

Research Article

A Portfolio Selection Model Based on the Interval Number

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Received 25 April 2021; Accepted 25 May 2021; Published 2 June 2021

Academic Editor: Rongwei Guo

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Traditional portfolio theory uses probability theory to analyze the uncertainty of financial market. The assets' return in a portfolio is regarded as a random variable which follows a certain probability distribution. However, it is difficult to estimate the assets return in the real financial market, so the interval distribution of asset return can be estimated according to the relevant suggestions of experts and decision makers, that is, the interval number is used to describe the distribution of asset return. Therefore, this paper establishes a portfolio selection model based on the interval number. In this model, the semiabsolute deviation risk function is used to measure the portfolio's risk, and the solution of the model is obtained by using the order relation of the interval number. At the same time, a satisfactory solution of the model is obtained by using the concept of acceptability of the interval number. Finally, an example is given to illustrate the practicability of the model.

1. Introduction

Portfolio selection refers to the way in which investors allocate a certain proportion of their wealth to a number of different assets so as to spread risks among multiple assets and obtain some stable returns. Markowitz [1] created the classic portfolio theory in 1952, which laid the foundation of modern finance. In this theory, the covariance of all the securities in the portfolio was required, which was a considerable amount of calculation at that time, but difficult to achieve in practical application. Therefore, later, scholars constantly proposed improved optimization methods and thus put forward new portfolio models (e.g., [2–5]). In 1963, the capital asset pricing model (abbreviated as CAPM) proposed by Sharp [2] divided risks into systematic risks and nonsystematic risks on the basis of the model proposed by Markowitz. Markowitz [3] proposed a semivariance model. Mao [4] and Swalm [5] used the risk that the uncertain return is lower than the expected return to measure the investment risk and established the mean-semivariance portfolio selection model. In 1991, Konno and Yamazaki [6] proposed the absolute deviation risk function and constructed a mean-absolute deviation portfolio optimization model. Since then, scholars have also proposed many

portfolio optimization models based on different risk measures, such as semiabsolute deviation model [7], value-at-risk (VaR) model [8–11], and conditional value-at-risk (CVaR) model [12]. Since each measure of risk performs best in its own area and not necessarily in others, it is still up in the air whether there is a single measure of risk that is best for all portfolios [13, 14]. In this paper, we will use the semiabsolute deviation absolute risk function to measure the risk of the portfolio.

Uncertainty exists everywhere, and scholars use various methods to study it [15–21]. Traditional portfolio theory uses probability theory to analyze the uncertainty in the financial market. However, due to the nonrandom factors such as social, economic, political, psychological, and other factors existing in the real financial market, other technologies are needed to deal with the uncertainty, such as possibility theory and fuzzy set theory. Possibility theory is an important theory of fuzzy sets that was first proposed by Zadeh [22] and developed by Dubois and Prade [23] (see [24–27], for more details). Portfolio models based on theory of possibility have been fruitful (see [28–35]). In these possibilistic portfolio selection models, it is assumed that the possibility distribution of asset return in the portfolio is known, but in reality, it is often not so. The interval number

is a relatively simple fuzzy number, which is easy for experts and decision makers to estimate the interval number of fuzzy parameters in a certain precision range on the basis of comprehensive analysis of the influence of various factors. So far, there have been many reports using interval number theory to study portfolio selection, such as literature [36–42].

Lai *et al.* [37] gave the noninferior solutions of linear programming problems with interval coefficients. In [38], Ida regarded the portfolio problem with interval and fuzzy objective function coefficients as a kind of multiobjective problem containing uncertainty and gave its optimistic and pessimistic solutions. Giove *et al.* [39] established a portfolio model by taking securities' price as the interval variable and solved the model by using the minimax regret method. Bhattacharyya *et al.* [40] constructed a mean-variance-skewness portfolio model using interval numbers and used a hybrid intelligent algorithm (HIA) to solve the model. Liu [41] used an interval number to represent the expected return of an asset. According to the concept of mean-absolute deviation function, a pair of two-level portfolio model was constructed, and the upper and lower bounds of investment returns in the portfolio selection problem were calculated. Based on the semiabsolute deviation risk function proposed by Mansini and Speranza [43], a mean-semi-absolute deviation portfolio selection model with respect to the interval number will be established. The order relation of the interval number is very important for obtaining the solution of the model, so the order relation of the interval in [44] will be used to obtain the solution of the constructed model. At the same time, the satisfactory solution of the model is given according to the acceptability [45].

The rest of this paper is organized as follows. In Section 2, we will do some preliminary work and give the basic concepts and some notations that will be used later. In Section 3, the portfolio selection model construction and the solution of the model will be introduced in detail. To obtain the solution for the interval-valued programming model, the order relation and the acceptability of interval numbers are used to transform the model into a general programming model. Section 4 provides a numerical example to illustrate the proposed approach. Section 5 provides the conclusion.

2. Preliminaries

In this paper, concepts and operations related to interval numbers will be used. This section will briefly review the relevant concepts.

Definition 1 (see [46]). Given two interval numbers $a = [a_-, a_+]$ and $b = [b_-, b_+]$ and a real number λ , then

$$(i) \ a \pm b = [a_- \pm b_-, a_+ \pm b_+].$$

$$(ii) \ \lambda a = \begin{cases} [\lambda a_-, \lambda a_+], & \text{for } \lambda \geq 0, \\ [\lambda a_+, \lambda a_-], & \text{for } \lambda < 0. \end{cases}$$

The interval number is a special fuzzy number whose membership function takes value 1 over the interval and 0 anywhere else, as discussed in detail by Hansen [47] and Alefeld and Herzberger [46]. The operations related to interval numbers are as follows.

Definition 2 (see [48–50]). Let $a = [a_-, a_+]$ and $b = [b_-, b_+]$ be two interval numbers. We define the order relation \leq between a and b as

$$(1) \ a \leq b \text{ if and only if } m(a) \leq m(b),$$

$$(2) \ a < b \text{ if and only if } a \leq b \text{ and } a \neq b,$$

where $m(a) = (1/2)(a_- + a_+)$ is the midpoints of the interval numbers.

Specifically, if $a_+ \leq b_-$, then the inequality relationship $a \leq b$ is optimistic and satisfactory. On the contrary, if $a_+ > b_-$, the inequality relationship $a \leq b$ is pessimistic and satisfactory.

Definition 3 (see [51]). Given $a = [a_-, a_+]$ and $b = [b_-, b_+]$, then $\lambda(a \leq b) = (m(b) - m(a)) / (\omega(b) + \omega(a))$ is the acceptability of $a \leq b$, where $m(a)$ and $\omega(a)$ are the midpoints and radius of the interval number a , respectively.

The notations used in this article are given below:

p_{kt0} : the opening price of the k th security at period t

p_{kct} : the closing price of the k th security at period t

r_{kt} : $(p_{kct} - p_{kt0}) / p_{kt0}$ = the return of asset k at period t

T : the total periods

r_k : $(1/T) \sum_{t=1}^T r_{kt}$ = the return of asset k

r_f : the return of the risk-free asset

x_k : the percentage of assets that are invested in k

$w_t(x)$: the portfolio's risk at period t

$W(x)$: the portfolio's risk

R : the portfolio's return

3. Model Foundation

Let us consider a market consisting of a riskless assets and n stocks. As usual, we assume that there are no costs or taxes on trading, all assets are infinitely divisible, and short sales is not allowed.

Thus, the portfolio's return R can be written as

$$R = \sum_{k=1}^n x_k r_k + r_f \left(1 - \sum_{k=1}^n x_k \right). \quad (2)$$

To set up a portfolio selection model, the following values need to be given.

First is the expected return of the portfolio's return \bar{R} .

The expected return on security k is $\bar{r}_k = [r_k, \bar{r}_k]$. Thus, \bar{R} is given by

$$\bar{R} = \sum_{k=1}^n \left[\frac{r_k, \bar{r}_k}{2} \right] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right). \quad (3)$$

Secondly, the risk of the portfolio is as follows.

As mentioned in Section 1, there is no single risk measure that is best for all portfolios, so the risk in this paper will be measured by the semiabsolute deviation function. The semiabsolute deviation of the portfolio in period t can be calculated as follows:

$$w_t(x) = \left| \min \left\{ 0, \sum_{k=1}^n (r_{kt} - r_k)x_k \right\} \right| = \max \left\{ 0, \sum_{k=1}^n (r_k - r_{kt})x_k \right\}, \quad t = 1, 2, \dots, T. \quad (4)$$

The risk of the portfolio is given by $(1/T) \sum_{t=1}^T w_t(x)$.

So,

$$\begin{aligned} W(x) &= \frac{1}{T} \sum_{t=1}^T \left| \min \left\{ 0, \sum_{k=1}^n (r_{kt} - r_k)x_k \right\} \right| \\ &= \frac{1}{T} \sum_{t=1}^T \left[\max \left\{ 0, \sum_{k=1}^n (r_k - r_{kt})x_k \right\}, \max \left\{ 0, \sum_{k=1}^n (\bar{r}_k - r_{kt})x_k \right\} \right] \\ &= \frac{1}{T} \sum_{t=1}^T \left[\frac{\sum_{k=1}^n (r_k - r_{kt})x_k + \left| \sum_{k=1}^n (r_k - r_{kt})x_k \right|}{2}, \frac{\sum_{k=1}^n (\bar{r}_k - r_{kt})x_k + \left| \sum_{k=1}^n (\bar{r}_k - r_{kt})x_k \right|}{2} \right] \\ &= \frac{1}{T} \sum_{t=1}^T [\underline{w}_t(x), \bar{w}_t(x)], \end{aligned} \quad (5)$$

where

$$\begin{aligned} \underline{w}_t(x) &= \frac{\left(\sum_{k=1}^n (r_k - r_{kt})x_k + \left| \sum_{k=1}^n (r_k - r_{kt})x_k \right| \right)}{2}, \\ \bar{w}_t(x) &= \frac{\left(\sum_{k=1}^n (\bar{r}_k - r_{kt})x_k + \left| \sum_{k=1}^n (\bar{r}_k - r_{kt})x_k \right| \right)}{2}. \end{aligned} \quad (6)$$

Therefore, a portfolio selection model based on risk-return trade-off can be established:

$$\begin{cases} \min W(x), \\ \text{s.t. } \bar{R} \geq \mu, \sum_{k=1}^n x_k \leq 1, 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n, \end{cases} \quad (7)$$

where μ is a minimum threshold at which investors can tolerate the expected rate of return on their portfolio and set $\mu = [\mu^-, \mu^+]$ and l_k and h_k represent, respectively, the lower and the upper bounds on investment in asset k , $k = 1, 2, \dots, n$.

As can be seen from equations (3) and (5), equation (7) can be transformed into

$$\begin{cases} \min W(x) = \frac{1}{T} \sum_{t=1}^T [\underline{w}_t(x), \bar{w}_t(x)], \\ \text{s.t. } \sum_{k=1}^n [r_k^-, r_k^+] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{cases} \quad (8)$$

The solution of model (8) is equivalent to the following equation:

$$\begin{cases} \max \bar{W}(x) = -W(x) = -\frac{1}{T} \sum_{t=1}^T [\underline{w}_t(x), \bar{w}_t(x)], \\ \text{s.t. } \sum_{k=1}^n [r_k^-, r_k^+] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{cases} \quad (9)$$

From Definition 1, we can have

$$\begin{cases} \max \bar{W}(x) = \frac{1}{T} \sum_{t=1}^T [-\bar{w}_t(x), -\underline{w}_t(x)], \\ \text{s.t. } \sum_{k=1}^n [r_k^-, r_k^+] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{cases} \quad (10)$$

This is an interval-valued linear programming problem. For the solution method of interval-valued linear programming, scholars have carried out a lot of research and put forward some solutions. For example, Yoon [52] proposed

the error analysis method. Bryson and Mobolurin [53] proposed the linear programming method. Romelfanger et al. [54] studied the solution method of linear programming with the interval number as the coefficient of objective function. Liu and Iwamura [55] transformed interval number linear programming into a two-objective programming problem:

$$\left\{ \begin{array}{l} \max \bar{W}(x) = -\frac{1}{T} \sum_{t=1}^T \bar{w}_t(x), \\ \max \underline{W}(x) = -\frac{1}{T} \sum_{t=1}^T \underline{w}_t(x), \\ \text{s.t. } \sum_{k=1}^n \left[\underline{r}_k, \bar{r}_k \right] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (11)$$

Chankong and Haimes [56] transformed the above-mentioned two-objective programming problem (13) into the following parameter programming problem:

$$\left\{ \begin{array}{l} \max \bar{W}(x) = \frac{1}{T} \sum_{t=1}^T [-(1-b)\bar{w}_t(x) - b\underline{w}_t(x)], \\ \text{s.t. } \bar{R} \geq [\mu^-, \mu^+], \sum_{k=1}^n x_k \leq 1, 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (12)$$

The solution of model (12) is also the solution of the following model:

$$\left\{ \begin{array}{l} \min H(x) = \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)], \\ \text{s.t. } \bar{R} \geq [\mu^-, \mu^+], \sum_{k=1}^n x_k \leq 1, 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (13)$$

Because

$$\begin{aligned} H(x) &= \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)] \\ &= \frac{1}{T} \sum_{t=1}^T \left[(1-b) \frac{\left(\sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k + \left| \sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k \right| \right)}{2} + b \frac{\left(\sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k + \left| \sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k \right| \right)}{2} \right] \\ &= \sum_{k=1}^n \frac{(1-b)\xi_k + b\eta_k}{2T} x_k + \frac{1}{2} \sum_{t=1}^T \left[(1-b) \frac{\left| \sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k \right|}{T} + b \frac{\left| \sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k \right|}{T} \right] \\ &= \sum_{k=1}^n \frac{(1-b)\xi_k + b\eta_k}{2T} x_k + \frac{1}{2} \sum_{t=1}^T \left[(1-b) \frac{u_t}{T} + b \frac{v_t}{T} \right], \end{aligned} \quad (14)$$

where

$$\left\{ \begin{array}{l} \xi_k = \sum_{t=1}^T (\bar{r}_k - r_{kt}), \\ \eta_k = \sum_{t=1}^T \left(\underline{r}_k - r_{kt} \right), \\ u_t = \left| \sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k \right|, \\ v_t = \left| \sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k \right|, \end{array} \right. \quad (15)$$

then we have

$$\left\{ \begin{array}{l} \min H(x) = \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)], \\ \text{s.t. } \sum_{k=1}^n \left[\underline{r}_k, \bar{r}_k \right] x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (16)$$

According to Definition 1, (16) can be transformed into

$$\left\{ \begin{array}{l} \min H(x) = \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)], \\ \text{s.t. } \left[\sum_{k=1}^n \underline{r}_k x_k + r_f \left(1 - \sum_{k=1}^n x_k \right), \sum_{k=1}^n \bar{r}_k x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \right] \geq [\mu^-, \mu^+], \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (17)$$

In order to obtain the solution of (17), the order relation of the interval number in Definition 2 and (14) can be used to convert (17) into

$$\left\{ \begin{array}{l} \min H(x) = \sum_{k=1}^n \frac{(1-b)\xi_k + b\eta_k}{2T} x_k + \frac{1}{2} \sum_{t=1}^T \left[(1-b) \frac{u_t}{T} + b \frac{v_t}{T} \right], \\ \text{s.t. } \frac{1}{2} \left(\sum_{k=1}^n \underline{r}_k x_k + \sum_{k=1}^n \bar{r}_k x_k \right) + r_f \left(1 - \sum_{k=1}^n x_k \right) \geq \frac{1}{2} (\mu^- + \mu^+), \\ u_t + \sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k \geq 0, \quad t = 1, 2, \dots, T, \\ u_t - \sum_{k=1}^n (\bar{r}_k - r_{kt}) x_k \geq 0, \quad t = 1, 2, \dots, T, \\ v_t + \sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k \geq 0, \quad t = 1, 2, \dots, T, \\ v_t - \sum_{k=1}^n \left(\underline{r}_k - r_{kt} \right) x_k \geq 0, \quad t = 1, 2, \dots, T, \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (18)$$

Then, (18) is a parameter-planning problem, which can be solved by Matlab, Lingo, and other software.

Definition 4. The optimal solution to (18) is called an interval-valued efficient portfolio.

The lower bounds of all the interval-valued efficient portfolios construct the interval-valued lower efficient frontier. The upper bounds of all the interval-valued efficient portfolios construct the interval-valued upper efficient frontier.

Meanwhile, based on the acceptability, (17) can also be transformed into

TABLE 1: The expected return of the five securities.

| Code | $\tilde{r}_k = [r_k^-, r_k^+]$ |
|------|--------------------------------|
| S1 | [-0.557, 0.341] |
| S2 | [-0.170, 0.306] |
| S3 | [-0.381, 0.953] |
| S4 | [-0.316, 0.258] |
| S5 | [-0.503, 0.156] |

$$\left\{ \begin{array}{l} \min H(x) = \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)], \\ \text{s.t. } \lambda \left(\left[\sum_{k=1}^n \underline{r}_k x_k + r_f \left(1 - \sum_{k=1}^n x_k \right), \sum_{k=1}^n \bar{r}_k x_k + r_f \left(1 - \sum_{k=1}^n x_k \right) \right] \geq [\mu^-, \mu^+] \right) \geq \alpha, \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n, \end{array} \right. \quad (19)$$

where α is the minimum value of acceptability.

According to Definition 3, we can obtain

$$\left\{ \begin{array}{l} \min H(x) = \frac{1}{T} \sum_{t=1}^T [(1-b)\bar{w}_t(x) + b\underline{w}_t(x)], \\ \text{s.t. } \frac{\sum_{k=1}^n \underline{r}_k x_k + \sum_{k=1}^n \bar{r}_k x_k + 2r_f(1 - \sum_{k=1}^n x_k) - (\mu^- + \mu^+)}{\left(\sum_{k=1}^n \bar{r}_k x_k - \sum_{k=1}^n \underline{r}_k x_k \right) - (\mu^+ - \mu^-)} \geq \alpha, \\ \sum_{k=1}^n x_k \leq 1, \\ 0 \leq l_k \leq x_k \leq h_k, \quad k = 1, 2, \dots, n. \end{array} \right. \quad (20)$$

Thus, a satisfactory solution of the model is obtained, and an acceptable efficient portfolio is obtained.

4. Numerical Example

In order to illustrate the practicality of this model, we select five securities and one risk-free asset from the Chinese stock market for investment. Annual data from 2016 to 2020 were selected. Table 1 shows the expected return.

Let the risk-free asset be a treasury bond. We use the one-year treasury bond rate as the return rate of the riskless asset. So, we get the return on risk-free asset $r = 2.8\%$ if the lower bound of the investment ratio x_k must be $l = \{0.01, 0.03, 0.01, 0.01, 0\}$ and the upper bound $h = \{0.3, 0.4, 0.1, 0.5, 0.3\}$.

Table 2 shows the effective portfolios of model (18) with different μ s when $b = 0.1$. Figure 1 gives some efficient portfolios for model (18).

As can be seen from Table 2,

- (1) The lower limit of the minimum expected return rate remains unchanged. With the increase of the upper limit, the investment proportion of S5 will increase first. When the investment proportion of S5 reaches its upper limit, the investment proportion of S1 will be increased again.
- (2) As the minimum expected return rate increases, so does portfolio risk.
- (3) When the minimum expected return rate increases to a certain value, the model will have no feasible solution.

Parameter b is the risk preference coefficient. The larger b is, the more investors are inclined to avoid risk; the smaller b is, the more investors are inclined to risk. As can be seen from Table 3, b reflects investors' risk preference. The bigger b is, the less risk the portfolio is and the more cautious the investor is. Figure 2 also reflects the relationship between portfolio risk and the risk preference coefficient b .

Model (19) presents the acceptable solution of the portfolio model based on the acceptability. The upper and lower limits of the portfolio are given as $h = \{0.3, 0.4, 0.2, 0.5, 0.3\}$ and $l = \{0.01, 0.03, 0.01, 0.01, 0\}$, respectively, and Tables 4 and 5, respectively, show the acceptable solution of model (20) and the risk of the portfolio for different μ s when $\alpha = 0.1$ and $\alpha = 0.2$.

As can be seen from Table 4,

- (1) With the increase of μ , the investment proportion of S3 increases first. When μ increases to a certain

TABLE 2: The investment proportion and risk for different μ s in (18) when $b = 0.1$

| μ | S1 | S2 | S3 | S4 | S5 | Risk | $\sum_{k=1}^5 x_k$ |
|--------------|-------|-------|-------|-------|-------|-------|--------------------|
| [0.04, 0.04] | 0.010 | 0.030 | 0.010 | 0.010 | 0.118 | 0.036 | 0.178 |
| [0.04, 0.05] | 0.010 | 0.030 | 0.010 | 0.010 | 0.153 | 0.042 | 0.213 |
| [0.04, 0.06] | 0.010 | 0.030 | 0.010 | 0.010 | 0.187 | 0.048 | 0.247 |
| [0.04, 0.07] | 0.010 | 0.030 | 0.010 | 0.010 | 0.221 | 0.054 | 0.281 |
| [0.04, 0.08] | 0.010 | 0.030 | 0.010 | 0.010 | 0.256 | 0.060 | 0.316 |
| [0.04, 0.09] | 0.010 | 0.030 | 0.010 | 0.010 | 0.290 | 0.066 | 0.350 |
| [0.04, 0.10] | 0.054 | 0.030 | 0.010 | 0.010 | 0.300 | 0.080 | 0.404 |
| [0.04, 0.11] | 0.116 | 0.030 | 0.010 | 0.010 | 0.300 | 0.097 | 0.466 |
| [0.04, 0.12] | 0.178 | 0.030 | 0.010 | 0.010 | 0.300 | 0.114 | 0.528 |
| [0.04, 0.14] | 0.300 | 0.030 | 0.010 | 0.220 | 0.300 | 0.182 | 0.860 |

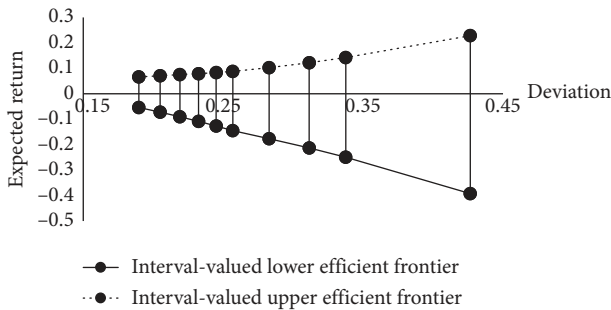


FIGURE 1: Some efficient portfolios for model (18).

TABLE 3: The investment risk for different b s in (18).

| b | Risk | b | Risk |
|-----|-------|------|-------|
| 0 | 0.190 | 0.55 | 0.107 |
| 0.1 | 0.182 | 0.6 | 0.097 |
| 0.2 | 0.134 | 0.7 | 0.086 |
| 0.3 | 0.128 | 0.8 | 0.078 |
| 0.4 | 0.115 | 0.9 | 0.056 |
| 0.5 | 0.109 | 1.0 | 0.045 |

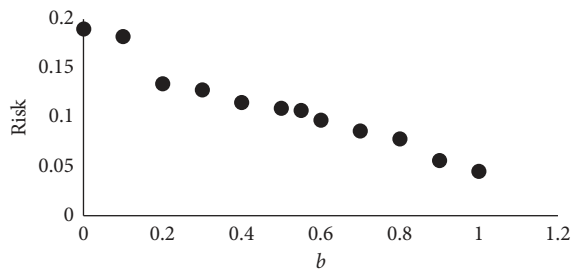


FIGURE 2: The scatter diagram of portfolio risk and the risk preference coefficient b .

extent, the investment proportion of S3 reaches the upper limit of investment, and then, the investment proportion of S2 increases.

TABLE 4: The investment proportion and risk for different μ s with $\alpha = 0.1$ in (20).

| μ | S1 | S2 | S3 | S4 | S5 | Risk | $\sum_{k=1}^5 x_k$ |
|--------------|-------|-------|-------|-------|-------|-------|--------------------|
| [0.03, 0.03] | 0.010 | 0.030 | 0.034 | 0.010 | 0.010 | 0.028 | 0.094 |
| [0.03, 0.04] | 0.010 | 0.030 | 0.058 | 0.010 | 0.010 | 0.040 | 0.118 |
| [0.03, 0.05] | 0.010 | 0.030 | 0.081 | 0.010 | 0.010 | 0.052 | 0.141 |
| [0.03, 0.06] | 0.010 | 0.030 | 0.105 | 0.010 | 0.010 | 0.063 | 0.165 |
| [0.03, 0.07] | 0.010 | 0.030 | 0.128 | 0.010 | 0.010 | 0.075 | 0.188 |
| [0.03, 0.08] | 0.010 | 0.030 | 0.152 | 0.010 | 0.010 | 0.087 | 0.212 |
| [0.03, 0.09] | 0.010 | 0.030 | 0.175 | 0.010 | 0.010 | 0.098 | 0.235 |
| [0.03, 0.10] | 0.010 | 0.030 | 0.199 | 0.010 | 0.010 | 0.110 | 0.259 |
| [0.03, 0.11] | 0.010 | 0.293 | 0.200 | 0.010 | 0.010 | 0.157 | 0.523 |

TABLE 5: The investment proportion and risk for different μ s with $\alpha = 0.2$ in (20).

| μ | S1 | S2 | S3 | S4 | S5 | Risk | $\sum_{k=1}^5 x_k$ |
|--------------|-------|-------|-------|-------|-------|-------|--------------------|
| [0.03, 0.03] | 0.010 | 0.030 | 0.067 | 0.010 | 0.010 | 0.044 | 0.127 |
| [0.03, 0.04] | 0.010 | 0.030 | 0.099 | 0.010 | 0.010 | 0.052 | 0.159 |
| [0.03, 0.05] | 0.010 | 0.030 | 0.131 | 0.010 | 0.010 | 0.076 | 0.191 |
| [0.03, 0.06] | 0.010 | 0.030 | 0.163 | 0.010 | 0.010 | 0.092 | 0.223 |
| [0.03, 0.07] | 0.010 | 0.030 | 0.195 | 0.010 | 0.010 | 0.108 | 0.255 |

(2) When μ increases to a certain extent, the model will have no feasible solution. When the acceptability of the expected return rate of the portfolio is not less than μ is 0.1, the proportion of risky assets invested will continue to increase as μ increases, and so will the risk of the portfolio. Since $\alpha = 0.1$, when μ reaches a certain level, the risk of the portfolio will no longer increase, and the investment proportion of risky assets will not change, that is, a part of the capital must be invested in the risk-free assets.

It can be seen from Table 5 that the law is similar to that in Table 4.

By comparing Tables 4 and 5, it can be found that when μ is set to the same value, α is larger, the objective function value is larger, and the investment proportion of risky assets is also higher. That is to say, the greater the acceptability of the expected return rate of the portfolio is less than μ , the greater the risk of the portfolio and the higher the proportion of risky assets' investment.

5. Conclusions

This paper takes the assets return as the interval number and uses the semiabsolute deviation function of the interval number to measure the portfolio's risk. Therefore, a portfolio selection model with mean-semiabsolute deviation based on the interval number is constructed. In this model, firstly, the lower bound of the investors' expected rate of return is also regarded as an interval number, which can better grasp investors' psychology and measure investors' expected return rate. Secondly, when solving the semiabsolute deviation portfolio selection model, a parameter which can reflect investors' risk preference is introduced, and this

parameter can reflect investors' risk preference more intuitively. Finally, an application of the portfolio diversification problem is given by using a portfolio consisting of 5 risky assets and 1 risk-free asset. The results show that the introduced risk preference parameter can well reflect the investors' attitude to risk, and the lower bound of the expected return rate of this method is more elastic. The model can be used more widely and can describe the expected return rate of investment portfolio and the investors' attitude to risk more flexibly [56].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was funded by the National Natural Science Foundation of China (11401438), the National Social Science Foundation Project of China (13CRK027), and the social science planning project of Shandong Province (21CTJJ04).

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