A PERTURBATION METAHEURISTIC FOR THE VEHICLE ROUTING PROBLEM WITH PRIVATE FLEET AND COMMON CARRIERS

Marie-Claude BOLDUC[1], Jacques RENAUD[*1], Fayez BOCTOR[1], Gilbert LAPORTE[2]

[1] Network Organization Technology Research Center (CENTOR),
Faculté des sciences de l'administration, Université Laval, Québec, G1K 7P4, Canada
[2] Canada Research Chair in Distribution Management,
HEC Montréal, 3000, chemin de la Côte-Sainte-Catherine, Montréal, H3T 2A7, Canada

Abstract

The purpose of this article is to propose a perturbation metaheuristic for the *Vehicle Routing Problem with Private fleet and Common carrier* (VRPPC). This problem consists of serving all customers such in a way that 1) each customer is served exactly once either by a private fleet vehicle or by a common carrier vehicle, 2) all routes associated with the private fleet start and end at the depot, 3) each private fleet vehicle performs only one route, 4) the total demand of any route does not exceed the capacity of the vehicle assigned to it, and 5) the total cost is minimized. This article describes a new metaheuristic for the VRPPC, which uses a perturbation procedure in the construction and improvement phases and also performs exchanges between the sets of customers served by the private fleet and the common carrier. Extensive computational results show the superiority of the proposed metaheuristic over previous methods.

Key words: Heterogeneous Vehicle Routing Problem, Common carrier, Private fleet, Metaheuristic, Perturbation.

Introduction

The purpose of this article is to propose a perturbation metaheuristic for the *Vehicle Routing Problem with Private fleet and Common carrier* (VRPPC) defined as follows. Let $G=(V,A)$ be a graph where $V=\{0,...,n\}$ is the vertex set and $A=\{(i,j):i,j\in V, i\neq j\}$ is the arc set. Vertex 0 is a depot, while the remaining vertices represent customers. A *private fleet* of m vehicles is available at the depot. The fixed cost of vehicle k is denoted by f_k, its capacity by Q_k, and the demand of customer i is denoted by q_i. A travel cost matrix (c_{ij}) is defined on A. If travel costs are vehicle

[*] Corresponding author.
E-mails addresses: mc@mcbolduc.com (M.-C. Bolduc), jacques.renaud@fsa.ulaval.ca (J. Renaud), fayez.boctor@fsa.ulaval.ca (F. Boctor), gilbert@crt.umontreal.ca (G. Laporte).

dependent, then c_{ij} can be replaced with c_{ijk}, where $k \in \{1,...,m\}$. Each customer i can be served by a vehicle of the private fleet, in which case it is called an *internal customer*, or by a *common carrier* at a cost equal to e_i, in which case it is called an *external customer*. The VRPPC consists of serving all customers such in a way that 1) each customer is served exactly once either by a private fleet vehicle or by a common carrier vehicle, 2) all routes associated with the private fleet start and end at the depot, 3) each private fleet vehicle performs only one route, 4) the total demand of any route does not exceed the capacity of the vehicle assigned to it, and 5) the total cost is minimized. In practice, several common carriers may be used to serve any of the customers unvisited by the private fleet. Typically, the one selected is the lowest cost carrier. It is not necessary to specify the routes followed by the common carrier because it charges a fixed amount e_i for visiting customer i, irrespective of visit sequence.

As far as we are aware, the VRPPC was introduced by Chu (2005) who modeled the problem and solved it heuristically through a savings based construction procedure, followed by intra-route and inter-route exchanges. Bolduc, Renaud & Boctor (2005) later improved on Chu's results by using more sophisticated exchanges. On ten instances $(n \leq 29)$, they reduced the average optimality gap from 13.36% to 0.69%. The single vehicle case was formulated by Volgenant & Jonker (1987) who showed that it can be transformed into a *Traveling Salesman Problem* (TSP), and solved exactly for $n \leq 200$ by Diaby & Ramesh (1995). A related but different one-to-one pickup and delivery problem arising in a chemical firm was formulated and solved heuristically by Ball *et al.* (1983). Klincewicz, Luss & Pilcher (1990) have analyzed a fleet sizing problem in a context where the customer set is partitioned into sectors and one must determine the private and common carrier fleet size for each sector.

This paper makes two main contributions. We first show that the VRPPC can be formulated as a *Heterogeneous Vehicle Routing Problem* (HVRP) (see, e.g., Gendreau *et al.* 1999; Li, Golden & Wasil 2006). Second, we introduce a new metaheuristic for the VRPPC. With respect to the method proposed by Bolduc, Renaud & Boctor (2005), called SRI, the metaheuristic uses a perturbation procedure in the construction and improvement phases and it also performs exchanges between the sets of customers served by the private fleet and the common carrier. Our computational results show marked improvements over those of Bolduc, Renaud & Boctor (2005) on a set of 78 test instances $(50 \le n \le 480)$.

The remainder of this article is organized as follow. Two formulations are presented in the next section, followed by the metaheuristic and computational results.

Formulation

Our first formulation uses the following variables:

$$x_{ijk} = \begin{cases} 1 \text{ if vehicle } k \text{ visits a vertex } j \text{ immediatly after vertex } i \\ 0 \text{ otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 \text{ if vehicle } k \text{ visits vertex } i \\ 0 \text{ otherwise} \end{cases}$$

$$z_i = \begin{cases} 1 \text{ if customer } i \text{ is assigned to a common carrier} \\ 0 \text{ otherwise} \end{cases}$$

u_{ik} : A upper bound on the load of vehicle k upon leaving customer i.

The formulation is as follows:

$$\text{minimize} \quad \sum_{k=1}^{m} f_k y_{0k} + \sum_{i=0}^{n} \sum_{\substack{j=0 \\ j \ne i}}^{n} \sum_{k=1}^{m} c_{ijk} x_{ijk} + \sum_{i=1}^{n} e_i z_i \tag{1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \sum_{k=1}^{m} x_{0jk} = \sum_{i=1}^{n} \sum_{k=1}^{m} x_{i0k} \le m \tag{2}$$

$$\sum_{\substack{j=0 \\ j \ne h}}^{n} x_{hjk} = \sum_{\substack{i=0 \\ i \ne h}}^{n} x_{ihk} = y_{hk} \qquad \left(h \in \{0,...,n\}; k \in \{1,...,m\}\right) \tag{3}$$

$$z_i + \sum_{k=1}^{m} y_{ik} = 1 \qquad \left(i \in \{1,...,n\}\right) \tag{4}$$

$$\sum_{i=1}^{n} q_i y_{ik} \le Q_k \qquad \left(k \in \{1,...,m\}\right) \tag{5}$$

$$u_{ik} - u_{jk} + Q_k x_{ijk} \le Q_k - q_j \qquad \left(i, j \in \{1,...,n\}, i \ne j; k \in \{1,...,m\}\right) \tag{6}$$

$$x_{ijk} \in \{0,1\} \qquad \left(i, j \in \{0,...,n\}, i \ne j; k \in \{1,...,m\}\right) \tag{7}$$

$$y_{ik} \in \{0,1\} \qquad \left(i \in \{0,...,n\}; k \in \{1,...,m\}\right) \tag{8}$$

$$z_i \in \{0,1\} \qquad \left(i \in \{1,...,n\}\right) \tag{9}$$

$$u_{ik} \ge 0 \qquad \left(i \in \{1,...,n\}; k \in \{1,...,m\}\right). \tag{10}$$

This formulation extended the classical Miller, Tucker & Zemlin (1960) formulation for the TSP. The objective function minimizes the sum of vehicle fixed costs, routing costs and common carrier costs. Constraints (2) specify that at most m private fleet vehicles can be used in the solution, while constraints (3) indicate that the same vehicle k must enter and leave customer h. Constraints (4) assign each customer either to an private vehicle or to a common carrier. Constraints (5) ensure that vehicle capacity is never exceeded. Constraints (6) eliminate subtours not including the depot. These were first introduced by Kulkarni & Bhave (1985) for the *Vehicle Routing Problem* (VRP). With respect to Chu's model, this formulation corrects a number of mistakes and uses subtour elimination constraints proper to the VRP as opposed to the TSP.

This formulation can be strengthened and simplified as follows. First, as shown by Kara, Laporte & Bektas (2004), constraints (6) can be lifted as follows:

$$u_{ik} - u_{jk} + Q_k x_{ijk} + \left(Q_k - q_i - q_j\right) x_{jik} \leq Q_k - q_j \quad \left(i, j \in \{1, \ldots, n\}, i \neq j; k \in \{1, \ldots, m\}\right). \tag{11}$$

Variables y_{ik} can be eliminated through the use of constraints (3). Variables z_i can also be eliminated through the introduction of a dummy vehicle 0 used to visit all customers assigned to the common carrier. This is done by eliminating constraints (4), setting $Q_0 = \sum_{i=1}^{n} q_i$ and redefining the travel cost matrix as $\bar{c} = \left(\bar{c}_{ijk}\right)$, where

$$\bar{c}_{ijk} = \begin{cases} c_{ijk} + f_k & \left(i = 0; j = \{1, \ldots, n\}; k = \{1, \ldots, m\}\right) \\ c_{ijk} & \left(j = 0; i = \{1, \ldots, n\}; k = \{1, \ldots, m\}\right) \\ e_j & \left(i = \{0, \ldots, n\}; j = \{1, \ldots, n\}; k = 0\right) \\ 0 & \left(i = \{1, \ldots, n\}; j = 0; k = 0\right). \end{cases}$$

The problem can then be formulated as follows:

$$\text{minimize} \quad \sum_{i=0}^{n} \sum_{\substack{j=0 \\ j \neq i}}^{n} \sum_{k=0}^{m} \bar{c}_{ijk} x_{ijk} \tag{12}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \sum_{k=0}^{m} x_{0jk} = \sum_{i=1}^{n} \sum_{k=0}^{m} x_{i0k} \leq m + 1 \tag{13}$$

$$\sum_{k=0}^{m}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ijk} = 1 \qquad \left(i \in \{1,...,n\}\right) \tag{14}$$

$$\sum_{\substack{j=0\\ j\neq h}}^{n} x_{hjk} = \sum_{\substack{i=0\\ i\neq h}}^{n} x_{ihk} \qquad \left(h \in \{0,...,n\}; k \in \{0,...,m\}\right) \tag{15}$$

$$\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} q_i x_{ijk} \leq Q_k \qquad \left(k \in \{0,...,m\}\right) \tag{16}$$

$$u_{ik} - u_{jk} + Q_k x_{ijk} + \left(Q_k - q_i - q_j\right) x_{jik} \leq Q_k - q_j \qquad \left(i, j \in \{1,...,n\}, i \neq j; k \in \{0,...,m\}\right) \tag{17}$$

$$x_{ijk} \in \{0,1\} \qquad \left(i, j \in \{0,...,n\}, i \neq j; k \in \{0,...,m\}\right) \tag{18}$$

$$u_{ik} \geq 0 \qquad \left(i \in \{1,...,n\}; k \in \{0,...,m\}\right). \tag{19}$$

This model is precisely that of the HVRP.

Metaheuristic

Our metaheuristic, called RIP (Randomized construction – Improvement – Perturbation) contains three main steps embedded within loops and five basic procedures: 1) a randomized savings construction phase; 2) a 4-opt* route improvement procedure (Renaud, Boctor & Laporte 1996); 3) a 2^*-interchange inter-route improvement procedure; 4) a 2-add-drop improvement procedure used to transfer customers between the private fleet and the common carrier, and 5) a switch procedure used to create a perturbation of a feasible solution. Our metaheuristic essentially combines a descent method (procedures 4-opt*, 2^*-interchange and 2-add-drop) with two diversification strategies which have contributed to the success of several recent metaheuristics, namely the use of a randomized process to construct an initial solution, and the idea of perturbating a solution within an improvement cycle. Successful vehicle routing implementations of similar ideas in the literature include perturbation heuristics (Shi, Olafsson & Sun 1999; Renaud, Boctor & Laporte 2002; Cordeau *et al.* 2005), and ant colony algorithms (Reimann, Doerner & Hartl 2004). We now describe our five procedures, followed by the overall algorithm.

Procedure 1. Randomized savings construction phase

Given a VRP with cost matrix (c_{ij}), the Clarke & Wright (1964) savings heuristic first computes a savings matrix (s_{ij}), where $s_{ij} = c_{i0} + c_{0j} - c_{ij}$. It then constructs back and forth routes $(0, i, 0)$ $(i \in \{1, ..., n\})$ and iteratively merges a route ending at i with a route starting at j as long as the merged route is feasible. In the parallel version of the algorithm the feasible merge yielding the largest s_{ij} is implemented. It has been demonstrated (Yellow 1970; Golden, Magnanti & Nguyen 1977) that the performance of this algorithm can be improved by redefining the savings as $s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$. Li, Golden & Wasil (2005) use $\lambda = 0.6$, 1.4 and 1.6 in their record-to-record heuristic. Because our construction phase is repeatedly applied in order to create a diversification effect, we define the savings as $s_{ij} = c_{i0} + c_{0j} - \lambda_{ij} c_{ij}$, where λ_{ij} is randomly selected in an interval $\left[\underline{\lambda}, \overline{\lambda}\right]$ according to a continuous uniform distribution. In our implementation, each customer is initially assigned to a common carrier. Routes are then merged as in the Clarke & Wright algorithm using the randomized savings, while routes containing at least two customers can be feasibly assigned to the private fleet. When no further merge is feasible, the solution cost is computed as in (1).

Procedure 2. 4-opt* route improvement

The 4-opt* route improvement procedure is applied to every individual route of the current solution. It was designed by Renaud, Boctor & Laporte (1996) as an improvement algorithm for the TSP. It implements eight of all 48 potential 4-opt moves (Lin 1965) having a high probability of yielding a positive cost reduction. This is achieved by executing a preliminary test comparing the cost of the removed edges to that of the inserted edges. Details can be found in the original reference.

Procedure 3. 2*-interchange inter-route improvement

In the λ-interchange procedure proposed by Osman (1993), every pair of routes is considered and up to λ customers from a route are exchanged with up to λ customers from another. It is common to use $\lambda = 2$. The case $\lambda = 1$ includes the relocation of a single customer to a different route and the exchange of two customers, each taken from a different route. We have implemented a restricted version of the 2-interchange procedure that operates on two chains $\left(i_s, i_{s+1}, i_{s+2}\right)$ and $\left(j_t, j_{t+1}, j_{t+2}\right)$ belonging to two different routes. This restricted procedure considers 25 possible moves, assuming each route contains at least 3 customers and neither of the two chains constrains the depot. These moves are described in Table 1.

If any of the two chains contains the depot, then any move inducing the transfer of depot is not implemented. If a route contains only two customers, then moves (16), (17) and (22) to (25) do not apply. Furthermore, the transfer of two consecutive vertices will empty one of the routes. Similarly, if one route contains only one customer only cases (1) to (11) apply and transferring the customer will empty its original route.

In the spirit of 4-opt*, we only apply the restricted 2-interchange procedure if it is likely to decrease the routing cost. More specifically, we implement it if the cost of the largest removed arc is at least as large as that of the smallest inserted one, i.e.,

$$\begin{aligned} &\max\left\{c_{i_s,i_{s+1}}, c_{i_{s+1},i_{s+2}}, c_{j_t,j_{t+1}}, c_{j_{t+1},j_{t+2}}\right\} \geq \\ &\min\left\{c_{i_s,j_t}, c_{i_s,j_{t+1}}, c_{i_s,j_{t+2}}, c_{i_{s+1},j_t}, c_{i_{s+1},j_{t+1}}, c_{i_{s+1},j_{t+2}}, c_{i_{s+2},j_t}, c_{i_{s+2},j_{t+1}}, c_{i_{s+2},j_{t+2}}\right\}. \end{aligned} \tag{20}$$

This test implicitly requires that all vehicles have the same variable routing cost. The restricted 2-interchange procedure containing this test is called the 2^*-interchange procedure. In our computational experiments, we have found that the introduction of the test only has a negligible effect on solution quality but yields a computing time reduction of about 70% (see Table 5).

The moves of the 2^*-interchange procedure are applied to every chain of every route and to every pair of routes until no more improvement can be reached. Even if the application of the 2^*-interchange procedure yields a routing cost reduction, it will not be implemented if no feasible assignment of vehicles to the resulting routes exists or if the overall cost of the modified solution exceeds that of the initial one due to larger vehicle fixed costs.

Table 1 – 25 possible moves for the restricted 2-interchange procedure

Original chains	Move	Chain from route 1 (i_s, i_{s+1}, i_{s+2})	Chain from route 2 (j_t, j_{t+1}, j_{t+2})
Transfer of one vertex	(1)	(i_s, i_{s+2})	$(j_t, j_{t+1}, i_{s+1}, j_{t+2})$
	(2)	$(i_s, i_{s+1}, j_{t+1}, i_{s+2})$	(j_t, j_{t+2})
Exchange of one vertex	(3)	(i_s, j_{t+1}, i_{s+2})	(j_t, i_{s+1}, j_{t+2})
Exchange of two consecutive vertices against one vertex	(4)	(j_{t+1}, i_{s+2})	$(j_t, i_s, i_{s+1}, j_{t+2})$
	(5)	$(i_s, j_t, j_{t+1}, i_{s+2})$	(i_{s+1}, j_{t+2})
	(6)	(j_{t+1}, i_{s+2})	$(j_t, i_{s+1}, i_s, j_{t+2})$
	(7)	$(i_s, j_{t+1}, j_t, i_{s+2})$	(i_{s+1}, j_{t+2})
	(8)	(j_t, i_{s+2})	$(j_{t+1}, i_s, i_{s+1}, j_{t+2})$
	(9)	$(i_{s+1}, j_t, j_{t+1}, i_{s+2})$	(i_s, j_{t+2})
	(10)	(j_t, i_{s+2})	$(j_{t+1}, i_{s+1}, i_s, j_{t+2})$
	(11)	$(i_{s+1}, j_{t+1}, j_t, i_{s+2})$	(i_s, j_{t+2})
Exchange of two consecutive vertices	(12)	(j_t, j_{t+1}, i_{s+2})	(i_{s+1}, i_s, j_{t+2})
	(13)	(j_{t+1}, j_t, i_{s+2})	(i_s, i_{s+1}, j_{t+2})
	(14)	(j_{t+1}, j_t, i_{s+2})	(i_{s+1}, i_s, j_{t+2})
	(15)	(j_t, j_{t+1}, i_{s+2})	(i_s, i_{s+1}, j_{t+2})
Exchange of two non-consecutive vertices	(16)	(j_t, i_{s+1}, j_{t+2})	(i_s, j_{t+1}, i_{s+2})
	(17)	(j_{t+2}, i_{s+1}, j_t)	(i_{s+2}, j_{t+1}, i_s)
Transfer of two consecutive vertices	(18)	(i_{s+2})	$(j_t, j_{t+1}, i_s, i_{s+1}, j_{t+2})$
	(19)	$(i_s, i_{s+1}, j_t, j_{t+1}, i_{s+2})$	(j_{t+2})
	(20)	(i_{s+2})	$(j_t, j_{t+1}, i_{s+1}, i_s, j_{t+2})$
	(21)	$(i_s, i_{s+1}, j_{t+1}, j_t, i_{s+2})$	(j_{t+2})
Transfer of two non-consecutive vertices	(22)	(i_{s+1})	$(j_t, j_{t+1}, i_s, i_{s+2}, j_{t+2})$
	(23)	$(i_s, i_{s+1}, j_t, j_{t+2}, i_{s+2})$	(j_{t+1})
	(24)	(i_{s+1})	$(j_t, j_{t+1}, i_{s+2}, i_s, j_{t+2})$
	(25)	$(i_s, i_{s+1}, j_{t+2}, j_t, i_{s+2})$	(j_{t+1})

Procedure 4. 2-add-drop improvement procedure

The 2-add-drop improvement procedure repeatedly executes the following moves until no further improvement can be achieved: 1) transferring up to two customers from the private fleet to the common carrier or conversely and 2) swapping an internal customer with an external one. Reinsertions are performed according to a cheapest feasible insertion criterion.

Procedure 5. Switch perturbation procedure

The switch perturbation procedure swaps $\lceil \tau n \rceil$ pairs of customers, where τ is a user-controlled parameter in $[0,1]$. Each pair consists either of two internal customers belonging to two different routes, or of an internal and an external customer. The first customer of a pair is randomly selected. If it is internal, it is swapped with its closest neighbor not belonging to the same route. If it is external, it is swapped with its closest internal customer. After all swaps have been implemented, a feasibility check is conducted. For each infeasible route, randomly select an internal customer and transfer it to common carriers until the route became feasible.

General description of the perturbation metaheuristic

The perturbation metaheuristic works with five parameters:

α	: the number of applications of the randomized savings construction phase;
$[\underline{\lambda};\overline{\lambda}]$	: the interval in which λ_{ij} is selected in the construction phase;
τ	: the percentage of customers moved in the switch perturbation procedure;
β	: the number of perturbation cycles;
γ	: the number of restarts of the entire procedure.

It can be summarized as follows.

Step 1 (Initialization)

Let S^* be the best known solution and let z^* be its cost. Initially, S^* is undefined and $z^* = \infty$ and set the iteration counter $\theta = 1$.

Step 2 (Randomized construction)

Apply the randomized savings construction procedure α times; retain the best solution T^*. Set the perturbation iteration counter $\sigma = 1$.

Step 3 (Improvement)

Apply in turn to T^* the 4-opt* route improvement procedure, the 2*-interchange inter-route improvement procedure and the 2-add-drop improvement procedure. Update T^*, S^* and z^*.

Step 4 (Perturbation)

If $\sigma > \beta$, go to Step 5. Otherwise, apply to T^* the switch perturbation procedure; update T^*, S^* and z^*. Set $\sigma = \sigma + 1$ and go to Step 3.

Step 5 (Restart)

If $\theta > \gamma$, stop. Otherwise, set $\theta = \theta + 1$ and go to Step 2.

Computational results

The RIP metaheuristic was coded in Visual Basic and run on a personal computer equipped with a Xeon 3.6 GHz processor and 1.00 Gb of RAM under Windows XP. All reported times are expressed in seconds and all statistics are averages over the tested instances. We first describe our test instances, followed by the obtained results.

Test instances

Two sets of instances were used to assess the performance of the tested heuristics. For the 34 instances of the first set (Table 2), the fleet is composed of a limited number of homogeneous vehicles, while the fleet for the 44 instances of the second set (Table 3) is limited and heterogeneous. The first set is divided into two subsets: the 14 instances subset of Christofides & Eilon (1969) and the 20 instances proposed by Golden *et al.* (1998). The name of these instances starts respectively with *CE* and *G* in Table 2. For all these instances, customer coordinates, customer demands and vehicle capacities are the same as in the original problems. The number of vehicles was set equal to $\lceil 0.8q/\bar{Q} \rceil$, where q is the sum of all customer demands and $\bar{Q}$ is the capacity of the vehicle fleet. The vehicle variable cost is set equal to 1 per unit of distance and the fixed cost f is set equal to the average route length within the best known solution of the original corresponding instance rounded to the nearest 20. The common carrier cost for customer i, denoted by e_i, is set equal to $f/\bar{n} + \mu_i c_{0i}$, where $\bar{n}$ is the average number of customers per route according to the best known solution of the corresponding original instance, and μ_i is a factor related to the demand of customer i. This factor allows the creation of a realistic transportation function cost similar to that used by common carriers. For a given instances, let $q_{\min}$ be the lowest demand, let $q_{\max}$ be the highest one, and $\eta = (q_{\max} - q_{\min})/3$, then :

$$\mu_i = \begin{cases} 1, \text{ if } q_i \in [q_{\min}, q_{\min} + \eta[\\ 1.5, \text{ if } q_i \in [q_{\min} + \eta, q_{\min} + 2\eta[\\ 2, \text{ if } q_i \in [q_{\min} + 2\eta, q_{\max}]. \end{cases} \tag{21}$$

The set of instances with heterogeneous fleet is divided into the three subsets (Table 3). The first subset contains the five small instances (instances beginning with Chu-H in Table 3) used by Chu (2005), and the five instances (instances beginning with B-H in Table 3), used by Bolduc, Renaud & Boctor (2005). Details can be found in these references. The second and third subsets were generated from the instances of Christofides & Eilon (1969) and from those of Golden *et al.* (1998). These begin respectively with CE-H and G-H. The fleet of these two subsets is composed of three vehicle types. The capacity and fixed cost of these three vehicle types are 80%, 100% and 120% of those used for the homogeneous fleet instances. The number of vehicles of each type was randomly generated in such a way that the total capacity is about 80% of the total demand. In Table 2 and Table 3, n is the number of customers, m is the number of vehicles, Q is the vehicle capacity, f is the vehicle fixed cost, and c is the cost of unit of distance. Table 3 gives the value of these parameters for the three vehicle types A, B and C. These instances are available at http://www.mcbolduc.com/VRPPC/tests.htm.

Small test instances Chu-H and B-H were solved to optimality using the commercial MIP code Cplex 9.0. In the following, solutions quality will be measured by the percentage deviation above the optimal solution value for instances Chu-H and B-H, and above the best solution value found in our computational tests for the other homogeneous and heterogeneous instances, set CE, G, CE-H and G-H.

Table 2 – Characteristics of instances with homogeneous limited fleet

Instances	*n*	*m*	*Q*	*f*	*c*
CE-01	50	4	160	120	1.00
CE-02	75	9	140	100	1.00
CE-03	100	6	200	140	1.00
CE-04	150	9	200	120	1.00
CE-05	199	13	200	100	1.00
CE-06	50	4	160	140	1.00
CE-07	75	9	140	120	1.00
CE-08	100	6	200	160	1.00
CE-09	150	10	200	120	1.00
CE-10	199	13	200	120	1.00
CE-11	120	6	200	180	1.00
CE-12	100	8	200	120	1.00
CE-13	120	6	200	260	1.00
CE-14	100	7	200	140	1.00
G-01	240	7	550	820	1.00
G-02	320	8	700	1060	1.00
G-03	400	8	900	1380	1.00
G-04	480	8	1000	1720	1.00
G-05	200	4	900	1620	1.00
G-06	280	5	900	1700	1.00
G-07	360	7	900	1460	1.00
G-08	440	8	900	1480	1.00
G-09	255	11	1000	60	1.00
G-10	323	13	1000	60	1.00
G-11	399	14	1000	80	1.00
G-12	483	15	1000	80	1.00
G-13	252	21	1000	60	1.00
G-14	320	23	1000	60	1.00
G-15	396	26	1000	60	1.00
G-16	480	29	1000	60	1.00
G-17	240	18	200	40	1.00
G-18	300	22	200	60	1.00
G-19	360	26	200	60	1.00
G-20	420	31	2000	60	1.00

Table 3 – Characteristics of the instances with heterogeneous limited fleet

Instances	n	Vehicle type											
		A				*B*				*C*			
		m_A	Q_A	f_A	c_A	m_B	Q_B	f_B	c_B	m_C	Q_C	f_C	c_C
Chu-H-01	5	1	40	60	1.50	1	30	50	1.50				
Chu-H-02	10	1	75	120	1.50	1	65	100	1.50				
Chu-H-03	15	1	110	150	1.50	1	100	140	1.50	1	90	130	1.50
Chu-H-04	22	1	4500	250	1.50	1	4000	200	1.50				
Chu-H-05	29	1	4500	250	1.50	1	4000	200	1.50	1	3500	180	1.50
B-H-01	5	1	40	60	1.50	1	30	50	1.50				
B-H-02	10	1	75	120	1.50	1	65	100	1.50				
B-H-03	15	1	110	150	1.50	1	100	140	1.50	1	90	130	1.50
B-H-04	22	1	4500	250	1.50	1	4000	200	1.50				
B-H-05	29	1	4500	250	1.50	1	4000	200	1.50	1	3500	180	1.50
CE-H-01	50	2	160	140	1.00	2	192	168	1.00				
CE-H-02	75	4	112	80	1.00	5	168	120	1.00				
CE-H-03	100	2	160	112	1.00	2	200	140	1.00	2	240	168	1.00
CE-H-04	150	2	160	96	1.00	4	200	120	1.00	3	240	144	1.00
CE-H-05	199	7	160	80	1.00	5	200	100	1.00	2	240	120	1.00
CE-H-06	50	1	128	112	1.00	2	160	140	1.00	1	192	168	1.00
CE-H-07	75	4	112	96	1.00	3	140	120	1.00	2	168	144	1.00
CE-H-08	100	1	160	128	1.00	1	200	160	1.00	4	240	192	1.00
CE-H-09	150	4	160	96	1.00	3	200	120	1.00	3	240	144	1.00
CE-H-10	199	2	160	96	1.00	5	200	120	1.00	6	240	144	1.00
CE-H-11	120	2	160	144	1.00	2	200	180	1.00	2	240	216	1.00
CE-H-12	100	2	160	96	1.00	3	200	120	1.00	3	240	144	1.00
CE-H-13	120	1	160	208	1.00	4	200	260	1.00	1	240	312	1.00
CE-H-14	100	1	160	96	1.00	1	200	120	1.00	5	240	144	1.00
G-H-01	240	3	440	656	1.00	1	550	820	1.00	3	660	984	1.00
G-H-02	320	2	560	848	1.00	2	700	1060	1.00	4	840	1272	1.00
G-H-03	400	3	720	1104	1.00	3	900	1380	1.00	2	1080	1656	1.00
G-H-04	480	2	800	1376	1.00	4	1000	1720	1.00	2	1200	2064	1.00
G-H-05	200	2	720	1296	1.00	2	900	1620	1.00				
G-H-06	280	3	720	1360	1.00	2	900	1700	1.00	1	1080	2040	1.00
G-H-07	360	3	720	1168	1.00	1	900	1460	1.00	3	1080	1752	1.00
G-H-08	440	1	720	1184	1.00	2	900	1480	1.00	5	1080	1776	1.00
G-H-09	255	6	800	48	1.00	3	1000	60	1.00	3	1200	72	1.00
G-H-10	323	3	800	48	1.00	3	1000	60	1.00	6	1200	72	1.00
G-H-11	399	6	800	64	1.00	8	1000	80	1.00	1	1200	96	1.00
G-H-12	483	6	800	64	1.00	6	1000	80	1.00	4	1200	96	1.00
G-H-13	252	6	800	48	1.00	4	1000	60	1.00	10	1200	72	1.00
G-H-14	320	11	800	48	1.00	2	1000	60	1.00	11	1200	72	1.00
G-H-15	396	7	800	48	1.00	9	1000	60	1.00	10	1200	72	1.00
G-H-16	480	12	800	48	1.00	6	1000	60	1.00	11	1200	72	1.00
G-H-17	240	4	160	32	1.00	7	200	40	1.00	6	240	48	1.00
G-H-18	300	7	160	48	1.00	9	200	60	1.00	6	240	72	1.00
G-H-19	360	9	160	48	1.00	7	200	60	1.00	10	240	72	1.00
G-H-20	420	16	160	48	1.00	6	200	60	1.00	10	240	72	1.00

Randomized savings construction and best parameters

We first conducted tests for the randomized savings construction procedure over all 78 instances with several values of the parameters $\overline{\lambda}$, $\underline{\lambda}$ and α. For Table 4 to Table 8, average results are presented for each set of instances. For the homogeneous instances, sets CE and G contain respectively 14 and 20 instances. For the heterogeneous instances, sets Chu-H, B-H, CE-H and G-H contain respectively 5, 5, 14 and 20 instances. Table 4 gives the average percentage of deviation (%) with respect to either the optimal solution (for small instances, set Chu-H and B-H) or the best known solution. We observe that the percentage deviation from the optimal or best known solutions decreases as α increases while computational time increases. In our RIP metaheuristic, we will use $\alpha = 10$ because it offers a good compromise between solution quality and computation time. The results obtained with $\alpha = 20$ are slightly better than those obtained with $\alpha = 10$ but require about twice the computation time. Also, the table indicates that for $\alpha = 10$, the best results were obtained with $\left(\underline{\lambda}, \overline{\lambda}\right) = \left(0.9, 1.1\right)$ for the homogeneous fleet instances (average deviation of 8.90% over the 34 instances), and with $\left(\underline{\lambda}, \overline{\lambda}\right) = \left(0.8, 1.2\right)$ for the heterogeneous fleet instances (average deviation of 6.99% over the 44 instances). These values will be used within our computational tests.

Table 4 – Results of the randomized savings construction procedure

HOMOGENEOUS INSTANCES									
		α = 1		α = 5		α = 10		α = 20	
Instances	$\left[\underline{\lambda}, \overline{\lambda}\right]$	%	sec.	%	sec.	%	sec.	%	sec.
CE	[0.7, 1.3]	7.92	0.21	6.78	0.93	6.28	1.86	5.66	3.57
G		13.60	1.75	12.56	8.10	11.99	16.20	11.51	31.90
All		**11.26**	**1.12**	**10.18**	**5.15**	**9.64**	**10.29**	**9.10**	**20.24**
CE	[0.8, 1.2]	7.92	0.21	5.91	0.86	5.61	2.07	5.18	3.64
G		13.60	1.70	11.72	8.40	11.34	16.60	10.92	32.90
All		**11.26**	**1.09**	**9.32**	**5.29**	**8.98**	**10.62**	**8.56**	**20.85**
CE	[0.9, 1.1]	7.92	0.14	5.66	0.79	5.33	2.00	5.00	3.50
G		13.60	1.65	11.81	8.25	11.40	16.55	11.14	33.50
All		**11.26**	**1.03**	**9.28**	**5.18**	**8.90**	**10.56**	**8.61**	**21.15**
HETEROGENEOUS INSTANCES									
		α = 1		α = 5		α = 10		α = 20	
Instances	$\left[\underline{\lambda}, \overline{\lambda}\right]$	%	sec.	%	sec.	%	sec.	%	sec.
Chu-H	[0.7, 1.3]	7.71	0.00	3.98	0.00	2.58	0.40	1.93	0.40
B-H		7.61	0.00	7.02	0.25	6.68	0.25	4.93	0.25
CE-H		6.95	0.07	5.67	0.93	5.27	1.64	4.76	3.64
G-H		12.00	1.65	11.04	8.15	10.24	16.15	9.97	31.95
All		**9.40**	**0.77**	**8.07**	**4.03**	**7.38**	**7.94**	**6.83**	**15.76**
Chu-H	[0.8, 1.2]	7.71	0.00	3.52	0.00	2.26	0.00	1.80	0.00
B-H		7.61	0.00	6.79	0.00	6.49	0.20	6.22	0.20
CE-H		6.95	0.14	5.09	0.79	4.81	1.50	4.61	3.29
G-H		12.00	1.90	10.43	8.55	9.83	16.40	9.64	32.85
All		**9.40**	**0.91**	**7.53**	**4.14**	**6.99**	**7.95**	**6.76**	**16.00**
Chu-H	[0.9, 1.1]	7.71	0.00	6.75	0.00	6.73	0.00	6.48	0.20
B-H		7.61	0.00	6.86	0.00	6.86	0.00	6.11	0.00
CE-H		6.95	0.00	4.96	0.79	4.48	1.93	3.95	3.50
G-H		12.00	1.90	10.35	8.65	10.08	17.05	9.33	33.75
All		**9.40**	**0.86**	**7.83**	**4.18**	**7.55**	**8.36**	**6.93**	**16.48**

2^*-interchange improvement and best parameters

We have conducted tests to compare the 2-interchange and the 2^*-interchange inter-route improvement procedures and to determine the effect of Equation 19 on both solution quality and computational times. Table 5 gives the average results obtained by applying once these two improvement procedures to the solution given by the randomized savings construction procedure. We observe that the percentage deviation given by the both interchange procedures is the same (4.43 %) for the 34 homogeneous instances and almost the same (3.42 % versus 3.43 %) for the 44 heterogeneous instances. However, the 2^*-interchange improvement procedure requires about 30% of the computation time required by the 2-interchange procedure. Thus, the 2^*-interchange inter-route improvement procedure will be used in our tests.

Table 5 – Comparison between inter-route interchange procedures

HOMOGENEOUS INSTANCES				
	2-interchange		2*-interchange	
Instances	**%**	**sec.**	**%**	**sec.**
CE	3.10	0.29	3.10	0.07
G	5.36	2.05	5.36	0.65
All	**4.43**	**1.32**	**4.43**	**0.41**
HETEROGENEOUS INSTANCES				
	2-interchange		2*-interchange	
Instances	**%**	**sec.**	**%**	**sec.**
Chu-H	2.08	0.00	2.08	0.00
B-H	0.13	0.20	0.13	0.00
CE-H	3.07	0.07	3.09	0.00
G-H	4.83	1.85	4.83	0.45
All	**3.42**	**0.89**	**3.43**	**0.20**

Switch perturbation and best parameters

Tests were conducted to determine the effect of β, the number of perturbation cycles, and τ, the percentage of customers to move in the switch procedure. Several values of β (5, 10, 20 and 30) and τ (4, 5 and 6%) were used within the RIP metaheuristic which was applied to solve all test instances with only one start $(\gamma = 1)$.

Table 6 shows the average percentage deviation from either the optimal or the best known solutions (%) and the corresponding computation times in seconds. It is worth noting that other values of these two parameters were tested. We only report the results for the most promising values. As expected, solution quality and computation times increase with β. The best results were obtained with $\beta = 30$ and $\tau = 5\%$ with a average deviation of $2.87\,\%$ and $2.18\,\%$ for homogeneous and heterogeneous instances, respectively. However, using $\beta = 20$ and $\tau = 5\%$ seems to offer a good compromise between solution quality and computation time. These values generated deviation of 3.02 % in 24.29 seconds for homogeneous instances and of 2.34 % in 18.64 seconds for heterogeneous instances. These parameter values will be used within our complete RIP metaheuristic.

Table 6 – Results of the switch perturbation procedure in function of β and τ

HOMOGENEOUS INSTANCES									
		β = 5		β = 10		β = 20		β = 30	
Instances	τ	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**
CE	4%	3.09	3.79	3.03	4.43	2.65	5.43	2.40	6.43
G		4.81	26.05	4.55	29.70	4.08	37.35	3.87	44.70
All		**4.10**	**16.88**	**3.92**	**19.29**	**3.49**	**24.21**	**3.27**	**28.94**
CE	5%	2.87	2.64	2.73	3.07	2.26	4.14	2.15	5.00
G		4.41	26.20	4.02	30.60	3.56	38.40	3.38	46.25
All		**3.78**	**16.50**	**3.49**	**19.26**	**3.02**	**24.29**	**2.87**	**29.26**
CE	6%	3.09	2.57	2.64	3.36	2.47	4.14	2.20	5.36
G		4.46	26.15	4.16	30.45	3.62	38.60	3.36	46.65
All		**3.89**	**16.44**	**3.54**	**19.29**	**3.15**	**24.41**	**2.88**	**29.65**
HETEROGENEOUS INSTANCES									
		β = 5		β = 10		β = 20		β = 30	
Instances	τ	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**
Chu-H	4%	2.06	0.20	2.06	0.20	2.06	0.20	2.06	0.20
B-H		0.04	0.00	0.04	0.00	0.04	0.00	0.04	0.20
CE-H		2.74	2.43	2.47	4.14	2.33	4.14	2.18	5.00
G-H		4.15	31.65	3.87	36.35	3.55	44.45	3.39	52.70
All		**2.99**	**15.18**	**2.78**	**17.86**	**2.60**	**21.55**	**2.47**	**25.59**
Chu-H	5%	0.11	0.00	0.11	0.20	0.11	0.20	0.11	0.20
B-H		0.04	0.20	0.04	0.40	0.04	0.40	0.04	0.40
CE-H		2.69	2.43	2.59	4.00	2.30	4.00	2.08	5.21
G-H		4.26	25.65	3.92	29.80	3.49	38.05	3.29	45.80
All		**2.81**	**12.45**	**2.62**	**14.89**	**2.34**	**18.64**	**2.18**	**22.55**
Chu-H	6%	2.06	0.00	2.06	0.00	2.06	0.00	2.06	0.20
B-H		0.08	0.00	0.04	0.00	0.00	0.00	0.00	0.00
CE-H		2.81	2.71	2.56	4.50	2.43	4.50	2.27	5.57
G-H		4.06	26.15	3.62	30.40	3.32	38.75	3.14	46.80
All		**2.98**	**12.75**	**2.70**	**15.25**	**2.52**	**19.05**	**2.38**	**23.07**

The RIP metaheuristic and number of restarts

The number of restarts γ has a significant effect, both on solution quality and on computation time. As shown in Table 7, the percentage deviation with respect to the best known solution decreases as γ increases. Also, as could be expected, computational time is almost a linear function of γ. With $\gamma = 50$, the RIP metaheuristic generated average deviation of 1.07 % in 1047.15 seconds for the homogeneous instances and of 0.78 % in 812.73 seconds for the heterogeneous instances.

Table 7 – Results of the RIP metaheuristic in function of the number of restarts γ

HOMOGENEOUS INSTANCES										
	γ = 1		γ = 5		γ = 10		γ = 30		γ = 50	
Instances	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**
CE	2.26	4.14	1.57	18.29	1.29	36.64	0.88	107.21	0.75	178.14
G	3.56	38.40	2.72	165.20	2.07	332.35	1.52	996.50	1.29	1 655.45
All	**3.02**	**24.29**	**2.25**	**104.71**	**1.75**	**210.59**	**1.26**	**630.32**	**1.07**	**1 047.15**
HETEROGENEOUS INSTANCES										
	γ = 1		γ = 5		γ = 10		γ = 30		γ = 50	
Instances	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**	**%**	**sec.**
Chu-H	0.11	0.20	0.07	0.40	0.07	1.00	0.07	2.80	0.07	4.40
B-H	0.04	0.40	0.04	1.00	0.04	2.00	0.04	4.80	0.04	7.80
CE-H	2.30	4.00	1.35	18.21	1.11	36.57	0.69	108.64	0.63	180.79
G-H	3.49	38.05	2.50	165.85	2.09	336.75	1.48	996.95	1.25	1 658.40
All	**2.34**	**18.64**	**1.58**	**81.34**	**1.31**	**165.05**	**0.91**	**488.59**	**0.78**	**812.73**

We also ran Chu's heuristic to solve the 10 small heterogeneous test instances. To do so, we used a computer code provide by Chu. Unfortunately, this code failed to solve the other test instances. We also compared our solutions to those obtained by the SRI heuristic (Bolduc, Renaud & Boctor 2005). For these instances, all computations use truncated distances as was done by Chu. Table 8 compares the solution value (z) obtained for the 10 small instances by Chu's heuristic, the SRI heuristic and the RIP metaheuristic. It shows that the RIP metaheuristic outperforms the other two heuristics with an average deviation from the optimum of 0.07 % on Chu's instances and of 0.04 % on B-H instances. However, it is clear that the RIP requires much longer computation times.

Table 8 – Results of small heterogeneous instances

Instances	Optimum	Chu		SRI		RIP	
		z	**sec.**	**z**	**sec.**	**z**	**sec.**
Chu-H-01	387.50	387.50	0.02	387.50	0.00	387.50	0.35
Chu-H-02	586.00	631.00	0.03	586.00	0.02	586.00	1.90
Chu-H-03	823.50	900.00	0.08	826.50	0.03	826.50	3.50
Chu-H-04	1 389.00	1 681.50	0.06	1 389.00	0.08	1 389.00	5.85
Chu-H-05	1 441.50	1 917.00	0.28	1 444.50	0.09	1 441.50	10.40
Average		**14.20%**	**0.09**	**0.11%**	**0.04**	**0.07%**	**4.40**
B-H-01	423.50	503.00	0.02	423.50	0.02	423.50	1.85
B-H-02	476.50	476.50	0.05	476.50	0.02	476.50	3.65
B-H-03	777.00	884.00	0.11	804.00	0.03	778.50	4.75
B-H-04	1 521.00	1 737.00	0.06	1 564.50	0.09	1 521.00	15.85
B-H-05	1 609.50	1 864.50	0.16	1 609.50	0.13	1 609.50	12.90
Average		**12.52%**	**0.08**	**1.27%**	**0.06**	**0.04%**	**7.80**
All		**13.36%**	**0.09**	**0.69%**	**0.05**	**0.06%**	**6.10**

Finally, Table 9 gives the solution values of instances with homogeneous fleet, while Table 10 contains results for the heterogeneous fleet instances. These tables compare the results obtained with the RIP metaheuristic to those obtained with the SRI heuristic and give the best known solution for each instance. The RIP metaheuristic produced an average percentage of deviation of 0.75% and 1.29% for the *CE* and *G* homogeneous instances, respectively. The SRI heuristic yields an average deviation of 4.20% and 4.18% for the same instances. For the heterogeneous instances, the RIP metaheuristic gives an average deviation of 0.63% and 1.25% for the CE-H and G-H instances, respectively, while the SRI yields an average deviation of 3.45% for both sets. The RIP metaheuristic is about 100 times slower than the SRI heuristic. This is mainly due to the repetition of the inter-route interchange procedure 1000 times because $\beta = 20$ and $\gamma = 50$.

Table 9 – Global results for instances with homogeneous limited fleet

Instances	SRI		RIP		
			Standard		Best
	z	**sec.**	**z**	**sec.**	**z**
CE-01	1 199.99	0	1 132.91	25	1 119.47
CE-02	1 890.33	0	1 835.76	73	1 814.52
CE-03	2 050.33	1	1 959.65	107	1 937.23
CE-04	2 694.72	1	2 545.72	250	2 528.36
CE-05	3 228.67	3	3 172.22	474	3 107.04
CE-06	1 282.94	0	1 208.33	25	1 207.47
CE-07	2 092.32	0	2 006.52	71	2 006.52
CE-08	2 163.32	1	2 082.75	110	2 052.05
CE-09	2 526.82	1	2 443.94	260	2 436.02
CE-10	3 511.02	3	3 464.90	478	3 407.13
CE-11	2 375.71	1	2 333.03	195	2 332.21
CE-12	2 037.54	0	1 953.55	128	1 953.55
CE-13	2 916.21	1	2 864.21	188	2 858.94
CE-14	2 220.77	1	2 224.63	110	2 216.68
Average	**4.20%**	**0.93**	**0.75%**	**178.14**	
G-01	14 675.33	4	14 388.58	651	14 160.77
G-02	20 108.84	9	19 505.00	1 178	19 234.03
G-03	26 046.80	16	24 978.17	2 061	24 646.79
G-04	36 234.51	27	34 957.98	3 027	34 607.12
G-05	15 751.31	5	14 683.03	589	14 249.82
G-06	23 255.65	8	22 260.19	1 021	21 703.54
G-07	25 298.48	13	23 963.36	1 628	23 549.53
G-08	30 899.74	18	30 496.18	2 419	30 173.53
G-09	1 378.67	4	1 341.17	832	1 336.91
G-10	1 646.91	8	1 612.09	1 294	1 598.76
G-11	2 238.57	14	2 198.45	2 004	2 179.71
G-12	2 597.14	17	2 521.79	2 900	2 503.71
G-13	2 339.93	5	2 286.91	802	2 268.32
G-14	2 825.76	8	2 750.75	1 251	2 704.01
G-15	3 269.96	12	3 216.99	1 862	3 171.20
G-16	3 784.63	19	3 693.62	2 778	3 654.20
G-17	1 732.70	5	1 701.58	806	1 677.22
G-18	2 821.82	8	2 765.92	1 303	2 742.72
G-19	3 614.59	11	3 576.92	1 903	3 528.36
G-20	4 439.45	15	4 378.13	2 800	4 352.95
Average	**4.18%**	**11.30**	**1.29%**	**1655.45**	
All	**4.19%**	**7.03**	**1.07%**	**1047.15**	

Table 10 – Global results for instances with heterogeneous limited fleet

Instances	SRI		RIP		
			Standard		Best
	z	sec.	z	sec.	z
CE-H-01	1 220.72	0	1 192.72	26	1 191.70
CE-H-02	1 858.24	0	1 798.26	72	1 790.67
CE-H-03	1 999.91	1	1 934.85	105	1 919.05
CE-H-04	2 615.95	1	2 493.93	251	2 475.16
CE-H-05	3 248.26	3	3 195.66	490	3 146.45
CE-H-06	1 264.72	0	1 210.23	25	1 204.48
CE-H-07	2 093.48	1	2 042.79	74	2 025.98
CE-H-08	2 058.81	0	2 015.72	112	1 984.36
CE-H-09	2 570.57	2	2 445.88	267	2 438.73
CE-H-10	3 391.25	3	3 304.69	482	3 267.85
CE-H-11	2 334.41	1	2 308.76	188	2 303.13
CE-H-12	1 924.92	0	1 908.74	130	1 908.74
CE-H-13	2 925.27	1	2 842.18	195	2 842.18
CE-H-14	1 957.63	1	1 920.36	114	1 907.74
Average	**3.45%**	**1.00**	**0.63%**	**180.79**	
G-H-01	14 599.16	4	14 408.31	647	14 251.75
G-H-02	18 945.77	13	18 663.15	1254	18 560.07
G-H-03	26 151.24	13	25 561.55	2053	25 356.63
G-H-04	36 519.42	22	35 495.66	2904	34 589.11
G-H-05	17 173.22	3	16 138.50	512	15 667.13
G-H-06	21 083.42	8	20 329.04	1005	19 975.32
G-H-07	24 854.96	14	24 184.83	1608	23 510.98
G-H-08	28 412.97	21	27 710.66	2584	27 420.68
G-H-09	1 371.98	5	1 346.03	814	1 331.83
G-H-10	1 599.77	8	1 575.82	1332	1 561.52
G-H-11	2 249.11	14	2 218.91	2140	2 195.31
G-H-12	2 573.81	19	2 510.07	2970	2 487.38
G-H-13	2 325.09	5	2 253.45	733	2 239.18
G-H-14	2 783.74	10	2 711.81	1246	2 682.85
G-H-15	3 224.50	13	3 156.93	1895	3 131.89
G-H-16	3 740.85	22	3 649.09	2785	3 629.41
G-H-17	1 741.66	4	1 705.48	762	1 695.75
G-H-18	2 787.10	7	2 759.99	1299	2 740.05
G-H-19	3 518.50	11	3 517.48	1892	3 464.70
G-H-20	4 362.31	15	4 413.82	2733	4 352.35
Average	**3.45%**	**11.55**	**1.25%**	**1658.40**	
All	**3.45%**	**7.21**	**1.00%**	**1049.97**	

Conclusion

We have described a new metaheuristic for the Vehicle Routing Problem with Private fleet and Common carrier. This metaheuristic uses a perturbation procedure during the construction and improvement phases and it also makes use of a streamlined family of edge exchanges. Extensive computational results obtained on several sets of benchmark instances show that the proposed metaheuristic generates much better solutions than two previous methods. In particular, on instances for which the optimum is known, it yields average optimality gaps of 0.07% or 0.04%, depending on the set of instances used in the comparison.

Acknowledgements

This work was partially supported by the Canadian Natural Sciences and Engineering Research Council (NSERC) under grants OGP0036509, OGP0039682 and OGP0172633. This support is gratefully acknowledged. The authors also thank Ching-Wu Chu for providing the computer code of his heuristic.

References

1. Ball M.O., Golden B.L., Assad A.A. & Bodin L.D. "Planning for truck fleet size in the presence of a common-carrier option." *Decision Sciences*, 14, 1983, 103-120.

2. Bolduc M.-C., Renaud J. & Boctor F.F. "A heuristic for the routing and carrier selection problem." Working paper, Network Organization Technology Research Center (CENTOR), Faculté des sciences de l'administration, Université Laval, Québec, 2005.

3. Chu C.-W. "A heuristic algorithm for the truckload and less-than-truckload problem." *European Journal of Operational Research*, 165, 2005, 657-667.

4. Christofides N. & Eilon S. "An Algorithm for the Vehicle-Dispatching Problem." *Operations Research*, 20, 1969, 309-318.

5. Clarke G. & Wright J.W. "Scheduling of vehicles from a central depot to a number of delivery points." *Operations Research*, 12, 1964, 568-581.

6. Cordeau J.-F., Gendreau M., Hertz A., Laporte G. & Sormany J.-S. "New heuristics for vehicle routing problem." In Logistics Systems: Design and Optimization (Eds A. Langevin & D. Riopel), Springer, Boston, 2005, 279-297.

7. Diaby M. & Ramesh R. "The distribution problem with carrier service: A dual based penalty approach." *ORSA Journal on Computing*, 7, 1, 1995, 24-35.

8. Gendreau M., Laporte G., Musaraganyi C. & Taillard É.D. "A tabu search heuristic for the heterogeneous fleet vehicle routing problem." *Computers & Operations Research*, 26, 1999, 1153-1173.

9. Golden B.L., Magnanti T.L. & Nguyen H.Q. "Implementing Vehicle Routing Algorithms." *Networks*, 7, 1977, 113-148.

10. Golden B.L., Wasil E.A., Kelly J.P. & Chao I.-M. "The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results." In: T.G. Crainic, G. Laporte (Eds.), Fleet Management and Logistics, Kluwer, Boston, 1998, 33-56.

11. Kara I., Laporte G. & Bektas T. "A note on the lifted Miller-Tucker-Zemlin subtour elimination constraints for the capacitated vehicle routing problem." *European Journal of Operational Research*, 158, 2004, 793-795.

12. Klincewicz J.G., Luss H. & Pilcher M.G. "Fleet size plannnig when outside carrier services are available." *Transportation Science*, 24, 1990, 169-182.

13. Kulkarni R.V. & Bhave P.R. "Integer programming formulations of vehicle routing problems ." *European Journal of Operational Research*, 20, 1985, 58-67.

14. Li F., Golden B.L. & Wasil E.A. "Very large-scale vehicle routing: new problems, algorithms, and results." *Computers & Operations Research*, 32, 2005, 1165-1179.

15. Li F., Golden B.L. & Wasil E.A. "A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem." *Computers & Operations Research*, 2006, to appear.

16. Lin S. "Computer solutions of the traveling salesman problem." *The Bell System Technical Journal*, 1965, 2245-2269.

17. Miller C.E., Tucker A.W. & Zemlin R.A. "Integer programming formulation of traveling salesman problems." *Journal of the ACM*, 7, 4, 1960, 326-329.

18. Osman I.H. "Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem." *Annals of Operations Research*, 41, 1993, 421-451.

19. Reimann M., Doerner K. & Hartl R.F. "D-Ants: Savings based ants divide and conquer the vehicle routing problem." *Computers & Operations Research*, 31, 2004, 563-591.

20. Renaud J., Boctor F.F. & Laporte G. "A fast composite heuristic for the symmetric traveling salesman problem." *INFORMS Journal on Computing*, 8, 2, 1996, 134-143.

21. Renaud J., Boctor F.F. & Laporte G. "Perturbation heuristics for the pickup and delivery traveling salesman problem." *Computers & Operations Research*, 29, 2002, 1129-1141.

22. Shi L., Olafsson S. & Sun N. "New parallel randomized algorithms for the traveling salesman problem." *Computers & Operations Research*, 26, 1999, 371-394.

23. Volgenant T. & Jonker R. "On some generalizations of the travelling-salesman problem." *Journal of the Operational Research Society*, 38, 1987, 1073-1079.

24. Yellow P.C. "A computational modification to the saving method of vehicle scheduling." *Operational Research Quarterly*, 21, 1970, 281-283.