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A Piloted Notch Time-Frequency Information Based Variable Step-Size Algorithm

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Abstract—In this paper, we have proposed a new steering direction determination mechanism and step-size update algorithm for piloted adaptive notch filter architecture. A time-domain averaging based gradient analysis of the piloted notch cost function has been utilized to determine the direction of the main notch with respect to the input sinusoid frequency. The steering direction will indicate the distance between the frequency of the input sinusoid and the zero of the main notch. This frequency domain information has been interleaved with time domain information to develop a novel algorithm for determination of variable step-sizes for improved speed of convergence with comparatively huge reduction in steady-state mean square error (MSE). The simulation results verifies the excellent performance exhibited by our proposed steering mechanism and the step-size update algorithm over original piloted adaptive notch filter with respect to the speed of convergence and MSE.

I. INTRODUCTION

The adaptive notch filters are popularly implemented using the least mean square (LMS) algorithm [1]-[2] in which the notch frequency is updated iteratively to eventually converge to the frequency of the input signal. The adaption step-size plays a vital role in determining the rate of convergence and the misadjustment at steady-state. A large value of step-size will increase the rate of convergence but will yield larger misadjustment, while a small value of step-size will decrease the rate of convergence but will yield smaller misadjustment. So, in view of this trade-off, a variable step-size LMS will always be preferred over a fixed step-size LMS as it will appropriately generate a larger step-size when the notch is away from the input frequency for faster convergence and a small step-size to be used when the notch frequency is close to the input frequency for smoother tracking.

In literature, various step-size update techniques [2]-[6] have been proposed which performs time-domain averaging on the output error to obtain suitable step-sizes. In [3], the sign changes between consequent gradient estimates have been used to update the step-size. While in [4], the authors have used the product of consecutive gradient estimates to update the step-size, the authors in [5] have updated the step-size as a function of the low pass filtered values of gradient estimates. In [6], the authors have replaced the linear update recursions of step-sizes by multiplicative update recursions. All this techniques were based on time domain information but the authors in [7] have proposed a novel method of estimating the distance between the notch frequency and the frequency of the input sinusoid with a filter architecture incorporating the piloted notches. This frequency domain information is used to choose an appropriate step-size value. Thus, it depends on the probability of correct

steering direction which shows degradation in performance under low SNR [8] with only two fixed step-size values.

In this paper, we have developed a more robust time-domain averaging based steering direction determination mechanism for the piloted notch filter architecture which will have higher probability of correct steering direction. The frequency information from this steering direction has been interleaved with time domain information to develop a step-size update algorithm which will improve the speed of convergence with very low MSE. Extensive computer simulations have been done to test the proposed method under various conditions.

The paper is structured as follows. In Section II, we have revisited the concept of piloted adaptive notch filter. In Section III, the new steering direction determination mechanism is introduced and in Section IV, the time-frequency information based step-size update algorithm has been formulated. In Section V, we have dealt with computer simulations for comparing the performance of the proposed scheme with original piloted adaptive notch filter. Finally, the conclusions have been drawn in Section VI.

II. THE PILOTED ADAPTIVE NOTCH FILTER ARCHITECTURE

In [7], an adaptive system was introduced consisting of a main notch and two piloted notches. These are second order IIR filters with transfer function as stated in (1), which have a pole followed by a zero and the zero of the main notch being sandwiched between that of the piloted notches as in Fig. 1.

$$H(z) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (1)$$

where, r is the pole radius which indicates the bandwidth of the notch filter. $W(n)$ is the adjustable filter weight updated adaptively using the LMS algorithm. The Widrow's technique [1] has been used to update $W(n)$ as shown in (2)

$$W(n+1) = W(n) - 2\mu(n)x(n-1)e(n) \quad (2)$$

where, μ is the adaptation step-size which influences the rate of convergence and the misadjustment. $x(n)$ is an intermediate filter state and $e(n)$ is the output of the filter. The input $y(n)$ is assumed to be a single sinusoid with additive white Gaussian noise as shown in (3)

$$y(n) = a \cos(\omega_0 n + \varphi) + s(n) \quad (3)$$

where, ω_0 is the frequency of the input signal, φ is a random phase constant, a is the magnitude of the input, and $s(n)$ is a zero mean white Gaussian noise.

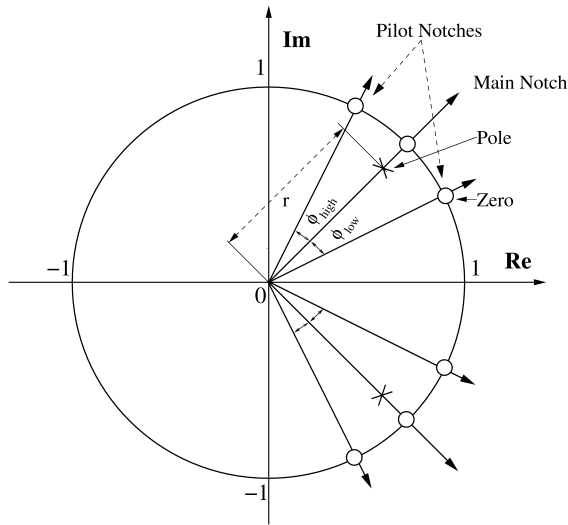


Fig. 1. Pole-zero plot of the piloted notch filter.

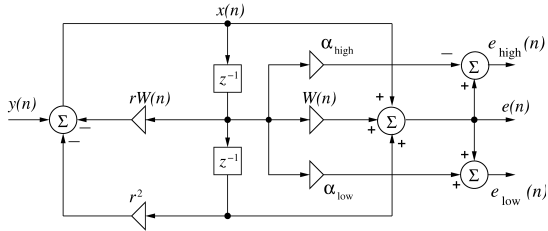


Fig. 2. Structure of the piloted notch filter.

The notch frequency at any time n can be calculated using (4)

$$\theta(n) = \cos^{-1}(-W(n)/2) \quad (4)$$

The notch frequency of one of the piloted notch is kept higher by ϕ_{high} and the other lower by ϕ_{low} . The complexity of the structure has been reduced by integrating the piloted notches with that of the main notch filter (as shown in Fig. 2) as they have the same denominator for all the notches. Thus, the transfer function of the piloted notches has been formulated as in (5) and (6) denoted by $H_{high}(z)$ and $H_{low}(z)$.

$$H_{high}(z) = \frac{1 + W_{high}(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (5)$$

$$H_{low}(z) = \frac{1 + W_{low}(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (6)$$

where, W_{high} and W_{low} has been defined as

$$W_{high}(n) = W(n) - \alpha_{high} \quad (7)$$

$$W_{low}(n) = W(n) + \alpha_{low} \quad (8)$$

The value of α_{high} and α_{low} corresponds to ϕ_{high} and ϕ_{low} respectively and can be calculated in advance for a particular application. This does not require high level of accuracy as the pilots have been used only to get the steering direction. Substituting the value of $W_{high}(n)$ and $W_{low}(n)$ in (5) and (6) would lead to

$$H_{high}(z) = H(z) - \frac{\alpha_{high}z^{-1}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (9)$$

$$H_{low}(z) = H(z) + \frac{\alpha_{low}z^{-1}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (10)$$

The output of the piloted notches being denoted as $e_{high}(n)$ and $e_{low}(n)$ can be calculated as

$$e_{high}(n) = e(n) - \alpha_{high}x(n-1) \quad (11)$$

$$e_{low}(n) = e(n) + \alpha_{low}x(n-1) \quad (12)$$

III. PROPOSED STEERING DIRECTION DETERMINATION MECHANISM FOR PILOTED NOTCH ARCHITECTURE

In this section, a time-domain averaging based steering direction determination mechanism has been proposed for piloted adaptive notch filter architecture. In [7], the information about the steering position has been determined using the signs of $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$. But, the probability of correct steering direction decreases with decrease in input SNR. This led to the development of a more robust steering mechanism to correctly steer the notch in the desired direction.

The input signal $y(n)$ is as shown in (3). The intermediate state $x(n)$ which will also be a sinusoid signal with change in amplitude and phase, can be expressed as

$$x(n) = A \cos(\omega_0 n + \beta) + \xi(n) \quad (13)$$

The error output $e(n)$ can be expressed as

$$e(n) = A \cos(\omega_0 n + \beta) + W(n)A \cos(\omega_0(n-1) + \beta) + A \cos(\omega_0(n-2) + \beta) + \eta(n) \quad (14)$$

where,

$$\eta(n) = \xi(n) + W(n)\xi(n-1) + \xi(n-2) \quad (15)$$

The product term $x(n-1)e(n)$ can be expressed as

$$\begin{aligned} x(n-1)e(n) &= [A \cos(\omega_0(n-1) + \beta) + \xi(n-1)][A \cos(\omega_0 n + \beta) \\ &+ W(n)A \cos(\omega_0(n-1) + \beta) + A \cos(\omega_0(n-2) + \beta) + \eta(n)] \\ &= A^2 \cos^2(\omega_0(n-1) + \beta)[W(n) + 2 \cos \omega_0] + \nu(n) \\ &= 2A^2 \cos^2(\omega_0(n-1) + \beta)[\cos \omega_0 - \cos(\theta(n))] + \nu(n) \end{aligned} \quad (16)$$

where, $\nu(n)$ is the noise term. This product term $x(n-1)e(n)$ is time averaged to obtain a process $\Psi(n)$, using a first-order filter to reduce the effect of noise term for the determination of correct steering direction as explained below:

$$\Psi(n) = \gamma\Psi(n-1) + x(n-1)e(n) \quad (17)$$

where, γ is the weighting parameter. Similarly, the processes $\Psi_{high}(n)$ and $\Psi_{low}(n)$ can also be obtained as

$$\Psi_{high}(n) = \gamma\Psi_{high}(n-1) + x(n-1)e_{high}(n) \quad (18)$$

$$\Psi_{low}(n) = \gamma\Psi_{low}(n-1) + x(n-1)e_{low}(n) \quad (19)$$

From (16) and (17), it can be inferred that if $\Psi(n)$ is positive at a particular instant n , then it indicates $\omega_0 < \theta(n)$. Inversely, if $\Psi(n)$ is negative at a particular instant n , then it indicates $\omega_0 > \theta(n)$. This is assumed to be true considering the noise has been reduced by the averaging process. Thus, the relative positions of ω_0 can be obtained from the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$. If the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$ are

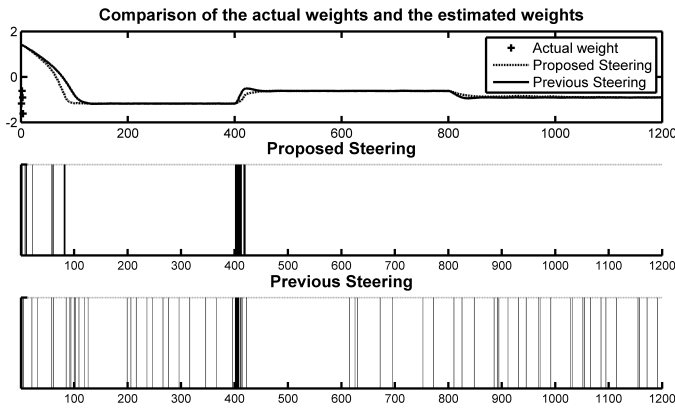


Fig. 3. Graphical analysis for steering mechanism.

same, then it can be concluded that the input frequency is away from the main notch. And, if they have different signs, then it will indicate that the input frequency has been sandwiched in between the piloted notch frequencies, that is, the input frequency is closer to the main notch. Simulations have been done to compare the proposed mechanism to the one used in [7]. Fig. 3 shows the relative improvement of the steering directions where the vertical dark coloured lines indicates the time-instants of wrong prediction of steering direction by the respective algorithms.

IV. PROPOSED TIME-FREQUENCY INFORMATION BASED STEP-SIZE UPDATE ALGORITHM

In this section, we have proposed a time-frequency information based step-size update adaptive algorithm for piloted adaptive notch filter. The information of the steering direction of the two-piloted notch structure can be mapped into frequency domain information as two levels being the input frequency at

- large distance from the main notch (denoted as L),
- small distance from the main notch (denoted as S).

The position of the notch structure at a particular level in the frequency domain helps us to define a suitable initial step-size for the weight update equation. As discussed earlier, a larger adaption step-size, μ_L , should be used when the distance between the zero of the main notch and the frequency of the input sinusoid is large. And a smaller step-size, μ_S , should be used when the zero of the notch is in the close vicinity of the frequency of the input sinusoid. It has been observed through simulations that the notch iterates for more than one iteration in a particular frequency domain level. This brings in a motivation to update the step-size for that region. Thus, for example, if it is observed that the notch remains in region L for consecutive iterations, then the step-size μ_L must be incremented by δ_L for faster convergence. The step-size $\mu(n)$ which is initialized by an experimentally obtained suitable value corresponding to a frequency domain level is updated at each sampling instance based on the process $\Phi(n)$, obtained by low pass filtering the product term $x(n-1)e(n)$, as in (16), to reduce the inherent noise. The $\Phi(n)$ can be expressed as

$$\Phi(n) = \alpha\Phi(n-1) + x(n-1)e(n) \quad (20)$$

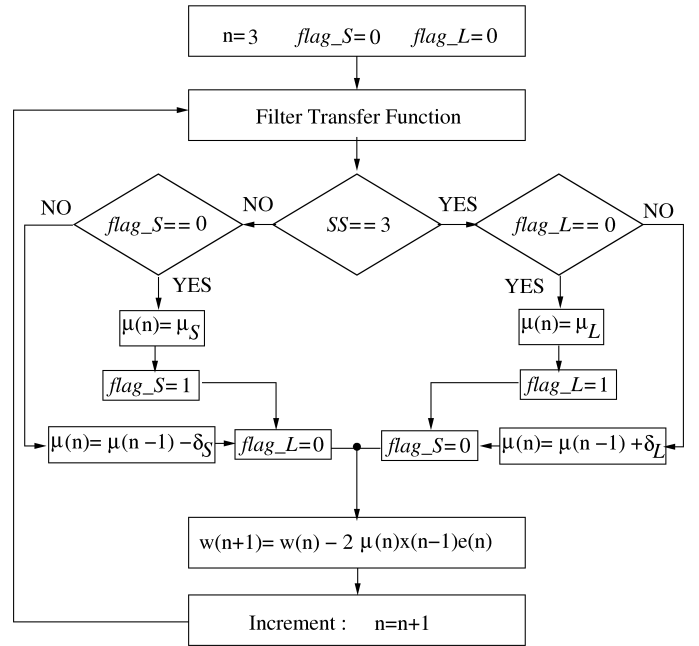


Fig. 4. Flow diagram for step-size update.

where, α is the weighting parameter and has value close to unity. The change in step-size for each level is formulated as

$$\delta_L = \rho_L * \{ |\Phi(n)| \} \quad (21)$$

$$\delta_S = \rho_S * \{ |\Phi(n)| \} \quad (22)$$

where, ρ_L and ρ_S are the scaling factors. The value of ρ_L and ρ_S are different for L and S as it will define the change in the adaption step-sizes in a particular region. While, ρ_L will effect the speed of convergence, the value of ρ_S will control the variance in the convergent frequency, once the zero of the notch is in the vicinity of the input frequency. The step-size is updated using

$$\mu(n) = \mu(n-1) + \delta_L \quad (23)$$

$$\mu(n) = \mu(n-1) - \delta_S \quad (24)$$

The values of δ_L and δ_S will be chosen according to the frequency domain information. Thus, when the notch is observed to be in level L , multiple large valued step-sizes will be used till the notch is steered in the desired direction. Once the notch is steered in the desired direction, that is, in the region of the frequency to be tracked, a smaller value of step-size will help in convergence which can be updated at each sampling instance based on the same process $\Phi(n)$. Thus, we are using multiple step-sizes instead of only two step-sizes for faster and smooth tracking of the input frequency. The step-size is checked at every iteration to stay within the bounds to ensure the stability of LMS algorithm [9] as shown below:

$$0 < \mu(n) < \frac{1}{3 \text{tr}\{\mathbf{R}\}} \quad (25)$$

where, $\text{tr}\{(\cdot)\}$ denotes trace of the matrix (\cdot) and \mathbf{R} is the autocorrelation matrix of the input signal.

Figure 4 shows the algorithm to be followed to update the step size. The variables $flag_L$ and $flag_S$ have been taken

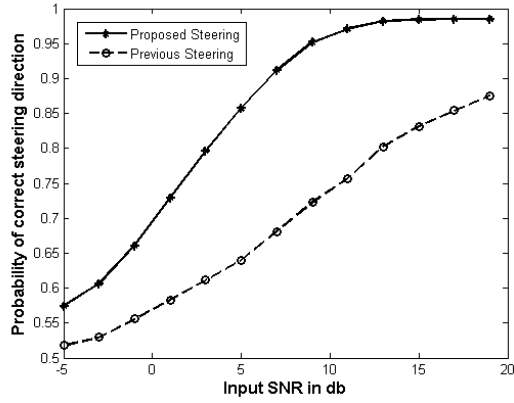


Fig. 5. Probability of correct steering direction for different SNR.

to check the position of the notch if it has continued to stay in the level L and S for consecutive iteration respectively. The variable SS is the absolute sum of the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$ to determine the steering direction for an iteration.

V. SIMULATION RESULTS

We have done computer simulations to demonstrate the excellent performance of the proposed new steering direction detection mechanism and step-size update algorithm over the original piloted notch filter in [7]. The simulations are performed with a data length of 1000 samples over 100 runs to generate the results of each plot. The bandwidth of all the notches has been kept same at $r = 0.9$. The parameters for piloted notch filter, α_{high} and α_{low} are kept symmetrical and equal to 0.4. The step-size μ_L and μ_S are taken as 3×10^{-2} and $(1/3) \times 10^{-2}$ respectively. The γ is taken to be 0.6 while the value of α is 0.95. The frequency of the input sinusoid is 0.3π . The simulation have been done over a range of SNR values to compare the robustness of the proposed method. Fig. 5 shows that the proposed methods have higher probability of correct steering direction as compared to the original piloted notch filter. Fig. 6 shows that the steady-state MSE error is highly reduced as compared to original piloted notch filter with the use of a variable step-size update algorithm. Fig. 7 shows the improvement in the speed of convergence with smoother tracking when the input SNR is 10 dB and the input sinusoid frequency is abruptly changed to 0.4π at 300^{th} iteration.

VI. CONCLUSIONS

In this paper, a new steering direction determination mechanism and step-size update algorithm have been proposed for the piloted adaptive notch architecture. The new steering direction determination mechanism is based on time-averaging of gradient analysis of the piloted notch cost function has proved to be superior with high probability of correct steering direction. This highly accurate steering direction information has been utilized to derive the frequency domain information which is used for the step-size update algorithm. A new step-size update algorithm has been introduced based on the interleaving of time-frequency domain information which has improved the speed of convergence with a comparatively huge reduction in steady-state mean square error (MSE).

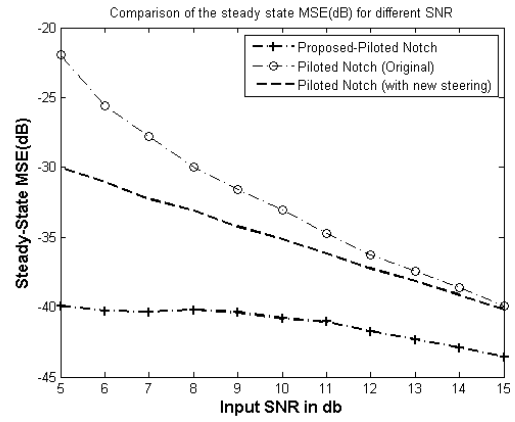


Fig. 6. Comparison of the steady state MSE (dB) for different SNR.

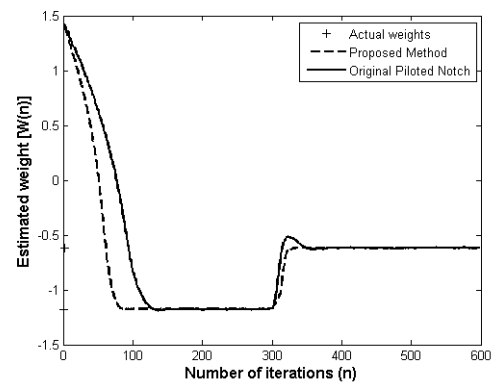


Fig. 7. Comparison of the actual weights and the estimated weights.

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