A personalized consensus feedback mechanism based on maximum harmony degree

Mingshuo Cao, Jian Wu, *Senior Member, IEEE*, Francisco Chiclana, and Raquel Ureña, Enrique Herrera-Viedma. *Senior Member, IEEE*

***Abstract*—This article proposes a framework of personalized feedback mechanism to help multiple inconsistent experts to reach consensus in group decision making by allowing to select different feedback parameters according to individual consensus degree. The general harmony degree (GHD) is defined to determine the before/after feedback difference between the original and revised opinions. It is proved that the GHD index is monotonically decreasing with respect to the feedback parameter, which means that higher parameters values will result in higher changes of opinions. An optimisation model is built with the GHD as the objective function and the consensus thresholds as constraints, with solution being personalized feedback advices to the inconsistent experts that keep a balance between consensus (group aim) and independence (individual aim). This approach is, therefore, more reasonable than the unpersonalized feedback mechanisms in which the inconsistent experts are forced to adopt feedback generated with only consensus target without considering the extent of the changes acceptable by individual experts. Furthermore, the following interesting theoretical results are also proved: (1) the personalized feedback mechanism guarantees that the increase of consensus level after feedback advices are implemented; (2) the GHD by the personalized feedback mechanism is higher than that of the unpersonalized one; and (3) the personalized feedback mechanism generalises the unpersonalized one as it is proved the latter is a particular type of the former. Finally, a numerical example is provided to model the feedback process and to corroborates these results when comparing both feedback mechanism approaches.**

***Index Terms*—Group decision making; Consensus; Personalized feedback mechanism; Harmony degree**

I. INTRODUCTION

GRoup decision making (GDM) problems involve a group of decision makers expressing their opinions on a finite set of alternatives on a set of criteria; the individual opinions are then fused or aggregated into a collective one to derive a common group solution [1]–[5]. In GDM, there usually exists inconsistency or disagreement among the group decision makers because of the individual decision makers' different styles and viewpoints on the decision problem [6]–[11]. Therefore, the elimination or lessening of inconsistency to an acceptable degree before implementing the mentioned aggregation process is key in GDM [12]–[17].

The Delphi method is regarded as an effective approach to tackle inconsistency in GDM because it includes an interactive process guided by a moderator, who collects the individual opinions as group opinion and encourages the individual experts to adjustment their opinions closer to the group opinion, that aims at achieving a high group consensus level [1], [18]–[21]. However, this interactive process does not generate personalized feedback advice [22]–[24]. In a real situation, the closer the individual is to the group opinion, the higher his/her consensus level will be, and therefore the less the expert adjustment opinion will be required. Therefore, it is preferable to generate personalized feedback advice according to individual current consensus status [25]–[27].

Several feedback mechanisms have been proposed for consensus in GDM, which usually implement a weighted approach based on a static feedback parameter usage [14], [19]. However, in most cases, the feedback parameter is discretionarily used in the interaction process of consensus without proper justification of its selected value [28]. In addition, the inconsistent experts have no idea of their resultant new consensus status when they adopt the provided feedback advices, which in many cases can lead to higher adjustment cost than required to achieve their aimed consensus threshold value [15]. Therefore, these '*traditional*' feedback mechanisms share the common limitation of 'forcing' inconsistent experts to adopt the feedback advices, which could end in a lack of willingness with regarding the adjustment of their individual original opinions [4], [15], [29].

To improve the validity of the feedback mechanism in consensus, Wu et al. [30] proposed a visual feedback mechanism by providing graphic representations of the new consensus status in every round of the interaction. Considering that the consensus reaching process is usually associated with adjustment cost (or limited budget) [10], [13], [31]–[35], Wu et al. [15] proposed a minimum cost optimization model to support the inconsistent experts to achieve their threshold of consensus subject to a cost limitation or constraint. However, it is still difficult to generate personalized feedback advices in these methods. In practice, although it is up to the experts to adopt or not the feedback advices given to them, it is preferable to generate personalized feedback advices than not. Recently,

Mingshuo Cao and Jian Wu are with the School of Economics and Management, Shanghai Maritime University, Shanghai 201306, China (caomingshuo0606@163.com; jyajian@163.com).

Francisco Chiclana is with the Institute of Artificial Intelligence (IAI), Faculty of Computing, Engineering and Media, De Montfort University, Leicester, UK (chiclana@dmu.ac.uk) and with the Andalusian Research Institute on Data Science and Computational Intelligence (DaSCI), University of Granada, Spain.

Raquel Ureña is with the the Institute of Artificial Intelligence (IAI), Faculty of Computing, Engineering and Media, De Montfort University, Leicester, UK (raquel.urena@dmu.ac.uk).

Enrique Herrera-Viedma is with the Andalusian Research Institute on Data Science and Computational Intelligence (DaSCI), University of Granada, Spain (viedma@decsai.ugr.es) and with the Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia.

Liu et al. [7] proposed a personalized recommendation mechanism by using trust relationship between group members to induce the inconsistent experts to implement the advice received. Inspired by these idea, this article aims to propose a personalized feedback mechanism by considering the distance between the individual opinion and the group opinion with the novelty goal of getting the inconsistent experts to willingly adopt the feedback advice received based on their personalized consensus status. Regarding this, Ureña et al. proposed in [36] a social network approach in which the experts receive feedback based on the opinions of others close experts with high confidence level, while other interesting approaches to feedback provision based on experts' trust were examined in [37].

Apart from group consensus, the inconsistent experts also tend to retain their original opinions and they aim at changing them as little as possible as to not compromise their individual dependence by the group consensus, i.e. individual aim at having a '*harmony degree*' (HD) as high as possible [4], [38], [39]. The HD index is used to measure the difference between the original opinion and the revised one after implementing feedback advices [4], [15]. Obviously, group consensus is in conflict with individual HDs [40] and, therefore, a reasonable policy would be to keep a balance between group consensus and HD for inconsistent experts. To do that, this article proposes an optimization model that maximizes general harmony degree (GHD) with the group consensus threshold as constraint. The proposed approach aims at generating personalized feedback advices for individual experts according to their original consensus status by allowing them to select the appropriate feedback parameter values that will adjust their opinions to precisely reach the required consensus boundary. By doing this, the personalized feedback mechanism can reach higher GHD than the traditional feedback mechanism. Moreover, the unpersonalized feedback mechanism is obtained from the proposed personalized feedback mechanism when feedback parameters are set equal for all inconsistent experts, and it is far to say that the proposed personalized feedback mechanism is more general and applicable the unpersonalized feedback mechanism.

Section II introduces some definitions regarding the trust decision making space used to represent formally experts trust/distrust opinions. A basic framework of group decision making problem is presented in section III, which includes consensus measures at three levels, visual consensus identification, feedback mechanism and selection process. In section IV, the proposed personalized feedback mechanism based on maximum HD is introduced, which is illustrated with a numerical example in section V, while an analysis of three feedback mechanisms is discussed in section V-B. Finally, the conclusions and future work are presented in section VI.

II. Trust decision making space

It is well known that in most of the decision making processes, the experts' opinions are vague and uncertain due to the limited of knowledge, the cost concern and the unpredictability of decision events [41]–[47]. While trust function and trust decision space is regard as an useful and basic tool to deal with uncertainty in GDM [7], [37], [48]. Considering that multiple decision makers might have more uncertainty opinions about alternatives, as previously said, this article introduces the trust scores space to express trust degree and distrust degree about alternatives by crisp as trust function allow to. To do that, the definition of trust functions are introduced by [49]:

Definition 1 (Trust Function (TF))**.** *"An array $\gamma = (\tilde{t}, \tilde{d})$ with the first unit $\tilde{t}$ representing a trust degree and the second unit $\tilde{d}$ meaning a distrust degree such that $0 \leq (\tilde{t}, \tilde{d}) \leq 1$ will be called trust function."*

The set of trust functions will be represented by

$$\Omega = \left\{ \gamma = (\tilde{t}, \tilde{d}) | \tilde{t}, \tilde{d} \in [0, 1] \right\} \tag{1}$$

From the definition of trust functions, trust decision making space ($TDMS$) can be defined to contain the possible different types of decision making information:

Definition 2 (Trust Decision Making Space ($TDMS$))**.** *"The trust decision making space consists by the following three components: the set of trust functions (Ω), a trust conflict space (TCS), and a trust hesitancy space (THS). It is generally represented as*

$$TDMS = (\Omega, TCS, THS) \tag{2}$$

with

$$TCS = \left\{ \gamma \in \Omega | \tilde{t} + \tilde{d} \geq 1 \right\} \tag{3}$$

and

$$THS = \left\{ \gamma \in \Omega | \tilde{t} + \tilde{d} \leq 1 \right\} \tag{4}$$

TCS includes the following type of decision information: trust degree, distrust degree and conflict degree; however, THS includes a different type of decision information: trust degree, distrust degree and hesitancy degree. Obviously, TDMS involves TCS and THS at the same time, and hence, the possible alternatives in GDM problem can be assessed by the four tuples of decision information: trust, distrust, hesitancy and conflict."

A ranking method is needed to determine the most optimal alternative in the trust decision making space. Next, the concept of trust score and knowledge degree are defined to rank the trust functions, which is proposed by Wu et al. [30].

Definition 3 (Trust Score (TS))**.** *"The trust score is a mapping on the set of TFs, Ω, that associates a value in $[0, 1]$ to each trust function value γ as follows:*

$$TS : \Omega \rightarrow [0, 1] \tag{5}$$

$$TS(\gamma) = \frac{\tilde{t} - \tilde{d} + 1}{2} \tag{6}$$

The larger the score $TS(\gamma)$ is, the greater the trust function value γ is."

The Knowledge Degree (KD) [3] supplement the TS in ranking trust functions.

Definition 4 (Knowledge Degree (KD))**.** *"The Knowledge*

Degree (KD) is a mapping on the set of TFs, Ω, that associates a value in $[0,1]$ to each trust function value γ as follows:

$$KD:\Omega\rightarrow[0,1] \tag{7}$$

$$KD(\gamma)=(1-\tilde{t}-\tilde{d})^2 \tag{8}$$

When $KD(\gamma)=0$, a perfect knowledge state is defined, i.e. a perfect trust or complete information state happens where there is no adverse selection and moral hazard. By contrast, an ignorance state happens when $KD(\gamma)=1$. Otherwise, there exists information uncertainty or information vague.

Indeed, by combining TS and KD a trust order space is possible to be defined as a model that can compare and preserve original information of trust functions in [3]:

Definition 5 (Order relation of TFs)**.** *"Let $\gamma_1=(\tilde{t}_1,\tilde{d}_1)$ and $\gamma_2=(\tilde{t}_2,\tilde{d}_2)$ be two trust values, $TS(\gamma_1)=\frac{\tilde{t}_1-\tilde{d}_1+1}{2}$ and $TS(\gamma_2)=\frac{\tilde{t}_2-\tilde{d}_2+1}{2}$ be the scores of γ_1 and γ_2 , respectively, and let $KD(\gamma_1)=(1-\tilde{t}_1-\tilde{d}_1)^2$ and $KD(\gamma_2)=(1-\tilde{t}_2-\tilde{d}_2)^2$ be the knowledge degree of γ_1 and γ_2, respectively, then:*

(1) If $TS(\gamma_1)<TS(\gamma_2)$, then γ_1 is smaller than γ_2, denote $\gamma_1<\gamma_2$.

(2) If $TS(\gamma_1)=TS(\gamma_2)$, then

(2.1) If $KD(\gamma_1)<KD(\gamma_2)$, then γ_1 is greater than γ_2, denoted by $\gamma_1>\gamma_2$.

(2.2) If $KD(\gamma_1)=KD(\gamma_2)$, then γ_1 and γ_2 represent the same information, denoted by $\gamma_1=\gamma_2$."

III. A BASIC FRAMEWORK TO SOLVE THE PROBLEM OF GDM

Group decision making can be divided into two parts: consensus reaching process (CRP) and selection process [18], [50]–[54]. Reaching an acceptable level of consensus in a group is key to obtain a stable and implementable decision [21], [55], [56]. The CRP consists of four steps: (1) Opinions representation; (2) Consensus Measure; (3) Inconsistency identification; and (4) Feedback Mechanism [4], [18], [57]. In this section, a new framework of personalized feedback mechanism based on maximum harmony degree in GDM is proposed, which is illustrated in Fig. 1. Feedback advices are based on experts' individual consensus states.

A. Determining the consensus degree at three levels

Consensus degrees, which are based on distance functions, measure the actual agreement level in group decision making process [58], [59]. In general, consensus degrees can be divide into two categories [18]: (i) based on distances to the aggregated group/collective preference[60]; and (ii) based on pairwise distances between decision makers' preferences. This article will define the consensus degree based on trust functions at three levels: (1) decision matrix; (2) alternatives; and (3) element values [3], [4], [61]. This measure structure will enable us to find out the consensus state of the process at these different levels.

Starting with a group of m experts, $\{e_k;k=1,\ldots,m\}$, providing decision matrices $\left\{\tilde{R}^k=\left(\tilde{r}^k_{ij}\right)_{p\times q},k=1,\ldots,m\right\}$, where $\tilde{r}^k_{ij}=(t^k_{ij},d^k_{ij})$ is the opinion/preference of expert e_k on the alternative A_i $(i=1,\ldots,p)$ with respect to the criterion c_j $(j=1,\ldots,q)$ [element level (A_i,c_j)], the following consensus degrees can be defined.

Level 1. *Element of alternative level.* The consensus level between experts e_k and e_s at the element of alternative level (A_i,c_j) is:

$$CE^{ks}_{ij}=1-d(\tilde{r}^k_{ij},\tilde{r}^s_{ij})=1-\frac{|t^k_{ij}-t^s_{ij}|+|d^k_{ij}-d^s_{ij}|}{2} \tag{9}$$

The consensus degree of expert e_k with respect to the group of experts at the element of alternative level (A_i,c_j) is:

$$ACE^k_{ij}=\frac{1}{m-1}\sum_{s=1,s\neq k}^{m}CE^{ks}_{ij} \tag{10}$$

Level 2. *Alternative level.* The consensus level between experts e_k and e_s on alternatives A_i is:

$$CA^{ks}_{i}=1-\frac{1}{q}\sum_{j=1}^{q}\frac{|t^k_{ij}-t^s_{ij}|+|d^k_{ij}-d^s_{ij}|}{2} \tag{11}$$

The consensus degree of expert e_k to the group of experts on the alternative A_i is:

$$ACA^k_{i}=\frac{1}{m-1}\sum_{s=1,s\neq k}^{m}CA^{ks}_{i} \tag{12}$$

Level 3. *Decision matrix level.* The consensus level between experts e_k and e_s is :

$$CD^{ks}=1-\frac{1}{p\cdot q}\sum_{i=1}^{p}\sum_{j=1}^{q}\frac{|t^k_{ij}-t^s_{ij}|+|d^k_{ij}-d^s_{ij}|}{2} \tag{13}$$

Therefore, the consensus degree of the expert e_k to the group of experts is:

$$ACD^k=\frac{1}{m-1}\sum_{s=1,s\neq k}^{m}CD^{ks} \tag{14}$$

When experts' ACD values are bigger than a predefined threshold value β, the selection process is applied to derive the solution of consensus. Otherwise, the feedback mechanism is activated to generate recommendation advice for decision makers with ACD values lower than β with the aim of increasing their consensus degree.

Example 1. *Assume group of experts $\{e_1,e_2,e_3,e_4,e_5\}$ from five shipping companies are to select the most appropriate ship-breaking plant to purchase from four alternatives $\{A_1,A_2,A_3,A_4\}$ according to four criteria $\{c_1,c_2,c_3,c_4\}$: debt ratio, size, fixed cost and historical operating conditions of ship-breaking plant. The five matrices with trust function given by the five experts are:*

$$\tilde{R}^1=\begin{pmatrix} & c_1 & c_2 & c_3 & c_4\\ A_1 & (0.4,0.6) & (0.2,0.6) & (0.2,0.5) & (0.6,0.8)\\ A_2 & (0.5,0.3) & (0.4,0.5) & (0.3,0.7) & (0.4,0.6)\\ A_3 & (0.6,0.8) & (0.5,0.7) & (0.3,0.5) & (0.1,0.3)\\ A_4 & (0.7,0.1) & (0.9,0.2) & (0.7,0.1) & (0.6,0.4)\end{pmatrix}$$

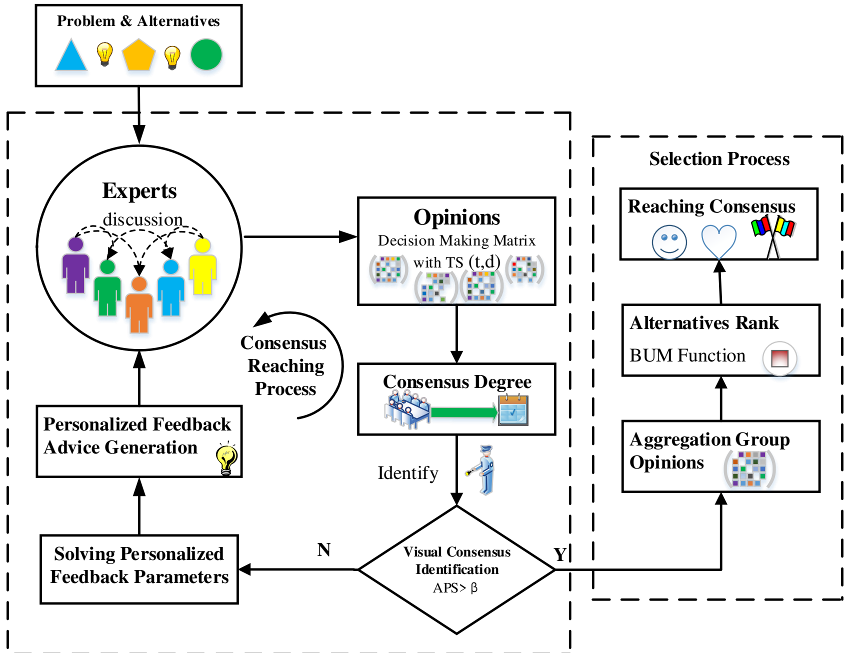


Fig. 1: A framework of personalized feedback mechanism based on maximum harmony degree in GDM

$$\tilde{R}^2=\begin{array}{c|cccc} & c_1 & c_2 & c_3 & c_4 \\ \hline A_1 & (0.4,0.5) & (0.5,0.7) & (0.3,0.7) & (0.4,0.6) \\ A_2 & (0.3,0.6) & (0.2,0.5) & (0.5,0.6) & (0.4,0.7) \\ A_3 & (0.3,0.5) & (0.2,0.4) & (0.4,0.6) & (0.2,0.5) \\ A_4 & (0.7,0.3) & (0.8,0.2) & (0.8,0.1) & (0.3,0.7) \end{array}$$

$$\tilde{R}^3=\begin{array}{c|cccc} & c_1 & c_2 & c_3 & c_4 \\ \hline A_1 & (0.2,0.5) & (0.3,0.8) & (0.5,0.9) & (0.4,0.5) \\ A_2 & (0.4,0.6) & (0.3,0.7) & (0.3,0.6) & (0.5,0.7) \\ A_3 & (0.3,0.6) & (0.4,0.8) & (0.4,0.7) & (0.2,0.4) \\ A_4 & (0.6,0.3) & (0.8,0.1) & (0.5,0.4) & (0.5,0.2) \end{array}$$

$$\tilde{R}^4=\begin{array}{c|cccc} & c_1 & c_2 & c_3 & c_4 \\ \hline A_1 & (0.3,0.6) & (0.2,0.5) & (0.6,0.9) & (0.3,0.5) \\ A_2 & (0.5,0.7) & (0.4,0.8) & (0.2,0.6) & (0.4,0.7) \\ A_3 & (0.4,0.6) & (0.4,0.8) & (0.5,0.8) & (0.3,0.5) \\ A_4 & (0.1,0.9) & (0.2,0.3) & (0.7,0.5) & (0.5,0.5) \end{array}$$

$$\tilde{R}^5=\begin{array}{c|cccc} & c_1 & c_2 & c_3 & c_4 \\ \hline A_1 & (0.8,0.1) & (0.5,0.3) & (0.1,0.7) & (0.4,0.3) \\ A_2 & (0.5,0.5) & (0.8,0.2) & (0.5,0.6) & (0.3,0.7) \\ A_3 & (0.5,0.4) & (0.3,0.7) & (0.2,0.2) & (0.3,0.6) \\ A_4 & (0.7,0.1) & (0.8,0.1) & (0.7,0.2) & (0.6,0.1) \end{array}$$

The consensus degrees of experts at the element of alternative level are:

$$ACE^1=\begin{pmatrix} 0.825 & 0.825 & 0.738 & 0.725 \\ 0.813 & 0.813 & 0.888 & 0.925 \\ 0.750 & 0.850 & 0.825 & 0.825 \\ 0.763 & 0.838 & 0.863 & 0.825 \end{pmatrix}$$

$$ACE^2=\begin{pmatrix} 0.838 & 0.800 & 0.825 & 0.875 \\ 0.850 & 0.763 & 0.900 & 0.963 \\ 0.850 & 0.725 & 0.850 & 0.913 \\ 0.788 & 0.880 & 0.825 & 0.675 \end{pmatrix}$$

$$ACE^3=\begin{pmatrix} 0.788 & 0.788 & 0.775 & 0.888 \\ 0.888 & 0.775 & 0.925 & 0.925 \\ 0.863 & 0.875 & 0.838 & 0.900 \\ 0.775 & 0.863 & 0.775 & 0.813 \end{pmatrix}$$

$$ACE^4=\begin{pmatrix} 0.813 & 0.813 & 0.738 & 0.850 \\ 0.863 & 0.750 & 0.888 & 0.963 \\ 0.875 & 0.875 & 0.763 & 0.900 \\ 0.363 & 0.613 & 0.813 & 0.825 \end{pmatrix}$$

$$ACE^5=\begin{pmatrix} 0.538 & 0.725 & 0.775 & 0.813 \\ 0.888 & 0.550 & 0.900 & 0.925 \\ 0.813 & 0.875 & 0.675 & 0.863 \\ 0.763 & 0.863 & 0.875 & 0.763 \end{pmatrix}$$

The consensus degrees of experts at the alternatives level are:

$$ACA^1 = (0.778, 0.859, 0.813, 0.822)$$
$$ACA^2 = (0.834, 0.869, 0.834, 0.791)$$
$$ACA^3 = (0.809, 0.878, 0.869, 0.806)$$
$$ACA^4 = (0.803, 0.866, 0.853, 0.653)$$
$$ACA^5 = (0.713, 0.816, 0.806, 0.816)$$

The consensus degree of experts at the decision matrix level are: $ACD^1 = 0.818$*;* $ACD^2 = 0.832$*;* $ACD^3 = 0.841$*;* $ACD^4 = 0.794$*;* $ACD^5 = 0.788$*.*

B. A Consensus Identification Process

In GDM problems, it is difficult to reach a complete consensus. Therefore, a threshold value $\beta < 1$ is easier to be accepted in practice [62]. Usually, consensus also implies that at least half of the experts reach agreement, and therefore it is expected that the threshold value to consider satisfies: $\beta \in (0.5, 1)$. The discordant experts, alternatives and elements of alternatives where consensus degree is below the threshold value are identified as per the subsequent application of the following rules:

Step 1. The set of experts with consensus degree at decision matrix level below the threshold β are identified

$$ECH = \{h | ACD^h < \beta\}.$$

This is the set of inconsistent experts.

Step 2. For the inconsistent experts identified above, their alternatives with consensus degree lower than the threshold β are identified

$$ACH = \{(h, i) | h \in ECH \wedge ACA_i^h < \beta\}.$$

Step 3. Finally, the elements of alternatives where consensus degree is below the threshold β are

$$APS = \left\{(h, i, j) | (h, i) \in ACH \wedge ACE_h^{ij} < \beta\right\}.$$

Given $h \in ECH$, then $APS^h = \{(i, j) | (h, i, j) \in APS\}$ is the set of inconsistent element in the decision matrix $\tilde{R}^h$ of inconsistent expert e_h.

Example 2 (Continuation from Example 1)**.** *Assuming a threshold value of consensus $\beta = 0.8$, then both experts e_4 and e_5 are below such consensus threshold. Applying the above identification rules, we obtained:*

$$APS = \{(4, 4, 1), (4, 4, 2), (5, 1, 1), (5, 1, 2), (5, 1, 3)\}.$$

Thus, expert e_4 opinions on alternative A_4 under criteria c_1 and c_2 and expert e_5 opinions alternative A_1 under criteria c_1, c_2 and c_3 are discordant with the corresponding opinions of the group.

C. Feedback Mechanism

A feedback mechanism is one of the key steps to reach consensus. As aforementioned, when there is inconsistency from multiple experts in GDM, it is necessary that the set of experts reach consensus before aggregating the individual opinions into a collective one [7], [10], [36]. The use of a feedback mechanism is very effective to generate advice for discordant experts to upgrade the group consensus degree. Tradition feedback mechanisms are used to generate feedback advice in many GDM problems by commonly adopting an ad hoc and fixed modification approach of the inconsistent opinions for discordant experts in every iterative round until all the experts' consensus degree are above the threshold of consensus [3], [7], [15]. In these traditional feedback mechanisms, generally, there is a moderator to guide decision makers to reach consensus and the feedback parameter cannot change once it is determined. Thus, given $(h, i, j) \in ASP$, the following rule is applied:

$$\tilde{r}_{ij}^{h'} = (1 - \delta)\tilde{r}_{ij}^h + \delta\bar{r}_{ij}^h, \quad (h, i, j) \in APS \tag{15}$$

Thus, $\tilde{R}^{h'} = \left(\tilde{r}_{ij}^{h'}\right)$ represents the new opinions of the inconsistent expert h, while $\tilde{R}^h = \left(\tilde{r}_{ij}^h\right)$ represents his/her original opinions; $\bar{r}_{ij}^h = \frac{1}{m-1}\sum_{k=1,k\neq h}^{m} \tilde{r}_{ij}^k$ is the average of the original opinions on the element of alternative (i, j) of the rest of experts; and $\delta \in [0, 1]$ is the unpersonalized feedback mechanism parameter to control the changing degree of the original opinions of inconsistent experts. Notice that when $\delta = 0$ the feedback advices coincide with the original opinions, i.e. there is no feedback process at all. Thus, in practice the restrictions of positiveness is imposed to the feedback parameter. This feedback mechanism improves the consensus degree after adopting the recommendation advices as the following Theorem 1 proves.

Theorem 1. *For all $(h, i, j) \in APS$, it is*

$$ACE_{ij}^h < ACE_{ij}^{h'} \tag{16}$$

where $ACE_{ij}^{h'}$ is the consensus degree for e_h at the element level after modifying his/her opinions according to the traditional feedback mechanism advices generated with Eq. (15).

Proof. Without loss of generality, it can be assumed that $h \in \{1, 2, \ldots, n\}$. We have:

$$ACE_{ij}^k = \frac{1}{m-1}\sum_{s=1,s\neq k}^{m} CE_{ij}^{ks}$$

where

$$CE_{ij}^{ks} = 1 - d(\tilde{r}_{ij}^k, \tilde{r}_{ij}^s) = 1 - \frac{|t_{ij}^k - t_{ij}^s| + |d_{ij}^k - d_{ij}^s|}{2}.$$

For simplicity, we denote $d(\tilde{r}_{ij}^k, \tilde{r}_{ij}^s) = \left|\tilde{r}_{ij}^k - \tilde{r}_{ij}^s\right|$. Thus, it is

$$ACE_{ij}^{h'} = 1 - \frac{1}{m-1}\left(\sum_{s=1,s\neq h}^{n}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'}\right| + \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l}\right|\right).$$

From Eq. (15):

$$\forall h \in \{1, 2, \ldots, n\} : \tilde{r}_{ij}^{h'} = (1 - \delta)\cdot\tilde{r}_{ij}^h + \delta\cdot\bar{r}_{ij}^h.$$

Thus,

$$\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'} = (1 - \delta)\cdot\left(\tilde{r}_{ij}^h - \tilde{r}_{ij}^s\right) + \delta\cdot\left(\bar{r}_{ij}^h - \bar{r}_{ij}^s\right).$$

Because

$$\begin{aligned}\bar{r}_{ij}^h &= \frac{1}{m-1}\cdot\sum_{o=1,o\neq h}^{m}\tilde{r}_{ij}^o \\ &= \frac{1}{m-1}\cdot\left(\sum_{o=1}^{m}\tilde{r}_{ij}^o - \tilde{r}_{ij}^h\right) \\ &= \frac{1}{m-1}\cdot\sum_{o=1}^{m}\tilde{r}_{ij}^o - \frac{\tilde{r}_{ij}^h}{m-1},\end{aligned}$$

It is

$$\bar{r}_{ij}^h - \bar{r}_{ij}^s = -\frac{1}{m-1}\cdot\left(\tilde{r}_{ij}^h - \tilde{r}_{ij}^s\right),$$

and

$$\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'} = \left(1 - \frac{m}{m-1}\cdot\delta\right)\cdot\left(\tilde{r}_{ij}^h - \tilde{r}_{ij}^s\right).$$

Thus,

$$\sum_{s=1,s\neq h}^{n}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'}\right| = \left(1 - \frac{m}{m-1}\cdot\delta\right)\cdot\sum_{s=1,s\neq h}^{n}\left|\tilde{r}_{ij}^h - \tilde{r}_{ij}^s\right|. \tag{17}$$

Also:

$$\tilde{r}_{ij}^{h'} = (1-\delta)\cdot\tilde{r}_{ij}^{h} + \delta\cdot\left(\frac{1}{m-1}\cdot\sum_{o=1}^{m}\tilde{r}_{ij}^{o} - \frac{\tilde{r}_{ij}^{h}}{m-1}\right).$$

Therefore,

$$\begin{aligned}\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l} &= \left(1-\frac{m\cdot\delta}{m-1}\right)\cdot\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l} + \frac{\delta}{m-1}\cdot\sum_{o=1}^{m}\tilde{r}_{ij}^{o}\\ &= \left(\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right) + \frac{\delta}{m-1}\cdot\left(\sum_{o=1}^{m}\tilde{r}_{ij}^{o} - \sum_{o=1}^{m}\tilde{r}_{ij}^{h}\right).\end{aligned}$$

Thus, applying the triangle inequality followed by the followed by the Cauchy-Schwarz inequality

$$\begin{aligned}\sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l}\right| &= \sum_{l=n+1}^{m}\left|\left(\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right) + \frac{\delta}{m-1}\cdot\sum_{o=1}^{m}\left(\tilde{r}_{ij}^{o} - \tilde{r}_{ij}^{h}\right)\right|\\ &\le \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right| + \frac{\delta}{m-1}\cdot\sum_{l=n+1}^{m}\left|\sum_{o=1}^{m}\left(\tilde{r}_{ij}^{o} - \tilde{r}_{ij}^{h}\right)\right|\\ &= \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right| + \frac{(m-n)\cdot\delta}{m-1}\cdot\left|\sum_{o=1}^{m}\left(\tilde{r}_{ij}^{o} - \tilde{r}_{ij}^{h}\right)\right|\\ &\le \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right| + \frac{(m-n)\cdot\delta}{m-1}\cdot\sum_{o=1}^{m}\left|\tilde{r}_{ij}^{o} - \tilde{r}_{ij}^{h}\right|\end{aligned}$$

Noticing that

$$\sum_{o=1}^{m}\left|\tilde{r}_{ij}^{o} - \tilde{r}_{ij}^{h}\right| = \sum_{l=1}^{m}\left|\tilde{r}_{ij}^{l} - \tilde{r}_{ij}^{h}\right| = \sum_{l=1,l\neq h}^{m}\left|\tilde{r}_{ij}^{l} - \tilde{r}_{ij}^{h}\right|,$$

it is

$$\begin{aligned}\sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l}\right| &\le \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right| \\ &+ \frac{(m-n)\cdot\delta}{m-1}\cdot\left(\sum_{l=1,l\neq h}^{n}\left|\tilde{r}_{ij}^{l} - \tilde{r}_{ij}^{h}\right| + \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{l} - \tilde{r}_{ij}^{h}\right|\right)\end{aligned} \tag{18}$$

Putting (17) and (18) together, and using the notation

$$S1 = \sum_{l=1,l\neq h}^{n}\left|\tilde{r}_{ij}^{l} - \tilde{r}_{ij}^{h}\right|$$

and

$$S2 = \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h} - \tilde{r}_{ij}^{l}\right|,$$

it is:

$$\begin{aligned}\sum_{s=1,s\neq h}^{n}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'}\right| + \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l}\right| &\le \left(1-\frac{n\cdot\delta}{m-1}\right)\cdot S1\\ &+ \left(1-\frac{(m-n)\cdot\delta}{m-1}\right)\cdot S2.\end{aligned}$$

Because $\delta > 0$, we have $\dfrac{n\cdot\delta}{m-1} > 0$ and $\dfrac{(m-n)\cdot\delta}{m-1} \ge 0$, and consequently

$$\sum_{s=1,s\neq h}^{n}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{s'}\right| + \sum_{l=n+1}^{m}\left|\tilde{r}_{ij}^{h'} - \tilde{r}_{ij}^{l}\right| < S1 + S2.$$

Thus,

$$ACE_{ij}^{h'} > 1 - \frac{1}{m-1}\cdot(S1+S2) = ACE_{ij}^{h}$$

□

From Theorem 1, the consensus degree at the element level improves following the implementation of the feedback mechanism advices with constant feedback parameter for all inconsistent experts on the element of alternative level. This result can be expanded to the consensus degree at the decision matrix level.

Proposition 1. *Let $\widetilde{R}^{1},\dots\widetilde{R}^{h},\dots\widetilde{R}^{m}$ be the original decision matrices of experts. Assume that expert e_h is inconsistent, i.e ($ACD^{h} < \beta$). Then*

$$ACD^{h} < ACD^{h'} \tag{19}$$

where $ACD^{h'}$ the consensus degree for e_h computing using the expert new decision matrix $\widetilde{R}^{h'}$ with Eq. (15).

Proof. Omitted □

The above two results suggest that the feedback mechanism with constant feedback parameter for all inconsistent experts is effective in increasing the consensus for the discordant experts, and in turn to reach consensus by the whole group. The effectiveness of this traditional feedback mechanism depends on the feedback parameter. However, it is still impractical that this approach still forces the inconsistent experts to adopt the feedback advice; it makes use of a constant and equal feedback parameter for all inconsistent experts; and it does not specify how the feedback parameter is chosen. The traditional feedback mechanism can obviously be modified by allowing to generate advices, to discordant experts and for the opinions identified in APS, using an individualized parameter value. This article proposes two feedback mechanism based on the maximum harmony degree model in section IV to overcome the above raised issues.

D. Selection process

When consensus among decision makers is reached, a selection process is usually applied by fusing the preferences of individual decision makers from which the final ordering of the considered alternatives is derived [7], [39], [40], [63]. The consensus degree could be used as a strong source to assign weight or importance values to decision makers in the aggregation process, which is possible with an induced ordered weighted average (OWA) operator in addition to the implementation as well of the concept of majority when guided by a linguistic quantifier [8], [64]–[66]. Given a basic unit monotonic (BUM) increasing function $P:[0,1]\to[0,1]$ with boundary conditions $P(0)=0$ and $P(1)=1$, the weights of the OWA operator can be derived as follows:

$$\omega_M = P\left(\frac{F(M)}{F(m)}\right) - P\left(\frac{F(M-1)}{F(m)}\right), M = 1,\dots,m \tag{20}$$

where $F(M) = \sum_{k=1}^{M} ACD^{\sigma(k)}$, and $\sigma(k)$ is the permutation used to induce the ordering of the values to aggregate by ordering from highest to lowest the consensus degrees of the decision makers in the group. The above OWA weight determination method guarantees associates a zero weight to those experts with zero consensus degree, and that higher weight degree for higher consensus degrees can be guaranteed if a concave BUM function is implemented [67].

IV. Feedback mechanism base on maximum harmony degree

In order to avoid the discordant experts be forced to modify their opinions, we propose the concept and measure method of harmony degree in this section, which helps discordant experts measure the level of maintaining their original opinions. Thus, this article builds an optimization model based on the harmony degree, where the maximum harmony degree is pursued, and therefore aims at lessen the possibility that discordant experts are forced to modify their opinions. The feedback mechanism is applied to two scenarios: one scenario of personalized feedback, and another scenario of unpersonalized feedback. It is found, as it will be later reported, that the personalized case produces a higher harmony degree than the unpersonalized scenario.

A. Harmony Degree of Personalized feedback

'*Harmony Degree*', as measured (measured by the HD index), is a tool to determine the level of keeping of the original opinions by the inconsistent experts in the process of consensus reaching, and in a way can be a measure of the maintaining of their individual independence before and after implementing feedback advices [4]. This article provides the measure method of the HD index based on the Hamming similarity between two elements expressed by trust functions. Inspired by the individual feedback and group feedback [28], the HD is divided into two categories: individual harmony degree (IHD) and general harmony degree of group (GHD). In order to realize the global optimization, this article uses GHD in the feedback mechanism.

Assuming that there are '*m*' experts in group decision making and '*n*' experts are inconsistent ($\#ECH = n$), then, the original decision making matrices are divided into two groups: the set of inconsistent decision matrix $\left\{\tilde{R}^h = \left(\tilde{r}_{ij}^h\right)_{p\times q}, h \in ECH\right\}$ and the set of consistent decision matrix $\left\{\tilde{R}^H = \left(\tilde{r}_{ij}^H\right)_{p\times q}, H \notin ECH\right\}$. In the following, $h \in ECH$ will denote the position of decision matrices in ECH in the permutation of their consensus degree from highest to lowest: $ACD^1 \geq \ldots \geq ACD^h \geq \ldots \geq ACD^n$.

Definition 6 (Individual Harmony Degree (IHD))**.** *"Suppose $h \in ECH$, i.e. e_h is inconsistent. Then, the individual harmony degree of the expert e_h is*

$$IHD_h = 1 - \frac{1}{\#APS^h} d\left(\tilde{R}^{h'}, \tilde{R}^h\right) = 1 - \frac{1}{\#APS^h} \sum_{(i,j)\in APS^h} \frac{\left|t_{ij}^{h'} - t_{ij}^h\right| + \left|d_{ij}^{h'} - d_{ij}^h\right|}{2} \tag{21}$$

where $\#APS^h$ is the cardinality of the set of inconsistent element in the decision matrix $\tilde{R}^h$ of e_h."

Clearly, it is $0 \leqslant IHD_h \leqslant 1$, and when the expert e_h is a consistent expert if and only if $IHD_h = 1$

Definition 7 (General Harmony Degree (GHD))**.** *"The harmony degree for all the discordant experts is*

$$GHD = \frac{1}{n} \sum_{h\in ECH} IHD_h = 1 - \frac{1}{n} \sum_{(h,i,j)\in APS} \frac{\left|t_{ij}^{h'} - t_{ij}^h\right| + \left|d_{ij}^{h'} - d_{ij}^h\right|}{2\left(\#APS^h\right)} \tag{22}$$

is referred to as the general harmony degree."

The bigger the GHD index is, the less the original opinion are changed for the IHD_h indices. Obviously, no expert need modify their opinions when $GHD = 1$. Indeed, $GHD = 1$ if and only if all experts are consistent.

From (15), personalized feedback regarding inconsistent element of alternatives in APS become:

Definition 8 (Personalized Feedback)**.** *"For $(h, i, j) \in APS$, the original value (t_{ij}^h,d_{ij}^h) is adviced to be modified according to the following moderator's feedback:*

$$\begin{cases} t_{ij}^{h'} = t_{ij}^h \cdot (1-\delta_h) + \bar{t}_{ij}^h \cdot \delta_h \\ d_{ij}^{h'} = d_{ij}^h \cdot (1-\delta_h) + \bar{d}_{ij}^h \cdot \delta_h \end{cases} \tag{23}$$

where $\bar{t}_{ij}^h = \frac{1}{m-1} \sum\limits_{k=1,k\neq h}^{m} t_{ij}^k$ and $\bar{d}_{ij}^h = \frac{1}{m-1} \sum\limits_{k=1,k\neq h}^{m} d_{ij}^k$; and δ_h is the personalized feedback parameter."

Substituting expression (23) into (22) results in the following personalized GHD expression:

Proposition 2. *The expression of the general harmony degree of personalized feedback GHD^p is:*

$$GHD^p = 1 - \frac{1}{n} \sum_{h=1}^{n} \delta_h \psi\left(\bar{R}^h, \tilde{R}^h\right) \tag{24}$$

where

$$\psi\left(\bar{R}^h, \tilde{R}^h\right) = \sum_{(h,i,j)\in APS} \frac{\left(\left|\bar{t}_{ij}^h - t_{ij}^h\right| + \left|\bar{d}_{ij}^h - d_{ij}^h\right|\right)}{2\left(\#APS^h\right)} \tag{25}$$

Proof. Omitted. □

The following property provides lower and upper bounds for the GHD^p index.

Proposition 3 (Boundedness)**.**

$$1 - \frac{1}{n}\varphi_{\max}\left(\tilde{R}^h, \bar{R}^h\right) \cdot \sum_{h=1}^{n} \delta_h \leq GHD^p \leq 1 - \frac{1}{n}\varphi_{\min}\left(\tilde{R}^h, \bar{R}^h\right) \cdot \sum_{h=1}^{n} \delta_h \tag{26}$$

where $\varphi_{\min}\left(\tilde{R}^h, \bar{R}^h\right) = \min\limits_{h\in ECH}\left\{\varphi\left(\tilde{R}^h, \bar{R}^h\right)\right\}$ *and* $\varphi_{\max}\left(\tilde{R}^h, \bar{R}^h\right) = \max\limits_{h\in ECH}\left\{\varphi\left(\tilde{R}^h, \bar{R}^h\right)\right\}$.

Proof. Omitted □

Considering all the variables are fixed except δ_h, then it can be included that GHD^p index is a multivariate function on $(\delta_1, \ldots, \delta_n)$. It is obvious that (GHD^p) is a decreasing function in each one of the personalized feedback parameters when the rest of personalized feedback parameters are fixed.

Proposition 4 (Monotonic decreasing)**.** *The general harmony degree of personalized feedback (GHD^p) is monotonic decreasing with respect to each of the personalized feedback parameter δ_h when the rest are fixed.*

Proof. Omitted □

B. Harmony Degree of Unpersonalized Feedback

When all the feedback parameters δ_h are the same, then the unpersonalized feedback is derived. In this case, the unpersonalized feedback regarding inconsistent element of alternatives in APS become:

Definition 9 (Unpersonalized Feedback)**.** *"The original opinion of discordant expert e_h, (t_{ij}^h,d_{ij}^h), is to be modified according to the following moderator's feedback advice:*

$$\begin{cases} t_{ij}^{h'} = t_{ij}^h \cdot (1-\delta) + \bar{t}_{ij}^h \cdot \delta \\ d_{ij}^{h'} = d_{ij}^h \cdot (1-\delta) + \bar{d}_{ij}^h \cdot \delta \end{cases} \tag{27}$$

where $\tilde{r}_{ij}^h = \left(t_{ij}^h, d_{ij}^h\right)$ is the original opinion and $\bar{r}_{ij}^h = \left(\bar{t}_{ij}^h, \bar{d}_{ij}^h\right)$ is the recommendation advice; and δ is the unpersonalized feedback parameter."

Substituting expression (27) into (22) results in the following unpersonalized GHD expression:

Proposition 5. *The general harmony degree of unpersonalized feedback GHD^{np} is:*

$$GHD^{np} = 1 - \frac{1}{n}\sum_{h=1}^{n} \delta \cdot \psi\left(\bar{R}^h, \tilde{R}^h\right) \tag{28}$$

where

$$\psi\left(\bar{R}^h, \tilde{R}^h\right) = \sum_{(h,i,j)\in APS} \frac{\left(\left|\bar{t}_{ij}^h - t_{ij}^h\right| + \left|\bar{d}_{ij}^h - d_{ij}^h\right|\right)}{2\left(\#APS^h\right)}.$$

Obviously, GHD^p becomes GHD^{np} when δ_h is equal to δ. Therefore, the unpersonalized feedback mechanism is a special case of personalized feedback mechanism. To compare GHD^p with GHD^{np}, we first introduce the following Lemma from [64].

Lemma 1. 'For $\forall x = (x_1, x_2, \ldots, x_n)$, and OWA weights $W = (\omega_1, \omega_2, \ldots, \omega_n)$, $W' = (\omega_1', \omega_2', \ldots, \omega_n')$, if $\frac{\omega_i}{\omega_{i+1}} \geqslant \frac{\omega_i'}{\omega_{i+1}'}$, $i = 1, 2, \ldots, n-1$, then $F_W(x) \geqslant F_{W'}(x)$. If $x_i \neq x_j$ $(i \neq j; i, j = 1, 2, \ldots n)$, and $\frac{\omega_i}{\omega_{i+1}} > \frac{\omega_i'}{\omega_{i+1}'}$, $i = 1, 2, \ldots, n-1$, then $F_W(x) > F_{W'}(x)$. '

Under the assumption of being the personalized feedback parameter δ_h and the average distance $\psi\left(\bar{R}^1, \tilde{R}^1\right)$ both ordered, the following result proves that the personalized feedback mechanism GHD value, GHD^p, is higher than the unpersonalized feedback mechanism GHD value, GHD^{np}.

Theorem 2. *Under the assumption $\psi\left(\bar{R}^1, \tilde{R}^1\right) \geq \psi\left(\bar{R}^2, \tilde{R}^2\right) \geq \ldots \geq \psi\left(\bar{R}^n, \tilde{R}^n\right)$ and $\delta_1 \geq \delta_2 \geq \ldots \geq \delta_n$ then $GHD^p \geq GHD^{np}$, with $GHD^p > GHD^{np}$ when $\psi\left(\bar{R}^1, \tilde{R}^1\right) > \psi\left(\bar{R}^2, \tilde{R}^2\right) > \ldots > \psi\left(\bar{R}^n, \tilde{R}^n\right)$ and $\delta_1 > \delta_2 > \ldots > \delta_n$.*

Proof. Without loss of generality we can assume that $\psi\left(\bar{R}^1, \tilde{R}^1\right) \geq \psi\left(\bar{R}^2, \tilde{R}^2\right) \geq \ldots \geq \psi\left(\bar{R}^n, \tilde{R}^n\right)$. Because $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_n$; defining $\omega_i = \dfrac{1-\delta_{n-i}}{n}$ and $\omega_i' = \dfrac{1-\delta}{n}$ we have, on the one hand, that the OWA operators with weighting vectors W and W' applied to $\left(\psi\left(\bar{R}^1, \tilde{R}^1\right), \psi\left(\bar{R}^2, \tilde{R}^2\right), \ldots, \psi\left(\bar{R}^n, \tilde{R}^n\right)\right)$ are:

$$F_W\left(\psi\left(\bar{R}^1, \tilde{R}^1\right), \psi\left(\bar{R}^2, \tilde{R}^2\right), \ldots, \psi\left(\bar{R}^n, \tilde{R}^n\right)\right) = GHD^p;$$

$$F_{W'}\left(\psi\left(\bar{R}^1, \tilde{R}^1\right), \psi\left(\bar{R}^2, \tilde{R}^2\right), \ldots, \psi\left(\bar{R}^n, \tilde{R}^n\right)\right) = GHD^{np}.$$

On the other hand, it is $\dfrac{\omega_i}{\omega_{i+1}} \geq 1 = \dfrac{\omega_i'}{\omega_{i+1}'}$. The application of Lemma 1 proves Theorem 2.

□

C. Unpersonalized feedback mechanism

In the traditional feedback mechanism, discordant experts are forced to modify their opinions. The main issue, in fact, resides in that feedback advices are based on information provided by the experts themselves. The consensus state for every expert is easily obtained, are able to be visually identified, and are regarded as a hard restriction in reaching group agreement. Simultaneously, individual maximum harmony degrees is a goal this article pursues. However, there is a conflict between the individual and the group goals. The individual goal is to maintain the original opinion to the greatest extent possible, while the group goal is to reach consensus or agreement. Therefore, the nature of this problem can be addressed via an optimal approach to eliminate the conflict between individual and group goals [68], [69]. Thus, a programming model is constructed, where the maximum harmony degree is regarded as its objective function and the consensus threshold is regarded as its restriction. Even though we wish to obtain the maximum harmony degree, the basic constraint must also be satisfied, I.e. the modified opinions have to be within the consensus boundaries, that is, leading to a consensus above the threshold value.

Let us denote by $(\bar{t}_{ij}, \bar{d}_{ij})$ the consensus element obtained by the group opinion and $\delta \in [0,1]$ the parameter to control the feedback process. Base on (27), the following optimal model to generate unpersonalized feedback advice to reach maximum harmony degree is built:

$$\begin{cases} Max: GHD^{np} \\ s.t. \begin{cases} ACD^h \geqslant \beta \\ ACD^k \geqslant \beta \ (k = 1, \dots, m; k \neq h) \\ 0 \leqslant \delta \leqslant 1 \ (h \in APS) \end{cases} \end{cases} \tag{29}$$

D. Personalized feedback mechanism

In most practical situations, consensus still relies on the experts implementing or not the feedback advices provided to him/her. Recently, Wu et al.[4] proposed a trust induced method to generate recommendation advice with the aim of making experts aware that they were not forced in modifying their opinions as these stemmed from their trusted relationships in the social network. This invariably means that feedback advices in reaching consensus of GDM are preferable to be personalized [7], [15], [30]. Thus, in the below a personalized feedback mechanism based on maximum harmony degree is proposed. Firstly, a personalized feedback model is constructed to obtain the personalized feedback parameters, which are subsequently used by the moderator to offers the feedback advice to discordant experts to allow for the group consensus to go above the threshold value of consensus.

1) Construction of Personalized Feedback Model: Based on (23), the following optimal model to generate personalized feedback advice to reach maximum harmony degree is built:

$$\begin{cases} Max: GHD^{p} \\ s.t. \begin{cases} ACD^h \geq \beta \ (h = 1, ..., n) \\ ACD^k \geq \beta \ (k = n+1, ..., m; k \neq h) \\ 0 \leq \delta_h \leq 1 \\ \delta_h \leq \delta_{h+1} \\ (h, i, j) \in APS \end{cases} \end{cases} \tag{30}$$

2) Consensus boundary and feedback paradigm: The following **Algorithm 1** provides a formal description of consensus reaching process with personalized feedback mechanism:

V. Comparative Analysis

Continuing from Examples 1 and 2, in this section we present a comparative analysis of the following three feedback mechanisms: traditional feedback mechanism; unpersonalized feedback mechanism; and personalized feedback mechanism.

a) Traditional Feedback Mechanism: A stochastic feedback parameter is generated in traditional feedback mechanism. This article simulates the traditional feedback mechanism with a feedback parameter of 0.5, i.e. a value that will advice discordant expert to change their opinions to a value average between their original opinion and the group one. Consequently, the traditional feedback advices would become:

- Expert e_4 advices:
  - Your opinion $(0.1, 0.9)$ about alternative A_4 under criterion c_1 should be changed to the value $(0.39, 0.55)$.
  - Your opinion $(0.2, 0.3)$ about alternative A_4 under criterion c_2 should be changed to the value $(0.51, 0.23)$.
- Expert e_5 advices:

Algorithm 1: Personalized Feedback Algorithm Based on Maximum Harmony Degree

---

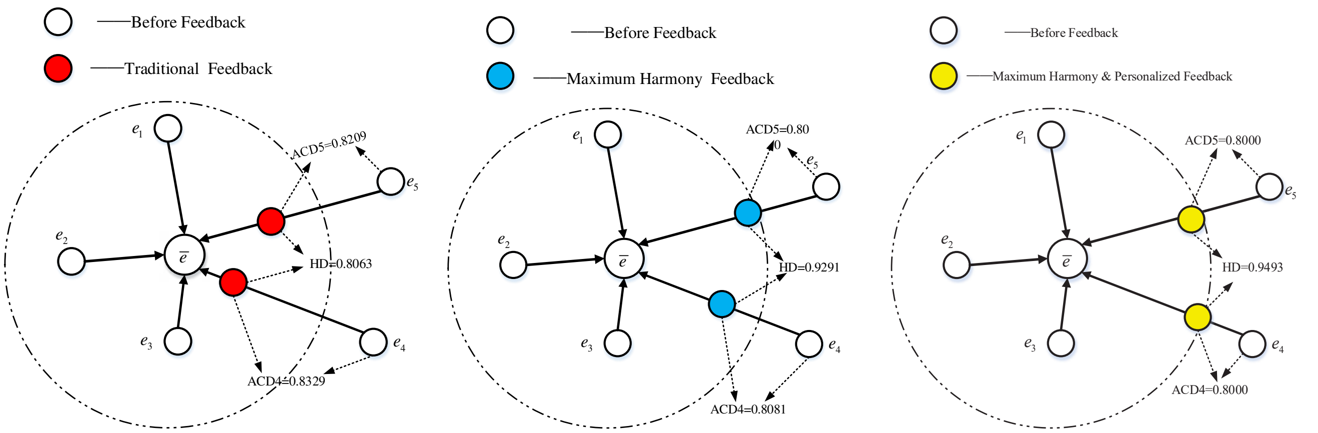
begin
  Input: $\{\tilde{R}^l = (\tilde{r}^l_{ij})_{p\times q}, l = 1, \dots, m\}$, $\tilde{r}^l_{ij} = (t^l_{ij}, d^l_{ij})$;
  criteria weighting vector $V = (v_1, v_2, \dots, v_q)$;
  consensus threshold β;
  Output: Ranking of alternatives;
  Compute consensus degrees at three levels for individual experts: ACE^l_{ij}, ACA^l_i and ACD^l;
  if $\exists ACD^l \leq \beta$ **then**
    2.a. Determine set APS;
    2.b. Solve personalized optimization model (30);
    2.c. $\forall (h, i, j) \in APS$; generate personalized feedback advice as per (23);
  else
    Switch on the Selection Procces.;
    3.a. Compute experts' weighting values by consensus degree based on the BUM function;
    3.b. Aggregate individual opinions into the collective opinions and compute trust scores for different alternatives;
    3.c. Rank the alternatives;
  end if
end

  - Your opinion $(0.8, 0.1)$ about alternative A_1 under criterion c_1 should be changed to a value closer to $(0.71, 0.18)$.
  - Your opinion $(0.5, 0.3)$ about alternative A_1 under criterion c_2 should be changed to a value closer to $(0.46, 0.36)$.
  - Your opinion $(0.1, 0.7)$ about alternative A_1 under criterion c_3 should be changed to a value closer to $(0.15, 0.71)$.

Figure 2a shows the consensus degree and harmony degree before and after the above advices have been implemented. It s clear that all experts are now above the consensus threshold value: $ACD^{1'} = 0.832, ACD^{2'} = 0.845, ACD^{3'} = 0.856, ACD^{4'} = 0.833, ACD^{5'} = 0.821$. The following traditional feedback mechanism GHD value is obtained: $GHD = 0.7958$.

b) Unpersonalized Feedback Mechanism: The unpersonalized feedback model based on maximum harmony degree has the following associated feedback parameter value [solution to model (29)]: $\delta = 0.18$. Consequently, in the unpersonalized feedback advices based on maximum harmony degree would become:

- Expert e_4 advices:
  - Your opinion $(0.1, 0.9)$ about alternative A_4 under criterion c_1 should be changed to the value $(0.21, 0.77)$.
  - Your opinion $(0.2, 0.3)$ about alternative A_4 under criterion c_2 should be changed to the value $(0.31, 0.27)$.
- Expert e_5 advices:



(a) Traditional Feedback Mechanism (b) Unpersonalized Feedback Mechanism (c) Personalized Feedback Mechanism

Fig. 2: Simulation of consensus degree and harmony degree before and after feedback for different feedback mechanism.

- Your opinion $(0.8, 0.1)$ about alternative A_1 under criterion c_1 should be changed to a value closer to $(0.56, 0.33)$.
- Your opinion $(0.5, 0.3)$ about alternative A_1 under criterion c_2 should be changed to a value closer to $(0.40, 0.48)$.
- Your opinion $(0.1, 0.7)$ about alternative A_1 under criterion c_3 should be changed to a value closer to $(0.25, 0.73)$.

Figure 2b shows the consensus degree and harmony degree before and after the above advices have been implemented. Again, all experts are now above the consensus threshold value: $ACD^{1'} = 0.823, ACD^{2'} = 0.837, ACD^{3'} = 0.846, ACD^{4'} = 0.806, ACD^{5'} = 0.800$. The following unpersonalized feedback mechanism GHD value is obtained: $GHD^{np} = 0.9253$.

c) Personalized Feedback Mechanism: In this case, model (30) becomes:

$$\begin{cases} Max : 1 - \frac{1}{2} \cdot (0.5125 \cdot \delta_4 + 0.3042 \cdot \delta_5) \\ \begin{cases} ACD^1 \geq 0.8 \\ ACD^2 \geq 0.8 \\ ACD^3 \geq 0.8 \\ ACD^4 \geq 0.8 \\ ACD^5 \geq 0.8 \\ 0 \leq \delta_4 \leq 1 \\ 0 \leq \delta_5 \leq 1 \\ \delta_4 \leq \delta_5 \end{cases} \end{cases}$$

Solving this model results in the following personalized feedback parameters: $\delta_4 = 0.05$ and $\delta_5 = 0.22$. Consequently, the personalized feedback advices based on maximum harmony degree would become:

- Expert e_4 advices:
  - Your opinion $(0.1, 0.9)$ about alternative A_4 under criterion c_1 should be changed to the value $(0.69, 0.20)$.
  - Your opinion $(0.2, 0.3)$ about alternative A_4 under criterion c_2 should be changed to the value $(0.23, 0.29)$.
- Expert e_5 advices:
  - Your opinion $(0.8, 0.1)$ about alternative A_1 under criterion c_1 should be changed to a value closer to $(0.69, 0.20)$.
  - Your opinion $(0.5, 0.3)$ about alternative A_1 under criterion c_2 should be changed to a value closer to $(0.46, 0.38)$.
  - Your opinion $(0.1, 0.7)$ about alternative A_1 under criterion c_3 should be changed to a value closer to $(0.17, 0.71)$.

Figure 2c shows the consensus degree and harmony degree before and after the above advices have been implemented. Again, all experts are now above the consensus threshold value: $ACD^{1'} = 0.822, ACD^{2'} = 0.835, ACD^{3'} = 0.845, ACD^{4'} = 0.800, ACD^{5'} = 0.800$. The following personalized feedback mechanism GHD value is obtained: $GHD^p = 0.9537$.

A. Personalized Resolution Process

After the personalized feedback process is completed, it is $ACD^{3'} > ACD^{2'} > ACD^{1'} > ACD^{4'} = ACD^{5'}$ the following weighting vector is obtained applying (20) with the fuzzy linguistic quantifier 'most of' represented by the BUM function $P(r) = r^{2/3}$: $\omega = (0.168, 0.202, 0.346, 0.148, 0.136)^T$. The corresponding collective decision matrix $\overline{E} = (\overline{E}_{ij})$ is:

$$\overline{E} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{pmatrix} (0.355, 0.491) & (0.331, 0.645) & (0.379, 0.767) & (0.419, 0.543) \\ (0.425, 0.551) & (0.379, 0.573) & (0.353, 0.617) & (0.421, 0.683) \\ (0.392, 0.586) & (0.363, 0.689) & (0.371, 0.593) & (0.212, 0.445) \\ (0.581, 0.324) & (0.732, 0.165) & (0.651, 0.277) & (0.490, 0.365) \end{pmatrix}$$

Using the criterion weighting vector $V = (0.3, 0.4, 0.1, 0.2)^T$, the weighted collective overall experts' trust scores associated to the alternatives A_I $(I = 1, 2, 3, 4)$, as per expression (6), are: $TS_{\tilde{A}1} = 0.3850$; $TS_{\tilde{A}2} = 0.4030$; $TS_{\tilde{A}3} = 0.3712$; and $TS_{\tilde{A}4} = 0.6833$. Consequently, it is concluded that $\tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_1 \succ \tilde{A}_3$, i.e. the best alternative is $\tilde{A}_4$.

B. Discussion

Table I summarised the consensus degree and harmony degree obtained following the three feedback mechanisms.

The traditional and unpersonalized feedback mechanisms generate the same feedback parameters for all discordant

TABLE I: Comparison of ACD index and HD index with three feedback parameters

	δ_4	δ_5	$\sum \delta_i$	$ACD^{4'}$	$ACD^{5'}$	GHD
Traditional Feedback	0.5	0.5	1	0.8329	0.8209	0.7958
Unpersonalized Feedback	0.18	0.18	0.36	0.8081	0.8000	0.9253
Personalized Feedback	0.05	0.22	0.27	0.8000	0.8000	0.9537

experts, which is not the case with the personalized feedback mechanism. On the one hand, the main difference between the traditional and unpersonalized feedback mechanisms resides in that the latter is driven by maximum harmony degree achievement, which is reflected in the lower unpersonalized feedback parameter used. Indeed, the traditional feedback mechanism only pursues the consensus (group aim) while neglecting harmony (individual aim). Furthermore, a fixed feedback parameter different to the one used in the unpersonalized feedback parameter would have resulted in a lower traditional harmony degree, i.e. in higher changes from the individual opinions with the same final end: reaching consensus. Thus, the unpersonalized feedback process would modify discordant experts opinions less than the traditional feedback process and still achieve the main goal of the consensus reaching process. On the other hand, because $ACD^4 = 0.794 > ACD^5 = 0.788$, it is reasonable that in order to be above the threshold consensus value of 0.8, expert e_4 would require changing his/her discordant opinions at an extent lower than the changes required by expert e_5. In contrast to the unpersonalized feedback mechanism, this is indeed well reflected in the different feedback parameters that the personalized feedback mechanism uses. In addition to this, the personalized feedback mechanism reaches consensus with a higher harmony degree than the unpersonalized feedback mechanism, i.e. it requires the lowest adjustment budget of the three feedback mechanisms.

Summarizing, the distinction of experts, which is expected to be the case in practice, by the personalized feedback process guarantees that the impact on the individual experts is minimized because the adjustment budget required from them is the lowest possible to achieve the group consensus. In other words, the personalized feedback process provided a balance between consensus (group aim) and independence (individual aim). This is a strong argument to support that the inconsistent experts will be more willing to accept the personalized advices than advices coming from an unpersonalized and/or a traditional feedback process.

VI. Conclusion and Future Work

This article contributes to create a personalized feedback mechanism advice for inconsistent experts to reach consensus in GDM. The proposed approach has the following main advantages with respect to existing approaches:

(i) It proposes the definition of the general harmony degree (GHD) to assess the deviation degree before and after revising the inconsistent opinions. The advantage of GHD is that it allows the inconsistent experts adopt their personalized feedback parameters. Also, GHD has the remarkable property of being monotonic decreasing with respect to feedback parameter. Moreover, the personalized feedback GHD value is higher than the unpersonalized feedback GHD value for the same adjustment budget, i.e. for equal sum of feedback parameters.

(ii) A maximum harmony degree model is built by combining the harmony degree as an objective function and the consensus threshold as restrictions. Then, a personalized feedback mechanism is developed to achieve maximum harmony degree with an acceptable compromise between group consensus and individual opinions. It generates personalized advices according to individual current consensus status of identified inconsistent experts in a reasonable way: the higher the consensus degree, the smaller the feedback parameter and the smaller the deviation from the original opinions. Moreover, it enables the inconsistent experts to exactly reach the boundary of group consensus degree, which means the adjustment degree of original opinions is minimized. Therefore, the personalized feedback mechanism can help the inconsistent expert to achieve a balance between the group consensus (group aim) and independence (individual aim).

(iii) The relationship between the personalized and unpersonalized feedback mechanism is studied. In the personalized feedback mechanism, the inconsistent experts' opinions are adjusted according to the personalized feedback parameters, while the feedback parameter are assumed to be the same in the unpersonalized feedback mechanism. Therefore, the personalized feedback mechanism is able to obtain higher harmony degree than the unpersonalized feedback mechanism. Furthermore, the unpersonalized feedback mechanism is proved to be a special case of the personalized one.

Personalized feedback mechanisms would also be important for group decision making within a social network framework. However, the effect of social network structural relationship in the personalized feedback mechanism is still to be studied. The mechanism presented in this paper focuses on inconsistent decision maker and, therefore, the personalized feedback mechanism that considers both consistent and inconsistent experts is to be designed in future work.

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Mingshuo Cao received the BSc in Accounting from Shanghai Maritime University, China, in 2018. He is currently enrolled in a Ph.D. degree at Shanghai Maritime University. His research interest is mainly in group decision making.

Jian Wu (SM'16) received the B.Sc. and Ph.D. degrees in Management Science and Engineering from Hefei University of Technology, Hefei, China, in 2000 and 2008, respectively. He is a Distinguished Professor with the School of Economics and Management, Shanghai Maritime University, Shanghai, China. From October 2012 to October 2013, he was an Academic Research Visitor with the Centre for Computational Intelligence, De Montfort University, Leicester, U.K. He has 60+ papers published in leading journals such IEEE Transactions on Fuzzy Systems, Information Fusion, Information Sciences, Knowledge-Based Systems, Expert Systems with Applications, Applied Soft Computing. Twelve papers have been classed as Highly Cited Papers by the Essential Science Indicators, five of them are HOT paper. One of his research papers was awarded the prestigious Emerald Citations of Excellence for 2017. His research interests include group decision making, social network, fuzzy preference modeling, and information fusion.

Prof. Wu is an Area Editor of the Journal of Computers & Industrial Engineering, Associate Editor of the Journal of Intelligent and Fuzzy Systems and a Guest Editor of Applied Soft Computing.

Francisco Chiclana received the BSc and PhD in Mathematics from the University of Granada, Spain, in 1989 and 2000, respectively.

He is a Professor of Computational Intelligence and Decision Making with the School of Computer Science and Informatics, Faculty of Computing, Engineering and Media, De Montfort University, Leicester, U.K. He is an Associate Editor for several ISI indexed journals such as IEEE Transactions on Systems, Man, and Cybernetics: Systems, Information Sciences, Applied Soft Computing, and Knowledge-Based Systems. He has organized and chaired special sessions/workshops in many major international conferences in research areas as fuzzy preference modeling, decision support systems, consensus, recommender systems, social networks, rationality/consistency, aggregation. He is currently a Highly Cited Researcher in Computer Sciences (according to Essential Science Indicators by Clarivate Analytics).

Raquel Ureña received the Bsc degree in Telecommunications Engineering and PhD degree in Computer Science, University of Granada, Spain, in 2009 and 2015, respectively. She is a Marie Curie Individual Fellow of Institute of Artificial Intelligence, De Montfort University, UK.(IP in the?H2020-MSCA-IF-2016-DeciTrustNET project). Her research line, entitled?Intelligent decision support systems in complex networks and e-health scenarios. She has published more than 44 indexed contributions, among them 16 papers in highly regarded international journals in the area of Artificial Intelligence and Computer Science, 3 of them highly cited according to the ISI WoS and more than 22 International Conference contributions.

Enrique Herrera-Viedma received the M.Sc. and Ph.D. degrees in computer science from the University of Granada, Granada, Spain, in 1993 and 1996, respectively. He is a Professor of computer science and the Vice-President for Research and Knowledge Transfer with University of Granada, Granada, Spain. His h-index is 68 with more than 17 000 citations received in Web of Science and 85 in Google Scholar with more than 29000 cites received. He has been identified as one of the worlds most influential researchers by the Shanghai Center and Thomson Reuters/Clarivate Analytics in both computer science and engineering in the years 2014, 2015, 2016, 2017, and 2018. His current research interests include group decision making, consensus models, linguistic modeling, aggregation of information, information retrieval, bibliometric, digital libraries, web quality evaluation, recommender systems, and social media. Dr. Herrera-Viedma is currently Vice-President for Publications in IEEE SMC Society and an Associate Editor in several journals such as IEEE Transactions on Fuzzy Systems, IEEE Transactions on Systems, Man, and Cybernetics: Systems, Information Sciences, Applied Soft Computing, Soft Computing, Fuzzy Optimization and Decision Making, and Knowledge-Based Systems.