$= (1 - b)\alpha(T - T_1)$$

As regarding the permissible delay in payment, there can be two possibilities: $M \le T_1$ and $M \ge T_1$.
We consider the two cases separately.

Case 1: $M \le T_1$

For $M \le T_1$, the inventory manager has stock on hand beyond M, and so he can use the sale revenue to earn interest at a rate I_e during $(0, T_1)$. The interest earned by the buyer is, therefore,

$$IE_1(T_1, T, b) = PI_e \int_0^{T_1} I(t)\,dt = \frac{PI_e \alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\}$$

Beyond the fixed settlement period, the unsold stock is financed with an interest rate I_r, so that the interest payable by the inventory manager is

$$IP_1(T_1, T, b) = PI_r \int_M^{T_1} I(t)\,dt = \frac{PI_r \alpha}{\beta}\{\frac{1}{\beta}(e^{\beta(T_1 - M)} - 1) - (T_1 - M)\}$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C_1(T_1,T,b) = \frac{1}{T}[K + hH(T_1,T,b) + s_1 S(T_1,T,b) + s_2 E(T_1,T,b) + IP_1(T_1,T,b) - IE_1(T_1,T,b)]$$

$$= \frac{1}{T}[K + h\frac{\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\} + s_1 \frac{b\alpha}{2}(T - T_1)^2 + s_2(1-b)\alpha(T - T_1)$$

$$+ \frac{PI_r\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta(T_1-M)} - 1) - (T_1 - M)\} - \frac{PI_e\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\}]$$

$$= \frac{N_1(T_1,T,b)}{T}, \text{ say.}$$

<u>Case 2</u>: $M \ge T_1$

    Since $M \ge T_1$, the inventory manager pays no interest, but earns interest in the interval $(0, M)$ at a rate I_e.

    The interest earned is given by

$$IE_2(T_1,T,b) = PI_e\int_0^{T_1} \frac{\alpha}{\beta}(e^{\beta(T_1-t)} - 1)dt + PI_e\int_{T_1}^{M} b\alpha(M - t)dt$$

$$= PI_e\frac{\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\} + PI_e\frac{b\alpha}{2}(M - T_1)^2$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C_2(T_1,T,b) = \frac{1}{T}[K + hH(T_1,T,b) + s_1 S(T_1,T,b) + s_2 E(T_1,T,b) - IE_2(T_1,T,b)]$$

$$= \frac{1}{T}[K + h\frac{\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\} + s_1 \frac{b\alpha}{2}(T - T_1)^2 + s_2(1-b)\alpha(T - T_1)$$

$$- \frac{PI_e\alpha}{\beta}\{\frac{1}{\beta}(e^{\beta T_1} - 1) - T_1\} - PI_e\frac{b\alpha}{2}(M - T_1)^2]$$

$$= \frac{N_2(T_1,T,b)}{T}, \text{ say.}$$

    The total expected cost per unit length of a replenishment cycle is, therefore, given by

$$C(T_1,T,b) = C_1(T_1,T,b), \text{ if } T_1 \ge M$$
$$= C_2(T_1,T,b), \text{ if } T_1 \le M.$$

    The optimal values of the decision variables (T_1,T,b) minimizing $C(T_1,T,b)$ will be the set of values minimizing $C_1(T_1,T,b)$ if $\min C_1(T_1,T,b) \le \min C_2(T_1,T,b)$, or the set of values minimizing $C_2(T_1,T,b)$ if $\min C_2(T_1,T,b) \le \min C_1(T_1,T,b)$.

    To find the optimal values of T_1, T and b, we note that for given b, (T_1,T) minimizing $C_1(T_1,T,b)$ satisfy

$$\frac{\alpha}{\beta}e^{\beta T_1}[h + P(I_r e^{-\beta M} - I_e)] + s_1 b\alpha T_1 = \frac{h\alpha}{\beta} + s_1 b\alpha T + s_2(1-b)\alpha + \frac{P\alpha}{\beta}(I_r - I_e) \quad (3.1)$$

$$s_1 b\alpha(T - T_1) + s_2(1-b)\alpha = C_1(T_1, T, b) \tag{3.2}$$

and (T_1, T) minimizing $C_2(T_1, T, b)$ satisfy

$$\frac{\alpha}{\beta} e^{\beta T_1}(h - PI_e) + (s_1 - PI_e)b\alpha T_1 = h\frac{\alpha}{\beta} + s_1 b\alpha T + s_2(1-b)\alpha - PI_e(\frac{\alpha}{\beta} + b\alpha M) \tag{3.4}$$

$$s_1 b\alpha(T - T_1) + s_2(1-b)\alpha = C_2(T_1, T, b) \tag{3.5}$$

Clearly, equations (3.1)-(3.2) and (3.3)-(3.4) give solutions to (T_1, T) that are non-linear in b. If these solutions are obtainable in closed form, one can substitute these in $C_1(T_1, T, b)$ and $C_2(T_1, T, b)$ respectively to get the cost functions as functions of b alone. Then, minimizing the cost functions with respect to b, one can find the optimal value of b, and hence of T_1, T. However, as closed form solutions are difficult to obtain, the following theorems may be helpful in finding the optimal solution to the problem.

Theorem 3.1: *For given T and b, $C_1(T_1, T, b)$ is a convex function of T_1 if $h + P(e^{-\beta M}I_r - I_e) \geq 0$, and is concave in T_1 if $h + P(e^{-\beta M}I_r - I_e) \leq 0$, while $C_2(T_1, T, b)$ is a convex function of T_1 if $min(s_1, h) - PI_e \geq 0$, and is concave in T_1 if $min(s_1, h) - PI_e \leq 0$.*

Proof: *We have*

$$\frac{\partial^2}{\partial T_1^2} C_1(T_1, T, b) = \frac{\alpha}{T}[e^{\beta T_1}\{h + P(I_r e^{-\beta M} - I_e)\} + s_1 b] \tag{3.6}$$

$$\frac{\partial^2}{\partial T_1^2} C_2(T_1, T, b) = \frac{\alpha}{T}[e^{\beta T_1}(h - PI_e) + (s_1 - PI_e)b]. \tag{3.7}$$

(3.6) is $\geq$ or ≤ 0 according as $h + P(e^{-\beta M}I_r - I_e) \geq 0$ or ≤ 0, while (3.7) is $\geq$ or ≤ 0 according as $min(s_1, h) - PI_e \geq 0$ or ≤ 0. Hence, the theorem.

Theorem 3.2: *For given T and b, optimal T_1 minimizing $C_1(T_1, T, b)$ is an increasing function of T if $h + P(e^{-\beta M}I_r - I_e) \geq 0$, and optimal T_1 minimizing $C_2(T_1, T, b)$ is an increasing function of T if $min(s_1, h) - PI_e \geq 0$.*

Proof: *Differentiating (3.1) w.r.t. T we get that if $h + P(e^{-\beta M}I_r - I_e) \geq 0$,*

$$\frac{\partial T_1}{\partial T} = \frac{s_1 b}{s_1 b + e^{\beta T_1}(h + P(e^{-\beta M}I_r - I_e))} > 0.$$

Again, differentiating (3.3) w.r.t. T we have that if $min(s_1, h) - PI_e \geq 0$,

$$\frac{\partial T_1}{\partial T} = \frac{s_1 b}{b(s_1 - PI_e) + e^{\beta T_1}(h - PI_e)} > 0.$$

Hence, the theorem.

4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

Since it is difficult to find closed form solutions to the sets of equations (3.1)-(3.2) and (3.3)-(3.4), we numerically find optimal solutions to the problem for given sets of model parameters, using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed. We assume that $\alpha = 70$, $\beta = 0.7$, $b_0 = 1$.

Table 1: Showing the optimal inventory policy for different values of s_1, when K=50, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, $M = 0.1$, $s_2 = 70$ and $h = 40$.

s_1	T_1	T	b	$C(T_1, T, b)$
40	1.0069	4.5069	0.9980	4230.03
45	0.9961	4.1072	0.9992	4166.68
50	0.9858	3.7858	0.5000	4106.49
60	0.9664	3.2997	0.9995	3994.60
70	0.9485	2.9485	0.9993	3892.55
80	0.9319	2.6819	0.9856	3798.93
100	0.9018	2.3018	0.9749	3632.60
120	0.8754	2.0420	0.9653	3488.72
125	0.8692	1.9892	0.9620	3455.66

Table 2: Showing the optimal inventory policy for different values of $s2$, when K=50, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, M=0.1, s_1=80 and $h = 40$.

s_2	T_1	T	b	$C(T_1, T, b)$
60	0.8288	2.3288	0.9977	3242.32
70	0.9319	2.6819	0.9856	3798.93
80	1.0289	3.0289	0.9961	4360.93
90	1.1205	3.3705	0.9923	4927.75
100	1.2072	3.7072	0.9904	5498.96
110	1.2895	4.0395	0.9885	6074.21
120	1.3678	4.3678	0.5000	6653.20
125	1.4056	4.5306	0.5000	6944.02

Table 3: Showing the optimal inventory policy for different values of h, when $K=50$, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, $M=0.1$, $s_1=80$ and $s_2 = 70$.

h	T_1	T	b	$C(T_1, T, b)$
25	1.2114	2.9614	0.9995	3525.52
30	1.0988	2.8488	0.9995	3632.45
40	0.9319	2.6819	0.9856	3798.93
50	0.8125	2.5625	0.9995	3924.07
60	0.7221	2.4721	0.9985	4022.43
70	0.6509	2.4009	0.9985	4102.18
80	0.5931	2.3431	0.9984	4168.38
100	0.5045	2.2545	0.9982	4272.35

Table 4: Showing the optimal inventory policy for different values of M, when $K=50$, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, $h=40$, $s_1=80$ and $s_2 = 70$.

M	T_1	T	b	$C(T_1, T, b)$
0.01	0.9240	2.6740	0.9980	3813.09
0.05	0.9275	2.6775	0.9983	3806.57
0.1	0.9319	2.6819	0.9856	3798.93
0.3	0.9484	2.6984	0.5000	3773.65
0.5	0.9638	2.7138	0.9982	3756.02
0.7	0.9781	2.7281	0.9983	3745.08
1	0.9977	2.7477	0.8556	3739.42
1.5	0.9745	2.7245	1.0000	3729.29
2	0.9466	2.6966	1.0000	3698.71
2.5	0.9137	2.6637	1.0000	3647.00

Table 5: Showing the optimal inventory policy for different values of I_e, when $K=50$, $P = 100$, $I_r = 0.05$, $h=40$, $s_1=80$ and $s_2 = 70$.

I_e	T_1	T	b	$C(T_1, T, b)$
0.010	0.9050	2.6550	0.9994	3826.67
0.015	0.9115	2.6615	0.9818	3819.88
0.020	0.9182	2.6682	0.9827	3812.99
0.025	0.9250	2.6750	0.9852	3806.01
0.030	0.9319	2.6819	0.9856	3798.93
0.035	0.9389	2.6889	0.9994	3791.75
0.040	0.9460	2.6960	0.9989	3784.47
0.045	0.9532	2.7032	0.9988	3777.08

Table 6: Showing the optimal inventory policy for different values of I_r, when $K=50$, $P = 100$, $I_e = 0.03$, $h=40$, $s_1=80$ and $s_2 = 70$.

I_r	T_1	T	b	$C(T_1, T, b)$
0.05	0.9319	2.6819	0.9856	3798.93
0.06	0.9199	2.6699	0.9828	3809.85
0.07	0.9083	2.6583	0.9992	3820.48
0.08	0.8971	2.6471	0.9993	3830.82
0.10	0.8756	2.6256	0.5000	3850.72
0.15	0.8270	2.5770	0.5000	3896.27
0.20	0.7845	2.5345	0.9990	3936.72
0.25	0.7469	2.4969	0.9986	3972.92

The above tables show that, for other parameters remaining constant,

(a) both T_1 and T are decreasing in s_1, h and I_r, but increase as s_2 and I_e increase;

(b) b, and hence π, decreases with increase in s_1, s_2 and h, but increases with M;

(c) the minimum cost per unit length of a reorder interval increases as h, s_2 and I_r increase, but decreases with increase in M, s_1 and I_e.

The above observations indicate that, with the aim to minimizing total cost, the policy should be to maintain high inventory level for low backorder and holding costs but high lost sales cost and interest earned. Also, higher the backorder cost, lower should be the price discount offered on a backorder.

5. CONCLUSIONS

The paper studies a periodic review inventory model with stock dependent demand, allowing shortages. When there is a stock out, the inventory manager offers a discount to each customer who is ready to wait till fulfillment of his demand. On the other hand, the replenishment source allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period but beyond it, the manager has to pay an interest. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that for low backorder cost, it is beneficial to the inventory manager to offer the customers high discount on backorders.

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