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Daniel A. Dirksz and Jacquelien M.A. Scherpen

Abstract— In this paper we take a port-Hamiltonian approach to address the problem of image-based visual servo control of a pick and place system. We realize a closed-loop system, including the nonlinear camera dynamics, which is port-Hamiltonian. Although the closed-loop system is nonlinear, the resulting controller is a PD-type controller which only requires the camera states. From the passivity property of port-Hamiltonian systems we then derive conditions for exponential stability of the closed-loop system, which are used to tune the PD controller gains.

I. INTRODUCTION

The current technological advances continuously increase the demand for robots and intelligent systems which are fast, accurate and are able to perform under different circumstances. A popular approach to deal with the different operation circumstances is to use visual information. As mentioned in [9], vision is a useful robotic sensor since it allows for non-contact measurement of the environment. With the increase of computational speed of modern computers, image processing is becoming faster, making vision systems more interesting for control applications.

The most popular classifications of visual servo control in the vision control literature are position-based visual servo (PBVS) control and image-based visual servo (IBVS) control [9]. In PBVS control, the pose of the target is estimated based on extracted features from the image. The error of the estimated pose is then used for computing the feedback control. In IBVS control, the control feedback is directly computed from the image features. Both classifications have received considerable attention and it is out of the scope of this paper to summarize all of them. For more details and other references we refer the interested reader to [9]. Lately, the vision control research has focused on control methods that deal with the shortcomings of both PBVS and IBVS control [2], [12], apply visual servo control to mobile robots and underactuated systems [3], [8], [13], [15] and exploit the passivity properties for visual servo control [6], [15].

In this paper a pick and place (P&P) system is considered, with a camera mounted on the gripper (eye-in-hand configuration). Similar to many publications on robot vision control, we apply perspective projection to model the camera dynamics. The main contribution of this paper is to realize a closed-loop PH system, via a coordinate transformation, which

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includes the nonlinear camera dynamics. The combination of the PH framework and the coordinate transformation leads to an IBVS PD control structure, which exponentially stabilizes the system.

The PH framework [14] has received a considerable amount of interest in the last decade because of its insightful physical structure. It is well known that a large class of (nonlinear) physical systems can be described in the PH framework. The popularity of PH systems can be accredited to its application for analysis and control design of physical systems, as shown in [4], [5], [17], [18], [19] and many others. To the authors knowledge, only [11] has taken a PH approach to visual servo control. Contrary to in our paper, in [11] they deal with an aerial robot and apply a spherical image representation, while we apply perspective projection.

In section II we briefly summarize the PH modeling framework and describe the P&P system and the camera system. We then present our main contributions in section III, where we show the PH approach to IBVS control. In section IV we analyze the robustness of the designed controller. Simulation results are then shown in section V and we give some concluding remarks in section VI.

II. PRELIMINARIES

In this section we briefly summarize the PH modeling framework and the important results on visual servo control necessary in this paper. We also give the model for the P&P system with vision.

A. Port-Hamiltonian systems

PH systems were introduced as a generalization of systems stemming from different physical domains [14]. They describe a large class of (nonlinear) systems including passive mechanical systems, electrical systems, electromechanical systems and mechanical systems with nonholonomic constraints [19]. A general (time-invariant) PH system is described by

$$\begin{aligned}
\dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\
y &= g(x)^\top \frac{\partial H}{\partial x}(x)
\end{aligned} \qquad (1)$$

with $J(x) \in \mathbb{R}^{n \times n}$ the skew-symmetric interconnection matrix, $R(x) \in \mathbb{R}^{n \times n}$ the symmetric, positive-semidefinite, damping matrix, $x \in \mathbb{R}^n$, the Hamiltonian $H(x)$, input u and output y, with $u, y \in \mathbb{R}^m$, $m \leq n$. PH systems are passive systems [19], [21], i.e.,

$$\dot{H} \leq u^\top y \qquad (2)$$

B. The pick and place system

Figure 1 shows the P&P system[1] we consider in this paper. The system consists of a gripper that can translate in three
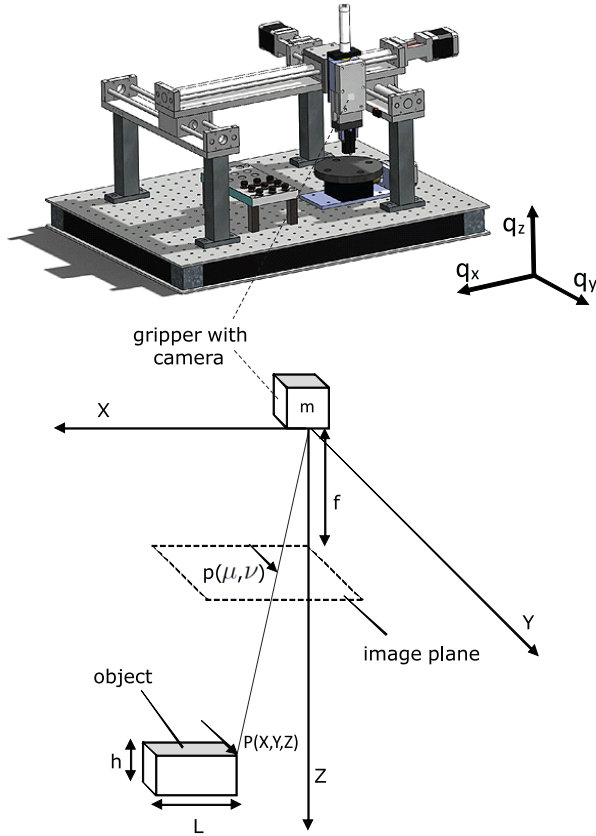

Fig. 1. Pick and place system with camera.

directions. Such a P&P system is widely used in industry, for both small and large objects. The gripper with camera of the P&P system is modeled as a mass m that can be translated in horizontal and vertical direction. We distinguish two coordinate frames: a base frame and a camera frame. The base frame (or world frame) [20], described by q_x, q_y, q_z , is the fixed coordinate system to which all objects, including the P&P system, are referenced. The dynamics of the mass m are described in the form of (1) by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & -D \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (3)$$

$$y = \frac{\partial H}{\partial p} = \dot{q}$$

with position $q = (q_x, q_y, q_z)^\top$, input vector $u = (u_x, u_y, u_z)^\top$, I the identity matrix and D the damping matrix. The Hamiltonian of the system is equal to the sum of kinetic and potential energy:

$$H(q,p) = \frac{1}{2} p^\top M^{-1} p + V(q) \quad (4)$$

where M is the system mass matrix and $V(q)$ the potential energy, i.e.,

$$M = \text{diag}\{m, m, m\}, \qquad V = mgq_z \quad (5)$$

Furthermore we have the vector of momenta $p = M\dot{q}$. We also assume a constant damping matrix $D = \text{diag}\{d_1, d_2, d_3\}$. The mass m can be translated in q_x direction by the input u_x, in q_y direction by the input u_y and in the q_z direction by u_z. Define $u_z = mg + \bar{u}_z$ with $\bar{u}_z$ the new input in q_z direction. The mg term is a precompensator to cancel the gravitational forces on the system.

In the image plane of the camera, objects are seen with respect to the camera coordinate frame. For the camera coordinate system the X and Y axes form a basis for the image plane, as described in [9]. The Z axis (optical axis) is perpendicular to the image plane with origin located at a distance f behind the image plane and where f is the focal length of the camera lens, see figure 1. The origin of the camera coordinate frame (also called viewpoint in [9]) is located at the focus of the camera lens. Since the gripper with camera is modeled as a mass, we assume this focus to be on the mass, as shown in figure 1. Notice for our P&P system that q_z is the vertical position of the gripper/camera (mass m) with respect to the base frame, while Z is the vertical distance from the camera to the object. Hence, $Z = q_z - h$ with h the object height. In this paper we assume that we cannot directly measure the position q of the system, and rely only on the camera information for control. The goal is to pick and place an object with length L and height h.

The vision literature [9] distinguishes *perspective* and *orthographic* projection models for the representation of the image formation process. In visual servo control perspective projection is mostly used, since orthographic projection is valid for scenes where the relative depth of the points in a scene are small compared to the distance from the camera to the scene, which is not the case here. For perspective projection, a point $\mathbf{P} = (X, Y, Z)^\top$, whose coordinates are expressed with respect to the camera coordinate frame, projects onto the image plane as shown in figure 1 with coordinates $\mathbf{p} = (\mu, \nu)^\top$, given by

$$\begin{bmatrix} \mu \\ \nu \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (6)$$

Notice that the measurement of one point of the object only gives us information on the X, Y positioning of the object, i.e., the horizontal position of the object with respect to the camera frame. To control the distance between the gripper/camera and the object we use the fact that the object has a constant length L, as shown in figure 1. The third state in our camera model then becomes the length l, which is the length of the object in the image plane[2] . The object has constant dimensions, so choosing the object thickness instead of length is also possible. Both increase (or decrease) in the image plane with the same factor, i.e., $\frac{f}{Z}$. In short, we use the μ and ν measurements to control the X and Y positions respectively, while the length l is used to control the vertical position. As shown in figure 2, we want to bring the

[2]The length l increases when the camera approaches the object, an decreases when the camera moves away.

mass m (the gripper with camera) to a position such that the object is seen in a desired form in the image plane (usually the center of the image plane). When this is achieved the
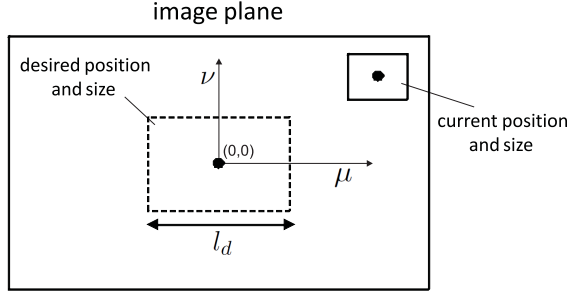

Fig. 2. Image plane view of the object. The origin of (μ, ν) lies in the center of the image plane.

gripper is ready to pick up the object. The three image plane coordinates used for control can be described by

$$\begin{bmatrix} \mu \\ \nu \\ l \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \\ L \end{bmatrix} \tag{7}$$

resulting in

$$\begin{bmatrix} \dot{\mu} \\ \dot{\nu} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} \frac{f}{Z} & 0 & -\frac{\mu}{Z} \\ 0 & \frac{f}{Z} & -\frac{\nu}{Z} \\ 0 & 0 & -\frac{l}{Z} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \tag{8}$$

The desired values for μ, ν and l are given by the constants μ_d, ν_d and l_d respectively.

We can now describe the relationship between the position of the gripper/camera (mass m) in the base frame and the position of the object in the camera frame. Let P_x and P_y be the (fixed) q_x and q_y position of the center of the object. We then have

$$\begin{array}{ll} X = P_x - q_x & \dot{X} = -\dot{q}_x \\ Y = P_y - q_y & \dot{Y} = -\dot{q}_y \\ Z = q_z - h & \dot{Z} = \dot{q}_z \end{array} \tag{9}$$

where q_x, q_y, q_z describe the position of the gripper/camera (mass m) in the base frame, and X, Y, Z the position of the object in the camera frame. It can be noticed from (9) and figures 1-2 that when the gripper/camera (mass m) moves in the positive q_x direction, the object in the image plane moves in the negative μ direction and vice versa. The same holds for movement in q_y and ν direction. This is because not the object is moving, but the camera. Furthermore, since L is constant in (7), we have $Z = \frac{fL}{l}$ and (8) becomes

$$\begin{bmatrix} \dot{\mu} \\ \dot{\nu} \\ \dot{l} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{l}{L} & 0 & -\frac{\mu l}{fL} \\ 0 & -\frac{l}{L} & -\frac{\nu l}{fL} \\ 0 & 0 & -\frac{l^2}{fL} \end{bmatrix}}_{\Delta(\mu, \nu, l)} \begin{bmatrix} \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix} \tag{10}$$

We can then describe the dynamics of the P&P system with

vision by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -I & -D & 0 \\ 0 & \Delta(\tau) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \tau} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{u} \\ 0 \end{bmatrix} \tag{11}$$

with $\tau = (\mu, \nu, l)^\top$, $\bar{u} = (u_x, u_y, \bar{u}_z)^\top$, $V(q) = 0$ in the Hamiltonian (4) and matrix $\Delta(\tau)$ as in (10). Notice that since l describes the length of the object, $l \geq 0$ always holds. The matrix $\Delta(\tau)$ is then always negative semi-definite.

III. A PORT-HAMILTONIAN APPROACH TO IBVS CONTROL

In this section we present how to realize a PH closed-loop system, such that the gripper of the P&P system moves to a desired position based on only the camera states. The main advantages of IBVS control is that it reduces computational time, eliminates the necessity for image interpretation and eliminates errors due to sensor modeling and camera calibration [9].

First, we define bounds on the 2-norm for the matrix $\Delta(\tau)$. We can write for $\Delta(\tau)$ in (10)

$$\Delta = \Lambda \frac{l}{L}, \qquad ||\Delta|| \leq ||\Lambda|| \frac{||l||}{L} \tag{12}$$

A matrix $\Lambda^{i \times j}$ satisfies [10] [3]

$$\frac{1}{\sqrt{i}} ||\Lambda||_\infty \leq ||\Lambda||_2 \leq \sqrt{j} ||\Lambda||_\infty \tag{13}$$

Let N be the maximum sensor dimension, such that $|\mu|, |\nu|, |l| \leq N$. The focal length f exclusively depends on the image sensor format, the working distance and the object size. Nevertheless, in most cameras the focal length is larger than the image sensor dimensions. We can then say that $l < f$ holds for most applications, such that $||\Lambda||_\infty \leq (1 + \frac{N}{f})$ and

$$\frac{||l||^2}{fL} < \frac{1}{\sqrt{3}} (1 + \frac{N}{f}) \frac{||l||}{L} \tag{14}$$

We then have for $\Delta(\tau)$

$$\frac{||l||^2}{fL} \leq ||\Delta(\tau)|| \leq \sqrt{3}(1 + \frac{N}{f}) \frac{||l||}{L} \tag{15}$$

Theorem *1:* Consider system (11) and a constant positive definite matrix K. Assume that $l < f$ and that

$$k_m > \frac{3f}{4Ld_m} (1 + \frac{N}{f})^2 \tag{16}$$

with k_m the smallest eigenvalue of K and $d_m = \min\{d_1, d_2, d_3\}$. Define the control input by

$$\bar{u} = MK\dot{\bar{\tau}} + DK\bar{\tau} + v \tag{17}$$

with new input v, $\bar{\tau} = \tau - \tau_d$ and $\tau_d = (\mu_d, \nu_d, l_d)^\top$. Then, the control input (17) with $v = 0$ exponentially stabilizes the system (11) in $\tau = \tau_d$.

---

[3]$||\cdot||_s$ denotes the matrix s-norm. Unless otherwise defined, $||\cdot||$ denotes the 2-norm.

Proof. Consider the coordinate transformation given by

$$\bar{p} = p - MK\bar{\tau} \tag{18}$$

Since

$$\dot{\bar{p}} = \dot{p} - MK\dot{\bar{\tau}} \tag{19}$$

the input (17) with $v = 0$ realizes a system which is described by the coordinate $\bar{p}$, i.e., [4]

$$
\begin{bmatrix} \dot{q} \\ \dot{\bar{p}} \\ \dot{\bar{\tau}} \end{bmatrix}
=
\begin{bmatrix} 0 & I & K \\ -I & -D & 0 \\ -K & \Delta(\tau) & \Delta(\tau)K \end{bmatrix}
\begin{bmatrix} \frac{\partial \bar{H}}{\partial q} \\ \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}
\tag{20}
$$

with

$$\bar{H} = \frac{1}{2}\bar{p}^{\top} M^{-1}\bar{p} + \frac{1}{2}\bar{\tau}^{\top}\bar{\tau} \tag{21}$$

For the system (20) we then have

$$
\dot{\bar{H}} = -
\begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}^{\top}
\begin{bmatrix} D & 0 \\ -\Delta(\tau) & -\Delta(\tau)K \end{bmatrix}
\begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}
\tag{22}
$$

Based on (15) equation (22) then satisfies

$$
\dot{\bar{H}} \leq -
\begin{bmatrix} ||\frac{\partial \bar{H}}{\partial \bar{p}}|| \\ ||\frac{\partial \bar{H}}{\partial \bar{\tau}}|| \end{bmatrix}^{\top}
\underbrace{\begin{bmatrix} d_m & -\frac{B}{2}||l|| \\ -\frac{B}{2}||l|| & \frac{k_m||l||^2}{fL} \end{bmatrix}}_{A}
\begin{bmatrix} ||\frac{\partial \bar{H}}{\partial \bar{p}}|| \\ ||\frac{\partial \bar{H}}{\partial \bar{\tau}}|| \end{bmatrix}
\tag{23}
$$

with

$$B = \frac{\sqrt{3}}{L}(1 + \frac{N}{f}) \tag{24}$$

The matrix A is positive definite when

$$\frac{d_m k_m}{fL}||l||^2 > \left(\frac{\sqrt{3}}{2L}(1 + \frac{N}{f})||l|| \right)^2 \tag{25}$$

from which (16) is derived such that $\dot{\bar{H}} \leq 0$. Then, $\bar{H}$ and $\dot{\bar{H}}$ satisfy

$$a_1||\omega||^2 \leq \bar{H} \leq a_2||\omega||^2 \tag{26}$$

$$\dot{\bar{H}} \leq -a_3||\omega||^2 \tag{27}$$

with a_1, a_2, a_3 positive constants and $\omega = (\bar{p}, \bar{\tau})^{\top}$. Following the proof of exponential stability in [10], all trajectories starting in $\{a_2||\omega||^2 \leq b\}$ for a sufficiently small b remain bounded. From (26) we see that $||\omega||$ satisfies the bound

$$||\omega||^2 \geq \frac{\bar{H}}{a_2} \tag{28}$$

We can then write

$$\dot{\bar{H}} \leq -\frac{a_3}{a_2}\bar{H} \tag{29}$$

Separation of $\bar{H}$ and t gives

$$\frac{d\bar{H}}{\bar{H}} \leq -\frac{a_3}{a_2}dt \tag{30}$$

resulting in

$$\bar{H}(\omega(t)) \leq \bar{H}(\omega(0))e^{-\lambda t}, \qquad \lambda = \frac{a_3}{a_2} \tag{31}$$

[4]Since τ_d is constant, $\dot{\bar{\tau}} = \dot{\tau}$.

Since in (26) we see that $||\omega|| \leq \sqrt{\frac{\bar{H}}{a_1}}$, we have that

$$
\begin{aligned}
||\omega(t)|| &\leq \left(\frac{\bar{H}(\omega(t))}{a_1} \right)^{\frac{1}{2}} \\
&\leq \left(\frac{a_2}{a_1} \right)^{\frac{1}{2}} ||\omega(0)||e^{-\lambda_1 t}, \quad \lambda_1 = \frac{a_3}{2a_2}
\end{aligned}
\tag{32}
$$

which shows that $\omega = (\bar{p}, \bar{\tau})$ converges to zero exponentially. If all the assumptions hold globally, b can be chosen arbitrarily large and (32) holds globally. Since any square matrix can be described by the sum of a symmetric matrix with a skew-symmetric matrix, system (20) can be described in the PH form (1). □

The total control input u is then given by

$$u = \rho(q) + MK\dot{\bar{\tau}} + DK\bar{\tau} + v, \qquad \rho(q) = (0, 0, mg)^{\top} \tag{33}$$

with $v = 0$.

IV. DESIGN ANALYSIS

A. Controller properties and robustness

In the previous section we presented a PH approach to IBVS control of a P&P system. The total control input (33) has the PD plus gravity cancelation structure presented in [16], since we also compensate for the gravitational force that pulls the mass m down. The controller however has the disadvantage that we need to know the matrices M and D. The assumption that the matrix D is known is not realistic. To analyze the robustness to uncertainty in the damping matrix we first make the following assumption.

A. 1: The matrix D is defined as the sum of a known constant matrix D_0 and an unknown/uncertain matrix $\hat{D}$, i.e.,

$$D = D_0 + \hat{D}, \quad -\delta_1 I \leq \hat{D} \leq \delta_2 I \tag{34}$$

with known constants δ_1, δ_2. ◁

Notice that when there is uncertainty in the matrix D, that

$$\bar{u} = MK\dot{\bar{\tau}} + DK\bar{\tau} - \underbrace{\hat{D}K\bar{\tau}}_{u_e} + v \tag{35}$$

instead of (17), with $u_e = \hat{D}K\bar{\tau}$ the error in the control input due to damping uncertainty. By following the same steps as in the proof of Theorem 1 with $v = 0$ we get the closed-loop system

$$
\begin{bmatrix} \dot{q} \\ \dot{\bar{p}} \\ \dot{\bar{\tau}} \end{bmatrix}
=
\begin{bmatrix} 0 & I & K \\ -I & -D & -\hat{D}K \\ -K & \Delta(\tau) & \Delta(\tau)K \end{bmatrix}
\begin{bmatrix} \frac{\partial \bar{H}}{\partial q} \\ \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}
\tag{36}
$$

For which we have

$$
\dot{\bar{H}} = -
\begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}^{\top}
\begin{bmatrix} D & \hat{D}K \\ -\Delta(\tau) & -\Delta(\tau)K \end{bmatrix}
\begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{\tau}} \end{bmatrix}
\tag{37}
$$

and which satisfies

$$\dot{\bar{H}} \leq -\frac{1}{2}\left[\begin{array}{c}||\frac{\partial\bar{H}}{\partial\bar{p}}||\\||\frac{\partial\bar{H}}{\partial\bar{\tau}}||\end{array}\right]^{\top}\underbrace{\left[\begin{array}{cc}d_m & -B||l||\\-B||l|| & \frac{k_m||l||^2}{fL}\end{array}\right]}_{A_1}\left[\begin{array}{c}||\frac{\partial\bar{H}}{\partial\bar{p}}||\\||\frac{\partial\bar{H}}{\partial\bar{\tau}}||\end{array}\right]$$

$$-\frac{1}{2}\left[\begin{array}{c}||\frac{\partial\bar{H}}{\partial\bar{p}}||\\||\frac{\partial\bar{H}}{\partial\bar{\tau}}||\end{array}\right]^{\top}\underbrace{\left[\begin{array}{cc}d_m & \delta_M k_M\\\delta_M k_M & \frac{k_m||l||^2}{fL}\end{array}\right]}_{A_2}\left[\begin{array}{c}||\frac{\partial\bar{H}}{\partial\bar{p}}||\\||\frac{\partial\bar{H}}{\partial\bar{\tau}}||\end{array}\right] \quad (38)$$

with B as in (24), $\delta_M = \max\{\delta_1, \delta_2\}$ and k_M the largest eigenvalue of matrix K. Matrix A_1 is positive definite when

$$\frac{d_m k_m}{fL}||l||^2 > \left(\frac{\sqrt{3}}{L}(1+\frac{N}{f})||l||\right)^2 \quad (39)$$

which results in the condition on k_m

$$k_m > \frac{3f}{Ld_m}(1+\frac{N}{f})^2 \quad (40)$$

Matrix A_2 is positive definite when

$$\frac{d_m k_m}{fL}||l||^2 > \delta_M^2 k_M^2 \quad (41)$$

For simplicity, it is possible to write $k_M = \alpha k_m$, with $\alpha \geq 1$. We can then rewrite (41) resulting in

$$||l|| > \sqrt{\frac{\delta_M^2 \alpha^2 k_m fL}{d_m}} \quad (42)$$

for matrix A_2 to be positive definite. Exponential stability can now be proven as in the proof of Theorem 1. It can be noticed that condition (40) is relatively straightforward, but that (42) imposes a lower bound on $||l||$, which is difficult to guarantee. The following proposition defines a region of attraction based on (42).

Proposition 1: Consider the closed-loop system (36) realized by the input with damping uncertainty (35) and $v = 0$. Assume that (40) and (42) are satisfied. Define constants $\alpha \geq 1$ and $\beta > 1$ such that

$$k_M = \alpha k_m \geq k_m \quad (43)$$
$$k_m = \beta\frac{3f}{Ld_m}(1+\frac{N}{f})^2 > \frac{3f}{Ld_m}(1+\frac{N}{f})^2 \quad (44)$$

in order to describe k_m and k_M in (40) and (42) in terms of the system parameters. Then, the closed-loop system (36) is exponentially stable with region of attraction given by

$$\{\xi \in \mathbb{R}^9 : ||\xi|| < \gamma\}, \qquad \gamma = \frac{Ld_m}{\delta_M\alpha\sqrt{3\beta}(1+\frac{N}{f})} \quad (45)$$

with redefined state $\xi = (\bar{q}, \bar{p}, \bar{\tau})^{\top}$, $\bar{q} = (q_x, q_y, q_z - h)^{\top}$.

Proof. We explained above that we can take the same steps as in the proof of Theorem 1 to prove exponential stability of the closed-loop system (36), which is realized by the input with damping uncertainty (35) and given that conditions (40)

and (42) are satisfied. Recall from section II that $l = \frac{f}{Z}L$ and from (9) that $Z = q_z - h$. We can then rewrite (42) into

$$\frac{fL}{||q_z - h||} > \sqrt{\frac{\delta_M^2 \alpha^2 k_m fL}{d_m}} \quad (46)$$

which then gives

$$||q_z - h|| < \frac{fL}{\sqrt{\frac{\delta_M^2 \alpha^2 k_m fL}{d_m}}} \quad (47)$$

With (44) we can simplify (47) into

$$||q_z - h|| < \frac{Ld_m}{\delta_M\alpha\sqrt{3\beta}(1+\frac{N}{f})} \quad (48)$$

Define $\bar{q} = (q_x, q_y, q_z - h)^{\top}$. Since

$$||q_z - h|| \leq ||\xi|| \quad (49)$$

with $\xi = (\bar{q}, \bar{p}, \bar{\tau})^{\top}$, we have the (conservative) region of attraction given by (45). $\qquad\square$

It can be seen that when there is an error due to uncertainty in the matrix D, that exponential stability holds for a specific region of attraction. Notice that the region of attraction is made smaller when increasing β, which means increasing k_m, see (44).

B. Implementation issues

So far we have assumed that the vision system works perfectly. However, there are a few issues that can cause problems in a practical setup. The fast motions of the gripper can cause motion blur. Techniques exist to compensate for motion blur, but are usually for constant speed motions [1]. Motion blur can also be reduced by a shorter exposure time of the camera, however, more light is then needed to have a good quality image. Images may also appear blurred if too close or too far away from the camera, unless the camera has autofocus capabilities. Furthermore, for vision systems it is very important to keep objects in the image plane. In the previous sections we showed that the gripper/camera of the P&P system converges exponentially to the desired position. Assuming that the system starts at $t = 0$ from a position at which it is at rest (zero velocity), then the Hamiltonian (21) with $\bar{p}$ as in (18) satisfies

$$\bar{H}(0) = \frac{1}{2}\bar{\tau}^{\top}(0)\left(KMK + I\right)\bar{\tau}(0) \quad (50)$$

since at rest $p(0) = 0$. The passivity property of PH systems implies that $\bar{H}(t) \leq \bar{H}(0)$, and we can then conclude that

$$||\bar{\tau}(t)|| \leq ||\bar{\tau}(0)||, \qquad t \geq 0 \quad (51)$$

V. SIMULATION EXAMPLE

In this section we test te designed controller of section III on the system shown in figure 1 and described by (11). For simulation purposes we also assume that all parameters are known. The goal is to bring the center of the object to the center of the camera, i.e., $\mu_d = 0, \nu_d = 0$, since then $q_x = P_x$ and $q_y = P_y$. The mass m (gripper/camera) is then right

above the object. The distance to the object is then controlled by bringing the object to a position such that the length of the object in the image l is equal to the desired value l_d. See also figure 2. For simplicity, take $m = d_1 = d_2 = d_3 = 0.5$. The camera is assumed to have an image sensor[5] with dimensions 5×4 mm and focal length $f = 5$ mm. The object is assumed to have dimensions 20×20 mm and height $h = 10$ mm. The desired object length in the image plane is set to $l_d = 2$ mm, which coincides with $q_z = 0.06$ m. Furthermore, we choose $K = \mathrm{diag}\{500, 500, 500\}$, which satisfies (16). Figure 3 shows the simulation results when there is no damping uncertainty, when $D_0 > D$ (dashed red) and $D_0 < D$. The first column shows the position error
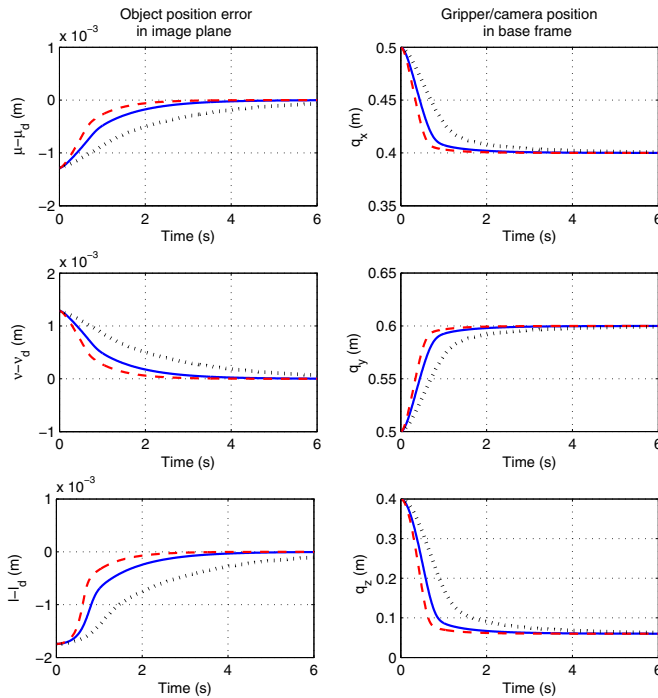

Fig. 3. Trajectories for the P&P system. Initial conditions: $[q(0)\ p(0)] = [0.5\ 0.5\ 0.4\ 0\ 0\ 0]$. Object position: $P_x = 0.4$ m, $P_y = 0.6$ m. Solid blue lines: $D_0 = D$, dashed red lines: $D_0 = 1.25D$, dotted black lines: $D_0 = 0.75D$.

of the object in the image plane, while the second column shows the position of the gripper with camera (mass m) in the base frame. The figure shows that the error trajectories for the camera states converge to zero, which means that the camera position q_x, q_y converge to the object position P_x, P_y (also shown in the results) and q_z converges to the specified 0.06 m. We emphasize again that positioning of the camera (and so of the mass) is realized by a feedback control depending only on the camera states μ, ν and l.

VI. FINAL REMARKS

We have presented a PH approach to visual servo control of a P&P system. The camera dynamics based on perspective projection modeling introduce nonlinearities in the whole system. Based on a coordinate transformation we realize a PD controller that exponentially stabilizes the system, using only the camera states. We also analyzed robustness of the results, since the PD controller requires exact knowledge of the damping coefficients. When there is damping uncertainty we prove that exponential stability holds for a region of attraction, and give an estimate of this region.

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[5]In practice a charged-coupled device (CCD) or complementary metal-oxide semiconductor (CMOS).