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A PHASE SENSITIVE INTERFEROMETER TECHNIQUE FOR THE MEASUREMENT OF THE FOURIER TRANSFORMS OF SPATIAL BRIGHTNESS DISTRIBUTIONS OF SMALL ANGULAR EXTENT

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Summary

A method is described whereby the amplitude and phase of the complex Fourier transform of a spatial brightness distribution of small angular extent may be uniquely determined from a series of measurements with a triple interferometer system. Absolute measurement of the amplitude function is available, whilst measurements of phase are relative to a datum obtained at short aerial spacings. A practical radio frequency interferometer incorporating the principle is described and its operation is discussed.

1. *Introduction.*—Unique measurements of the brightness distribution across the cosmic radio sources have hitherto been limited to those sources which subtend a large angular diameter. It has been possible to obtain orders of magnitude for the angular diameter of some of the smaller intense sources and in a few cases to give models for their general intensity distributions (Hanbury Brown, Jennison and Das Gupta 1952, Mills 1952, Jennison and Das Gupta 1953, 1956). These models have been obtained from the amplitude functions of the Fourier transforms of the source brightness distributions without corresponding information with regard to the phase. As such, the solutions are not unique and may not correspond in detail to the actual fine structure of the sources.

The amplitude function of the Fourier transform of the brightness distribution across a source may be determined from the visibility of the fringes in the reception pattern of an interferometer. The phase of the transform may be obtained from the phase of the fringes relative to a known datum. This datum may be obtained by accurately measuring the position of the source over various interferometer baselines.

The practical limitations imposed on these measurements are severe and render conventional techniques untenable when the baseline of the interferometer system exceeds about one thousand wave-lengths. At greater distances, when it is necessary to use a radio link between the channels, the measurement of fringe amplitude may be accomplished with an uncertain error if a powerful automatic gain control system is used on the receiving equipment to maintain the mean level of the signal constant over the period of the observations. The aerial gain, noise factor, bandwidth and dispersion are assumed constant or must be continuously calibrated (Mills 1953). If both the receiving aerials are sufficiently large the uncertainty may be removed by dispensing with the automatic gain control and measuring the increment in total power due to the source in each channel (Jennison and Das Gupta 1956). Neither of these methods allows for error due to phase dispersion in the systems and the latter method has the practical disadvantage of requiring a very large portable aerial. The method used by Smith (1952) is most reliable on short baselines but is not applicable where the distance between aerials is considerable. In these circumstances intermediate checks cannot rapidly be made and the gain of the most distant station cannot necessarily be relied upon to remain constant over long periods or on removal to a new site.

The conventional method for the measurement of the phase of the fringe system by timing the transit of the central fringe is not applicable at long baselines, especially when radio links are used to span the baseline. The combinations of variable phase errors in the electronic equipment and in the local oscillator and intermediate frequency signal link paths, together with the difficulties arising from unequal signal paths when the terrain is not level, render the phase of the transform indeterminate at spacings in excess of about one thousand wave-lengths.

An interferometer system designed to overcome the above defects was constructed in 1954 and its principle of operation is discussed below.

2. *Principle of operation.*—The main features of the apparatus are shown in Fig. 1. It consisted of three aerial systems, tuned to a frequency of 127 Mc/s, spaced in a line along which the measurements were to be made. One of the aerials was a large fixed array whilst the other two were small and portable. Associated with the portable systems were radio link transmitters to relay the received signals back to the fixed station. Provision was made to rotate, continuously and synchronously, the phase of the two relayed signals prior to equalizing the time delays, and combining all three signals in pairs in three switched interferometers, the outputs of which were displayed on a three-pen recorder. The total power received from the fixed aerial was displayed on a separate recorder.

The measurement of phase

The relative phases of the signals induced in the three aerials are indeterminate by the time they reach the switched interferometer units, due to the combinations of errors previously referred to. Thus the fringe systems produced at the outputs of these units and displayed upon the pen recorder are also of uncertain phase relative to an absolute datum based upon the time of transit of the source. It will be shown that the relative phase contribution due only to the brightness distribution of the source may be obtained by a comparison of the arguments of these fringe systems.

Treating the signals from aerial A as the phase reference and taking the origin of the transform at zero within the source, let ξ_{AB}, ξ_{BC} and ξ_{AC} represent the phase of the transform at spacings AB, BC and AC respectively; let $\omega_1(t)$ and $\omega_2(t)$ represent the time variable phase rotation of the scan at B and C respectively; let ψ_1 and ψ_2 represent the phase angle introduced by the position of the source in relation to the collimation plane at B and C respectively; and let δ_1 and δ_2 represent the phase error in the equipment at B and C respectively.

The argument of the fringes when channel B interferes with A will be

$$\xi_{AB}+\psi_1+\delta_1+\omega_1(t) \tag{1}$$

and the argument of the fringe when channel C interferes with channel B

$$=\xi_{BC}+(\psi_2-\psi_1)+(\delta_2-\delta_1)+\omega_2(t)-\omega_1(t). \tag{2}$$

The argument of the fringes formed by channel C interfering with channel A

$$=\xi_{AC}+\psi_2+\delta_2+\omega_2(t). \tag{3}$$

Adding the arguments of the two fringe systems AB and BC, we obtain

$$\xi_{AB}+\xi_{BC}+\psi_2+\delta_2+\omega_2(t). \tag{4}$$

Comparing equations (3) and (4) it will be seen that they represent the arguments of two fringe systems of identical frequency and relative phase

$$\phi=\xi_{AC}-(\xi_{AB}+\xi_{BC}). \tag{5}$$

This result is independent of the phase errors and the angular velocity of the phase rotation. It enables an estimate to be made of relative phase contributions due to the source structure only, and is applicable in the general case where the aerials are spaced by arbitrary distances. It will be observed that the position of the source relative to the collimation plane does not appear in the expression $\xi_{AC}-(\xi_{AB}+\xi_{BC})$ and hence absolute phase is not directly obtainable.

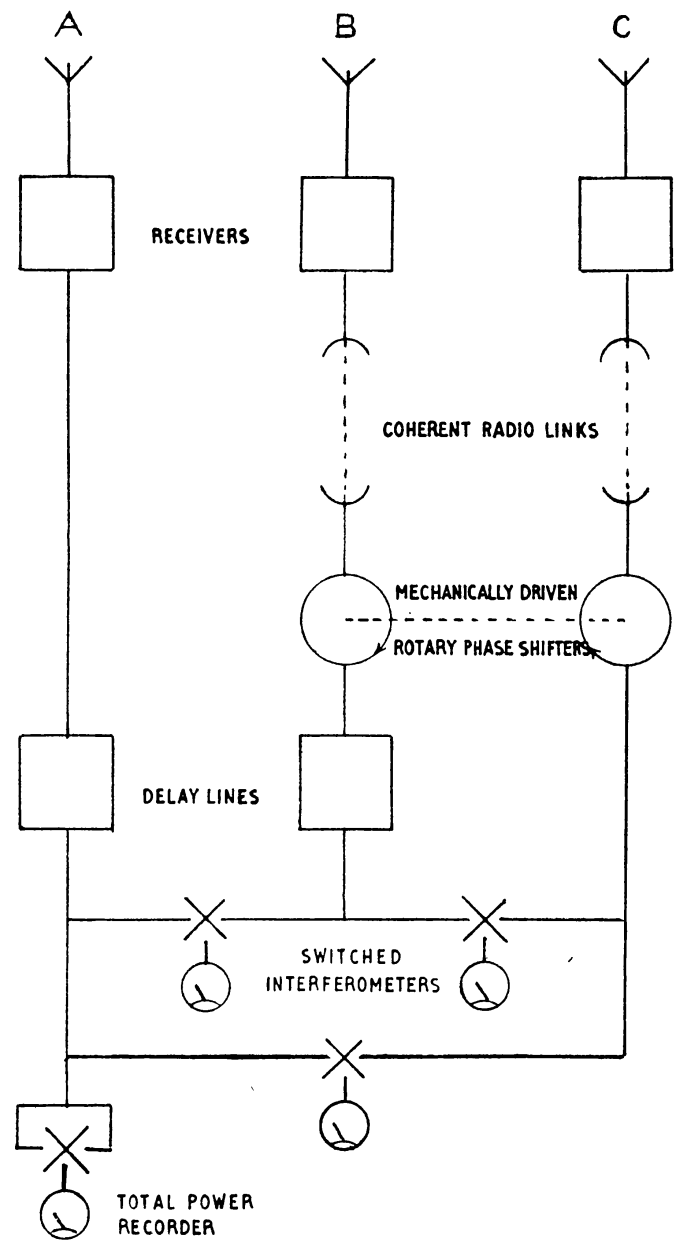

FIG. 1.—*The principal features of the phase-sensitive interferometer.*

It is apparent that if the spacings AB and BC both lie within the first maximum of the Fourier transform of a simple symmetrical distribution, then $\xi_{AB}=\xi_{BC}=0$. In these circumstances $\phi=\xi_{AC}=0$ if AC lies within the first maximum and $\phi=\xi_{AC}=\pi$ if AC lies within the second maximum.

If the transform is complex but the positions of the aerials are adjusted so that AB = BC, equation (5) reduces to

$$\phi=\xi_{AC}-2\xi_{AB}$$

and a direct check on the relative phase over twice the spacing may thus be obtained.

In the more general case, when the transform is complex and the aerial spacings unequal, the expression (5) may be used for the measurements in the following manner.

The two portable aerials B and C are moved away from the home station A with their baselines in any ratio until a phase displacement is recorded. This may be most simply done by keeping one station, B, at some convenient distance from A and moving the other until a displacement is observed. Within this range of spacings we may put

$$\xi_{AB}=\xi_{BC}=\text{constant.}$$

It is apparent that the phase function does not vary, within observational error, over intermediate spacings within the maximum separation AC, and hence both portable stations may be moved to more distant sites B′ and C′ subject only to the limitation that AB′ < AC and B′C′ < AC.

The relative phase occurring over the extension of the baseline to points in the range AC′ may now be established. The process may be repeated until the whole phase function is eventually mapped. It will be observed that this general method gives rise to a cumulative error, and the accuracy of the determination of the transform depends upon the ratio of the maximum aerial spacings used to the spacings at which a displacement is first observable, and the error in the assessment of the phase at this spacing.

The measurement of amplitude

From direct measurement on the pen recordings it is possible to form the function η such that

$$\eta=\frac{|AB|\times|AC|}{|BC|\times|A|^2},$$

where $|AB|$, $|BC|$ and $|AC|$ are the moduli of the fringe systems between channels A and B, B and C and A and C respectively, and $|A|^2$ is the total power due to the source recorded in channel A.

It will be shown that the function η is a measure of the relative amplitude and, in a special case, the absolute amplitude, of the Fourier transform of the intensity distribution of the source.

Let $g_1\alpha$, $g_2\beta$ and $g_3\gamma$ be the mean amplitudes of the signal at the outputs of channels A, B and C, where g_1, g_2, g_3 and α, β, γ refer to the overall voltage gains and input signal levels respectively. Let $f(ab)$, $f(bc)$ and $f(ac)$ be the amplitudes of the transform for baselines AB, BC and AC. Then

$$\eta=\frac{|AB|\times|AC|}{|BC|\times|A^2|}=\frac{G_1g_1\alpha g_2\beta f(ab)\times G_3g_1\alpha g_3\gamma f(ac)}{G_2g_2\beta g_3\gamma f(bc)\times G_4(g_1\alpha)^2}=\frac{f(ab)\times f(ac)}{f(bc)}\times\frac{G_1G_3}{G_2G_4}, \tag{6}$$

where G_1, G_2 and G_3 are the conversion gains for the cross multiplying recorder systems between channel A and B, B and C, and A and C respectively and G_4 is the conversion gain of the total power recorder system in channel A. The expression for η in equation (6) is not dependent on the gain of any part of the equipment other than the final cross multiplying and total power recording units. The equipment was designed so that these units were operated at high level and were extremely stable. Sockets were provided so that, immediately prior to and following a run, the inputs normally fed from the separate channels A, B and C could be connected together and provided with a single noise signal. From the constant deflection of the recording pens this yielded a normalizing function:

$$K = \frac{G_1 G_3}{G_2 G_4} \tag{7}$$

corresponding to a correlation coefficient of unity between the channels.

Substituting the normalizing function K in equation (6), the general operational function η' may be obtainable, where

$$\eta' = \frac{|AB| \times |AC|}{K \times |BC| \times |A|^2} = \frac{f(ab) \times f(ac)}{f(bc)}. \tag{8}$$

This expression only involves terms in the amplitude function of the Fourier transform of the brightness distribution across the source. It will be observed that in the special case where station B is midway between stations A and C, $f(ab) = f(bc)$ and equation (8) reduces to

$$\eta' = f(ac).$$

The absolute value of the amplitude of the transform may therefore be determined.

In general, when $AB \neq BC$, equation (8) may be used to obtain a relative measurement of the amplitude at one spacing from a knowledge of the amplitude at the other two. Measurements may therefore be made concurrently with the phase measurements, commencing at short spacings and extending the baselines as each value of the amplitude function is established.

3. *A practical radio frequency interferometer.*—The principles outlined above were incorporated in an interferometer constructed at Jodrell Bank and operating on a frequency of 127 Mc/s.

The aerial A consisted of 160 full-wave dipoles in a collinear broadside array. This aerial had a beam width of approximately 3° × 10°. Aerial B usually consisted of 20 full-wave dipoles in a collinear broadside array giving a beam pattern 24° × 10°. Aerial C was either a single folded dipole with reflecting screen or an array of four similar dipoles.

The basic lay-out of the electronic equipment is shown in Fig. 2. The out-stations each employed two crystal oscillators to feed their first and second mixers. The beat between these two oscillators was transmitted as a coherent local oscillator reference. The noise signal was transmitted from the broadband class A amplifier following the second mixer. Transmitter powers were normally about 0·25 and 1 watt from the signal and local oscillator respectively. The relay transmitting and receiving aerials consisted of single dipoles with parasitic reflectors, the polarization of the signal and local oscillator aerials was crossed and orthogonal to the corresponding aerials of the other radio link.

The receiving equipment for the radio links consisted of separate preamplifiers for the local oscillator and signal channels. The low level of the signal from the received local oscillator was converted coherently to a signal of the order of volts by being mixed with the tenth harmonic of a master local oscillator on 11·5 Mc/s, and subsequently passed through a narrow band amplifier at approximately 30 Mc/s. The output of this unit was applied to a mixer following the signal link preamplifier and yielded an output on an intermediate frequency of 12 Mc/s.

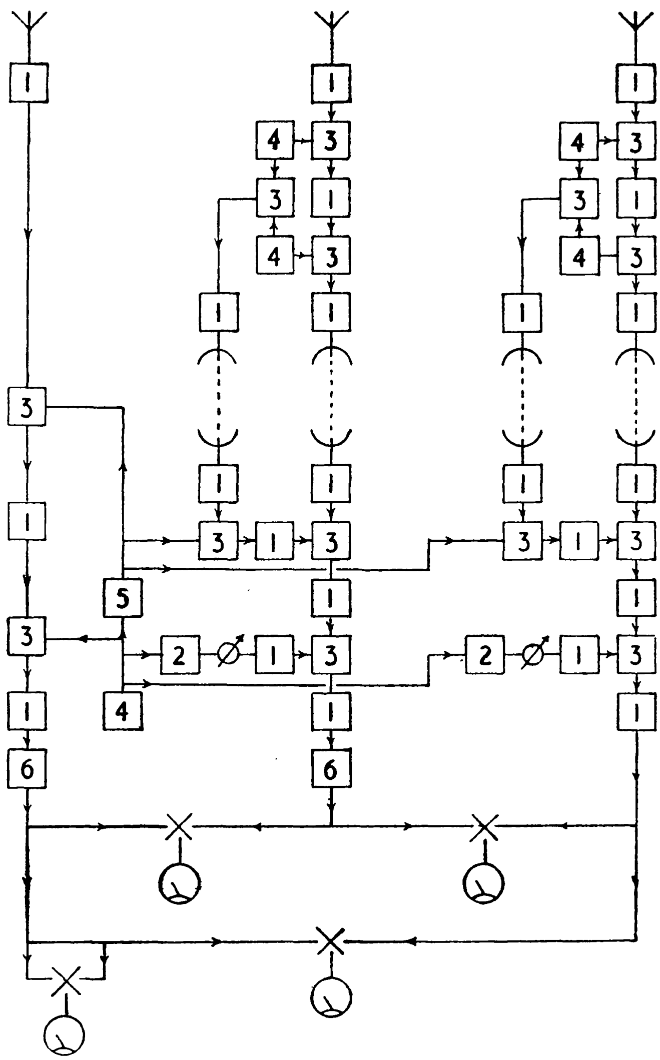

FIG. 2.—*Basic block diagram of the phase-sensitive interferometer. The numbers in the blocks correspond to:* 1, *amplifier;* 2, *buffer;* 3, *mixer;* 4, *oscillator;* 5, *frequency multiplier;* 6, *delay line.*

The corresponding equipment in channel A consisted only of a preamplifier, a mixer combining the 115 Mc/s master local oscillator frequency with the signal frequency, and an I.F. amplifier on 12 Mc/s.

The 12 Mc/s I.F. amplifiers in all three channels were followed by mixer valves converting the frequency to 500 kc/s. The conversion was performed by mixing with the master local oscillator on 11·5 Mc/s, directly in the case of channel A, but via continuously rotating phase shift networks in the case of channels B and C. These phase shift networks consisted of resistance capacity quadrature circuits applied to a four pole condenser system having a rotating search vane of high dielectric constant. The amplitude of the output of this unit was substantially constant, and the tracking error in the phase angle was better than 2°. The phase rotating systems in channels B and C were mechanically coupled to friction rollers adjustable in position on a single conical shaft driven from a synchronous electric motor. A large but constant ratio between the rates of phase rotation of the two channels was thus available.

The three signals at 500 kc/s were confined to matched bandwidths of 200 kc/s, or, in certain measurements, 50 kc/s. The signals from channels A and B were then delayed by suitable lengths of artificial lines before being applied to the cross multiplying and total power recording unit.

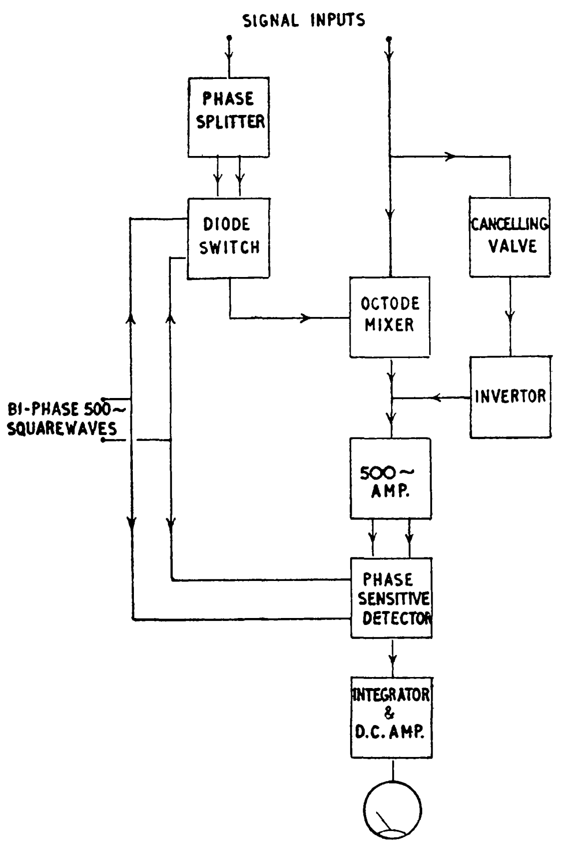

FIG. 3.—*Block diagram of one of the cross-multiplier units shown as a cross in Figs.* 1 *and* 2.

The three cross multiplying circuits and the total power recording circuits were identical except that the two input leads of the total power recorder were connected together. A basic block diagram of a single section is shown in Fig. 3. 500-cycle square waves were used for switching, whilst the actual multiplication was performed in an octode valve; an auxiliary circuit with a balancing valve eliminated single channel noise by cancelling the effect of non-linearity on the variable-μ signal grid. Two phase splitters and switching circuits only were required to serve the three multipliers and one square law detector which operated on a similar principle.

4. *The equipment in operation.*—The equipment was extensively tested on short baselines within the range of readings previously obtained with conventional

techniques. During the tests a slight dispersion in the radio links was observed. This was reduced by improved design and finally removed entirely by the introduction of compensating networks.

On the completion of the initial tests the equipment was applied to the measurement of the Cygnus A and Cassiopeia A radio sources. A typical record is shown in Fig. 4. The different declinations of the two sources necessitated the use of slightly different settings of the artificial delay lines and rotating phase shifters. At most aerial spacings it was possible to use a setting for the rotary phase shifters such that both sources gave fringes of suitable period but of opposite angular velocity. The direction of the velocity vector determined the interpretation of an observed phase shift as leading or lagging, and hence the sense of the asymmetry of the source.

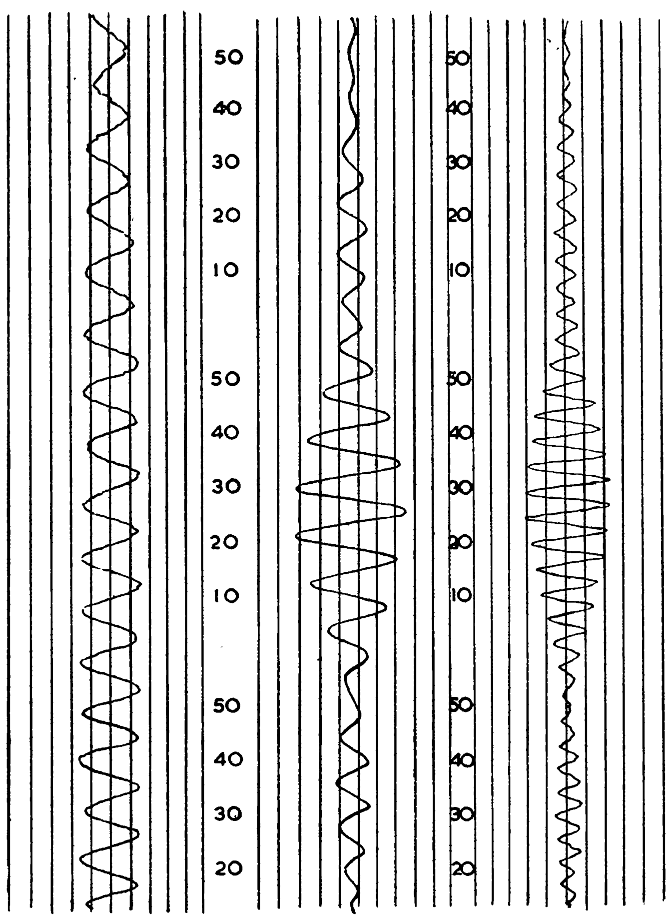

Fig. 4.

It was found that the measurement of the argument of the fringes was a considerably more powerful method for the determination of the order of the maxima of the transform patterns than a similar small number of measurements of the fringe modulus. The positions of the minima of the Cassiopeia source could be most accurately assessed by observing the change of phase in their immediate vicinity.

The measurements made on the Cygnus source confirmed the bifurcation of this object (I.A.U. Symposium on Radio Astronomy, 1955), whilst the measurements on the Cassiopeia source established three maxima of the Fourier transform of its intensity distribution and showed the existence of a faint component of larger angular diameter off-set from the principal component of the source. The details of this work will be described in subsequent papers.

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