A Phase Noise Suppression Algorithm for OFDM-Based WLANs

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***Abstract*—Orthogonal frequency-division multiplexing (OFDM) has been specified by IEEE 802.11a standard as the transmission technique for high-rate wireless local area networks (WLANs). Performance of an OFDM system, however, is heavily degraded by random Wiener phase noise, which causes both common phase error (CPE) and inter-carrier interference (ICI). To mitigate this problem, a new phase-noise suppression (PNS) algorithm is proposed in this letter to efficiently eliminate the effect of phase noise on OFDM based WLANs. Numerical results are presented to illustrate the effectiveness of the proposed algorithm.**

***Index Terms*—IEEE 80211a standard, orthogonal frequency-division multiplexing (OFDM), phase noise, wireless local area network (WLAN).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) is a bandwidth efficient transmission technique which easily handles time dispersion of channel. It has been adopted by IEEE 802.11a standard as the transmission technique for high-rate wireless local area networks (WLANs) [1]. The data packet of IEEE 802.11a standard consists of two parts: the preamble and the data. The preamble includes short and long pilots which are used, e.g., for synchronization, frequency offset and channel estimation. Pilot-aided OFDM channel estimation can be found in [2]. Frequency offset corrections have been presented in many papers [3], [4]. However, Wiener phase noise, whose effect has also been examined in [5], proves to be a much more complex phenomenon than frequency offset. This paper proposes, what is termed phase-noise suppression (PNS) algorithm to eliminate the effect of this noise.

II. Problem Formulation

Assume perfect frequency and timing synchronization, i.e., we only take phase noise into consideration. The received nth sample of the mth OFDM symbol can be expressed by

$$r_m(n) = x_m(n) \otimes h_m(n) \cdot e^{j\varphi_m(n)} + \xi_m(n) \tag{1}$$

where $x_m(n)$, $h_m(n)$ and $\varphi_m(n)$ denote the transmitted signal, the channel-impulse response and the phase noise, respectively, while $\xi_m(n)$ is the AWGN noise with variance σ^2. After removing the cyclic prefix and taking the DFT, the resulting frequency-domain signal is given by

$$R_m(k) = X_m(k)H_m(k)I_m(0) + \sum_{\substack{l=-N/2 \\ l\neq k}}^{N/2-1} X_m(l)H_m(l)I_m(l-k) + \zeta_m(k) \tag{2}$$

where $X_m(k)$, $H_m(k)$ and $\zeta_m(k)$ are the corresponding frequency domain expressions of $x_m(n)$, $h_m(n)$ and $\xi_m(n)$ respectively. $I_m(i)$ is a function of $\varphi_m(n)$ given by

$$I_m(i) = \frac{1}{N}\sum_{n=-N/2}^{N/2-1} e^{j2\pi ni/N} e^{j\varphi_m(n)}, \quad i = -\frac{N}{2}, \ldots, \frac{N}{2}-1. \tag{3}$$

From (2), we notice that random phase noise not only causes common phase error (CPE), i.e., the phase rotation of the desired sample, but also introduces inter-carrier interference (ICI). Therefore, it degrades receiver performance.

For IEEE 802.11a standard, there are 64 $(N = 64)$ samples per symbol, including data sample set S_D with $N_D = 48$ samples, pilot sample set S_P with $N_P = 4$ samples and null sample set S_N with $N_N = 12$ samples. An accurate channel estimate can be obtained using pilot preambles of the data packet [2], which can be further improved using two consecutive pilots with channel invariant during a packet period. In this paper, we assume that channel frequency response is known within the whole packet.

The ICI, indicated by the second term in (2), is a random variable which is independent of $\zeta_m(k)$. $X_m(k)$ can be treated as mutually independent random variables independent of $H_m(k)$ with zero mean and variance E_x. Therefore, the ICI term in (2) has a zero mean. By choosing the appropriate exponential power delay profile [2, Appendix A], the channel correlation $E\left[|H_m(k)|^2\right]$ is equal to 1. Furthermore, with the same method of [2], it can be shown that, for any power delay profiles, $E\left[|H_m(k)|^2\right]$ is constant which is independent of k. Hence, without loss of generality, we take throughout this paper $E\left[|H_m(k)|^2\right]$ as 1. We can also use from [5] the following approximation which is improved when N is large enough

$$\sum_{\substack{l=0 \\ l\neq k}}^{N-1} E\left[|I_m(l-k)|^2\right] = 1 - E\left[|I_m(0)|^2\right] = \frac{2\pi\beta T}{3} \tag{4}$$

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where β is the one-sided 3-dB linewidth of the Lorentzian power density spectrum of the free running carrier generator and T indicates the symbol period.

From (4) and the aforementioned discussion, we can get the approximation of the variance of the ICI term

$$E\left[\left|\sum_{\substack{l=-N/2\\l\neq k}}^{N/2-1} X_m(l)H_m(l)I_m(l-k)\right|^2\right] = \sum_{\substack{l=-N/2\\l\neq k}}^{N/2-1} E\left[|X_m(l)|^2\right] E\left[|H_m(l)|^2\right] E\left[|I_m(l-k)|^2\right] \cong 2\pi\beta T\left(N-N_N\right)\frac{E_x}{(3N)}. \quad (5)$$

Thus we can write (2) as

$$R_m(k) = X_m(k)H_m(k)I_m(0) + \varepsilon_m(k) \quad (6)$$

where $\varepsilon_m(k)$, the summation of the noise and ICI terms, is a random variable with zero mean and variance $\sigma_\varepsilon^2(m,k)$. With approximation in (5), we note that $\sigma_\varepsilon^2(m,k)$ keeps the same for $\forall k \in S_D$ and can be approximated by $\sigma_\varepsilon^2(m)$. We also notice that $\sigma_\varepsilon^2(m,k)$ can be different for $k \in S_N$, i.e., for the null subcarriers which acts as the guard band, since the analog bandpass filter before RF down conversion will color the AWGN noise within these subcarriers, but would hardly affect the ICI term caused by phase noise within these subcarriers. This is because phase noise occurs mainly due to the receiver oscillator after RF down conversion, rather than that caused by the transmitter oscillator.

By using the MMSE equalization, the transmitted data samples can be estimated by

$$\hat{X}_m(k) = R_m(k)^* C_m(k) \quad (7)$$

where $C_m(k)$ is obtained by the MMSE criterion as follows:

$$C_m(k) = \frac{I_m^*(0)H_m^*(k)}{|I_m(0)H_m(k)|^2 + \frac{\sigma_\varepsilon^2(m)}{E_x}} \quad (8)$$

where $(\cdot)^*$ represents the conjugate operation. In the absence of phase noise, the MMSE equalizer for OFDM receiver can be further reduced to

$$C_m(k) = \frac{H_m^*(k)}{|H_m(k)|^2 + \frac{\sigma^2}{E_x}}. \quad (9)$$

III. Phase Noise Suppression (PNS) Algorithm

In order to implement (8), we have to know $I_m(0)$ and σ_ε^2 first. It is very clear from (3) that, although $I_m(0)$ changes from symbol to symbol, it is the same for all the samples of symbol m and thus can be estimated by using pilot symbols [6]. Note that in [6], the phase of $I_m(0)$ was estimated separately from each pilot sample and averaged to get the final estimate, which is then used for CPE compensation. To avoid extra computation for obtaining the phase of $I_m(0)$, instead of its phase, we directly estimate $I_m(0)$ from each sample and use them to obtain the final estimate of $I_m(0)$. For 802.11a standard, e.g., we can take advantage of 4 pilot samples within a symbol, at position -21, -7, 7 and 21 [1]. The least-squares (LS) method is applied to minimize the cost function

$$\min_{I_m(0)} \sum_{k\in S_P} |R_m(k) - I_m(0)X_m(k)H_m(k)|^2 \quad (10)$$

which leads to the estimate

$$\tilde{I}_m(0) = \frac{\sum_{k\in S_P} R_m(k)X_m^*(k)H_m^*(k)}{\sum_{k\in S_P} |X_m(k)H_m(k)|^2}. \quad (11)$$

One may argue that (11) may not be accurate with so few numbers of pilot symbols. However, we can first use (11) to estimate $I_m(0)$; and, after equalization and detection, decision feedback is used for further enhancement of the performance of (11) by using

$$\hat{I}_m(0) = \gamma\tilde{I}_m(0) + (1-\gamma)\tilde{I}'_m(0) \quad (12)$$

where γ is the forgetting factor. $\tilde{I}'_m(0)$ takes the same form of (11) except that the observations are replaced by the detection results of the data sample set S_D.

Before implementing the MMSE equalizer of (8), we have to know the ICI plus noise energy $\sigma_\varepsilon^2(m)$. Using (5), the ICI energy (thus the ICI plus noise energy $\sigma_\varepsilon^2(m)$) is approximated as being the same for different subcarriers, but difficult to be obtained in practice since we don't know β and σ^2. The question is then whether it is possible to estimate from null subcarriers the part of the ICI plus noise energy corresponding to data subcarriers. First, the approximation of the ICI energy, derived in (5), is independent of k. Second, in spite of the colored noise due to the analog bandpass filter, for sufficiently high signal-to-noise ratio (SNR) level, the ICI term at the null subcarriers is dominant over the noise. Therefore, despite the existence of the colored noise, the estimation of ICI plus noise energy of null subcarriers can be used to approximate that of data subcarriers and hence used in the MMSE equalizer of (8).

By evaluating the energy of those null samples, we can get an estimate of $\sigma_\varepsilon^2(m)$ by

$$\hat{\sigma}_\varepsilon^2(m) = \frac{1}{N_N}\sum_{k\in S_N} |R_m(k)|^2 \quad (13)$$

but not without some estimation errors that may affect the algorithm performance. This will be checked by comparing the analytical results with those obtained through computer simulation.

Based on the discussion above, the proposed post-FFT PNS algorithm is described by the following steps.

1) Obtain the estimate $\hat{I}_m(0)$ of CPE by (11) as well as the estimate $\hat{\sigma}_\varepsilon^2(m)$ of ICI plus noise energy by (13).
2) Use (8) to calculate the equalizer coefficients for N samples of each symbol, where the unknown parameters are replaced by the estimated values from step 1.
3) Use (7) to get the estimated signals for data detection. Decision feedback is used to update the estimate of $I_m(0)$ by implementing (12).

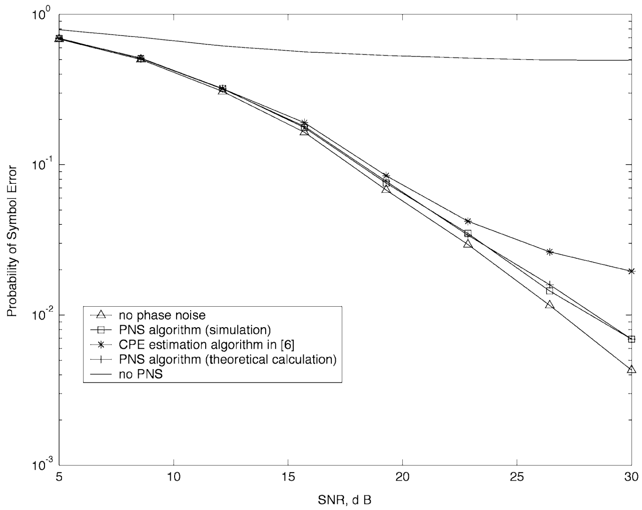

Fig. 1. PNS algorithm performance for 16 QAM, with phase noise energy $4\pi\beta T$ equal to 0.0384 rad^2.

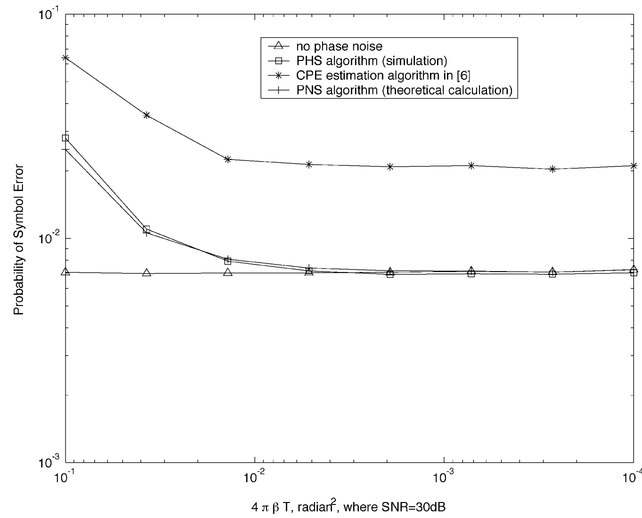

Fig. 2. PNS algorithm, symbol error probability versus phase noise energy $4\pi\beta T$.

4) Go through steps 1–3 until all symbols have been processed.

To judge on the computational complexity of the proposed algorithm, we note that, compared with the conventional MMSE equalizer of (10), the PNS algorithm is quite cost effective by requiring only $1+2(N_P+N_D)/N$ extra complex multiplication per sample (subcarrier).

IV. Numerical Results

The PNS algorithm is evaluated for a normalized frequency-selective Rayleigh fading channel by Monte Carlo trials. Six multiple radio paths have been chosen for simulation. Channel impulse response remains static within a frame containing 16 symbols, but varies independently from frame to frame. Transmitted data is constructed according to IEEE 802.11a WLAN standard in [1]. The receiver filter is matched to the transmit filter defined in [1, clause 17.3.9.6.2]. 16 QAM, which is more sensitive to phase noise than M-PSK, is used in the simulation to evaluate the performance of the PNS algorithm under the modulation. Phase noise is simulated using an independent Gaussian increment between adjacent samples (subcarriers) as proposed in [6]. The forgetting factor γ equals 0.1. Given phase noise and the AWGN noise, the theoretical values of $I_m(0)$ and $\sigma_\varepsilon^2(m,k)$ based on (3) and (5) is calculated to examine the effectiveness of the proposed algorithm. Simulations results with the PNS algorithm are compared with the theoretical calculation as well as the result obtained with the CPE estimation algorithm of [6] in Fig. 1.

It is easy to conclude from Fig. 1 that phase noise causes an irreducible error floor of OFDM receiver performance, which is unacceptable in practice. The proposed PNS algorithm, however, exhibits excellent performance. In this regard, it outperforms the CPE estimation algorithm proposed in [6], having the performance that is very close to the theoretical calculation and nonphase-noise case.

From Fig. 2, we note that the proposed PNS algorithm always outperforms the CPE estimation in [6] for different phase noise conditions. It is well known that the variance of phase noise is usually much less than 1 rad^2 (or $4\pi\beta T$ is much less than 1 rad^2). From Fig. 2, if $4\pi\beta T$ is of the order of 10^{-2} or lower, its performance is quite comparable with that of nonphase-noise case.

V. Conclusions

In this letter, we proposed a new and simple PNS algorithm specifically for IEEE 802.11a standard, which has an outstanding performance when dealing with phase noise. This algorithm takes advantage of pilot and null samples given in the IEEE 802.11a standard, as well as decision feedback and successfully suppresses phase noise. It has been shown that this algorithm has much better performance than other algorithms while keeping computational complexity low. The algorithm can be further extended to any OFDM systems.

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