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ABSTRACT

In the global, rather simple model of the synchronous machine with rectifier developed before, saturation phenomena were not taken into account. Here, a possible way of extending the machine model with saturation is described.
Starting with the derivation of the separate models of the synchronous machine and the rectifier, a steady-state model and a dynamic model are derived. The dynamic model is a global model, in which very fast phenomena, such as the ripple on the direct current, are neglected.

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CONTENTS

1 INTRODUCTION

The series system synchronous machine - rectifier - smoothing coil - inverter as depicted in figure 1.1 is a favourite system for variable-speed wind-energy conversion. At the Eindhoven University of Technology a research project is done on the field of modelling this system for control and simulation purposes [Bon 82; Bon 87; Hoe 84a; Hoe 84b; Hoe 86; Hoe 87a; Hoe87b; Hoe88a; Hoe88b; Hoe 89; Vle 87; Vle 88]. One of the aspects of this modelling is taking into account saturation phenomena in the synchronous machine. In this report, which is the result of a project in the frame work of the European Community project ENW3-044-NL, a possible way of extending the model developed before [Hoe 89] with saturation is described. This extension is based on the theory and the experimental results given in [Mel 86]. Since the report [Hoe 89] has been written in Dutch, its basic theory is given in this report too.

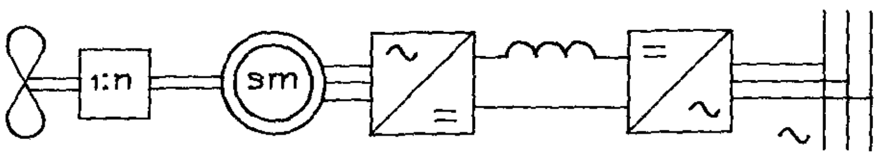

Figure 1.1 A wind-energy conversion-system with a synchronous machine and a dc-link

In chapter 2 general equations for the synchronous machine will be derived. In chapter 3 these equations will be adapted for the situation in which the synchronous machine is connected to a rectifier. After having described this rectifier in chapter 4, in chapter 5 some attention will be paid to the coupling of the synchronous machine model with the rectifier model. Next, in the chapters 5 and 6, the equations for the steady-state and the dynamic model of the synchronous machine with rectifier will be dealt with. Finally, in chapter 7, some attention will be paid to the parameters used in the model.

2 THE DERIVATION OF THE SYNCHRONOUS MACHINE EQUATIONS

2.1 Introduction

In this chapter the general equations of the synchronous machine will be derived. Without endangering the general usefulness, at first instance this description is restricted to the two-pole synchronous machine as shown in figure 2.1. Some of the quantities used in this description are defined in this figure.

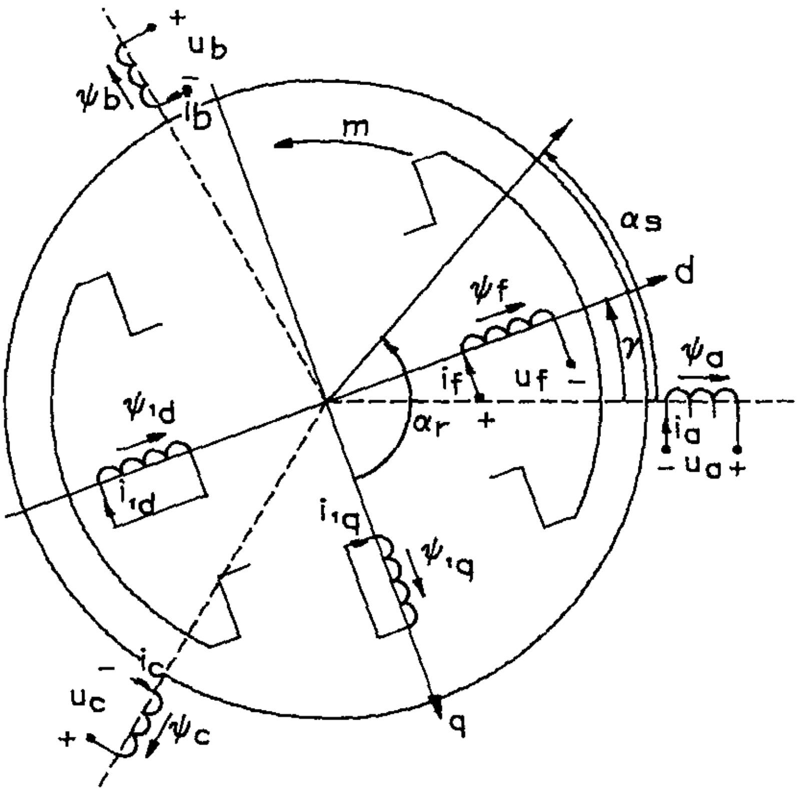

Figure 2.1 Schematic diagram of the synchronous machine

The following suppositions will be used in the description:

- The stator inner side is circular cylindrical: the bore diameter equals 2r and the core length equals 1.
- The stator innerside is smooth: the effects of slots are neglected.
- Hysteresis, eddy-currents, and skin-effect are neglected.

- The reference directions of current and flux correspond as do the direction of rotation and the direction of advance of a right-handed screw.
- The magnetic circuit of the rotor is symmetrical to two mutually perpendicular axes: the direct axis and the quadrature axis.
- The stator windings are sinusoidally distributed along the stator circumference. These distributions are described by:

$$Z_a(\alpha_s) = \hat{Z}_a \sin(\alpha_s) \tag{2.1a}$$

$$Z_b(\alpha_s) = \hat{Z}_a \sin(\alpha_s - \tfrac{2}{3}\pi) \tag{2.1b}$$

$$Z_c(\alpha_s) = \hat{Z}_a \sin(\alpha_s - \tfrac{4}{3}\pi) \tag{2.1c}$$

  where Z is the number of conductors per meter with $\hat{Z}_a$ as its maximum.
- The excitation winding, the field axis of which is the direct axis, is the only accessible rotor winding. The rotor damping circuits are represented by one damper circuit on the direct axis and one damper circuit on the quadrature axis. All rotor windings are sinusoidally distributed along the rotor circumference:

$$Z_f(\alpha_r) = \hat{Z}_f \sin(\alpha_r - \tfrac{\pi}{2}) \tag{2.2a}$$

$$Z_{1d}(\alpha_r) = \hat{Z}_{1d} \sin(\alpha_r - \tfrac{\pi}{2}) \tag{2.2b}$$

$$Z_{1q}(\alpha_r) = \hat{Z}_{1q} \sin(\alpha_r) \tag{2.3}$$

  where Z is the number of conductors per meter with $\hat{Z}$ as its maximum.

The assumption of sinusoidal distribution of the rotor windings may seem to be unreasonable. However, other distributions may be considered as a Fourier series and may often be approximated sufficiently by the first term of the series.

In the sections 2.2 and 2.3 attention is paid to the relations between currents and flux linkages in the machine; there, the basic assumptions for modelling saturation are introduced. These relations may be used for the voltage equations in section 2.4. The mechanical side of the machine will be dealt with in section 2.5.

Although in synchronous machine description the term "reactance" is widely used, it will not be used here, because a reactance is tied to a fixed frequency. Since a synchronous machine used in a combination with a rectifier does not generally operate at a fixed frequency, the term "inductance" is preferred here.

2.2 The derivation of the flux current relations

Starting from the armature current distribution, the magnetic induction in the air gap will be computed. From the air gap induction expressions for the main fluxes in the windings will be derived. While deriving these expressions, the Park transformation will be introduecd implicitly in order to eliminate the dependance of the inductance coefficients on the position angle γ (figure 2.1). For this purpose, two imaginary windings (one on the direct and one on the quadrature axis), will be introduced also. Finally, for each winding, the leakage flux is added to the main flux.

The armature current distribution

With (2.1) the sinusoidal current distributions (sheet) caused by the currents i_a, i_b, and i_c can be given:

$$A_a(\alpha_s,i_a) = i_a \hat{Z}_a\sin(\alpha_s) \tag{2.4a}$$

$$A_b(\alpha_s,i_b) = i_b \hat{Z}_a\sin(\alpha_s-\tfrac{2}{3}\pi) \tag{2.4b}$$

$$A_c(\alpha_s,i_c) = i_c \hat{Z}_a\sin(\alpha_s-\tfrac{4}{3}\pi) \tag{2.4c}$$

The total effect of these current distributions is the same as the effect of the superposition of these distributions:

$$A(\alpha_s,i_a,i_b,i_c) = \hat{Z}_a\{i_a\sin(\alpha_s) + i_b\sin(\alpha_s-\tfrac{2}{3}\pi) + i_c\sin(\alpha_s-\tfrac{4}{3}\pi)\} \tag{2.5}$$

Using

$$\alpha_s = \alpha_r + \gamma - \tfrac{\pi}{2} \tag{2.6}$$

this expression becomes:

$$\begin{aligned} A(\alpha_r,i_a,i_b,i_c) &= \hat{Z}_a\{i_a\sin(\alpha_r + \gamma - \tfrac{\pi}{2}) + \\ &\quad + i_b\sin(\alpha_r + \gamma - \tfrac{\pi}{2} - \tfrac{2}{3}\pi) + \\ &\quad + i_c\sin(\alpha_r + \gamma - \tfrac{\pi}{2} - \tfrac{4}{3}\pi)\} = \\ &= \hat{Z}_a\{i_a\sin(\alpha_r-\tfrac{\pi}{2})\cos(\gamma) + i_a\cos(\alpha_r-\tfrac{\pi}{2})\sin(\gamma) + \\ &\quad + i_b\sin(\alpha_r-\tfrac{\pi}{2})\cos(\gamma-\tfrac{2}{3}\pi) + i_b\cos(\alpha_r-\tfrac{\pi}{2})\sin(\gamma-\tfrac{2}{3}\pi) + \\ &\quad + i_c\sin(\alpha_r-\tfrac{\pi}{2})\cos(\gamma-\tfrac{4}{3}\pi) + i_c\cos(\alpha_r-\tfrac{\pi}{2})\sin(\gamma-\tfrac{4}{3}\pi)\} = \\ &= \hat{Z}_a\sin(\alpha_r-\tfrac{\pi}{2})\{i_a\cos(\gamma)+i_b\cos(\gamma-\tfrac{2}{3}\pi)+i_c\cos(\gamma-\tfrac{4}{3}\pi)\} + \\ &\quad + \hat{Z}_a\sin(\alpha_r)\{i_a\sin(\gamma)+i_b\sin(\gamma-\tfrac{2}{3}\pi)+i_c\sin(\gamma-\tfrac{4}{3}\pi)\} \end{aligned} \tag{2.7}$$

As can be seen in this expression, the current distibution

$A(\alpha_r, i_a, i_b, i_c)$ may also be caused by the currents

$$i_d = \frac{\sqrt{2}}{\sqrt{3}} \{i_a\cos(\gamma) + i_b\cos(\gamma-\tfrac{2}{3}\pi) + i_c\cos(\gamma-\tfrac{4}{3}\pi)\} \tag{2.8a}$$

and

$$i_q = \frac{\sqrt{2}}{\sqrt{3}} \{i_a\sin(\gamma) + i_b\sin(\gamma-\tfrac{2}{3}\pi) + i_c\sin(\gamma-\tfrac{4}{3}\pi)\} \tag{2.8b}$$

in, respectively, an imaginary winding on the direct axis with the distribution

$$Z_d(\alpha_r) = \frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a\sin(\alpha_r-\tfrac{\pi}{2}) \tag{2.9a}$$

and an imaginary winding on the quadrature axis with the distribution

$$Z_q(\alpha_r) = \frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a\sin(\alpha_r) \tag{2.9b}$$

together. The factor $\sqrt{2}/\sqrt{3}$ in (2.8) has been introduced in order to make the Park transformation, which will be dealt with later, orthogonal. The armature current distribution may now be described by

$$A(\alpha_r, i_d, i_q) = Z_d(\alpha_r)i_d + Z_q(\alpha_r)i_q \tag{2.10}$$

The current distribution in the air gap

Using (2.2), (2.3), (2.9), and (2.10), the total current distribution in the air gap may be expressed as:

$$A(\alpha_r) = A_d(\alpha_r) + A_q(\alpha_r) \tag{2.11a}$$

where

$$A_d(\alpha_r) = \left[\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a i_d + \hat{Z}_f i_f + \hat{Z}_{1d} i_{1d}\right]\sin(\alpha_r-\tfrac{\pi}{2}) \tag{2.11b}$$

$$A_q(\alpha_r) = \left[\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a i_q + \hat{Z}_{1q} i_{1q}\right]\sin(\alpha_r) \tag{2.11c}$$

After introducing the magnetizing currents

$$i_{dm} = i_d + \frac{\hat{Z}_f}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} i_f + \frac{\hat{Z}_{1d}}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} i_{1d} \tag{2.12a}$$

$$i_{qm} = i_q + \frac{\hat{Z}_{1q}}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} i_{1q} \tag{2.12b}$$

(2.11b) and (2.11c) become

$$A_d(\alpha_r) = \frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a\sin(\alpha_r-\tfrac{\pi}{2})i_{dm} \tag{2.13b}$$

$$A_q(\alpha_r) = \frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a\sin(\alpha_r)i_{qm} \tag{2.13c}$$

Using the winding ratios (with respect to the imaginary windings on the quadrature and on the direct axis)

$$K_f = \frac{\hat{Z}_f}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} \quad ; \quad K_{1d} = \frac{\hat{Z}_{1d}}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} \tag{2.14a}$$

$$K_{1q} = \frac{\hat{Z}_{1q}}{\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a} \tag{2.14b}$$

(2.12) may be written as

$$i_{dm} = i_d + K_f i_f + K_{1d} i_{1d} \tag{2.15a}$$

$$i_{qm} = i_q + K_{1q} i_{1q} \tag{2.15b}$$

The magnetic induction in the air gap

The current distribution A_d (alone) results in an induction distribution in the air gap $B_d(\alpha_r, i_{dm})$. On the basis of the symmetry with respect to the direct and to the quadrature axis, this distribution can be represented by:

$$B_d(\alpha_r, i_{dm}) = \sum_{k=0}^{\infty} B_{dk}(i_{dm}) \cos\{(2k+1)(\alpha_r - \tfrac{\pi}{2})\} \tag{2.16}$$

The current distribution A_q (alone) results in an induction distribution in the air gap $B_q(\alpha_r, i_{qm})$. On the basis of the symmetry with respect to the direct and to the quadrature axis, this distribution can be represented by:

$$B_q(\alpha_r, i_{qm}) = \sum_{k=0}^{\infty} B_{qk}(i_{qm}) \cos\{(2k+1)\alpha_r\} \tag{2.17}$$

As long as the magnetic circuit is supposed to be linear, the total magnetic induction in the air gap may be computed by means of superposition:

$$B(\alpha_r, i_{dm}, i_{qm}) = B_d(\alpha_r, i_{dm}) + B_q(\alpha_r, i_{qm}) \tag{2.18}$$

Using (2.16) and (2.17), this expression becomes

$$B(\alpha_r, i_{dm}, i_{qm}) = \sum_{k=0}^{\infty} [B_{dk}(i_{dm}) \cos\{(2k+1)(\alpha_r - \tfrac{\pi}{2})\} + B_{qk}(i_{qm})) \cos\{(2k+1)\alpha_r\}] \tag{2.19}$$

The flux linked with an arbitrary sinusoidally distributed armature winding caused by the air gap induction

Next, the flux associated with a sinusoidally distributed winding and corresponding with the air gap induction will be determined. For this purpose a winding with an arbitrary axis $(\alpha_r = \alpha_o)$ and with a distribution:

$$Z = \hat{Z}\sin(\alpha_r - \alpha_o) \tag{2.20}$$

will be considered (figure 2.2).

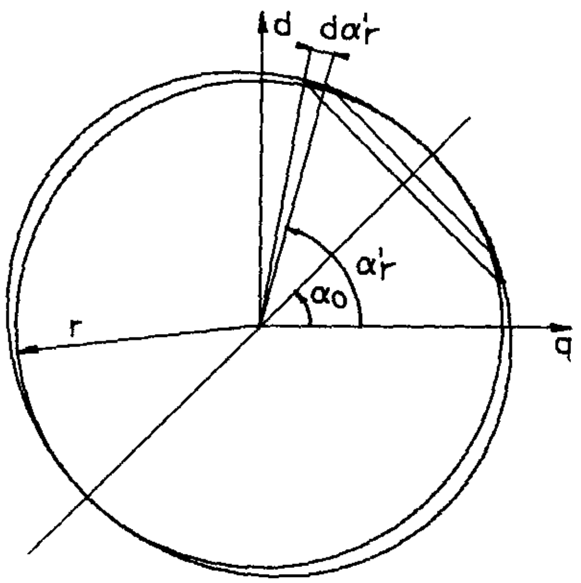

Figure 2.2 The determination of the flux associated with an armature winding

The flux linking a turn whose sides are located at the positions α'_r and $2\alpha_o - \alpha'_r$ is:

$\int_{2\alpha_o - \alpha'_r}^{\alpha'_r} B(\alpha_r) lr d\alpha_r$. On the part indicated by $d\alpha'_r$ at the positions α'_r and $2\alpha_o - \alpha'_r$, $r\hat{Z}\sin(\alpha'_r - \alpha_o) d\alpha'_r$ windings are present. So, the flux linked with this coil is

$$r\hat{Z}\sin(\alpha'_r - \alpha_o) \left\{ \int_{2\alpha_o - \alpha'_r}^{\alpha'_r} B(\alpha_r) lr d\alpha_r \right\} d\alpha'_r$$

The flux linked with the complete winding considered is

$$\psi = \int_{\alpha_o}^{\alpha_o + \pi} r\hat{Z}\sin(\alpha'_r - \alpha_o) \left\{ \int_{2\alpha_o - \alpha'_r}^{\alpha'_r} B(\alpha_r) lr d\alpha_r \right\} d\alpha'_r \tag{2.21}$$

Substituting (2.19) in (2.21) gives

$$\psi = r^2 \hat{Z} l \pi \{ B_{d1}(i_{dm}) \cos(\alpha_o - \tfrac{\pi}{2}) + B_{q1}(i_{qm}) \cos(\alpha_o) \} \tag{2.22}$$

As can be seen in this expression, only the fundamental component of the magnetic induction in the air gap $B(\alpha_r)$ contributes to the flux associated with a sinusoidally distributed winding: the sinusoidally distributed winding acts as a filter.

The main flux of the windings on the quadrature axis

When $\alpha_o=0$, (2.20) describes a winding on the quadrature axis. The flux linked with this winding caused by the air gap induction, the *main* flux, is according to (2.22):

$$\psi_{mq}(i_{qm}) = r^2\hat{Z}l\pi B_{q1}(i_{qm}) \tag{2.23}$$

When the imaginary armature winding on the quadrature axis according to (2.9b) is considered, $\hat{Z}$ is equal to $\sqrt{3}/\sqrt{2}\hat{Z}_a$. The flux linked with this winding is:

$$\psi_{mq}(i_{qm}) = r^2\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a l\pi B_{q1}(i_{qm}) \tag{2.24}$$

Now, we introduce the induction coefficient L_{mq}, which is a function of the current i_{qm} in order to incorporate saturation, as (with (2.24)):

$$L_{mq}(i_{qm}) = \frac{\psi_{mq}}{i_{qm}} = \frac{r^2\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a l\pi B_{q1}(i_{qm})}{i_{qm}} \tag{2.25}$$

Using (2.14b), (2.23), and (2.25), we may find for the main flux of the windings on the quadrature axis (with (2.15b)):

$$\psi_{mq} = L_{mq}(i_{qm})\, i_{qm} = L_{mq}(i_{qm})\,(i_q + K_{1q}i_{1q}) \tag{2.26a}$$

$$\psi_{m1q} = K_{1q}\, L_{mq}(i_{qm})\, i_{qm} = K_{1q}\, L_{mq}(i_{qm})\,(i_q + K_{1q}i_{1q}) \tag{2.26b}$$

The main flux of the windings on the direct axis

When $\alpha_o=\pi/2$, (2.20) describes a winding on the direct axis. The flux linked with this winding caused by the air gap induction is according to (2.22):

$$\psi(i_{dm}) = r^2\hat{Z}l\pi B_{d1}(i_{dm}) \tag{2.27}$$

When the imaginary armature winding on the direct axis according to (2.9a) is considered, $\hat{Z}$ is equal to $\sqrt{3}/\sqrt{2}\hat{Z}_a$. The flux linked with this winding is:

$$\psi_{md}(i_{dm}) = r^2\frac{\sqrt{3}}{\sqrt{2}}\hat{Z}_a l\pi B_{d1}(i_{dm}) \tag{2.28}$$

Now, we introduce the induction coefficient L_{md}, which is a function of the current i_{dm} in order to incorporate *saturation*, as (with (2.28)):

$$L_{md}(i_{dm}) = \frac{\psi_{md}}{i_{dm}} = \frac{r\frac{2\sqrt{3}}{\sqrt{2}}\hat{Z}_a l \pi B_{d1}(i_{dm})}{i_{dm}} \tag{2.29}$$

Using (2.14a), (2.27), and (2.29), we may find for the main flux of the windings on the direct axis (with (2.15a)):

$$\psi_{md} = L_{md}(i_{dm})\, i_{dm} = L_{md}(i_{dm})\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.30a}$$

$$\psi_{mf} = K_f\, L_{md}(i_{dm})\, i_{dm} = K_f\, L_{md}(i_{dm})\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.30b}$$

$$\psi_{m1d} = K_{1d}\, L_{md}(i_{dm})\, i_{dm} = K_{1d}\, L_{md}(i_{dm})\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.30c}$$

On the saturation model

In the previous two parts of this section, the direct and the quadrature axis have been dealt with separately: the coefficient L_{md} only depended on the currents in the windings on the direct axis, while L_{mq} only depended on the currents in the windings on the quadrature axis. As has been explained in [Mel 86], it is probably better to suppose that L_{md} and L_{mq} depend on the total flux in the machine. The main flux

$$\psi_m = \sqrt{\psi_{md}^2+\psi_{mq}^2} \tag{2.31}$$

may be used to represent this total flux.

Hence, $L_{md}(\psi_m)$ and $L_{mq}(\psi_m)$ will be used instead of, respectively, $L_{md}(i_{dm})$ and $L_{mq}(i_{qm})$. So, the equations (2.26) and (2.30) become:

$$\psi_{mq} = L_{mq}(\psi_m)\,(i_q + K_{1q} i_{1q}) \tag{2.32a}$$

$$\frac{\psi_{m1q}}{K_{1q}} = L_{mq}(\psi_m)\,(i_q + K_{1q} i_{1q}) \tag{2.32b}$$

$$\psi_{md} = L_{md}(\psi_m)\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.33a}$$

$$\frac{\psi_{mf}}{K_f} = L_{md}(\psi_m)\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.33b}$$

$$\frac{\psi_{m1d}}{K_{1d}} = L_{md}(\psi_m)\,(i_d+K_f i_f+K_{1d} i_{1d}) \tag{2.33c}$$

The rotor leakage flux

The flux linked with a rotor winding consists of a main flux (according to (2.32b), (2.33b), and (2.33c)) and a leakage flux. The leakage flux of the damper winding on the quadrature axis is incorporated by using the (on the stator reduced) coefficient $L_{l1q\sigma}$. Now, (2.32b) is extended as follows:

$$\frac{\psi_{1q}}{K_{1q}} = L_{mq}(\psi_m)\ (i_q + K_{1q}i_{1q}) + L_{11q\sigma}\ K_{1q}i_{1q} \tag{2.34}$$

The leakage fluxes of the windings on the direct axis are incorporated by using the factors $L_{f\sigma}$ (self inductivity reduced on the stator), $L_{11d\sigma}$ (self inductivity reduced on the stator), and $L_{f1d\sigma}$ (mutual inductivity reduced on the stator). Hence, (2.33b) and (2.33c) may be extended according to

$$\frac{\psi_f}{K_f} = L_{md}(\psi_m)\ (i_d+K_fi_f+K_{1d}i_{1d}) + L_{f\sigma}\ K_fi_f + L_{f1d\sigma}\ K_{1d}i_{1d} \tag{2.35a}$$

$$\frac{\psi_{1d}}{K_{1d}} = L_{md}(\psi_m)\ (i_d+K_fi_f+K_{1d}i_{1d}) + L_{f1d\sigma}\ K_fi_f + L_{11d\sigma}\ K_{1d}i_{1d} \tag{2.35b}$$

Generally, the (reduced) leakage coefficients (subscript σ) are much smaller than the main coefficients (subscript m) and hardly depend on saturation phenomena.

The main flux linked with the armature phase windings

Using (2.22), (2.24), (2.27), and the position of the axis of the armature phase winding a ($\alpha_r=\pi/2-\gamma$), an expression for the (m̲ain) flux linked with this winding caused by the air gap induction may be derived:

$$\psi_{ma} = \frac{\sqrt{2}}{\sqrt{3}}\ (\psi_{md}\cos\gamma + \psi_{mq}\sin\gamma) \tag{2.36a}$$

Similar expressions may be derived for the phase winding b and c:

$$\psi_{mb} = \frac{\sqrt{2}}{\sqrt{3}}\ (\psi_{md}\cos(\gamma-\tfrac{2}{3}\pi) + \psi_{mq}\sin(\gamma-\tfrac{2}{3}\pi)) \tag{2.36b}$$

$$\psi_{mc} = \frac{\sqrt{2}}{\sqrt{3}}\ (\psi_{md}\cos(\gamma-\tfrac{4}{3}\pi) + \psi_{mq}\sin(\gamma-\tfrac{4}{3}\pi)) \tag{2.36c}$$

The armature leakage flux

The flux associated with an armature winding consists of a main flux (according to (2.36)) and a leakage flux. This leakage flux is supposed to be independent of the rotor position. On the basis of armature symmetry the leakage fluxes associated with the armature windings can be represented by:

$$\psi_{a\sigma} = L_{ao\sigma}i_a - L_{abo\sigma}i_b - L_{abo\sigma}i_c \tag{2.37a}$$

$$\psi_{b\sigma} = L_{ao\sigma}i_b - L_{abo\sigma}i_a - L_{abo\sigma}i_c \tag{2.37b}$$

$$\psi_{c\sigma} = L_{ao\sigma}i_c - L_{abo\sigma}i_a - L_{abo\sigma}i_b \tag{2.37c}$$

where the inductances $L_{ao\sigma}$ and $L_{abo\sigma}$ are introduced. Although $L_{abo\sigma}$ will be positive in most cases, it might be negative.

Using the homopolar component of the three-phase system of armature currents

$$i_0 = \frac{1}{\sqrt{3}} (i_a + i_b + i_c) \tag{2.38}$$

the expressions (2.37) may be written as

$$\psi_{a\sigma} = (L_{ao\sigma}+L_{abo\sigma})i_a - \sqrt{3}L_{abo\sigma}i_0 \tag{2.39a}$$

$$\psi_{b\sigma} = (L_{ao\sigma}+L_{abo\sigma})i_b - \sqrt{3}L_{abo\sigma}i_0 \tag{2.39b}$$

$$\psi_{c\sigma} = (L_{ao\sigma}+L_{abo\sigma})i_c - \sqrt{3}L_{abo\sigma}i_0 \tag{2.39c}$$

In many cases the homopolar component i_0 is zero; in those cases the leakage inductance

$$L_{a\sigma} = L_{ao\sigma} + L_{abo\sigma} \tag{2.40}$$

is effective in each phase winding.

Using (2.36), (2.39), and (2.40), the flux linked with the armature windings may be expressed as:

$$\psi_a = \psi_{ma} + \psi_{\sigma a} = \frac{\sqrt{2}}{\sqrt{3}} (\psi_{md}\cos(\gamma)+\psi_{mq}\sin(\gamma)) + L_{a\sigma}i_a - \sqrt{3}L_{abo\sigma}i_0 \tag{2.41a}$$

$$\psi_b = \psi_{mb} + \psi_{\sigma b} = \frac{\sqrt{2}}{\sqrt{3}}(\psi_{md}\cos(\gamma-\tfrac{2}{3}\pi)+\psi_{mq}\sin(\gamma-\tfrac{2}{3}\pi))+L_{a\sigma}i_b-\sqrt{3}L_{abo\sigma}i_0 \tag{2.41b}$$

$$\psi_c = \psi_{mc} + \psi_{\sigma c} = \frac{\sqrt{2}}{\sqrt{3}}(\psi_{md}\cos(\gamma-\tfrac{4}{3}\pi)+\psi_{mq}\sin(\gamma-\tfrac{4}{3}\pi))+L_{a\sigma}i_c-\sqrt{3}L_{abo\sigma}i_0 \tag{2.41c}$$

The Park transformation

In (2.8) and (2.38) we implicitly introduced the Park transformation for the armature currents according to

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \mathbf{P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2.42}$$

where $\mathbf{P}$ is the Park transformation matrix:

$$\mathbf{P} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} \cos(\gamma) & \cos(\gamma-\frac{2}{3}\pi) & \cos(\gamma-\frac{4}{3}\pi) \\ \sin(\gamma) & \sin(\gamma-\frac{2}{3}\pi) & \sin(\gamma-\frac{4}{3}\pi) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{2.43}$$

In a similar way we may introduce the Park transformation for the armature flux linkages:

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} = \mathbf{P} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} \tag{2.44}$$

Using the matrix $\mathbf{P}$ according to (2.43) and equation (2.42), we may write (2.41) in matrix form:

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \mathbf{P}^{-1} \begin{bmatrix} \psi_{md} \\ \psi_{mq} \\ 0 \end{bmatrix} + L_{a\sigma}\mathbf{P}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - \sqrt{3}L_{abo\sigma} \begin{bmatrix} i_0 \\ i_0 \\ i_0 \end{bmatrix} \tag{2.45}$$

The dependence on the position angle γ in (2.41) has been incorporated in the matrix $\mathbf{P}$. Using (2.42) for the last term in (2.45), we also may write this equation as:

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \mathbf{P}^{-1} \begin{bmatrix} \psi_{md} + L_{a\sigma}i_d \\ \psi_{mq} + L_{a\sigma}i_q \\ L_{a\sigma}i_0 \end{bmatrix} - \sqrt{3}L_{abo\sigma}\mathbf{P}^{-1}\sqrt{3} \begin{bmatrix} 0 \\ 0 \\ i_0 \end{bmatrix}$$

$$= \mathbf{P}^{-1} \begin{bmatrix} \psi_{md} + L_{a\sigma}i_d \\ \psi_{mq} + L_{a\sigma}i_q \\ (L_{a\sigma} - 3L_{abo\sigma})i_0 \end{bmatrix} \tag{2.46}$$

Substituting (2.46) into (2.44) gives:

$$\psi_d = \psi_{md} + L_{a\sigma}i_d \tag{2.47a}$$

$$\psi_q = \psi_{mq} + L_{a\sigma}i_q \tag{2.47b}$$

and

$$\psi_0 = L_0 i_0 \tag{2.48a}$$

where

$$L_0 = L_{ao\sigma} - 2L_{abo\sigma} \tag{2.48b}$$

As may be seen with the help of (2.47), $L_{a\sigma}$ may be considered as the leakage flux inductance of the imaginary windings on the direct and on the quadrature axis according to (2.9).

The quadrature axis windings

Using (2.47b), (2.32a), and (2.34) the equivalent circuit for the fluxes in the quadrature axis in figure 2.3 may be drawn.

After introducing the inductance coefficients

$$L_q = L_{a\sigma} + L_{mq} \tag{2.49}$$

and

$$L_{a1q} = K_{1q}L_{mq} \quad ; \quad L_{11q} = K_{1q}^2(L_{mq}+L_{11q\sigma}) \tag{2.50}$$

flux equations (2.34) and (2.47b) with (2.32a) become:

$$\psi_q = L_q i_q + L_{a1q}i_{1q} \tag{2.51a}$$

$$\psi_{1q} = L_{a1q}i_q + L_{11q}i_{1q} \tag{2.51b}$$

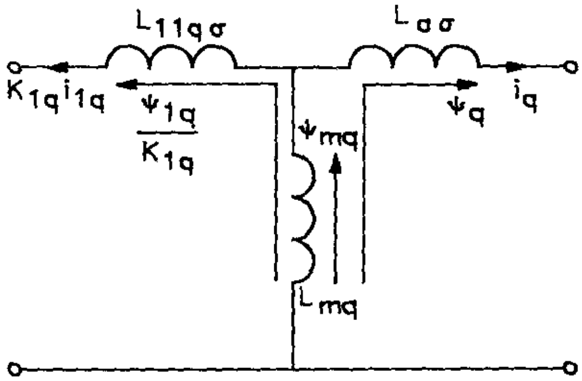

Figure 2.3 An equivalent circuit for the fluxes in the quadrature axis

The direct axis windings

Using (2.47a), (2.33a), and (2.35) the equivalent circuit for the fluxes in the direct axis in figure 2.4 may be drawn.

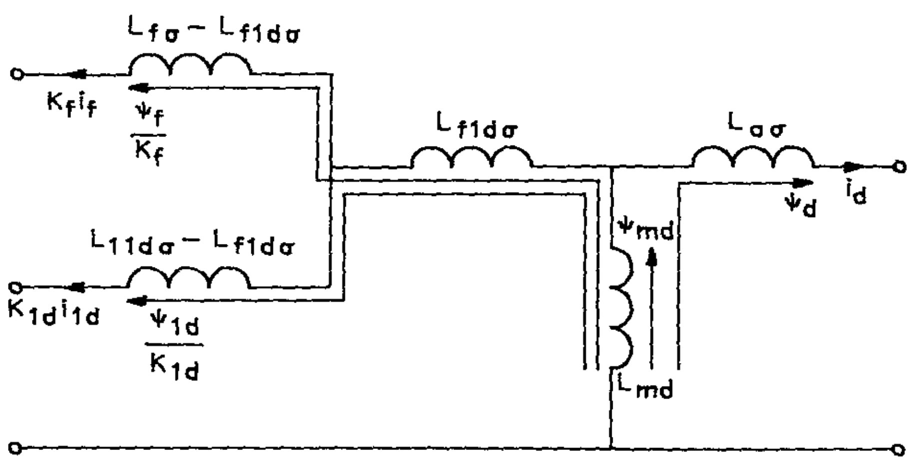

Figure 2.4 An equivalent circuit for the fluxes in the direct axis

After introducing the inductance coefficients

$$L_d = L_{a\sigma} + L_{md} \tag{2.52}$$

and

$$L_{afd}=K_f L_{md} \;;\; L_{a1d}=K_{1d}L_{md} \;;\; L_f=K_f^2(L_{md}+L_{f\sigma}) \;;$$
$$L_{f1d}=K_f K_{1d}(L_{md}+L_{f1d\sigma}) \;;\; L_{11d}=K_{1d}^2(L_{md}+L_{11d\sigma}) \tag{2.53}$$

the flux equations (2.35) and (2.47a) with (2.33a) become:

$$\psi_d = L_d i_d + L_{afd} i_f + L_{a1d} i_{1d} \tag{2.54a}$$

$$\psi_f = L_{afd} i_d + L_f i_f + L_{f1d} i_{1d} \tag{2.54b}$$

$$\psi_{1d} = L_{a1d} i_d + L_{f1d} i_f + L_{11d} i_{1d} \tag{2.54c}$$

2.3 The voltage equations

The armature voltage equations

The armature winding voltage equations are:

$$u_a = - R_a i_a - \frac{d\psi_a}{dt} \tag{2.55a}$$

$$u_b = - R_a i_b - \frac{d\psi_b}{dt} \tag{2.55b}$$

$$u_c = - R_a i_c - \frac{d\psi_c}{dt} \tag{2.55c}$$

where R_a is the armature winding resistance.

After introducing the Park transform for the armature winding voltages

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = \mathbf{P} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \tag{2.56}$$

these voltage equations may be Park transformed by using (2.42), (2.44), and (2.56) (in matrix form):

$$\mathbf{P}^{-1} \begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = - R_a \mathbf{P}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - \frac{d}{dt} \left\{ \mathbf{P}^{-1} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} \right\}$$

Multiplication with $\mathbf{P}$ and further calculation give

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = - R_a \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} - \mathbf{P} \frac{d\mathbf{P}^{-1}}{dt} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix}$$

Using (2.43) the factor $\mathbf{P} \frac{d\mathbf{P}^{-1}}{dt}$ may be evaluated::

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = - R_a \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} + \frac{d\gamma}{dt} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix} \tag{2.57}$$

The rotor voltage equations

The rotor winding voltage equations are:

$$0 = R_{11q} i_{1q} + \frac{d\psi_{1q}}{dt} \tag{2.58}$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \tag{2.59}$$

$$0 = R_{11d} i_{1d} + \frac{d\psi_{1d}}{dt} \tag{2.60}$$

The electric power

Using (2.42) and (2.56), the expression for the electric power withdrawn from the armature may be written as:

$$p = u_a i_a + u_b i_b + u_c i_c = u_d i_d + u_q i_q + u_0 i_0 \tag{2.61}$$

Using (2.57) this expression becomes:

$$p = -\left[i_d \frac{d\psi_d}{dt} + i_q \frac{d\psi_q}{dt} + i_0 \frac{d\psi_0}{dt}\right] - \left[i_d \psi_q - i_q \psi_d\right] \frac{d\gamma}{dt} - R_a \left[i_d^2 + i_q^2 + i_0^2\right] \tag{2.62}$$

The homopolar components

In this report we consider a star connected synchronous machine, the star connection of which is not used. Hence, according to (2.38), the homopolar current component equals zero: $i_0 = 0$. Using (2.48a) and (2.57), it may be seen that ψ_0 and u_0 are zero too.

2.4 The mechanical side of the machine

In the previous sections the electrical equations of a two-pole synchronous machine have been derived (the number of pole pairs equals 1). In this section these equations will be adapted for machines with p larger than 1. Besides, an expression for the electromagnetic torque will be given.

The number of poles

A machine with p larger than 1 may be seen as arised from p identical two-pole machines which have been cut open at the same place. Next, they are bended and put together in order to get a new machine with a larger diameter. The phase windings of the separate machines may be connected in series or in parallel. In figure 2.5, it is depicted how a machine with four poles (p=2) may arise from two machines with two poles each (p=1). In this figure the rotor windings have not been drawn and the poles are indicated with N (North) and S (South).

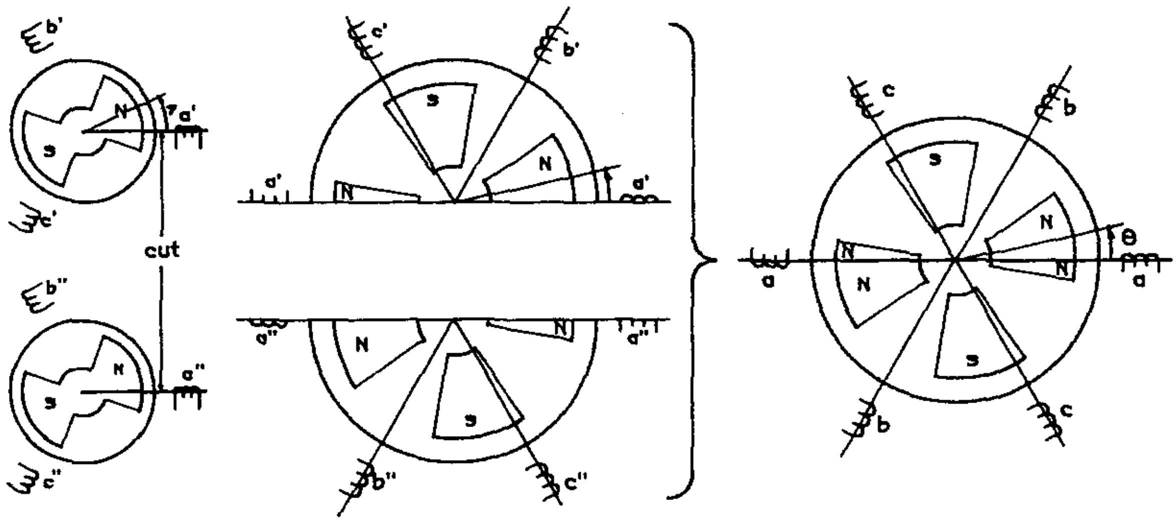

Figure 2.5 A machine with four poles arising from two machines with two poles each

If a machine with p larger than 1 is looked upon as been described in the previous paragraph, it may easily be seen that for the description of a machine with p larger than 1 a description of a machine with p=1 is sufficient. However, we have to take into account that the spatial angles used in the previous sections have to be multiplied by 1/p in reality. Hence, the rotor position angle, the angle between the positive direct axis and the axis of armature winding a, is γ/p in reality. This real position angle will be indicated by θ:

$$\gamma = p\theta \tag{2.63}$$

So, the relation between the (mechanical) angular speed ω_m and γ is:

$$\omega_m = \frac{d\theta}{dt} = \frac{1}{p}\frac{d\gamma}{dt} \tag{2.64}$$

The electromagnetic torque

The electromagnetic torque follows from the second term in the power equation (2.62). Hence, using (2.64) results into:

$$m = p(i_d\psi_q - i_q\psi_d) \tag{2.65}$$

3 ADAPTING THE MACHINE MODEL FOR THE SIMULATION OF A SYNCHRONOUS MACHINE WITH RECTIFIER

3.1 Introduction

Since the rotor winding fluxes may be considered as constant for very fast phenomena like the commutation in a rectifier, it is advantageous to use these fluxes as state variables in the machine model for the synchronous machine with rectifier.

In this chapter the equations of the synchronous machine derived in chapter 2 will be adapted for this purpose. First, this will be done for the flux current relations. Next, the saturation model will be simplified, and finally, the total set of machine equations will be adapted and slightly simplified.

3.2 The flux current relations

The quadrature axis

For surveyability, the expressions (2.51) are repeated here:

$$\psi_q = L_q i_q + L_{a1q} i_{1q} \tag{3.1a}$$

$$\psi_{1q} = L_{a1q} i_q + L_{11q} i_{1q} \tag{3.1b}$$

After reducing the quantities which are related to the quadrature-axis damper winding with

$$C_{1Q} = \frac{L_{a1q}}{L_{11q}} \tag{3.2}$$

and introducing

$$\psi_{1Q} = C_{1Q}\, \psi_{1q} \tag{3.3a}$$

$$i_{1Q} = \frac{1}{C_{1Q}}\, i_{1q} \tag{3.3b}$$

$$L_{1Q} = C_{1Q}^2\, L_{11q} \tag{3.3c}$$

the quadrature axis equations (3.1) may be written as

$$\psi_q = L_q i_q + L_{1Q} i_{1Q} = (L_q - L_{1Q}) i_q + L_{1Q} i_q + L_{1Q} i_{1Q} \tag{3.4a}$$

$$\psi_{1Q} = L_{1Q} i_q + L_{1Q} i_{1Q} \tag{3.4b}$$

After introducing

$$L''_q = L_q - L_{1Q} \tag{3.5}$$

these equations become:

$$\psi_q = L''_q i_q + L_{1Q}(i_q+i_{1Q}) \tag{3.6a}$$

$$\psi_{1Q} = L_{1Q}(i_q+i_{1Q}) \tag{3.6b}$$

These equations are represented in figure 3.1.

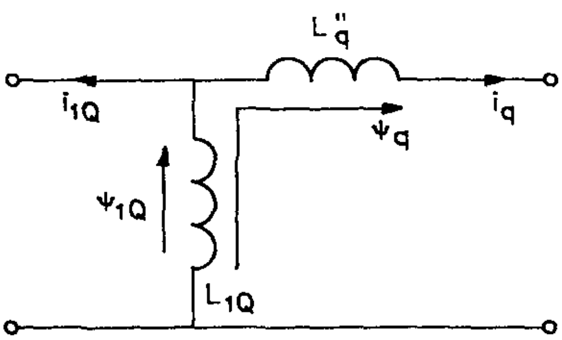

Figure 3.1 The flux linkages in the quadrature axis

In order to model saturation, the parameters C_{1Q}, L''_q, and L_{1Q} are expressed as functions of the parameters introduced in section 2.2. Using (3.2), (3.3c), (3.5), (2.49), and (2.50), the parameters C_{1Q}, L''_q, and L_{1Q} become

$$C_{1Q} = \frac{1}{K_{1q}\left(1+\frac{L_{11q\sigma}}{L_{mq}}\right)} \tag{3.7a}$$

$$L''_q = \frac{L_{a\sigma}+L_{11q\sigma}+\frac{L_{a\sigma}L_{11q\sigma}}{L_{mq}}}{1+\frac{L_{11q\sigma}}{L_{mq}}} \tag{3.7b}$$

$$L_{1Q} = \frac{L_{mq}}{1+\frac{L_{11q\sigma}}{L_{mq}}} \tag{3.7c}$$

Although it is possible to calculate these parameters as functions of L_{mq}, which depend on the flux level in the machine, this method is very long-winded. Here, in the factors in the parameters in which the influence of L_{mq} is small (the leakage inductance is always very small compared to the main inductance), the main inductance L_{mq} is replaced by its unsaturated values L_{mqu}. With this supposition, the parameters C_{1Q} and L''_q do not depend on saturation. The parameter L_{1Q} may now be written as

$$L_{1Q} = \frac{L_{mq}}{1+\frac{L_{11q\sigma}}{L_{mqu}}} = \frac{L_{mq}}{L_{mqu}} L_{1Qu} \tag{3.8}$$

In this expression L_{1Qu} is the unsaturated value of L_{1Q}.
For further use, we introduce the saturation factor

$$S_q = \frac{L_{mq}}{L_{mqu}} \tag{3.9}$$

which is supposed to depend on the main flux ψ_m according to (2.31). Using the suppositions mentioned before and the equations (3.8) and (3.9), the flux expressions (3.6) become

$$\psi_q = L''_q i_q + S_q L_{1Qu}(i_q+i_{1Q}) \tag{3.10a}$$

$$\psi_{1Q} = S_q L_{1Qu}(i_q+i_{1Q}) \tag{3.10b}$$

The direct axis

For surveyability, the expressions (2.54) are repeated here:

$$\psi_d = L_d i_d + L_{afd} i_f + L_{a1d} i_{1d} \tag{3.11a}$$

$$\psi_f = L_{afd} i_d + L_f i_f + L_{f1d} i_{1d} \tag{3.11b}$$

$$\psi_{1d} = L_{a1d} i_d + L_{f1d} i_f + L_{11d} i_{1d} \tag{3.11c}$$

After reducing the quantities which are related to the direct-axis damper winding with

$$C_{1D} = \frac{L_{a1d}}{L_{11d}} \tag{3.12}$$

and introducing

$$\psi_{1D} = C_{1D}\, \psi_{1d} \tag{3.13a}$$

$$i_{1D} = \frac{1}{C_{1D}}\, i_{1d} \tag{3.13b}$$

$$L_{1D} = C_{1D}^2\, L_{11d} \tag{3.13c}$$

$$K_{f1D} = \frac{L_{f1d}}{L_{a1d}} \tag{3.14}$$

the equations (3.11) may be written as

$$\psi_d = L_d i_d + L_{afd} i_f + L_{1D} i_{1D} \tag{3.15a}$$

$$\psi_f = L_{afd} i_d + L_f i_f + K_{f1D} L_{1D} i_{1D} \tag{3.15b}$$

$$\psi_{1D} = L_{1D} i_d + L_{1D} K_{f1D} i_f + L_{1D} i_{1D} \tag{3.15c}$$

The set of equations (3.15) may be written in the form:

$$\psi_d = (L_d - L_{1D})i_d + (L_{afd} - K_{f1D}L_{1D})i_f + L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.16a)$$

$$\psi_f = (L_{afd} - K_{f1D}L_{1D})i_d + (L_f - K_{f1D}^2 L_{1D})i_f + K_{f1D}L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.16b)$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.16c)$$

After introducing

$$L'_f = L_f - K_{f1D}^2 L_{1D} \quad (3.17a)$$

and

$$C_F = \frac{L_{afd} - K_{f1D}L_{1D}}{L'_f} \quad (3.17b)$$

(3.16) may be written as

$$\psi_d = (L_d - L_{1D} - C_F^2 L'_f)i_d + C_F L'_f(C_F i_d + i_f) + L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.18a)$$

$$\psi_f = L'_f(C_F i_d + i_f) + K_{f1D}L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.18b)$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.18c)$$

Using

$$L''_d = L_d - L_{1D} - C_F^2 L'_f \quad (3.19)$$

the equations (3.18) become:

$$\psi_d = L''_d i_d + C_F L'_f(C_F i_d + i_f) + L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.20a)$$

$$\psi_f = L'_f(C_F i_d + i_f) + K_{f1D}L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.20b)$$

$$\psi_{1D} = L_{1D}(i_d + K_{f1D}i_f + i_{1D}) \quad (3.20c)$$

These equations are represented in figure 3.2.

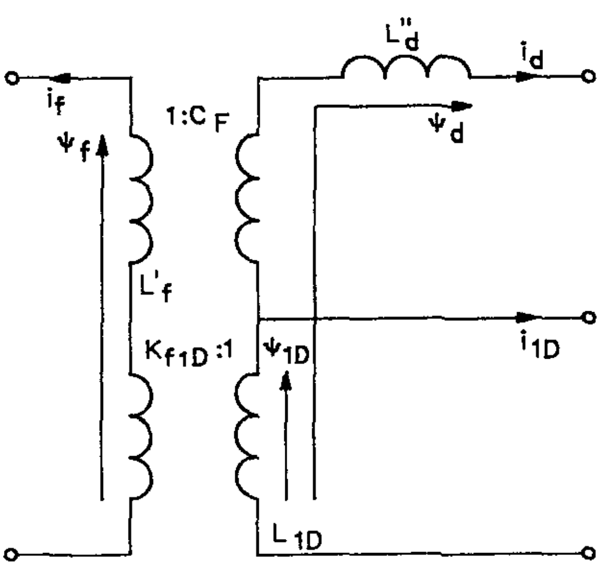

Figure 3.2 The flux linkages in the direct axis

In order to model saturation, the parameters C_{1D}, L_d'', C_F, L_f', K_{f1D} and L_{1D} are expressed as functions of the parameters introduced in section 2.2. Using (3.12), (3.13c), (3.15), (3.17), (3.10), (2.52), and (2.53), the parameters C_{1D}, L_d'', C_F, L_f', K_{f1D} become

$$C_{1D} = \frac{1}{K_{1d}\left(1+\frac{L_{11d\sigma}}{L_{md}}\right)} \tag{3.21a}$$

$$L_d'' = L_{a\sigma} + \frac{1}{1+\frac{L_{f1d\sigma}}{L_{md}}} \frac{L_{f\sigma}L_{11d\sigma} - L_{f1d\sigma}^2}{(L_{f\sigma}-L_{f1d\sigma})+(L_{11d\sigma}-L_{f1d\sigma})+\frac{(L_{f\sigma}-L_{f1d\sigma})(L_{11d\sigma}-L_{f1d\sigma})}{L_{md}+L_{f1d\sigma}}} \tag{3.21b}$$

$$C_F = \frac{1}{K_f} \frac{1}{1+\frac{L_{f1d\sigma}}{L_{md}}} \frac{L_{11d\sigma} - L_{f1d\sigma}}{L_{f\sigma}-L_{f1d\sigma}+L_{11d\sigma}-L_{f1d\sigma}+\frac{(L_{f\sigma}-L_{f1d\sigma})(L_{11d\sigma}-L_{f1d\sigma})}{L_{md}+L_{f1d\sigma}}} \tag{3.21c}$$

$$L_f' = K_f^2 \frac{(L_{f\sigma}-L_{f1d\sigma})+(L_{11d\sigma}-L_{f1d\sigma})+\frac{(L_{f\sigma}-L_{f1d\sigma})(L_{11d\sigma}-L_{f1d\sigma})}{L_{md}+L_{f1d\sigma}}}{1+\frac{L_{11d\sigma}-L_{f1d\sigma}}{L_{md}+L_{f1d\sigma}}} \tag{3.21d}$$

$$K_{f1D} = K_f \left(1 + \frac{L_{f1d\sigma}}{L_{md}}\right) \tag{3.21e}$$

$$L_{1D} = \frac{L_{md}}{1+\frac{L_{11d\sigma}}{L_{md}}} \tag{3.21f}$$

Although it is possible to calculate these parameters as functions of L_{md}, which depend on the flux level in the machine, this method is very long-winded. Here, in the factors in the parameters in which the influence of L_{md} is small (the leakage inductance is always very small compared to the main inductance), the main inductance L_{md} is replaced by its unsaturated values L_{mdu}. With this supposition, the parameters C_{1D}, L_d'', C_F, L_f', and K_{f1D} do not depend on saturation. The parameter L_{1D} may now be written as

$$L_{1D} = \frac{L_{md}}{1+\frac{L_{11d\sigma}}{L_{mdu}}} = \frac{L_{md}}{L_{mdu}} L_{1Du} \tag{3.22}$$

In this expressions L_{1Du} is the unsaturated values of L_{1D}.
For further use, we introduce the saturation factor

$$S_d = \frac{L_{md}}{L_{mdu}} \tag{3.23}$$

which is supposed to depend on the main flux ψ_m according to (2.31). Using the suppositions mentioned before and the equations (3.22) and (3.23), the flux expressions (3.20) become

$$\psi_d = L''_d i_d + C_F L'_f (C_F i_d + i_f) + S_d L_{1Du}(i_d + K_{f1D} i_f + i_{1D}) \tag{3.24a}$$

$$\psi_f = L'_f (C_F i_d + i_f) + K_{f1D} S_d L_{1Du}(i_d + K_{f1D} i_f + i_{1D}) \tag{3.24b}$$

$$\psi_{1D} = S_d L_{1Du}(i_d + K_{f1D} i_f + i_{1D}) \tag{3.24c}$$

3.3 Modelling saturation

As suggested in, for example, [Jon 82], the saturation factors are supposed to be equal according to

$$S_q = S_d = \frac{1}{1 + a\psi_m^6} \tag{3.25}$$

The main flux ψ_m may be found by means of (2.31).
Instead of the armature flux behind (see the the figures 2.3 and 2.4) the leakage inductance (main flux; (2.31) and (2.47)):

$$\psi_m = \sqrt{\psi_{md}^2 + \psi_{mq}^2}$$

$$\psi_{md} = \psi_d - L_{a\sigma} i_d$$

$$\psi_{mq} = \psi_q - L_{a\sigma} i_q$$

the flux behind (see the figures 3.1 and 3.2) the subtransient inductance will be used here:

$$S_q = S_d = \frac{1}{1 + a\psi''^6} \tag{3.26}$$

$$\psi'' = \sqrt{\psi''^2_d + \psi''^2_q} \tag{3.27}$$

$$\psi''_d = \psi_d - L''_d i_d \tag{3.28a}$$

$$\psi''_q = \psi_q - L''_q i_q \tag{3.28b}$$

3.4 The machine equations

As a result of the suppositions in section 3.2, only the parameters S_d and S_q depend on the saturation level; the other parameters are supposed to be constant.

The voltage equations

Using (2.64), the armature voltage equation (2.57) may be written as (homopolar components are ignored):

$$u_q = - R_a i_q - \frac{d\psi_q}{dt} + p\omega_m \psi_d \tag{3.29a}$$

$$u_d = - R_a i_d - \frac{d\psi_d}{dt} - p\omega_m \psi_q \tag{3.29b}$$

The rotor winding voltage equations (2.58), (2.59), and (2.60) are repeated here:

$$0 = R_{11q} i_{1q} + \frac{d\psi_{1q}}{dt} \tag{3.30a}$$

$$u_f = R_f i_f + \frac{d\psi_f}{dt} \tag{3.30b}$$

$$0 = R_{11d} i_{1d} + \frac{d\psi_{1d}}{dt} \tag{3.30c}$$

After reducing the damper winding quantities by means of (3.3a), (3.3b), (3.13a), and (3.13b) and introducing the resistances

$$R_{1Q} = c_{1Q}^2 R_{11q} \tag{3.31a}$$

and

$$R_{1D} = c_{1D}^2 R_{11d} \tag{3.31b}$$

the damper winding voltage equations become

$$0 = R_{1Q} i_{1Q} + \frac{d\psi_{1Q}}{dt} \tag{3.32a}$$

$$0 = R_{1D} i_{1D} + \frac{d\psi_{1D}}{dt} \tag{3.32b}$$

Choosing a set of state variables

The quantities i_q and ψ_{1Q} will be used as state variables for the quadratue axis. Using (3.10), ψ_q and i_{1Q} may be found:

$$\psi_q = L''_q i_q + \psi_{1Q} \tag{3.33a}$$

$$i_{1Q} = \frac{\psi_{1Q}}{S_q L_{1Qu}} - i_q \tag{3.33b}$$

Using these expressions, the voltage equations (3.29a) and (3.32a) become

$$u_q = -(R_a+R_{1Q})i_q - L''_q\frac{di_q}{dt} + \frac{R_{1Q}}{S_q L_{1Qu}}\psi_{1Q} + p\omega_m\psi_d \tag{3.34a}$$

$$\frac{d\psi_{1Q}}{dt} = -R_{1Q}\left(\frac{\psi_{1Q}}{S_q L_{1Qu}} - i_q\right) \tag{3.34b}$$

The quantities i_d, ψ_{1D}, and

$$\psi'_f = \psi_f - K_{f1D}\psi_{1D} \tag{3.35}$$

will be used as state variables for the direct axis. Using (3.24) and (3.35), ψ_d, ψ_f, i_f, and i_{1D} may be found:

$$\psi_d = L''_d i_d + C_F\psi'_f + \psi_{1D} \tag{3.36a}$$

$$\psi_f = \psi'_f + K_{f1D}\psi_{1D} \tag{3.36b}$$

$$i_f = \frac{\psi'_f}{L'_f} - C_F i_d \tag{3.36c}$$

$$i_{1D} = \frac{\psi_{1D}}{S_d L_{1Du}} - (1-K_{f1D}C_F)i_d - K_{f1D}\frac{\psi'_f}{L'_f} \tag{3.36d}$$

Using these expressions, the voltage equations (3.29b), (3.30b) and (3.32b) may be written as:

$$u_d = -\{R_a+(1-C_F K_{f1D})^2 R_{1D}+C_F^2 R_f\}i_d - L''_d\frac{di_d}{dt} - C_F u_f + (1-C_F K_{f1D})\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} +$$

$$+ \{C_F R_f + (C_F K_{f1D}-1)R_{1D}K_{f1D}\}\frac{\psi'_f}{L'_f} - p\omega_m\psi_q \tag{3.37a}$$

$$\frac{d\psi'_f}{dt} = u_f - (R_f+K_{f1D}^2 R_{1D})\frac{\psi'_f}{L'_f} + \{R_f C_F - K_{f1D}R_{1D}(1-K_{f1D}C_F)\}i_d + K_{f1D}\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} \tag{3.37b}$$

$$\frac{d\psi_{1D}}{dt} = -R_{1D}\left\{\frac{\psi_{1D}}{S_d L_{1Du}} - (1-K_{f1D}C_F)i_d - K_{f1D}\frac{\psi'_f}{L'_f}\right\} \tag{3.37c}$$

Substituting (3.36a) into (3.34a) and (3.33a) into (3.37a) gives

$$u_q = -(R_a+R_{1Q})i_q - L''_q\frac{di_q}{dt} + \frac{R_{1Q}}{S_q L_{1Qu}}\psi_{1Q} + p\omega_m(L''_d i_d + C_F\psi'_f + \psi_{1D}) \tag{3.38a}$$

$$u_d = -(R_a+(1-C_F K_{f1D})^2 R_{1D}+C_F^2 R_f)i_d - L''_d\frac{di_d}{dt} - C_F u_f + (1-C_F K_{f1D})\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} +$$
$$+ (C_F R_f + (C_F K_{f1D}-1)R_{1D}K_{f1D})\frac{\psi'_f}{L'_f} - p\omega_m(L''_q i_q + \psi_{1Q}) \tag{3.38b}$$

Now, the internal voltages

$$e_q = \frac{R_{1Q}}{S_q L_{1Qu}}\psi_{1Q} + p\omega_m(C_F\psi'_f + \psi_{1D}) \tag{3.39a}$$

and

$$e_d = -C_F u_f + (1-C_F K_{f1D})\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} + (C_F R_f + (C_F K_{f1D}-1)R_{1D}K_{f1D})\frac{\psi'_f}{L'_f} - p\omega_m\psi_{1Q} \tag{3.39b}$$

are introduced. These voltages are constant when the armature currents are changing very rapidly (with constant u_f and ω_m), because ψ_{1Q}, ψ'_f, and ψ_{1D} may be seen as constants in this case. Using (3.39), (3.38) may be written as:

$$u_q = e_q - (R_a+R_{1Q})i_q - L''_q\frac{di_q}{dt} + p\omega_m L''_d i_d \tag{3.40a}$$

$$u_d = e_d - (R_a+(1-C_F K_{f1D})^2 R_{1D}+C_F^2 R_f)i_d - L''_d\frac{di_d}{dt} - p\omega_m L''_q i_q \tag{3.40b}$$

Using (3.28), (3.33a), and (3.36a), the expressions (3.39) for e_d and e_q may also be written as:

$$e_q = \frac{R_{1Q}}{S_q L_{1Qu}}\psi_{1Q} + p\omega_m\psi''_d \tag{3.41a}$$

$$e_d = -C_F u_f + (1-C_F K_{f1D})\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} + (C_F R_f + (C_F K_{f1D}-1)R_{1D}K_{f1D})\frac{\psi'_f}{L'_f} - p\omega_m\psi''_q \tag{3.41b}$$

The mechanical side of the machine

Using (3.28), the expression for the electromagnetic torque (2.65) becomes

$$m = p(i_d(L''_q i_q + \psi''_q) - i_q(L''_d i_d + \psi''_d)) \tag{3.42}$$

3.5 Some simplifications

When modelling the synchronous machine for the case it is loaded via a rectifier, it is practical to transform a number of dq quantities back to the armature reference system (abc). At first instance, this results into a number of intricate expressions, which, however, may be simplified. Using (2.56) and $u_0=0$, (3.40) may be transformed back:

$$u_a = \frac{\sqrt{2}}{\sqrt{3}}\,[\{e_d-(R_a+C_F^2R_f+(1-C_FK_{f1D})^2R_{1D})i_d-L''_d\frac{di_d}{dt}-p\omega_mL''_qi_q\}\cos(\gamma) +$$

$$+\{e_q-(R_a+R_{1Q})i_q-L''_q\frac{di_q}{dt}+p\omega_mL''_di_d\}\sin(\gamma)] \tag{3.43a}$$

$$u_b = \frac{\sqrt{2}}{\sqrt{3}}\,[\{e_d-((R_a+C_F^2R_f+(1-C_FK_{f1D})^2R_{1D})i_d-L''_d\frac{di_d}{dt}-p\omega_mL''_qi_q\}\cos(\gamma-\tfrac{2}{3}\pi) +$$

$$+\{e_q-(R_a+R_{1Q})i_q-L''_q\frac{di_q}{dt}+p\omega_mL''_di_d\}\sin(\gamma-\tfrac{2}{3}\pi)] \tag{3.43b}$$

$$u_c = \frac{\sqrt{2}}{\sqrt{3}}\,[\{e_d-(R_a+C_F^2R_f+(1-C_FK_{f1D})^2R_{1D})i_d-L''_d\frac{di_d}{dt}-p\omega_mL''_qi_q\}\cos(\gamma-\tfrac{4}{3}\pi) +$$

$$+\{e_q-(R_a+R_{1Q})i_q-L''_q\frac{di_q}{dt}+p\omega_mL''_di_d\}\sin(\gamma-\tfrac{4}{3}\pi)] \tag{3.43c}$$

Next, i_q and i_d in these equations may be replaced by i_a, i_b, and i_c by using (2.42). When $i_a+i_b+i_c=0$ and $d\gamma/dt=p\omega_m$ ((2.64)) are used as well, we may find:

$$u_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma)+e_q\sin(\gamma)\} +$$

$$-\Big(R_a+\frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}+R_{1Q}}{2}\Big)i_a-\frac{L''_d+L''_q}{2}\frac{di_a}{dt} +$$

$$+\tfrac{2}{3}p\omega_m(L''_d-L''_q)\{\sin(2\gamma)i_a+\sin(2\gamma-\tfrac{2}{3}\pi)i_b+\sin(2\gamma-\tfrac{4}{3}\pi)i_c\} +$$

$$-\frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}-R_{1Q}}{3}\{\cos(2\gamma)i_a+\cos(2\gamma-\tfrac{2}{3}\pi)i_b+\cos(2\gamma-\tfrac{4}{3}\pi)i_c\} +$$

$$-\frac{L''_d-L''_q}{3}\{\cos(2\gamma)\frac{di_a}{dt}+\cos(2\gamma-\tfrac{2}{3}\pi)\frac{di_b}{dt}+\cos(2\gamma-\tfrac{4}{3}\pi)\frac{di_c}{dt}\} \tag{3.44a}$$

$$u_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi)+e_q\sin(\gamma-\tfrac{2}{3}\pi)\} +$$

$$-\Big(R_a+\frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}+R_{1Q}}{2}\Big)i_b-\frac{L''_d+L''_q}{2}\frac{di_b}{dt} +$$

$$+\tfrac{2}{3}p\omega_m(L''_d-L''_q)\{\sin(2\gamma-\tfrac{2}{3}\pi)i_a+\sin(2\gamma-\tfrac{4}{3}\pi)i_b+\sin(2\gamma)i_c\} +$$

$$-\frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}-R_{1Q}}{3}\{\cos(2\gamma-\tfrac{2}{3}\pi)i_a+\cos(2\gamma-\tfrac{4}{3}\pi)i_b+\cos(2\gamma)i_c\} +$$

$$-\frac{L''_d-L''_q}{3}\{\cos(2\gamma-\tfrac{2}{3}\pi)\frac{di_a}{dt}+\cos(2\gamma-\tfrac{4}{3}\pi)\frac{di_b}{dt}+\cos(2\gamma)\frac{di_c}{dt}\} \tag{3.44b}$$

$$u_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi)+e_q\sin(\gamma-\tfrac{4}{3}\pi)\} +$$
$$-\ (R_a+\frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}+R_{1Q}}{2})i_c - \frac{L_d''+L_q''}{2}\frac{di_c}{dt} +$$
$$+\ \tfrac{2}{3}p\omega_m(L_d''-L_q'')\{\sin(2\gamma-\tfrac{4}{3}\pi)i_a + \sin(2\gamma)i_b + \sin(2\gamma-\tfrac{2}{3}\pi)i_c\} +$$
$$-\ \frac{C_F^2R_f+(1-C_FK_{f1D})^2R_{1D}-R_{1Q}}{3}\{\cos(2\gamma-\tfrac{4}{3}\pi)i_a+\cos(2\gamma)i_b+\cos(2\gamma-\tfrac{2}{3}\pi)i_c\} +$$
$$-\ \frac{L_d''-L_q''}{3}\{\cos(2\gamma-\tfrac{4}{3}\pi)\frac{di_a}{dt} + \cos(2\gamma)\frac{di_b}{dt} + \cos(2\gamma-\tfrac{2}{3}\pi)\frac{di_c}{dt}\} \tag{3.44c}$$

Fortunately, these equations may be simplified for most practical situations. The first simplification is neglecting the terms with $L_d''-L_q''$ with respect to the terms with $L_d''+L_q''$. The second simplification is neglecting the resistive terms with respect to the inductive terms. Using these simplifications (3.44) becomes:

$$u_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma)+e_q\sin(\gamma)\} - \frac{L_d''+L_q''}{2}\frac{di_a}{dt} \tag{3.45a}$$

$$u_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi)+e_q\sin(\gamma-\tfrac{2}{3}\pi)\} - \frac{L_d''+L_q''}{2}\frac{di_b}{dt} \tag{3.45b}$$

$$u_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi)+e_q\sin(\gamma-\tfrac{4}{3}\pi)\} - \frac{L_d''+L_q''}{2}\frac{di_c}{dt} \tag{3.45c}$$

These expressions are depicted schematically in figure 3.3.

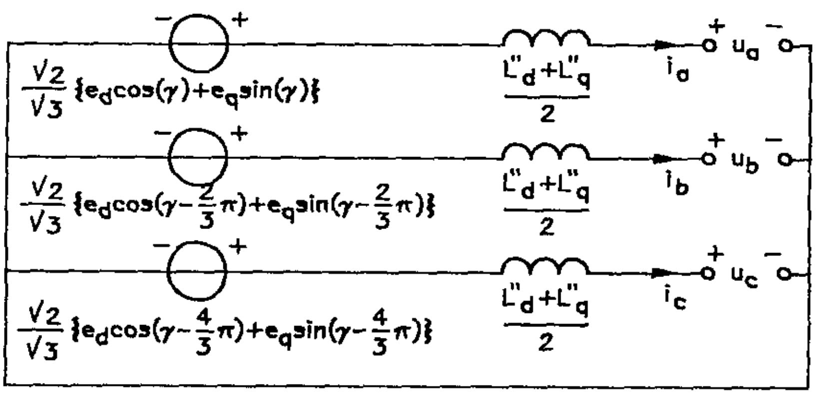

Figure 3.3 The simplified armature circuit

Now, the simplified model of the synchronous machine is described by the equations (2.42), (2.64), (2.43), (3.26), (3.27), (3.28), (3.33a), (3.34b), (3.36a), (3.37b), (3.37c), (3.41), (3.42), and (3.45). The resistive terms and the terms with $L_d''-L_q''$ have only been neglected in equation (3.45).

3.6 Summary of the machine equations

After combining the equations (2.42) and (2.43), (3.33a) and (3.28b), and, (3.36a) and (3.28a), the total set of machine equations (2.42), (2.43), (3.26), (3.27), (2.64), (3.28), (3.33a), (3.34b), (3.36a), (3.37b), (3.37c), (3.41), (3.42), and (3.45) as needed for modelling a synchronous machine with rectifier may be written as:

$$i_d = \frac{\sqrt{2}}{\sqrt{3}} \{i_a\cos(\gamma) + i_b\cos(\gamma-\tfrac{2}{3}\pi) + i_c\cos(\gamma-\tfrac{4}{3}\pi)\} \qquad (3.46a)$$

$$i_q = \frac{\sqrt{2}}{\sqrt{3}} \{i_a\sin(\gamma) + i_b\sin(\gamma-\tfrac{2}{3}\pi) + i_c\sin(\gamma-\tfrac{4}{3}\pi)\} \qquad (3.46b)$$

$$\psi''_q = \psi_{1Q} \qquad (3.46c)$$

$$\psi''_d = C_F\psi'_f + \psi_{1D} \qquad (3.46d)$$

$$\psi'' = \sqrt{\psi''^2_d+\psi''^2_q} \qquad (3.46e)$$

$$S_q = S_d = \frac{1}{1 + a\psi''^6} \qquad (3.46f)$$

$$e_q = \frac{R_{1Q}}{S_qL_{1Qu}}\psi_{1Q} + p\omega_m\psi''_d \qquad (3.46g)$$

$$e_d = - C_Fu_f + (1-C_FK_{f1D})\frac{R_{1D}}{S_dL_{1Du}}\psi_{1D} + \{C_FR_f+(C_FK_{f1D}-1)R_{1D}K_{f1D}\}\frac{\psi'_f}{L'_f} - p\omega_m\psi''_q \qquad (3.46h)$$

$$u_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma)+e_q\sin(\gamma)\} - \frac{L''_d+L''_q}{2}\frac{di_a}{dt} \qquad (3.46i)$$

$$u_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi)+e_q\sin(\gamma-\tfrac{2}{3}\pi)\} - \frac{L''_d+L''_q}{2}\frac{di_b}{dt} \qquad (3.46j)$$

$$u_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi)+e_q\sin(\gamma-\tfrac{4}{3}\pi)\} - \frac{L''_d+L''_q}{2}\frac{di_c}{dt} \qquad (3.46k)$$

$$\omega_m = \frac{1}{p}\frac{d\gamma}{dt} \qquad (3.46l)$$

$$\frac{d\psi_{1Q}}{dt} = - R_{1Q}\left\{\frac{\psi_{1Q}}{S_qL_{1Qu}} - i_q\right\} \qquad (3.46m)$$

$$\frac{d\psi'_f}{dt} = u_f - (R_f+K^2_{f1D}R_{1D})\frac{\psi'_f}{L'_f} + \{R_fC_F-K_{f1D}R_{1D}(1-K_{f1D}C_F)\}i_d + K_{f1D}\frac{R_{1D}}{S_dL_{1Du}}\psi_{1D} \qquad (3.46n)$$

$$\frac{d\psi_{1D}}{dt} = - R_{1D}\{\frac{\psi_{1D}}{S_d L_{1Du}} - (1-K_{f1D}C_F)i_d - K_{f1D}\frac{\psi'_f}{L'_f}\} \qquad (3.46o)$$

$$m = p\{i_d(L''_q i_q+\psi''_q) - i_q(L''_d i_d+\psi''_d)\} \qquad (3.46p)$$

4 THE THREE-PHASE BRIDGE RECTIFIER

4.1 The description of the rectifier

In the description of the rectifier, the circuit shown in figure 4.1 will be used. The rectifier is fed by a three-phase voltage source with internal self-inductance L_c and internal voltages e_a, e_b, and e_c according to

$$e_a=\hat{e}\cos(\omega t) \; ; \; e_b=\hat{e}\cos(\omega t-\tfrac{2}{3}\pi) \; ; \; e_c=\hat{e}\cos(\omega t-\tfrac{4}{3}\pi) \qquad (4.1)$$

where ω is a constant angular frequency and $\hat{e}$ is a constant amplitude. The rectifier is loaded by a constant current source I_g. The thyristors will be considered as ideal switches; resistances in the circuit are neglected.

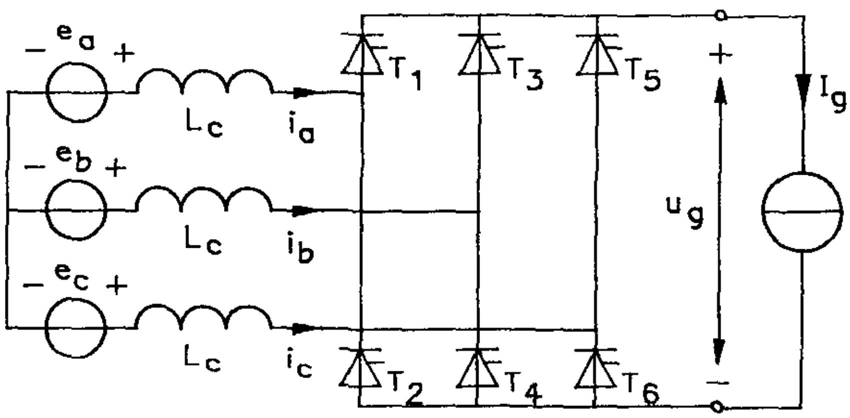

Figure 4.1 Base circuit for the rectifier description

Each $\pi/3$ rad a thyristor is triggered. The rectifier is controlled by varying the delay angle α: the angle by which the triggering instant is delayed with respect to the starting instant of the conduction of this thyristor in the case all thyristors are continuously triggered, i.e. the thyristors act like diodes. Hence a diode bridge rectifier corresponds with a rectifier with $\alpha=0$.

Thanks to the symmetry of the circuit and of the currents and voltages in this circuit (in the supposed steady state), the description of the rectifier can be restricted to an interval of $\pi/3$ rad. Here the interval between the triggering instant of thyristor T_1 and the triggering instant of thyristor T_6 will be used: $-\pi/3+\alpha<\omega t<\alpha$. This interval is indicated by means of a thick line piece in figure 4.2. The angle of overlap μ, which will be defined later on, is supposed to be smaller than $\pi/3$ rad.

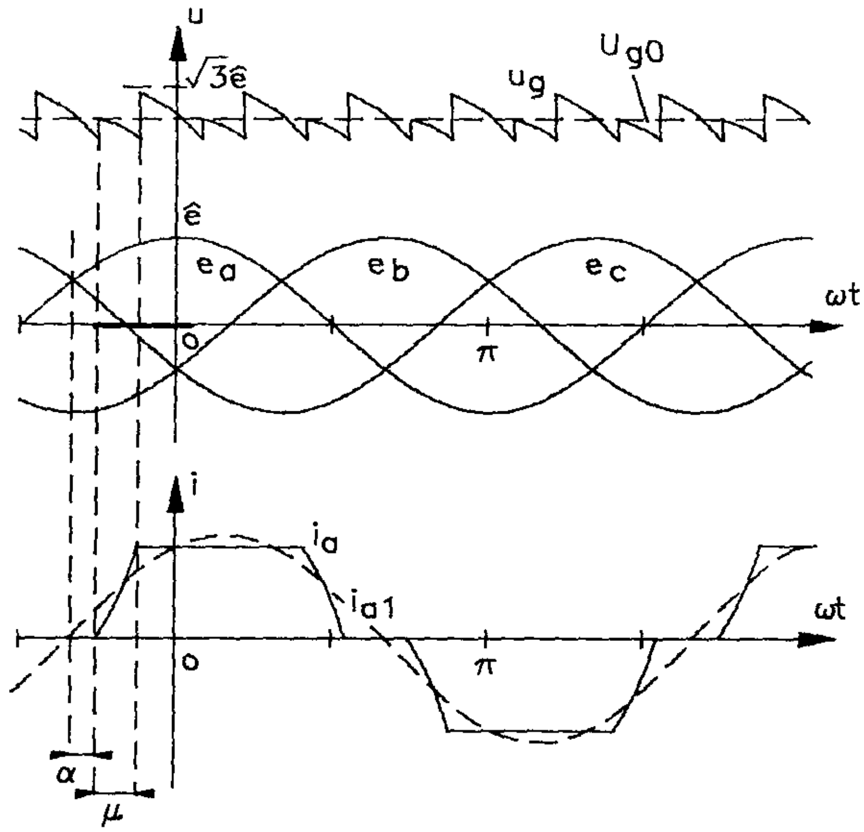

Figure 4.2 Some quantities as functions of ωt (α=0.3 ; μ=0.4)

Just before the considered interval, the thyristors T_4 and T_5 are conducting; at the beginning of this interval thyristor T_1 will turn on and the current I_g starts to transfer from thyristor T_5 to thyristor T_1 (the starting instant of the commutation). During this commutation only the thyristors T_1, T_4, and T_5 are conducting. Hence, using (4.1) and the initial condition $i_a(-\pi/3+\alpha)=0$, the following relations can be given for the commutation interval considered:

$$i_a=\frac{\sqrt{3}\hat{e}}{2\omega L_c}(\cos\alpha-\cos(\omega t+\tfrac{\pi}{3})) \quad ; \quad i_b=-I_g$$
$$i_c=I_g-\frac{\sqrt{3}\hat{e}}{2\omega L_c}(\cos\alpha-\cos(\omega t+\tfrac{\pi}{3})) \quad ; \quad u_g=\tfrac{3}{2}\hat{e}\cos(\omega t+\tfrac{\pi}{3}) \tag{4.2}$$

The commutation is finished when the current through thyristor T_5 (i_c) becomes zero. The time expressed in angular measure, elapsed from the beginning of the commutation until the end of the commutation is called the angle of overlap μ. In the considered interval the commutation is finished at the instant corresponding to $\omega t=-\pi/3+\alpha+\mu$. From the condition $i_c(-\pi/3+\alpha+\mu)=0$ and (4.2), it follows:

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_c I_g}{\sqrt{3}\hat{e}} \tag{4.3}$$

The commutation has to be finished before e_{ac} becomes negative: $-\pi/3+\alpha+\mu < 2\pi/3$. This results into the condition: $\alpha < \pi-\mu$.

After the commutation being finished, only the thyristors T_1 and T_4 are conducting. Using figure 4.1 and the voltage expressions (4.1), the following expressions can be given (only valid in the second part of the interval considered):

$$i_a = -i_b = I_g \;;\quad i_c = 0 \;;\quad u_g = e_a - e_b = \sqrt{3}\hat{e}\cos(\omega t + \tfrac{\pi}{6}) \tag{4.4}$$

The average value of the voltage u_g can be found by means of the expressions (4.2), (4.4), and (4.3):

$$U_{g0} = \frac{3}{\pi}\int_{\alpha-\frac{\pi}{3}}^{\alpha} u_g \, d\omega t = \frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - \frac{3}{\pi}\omega L_c I_g \tag{4.5}$$

By means of Fourier analysis and the equations (4.2), (4.3), and (4.4), the fundamental components of the phase currents may be expressed as

$$i_{a1}(\omega t) = i_{act}\cos(\omega t) + i_{rea}\sin(\omega t) \tag{4.6a}$$

$$i_{b1}(\omega t) = i_{act}\cos(\omega t - \tfrac{2}{3}\pi) + i_{rea}\sin(\omega t - \tfrac{2}{3}\pi) \tag{4.6b}$$

$$i_{c1}(\omega t) = i_{act}\cos(\omega t - \tfrac{4}{3}\pi) + i_{rea}\sin(\omega t - \tfrac{4}{3}\pi) \tag{4.6c}$$

where the active and the reactive component coefficients are given by

$$i_{act} = \frac{\sqrt{3}}{\pi} I_g \{\cos\alpha + \cos(\alpha+\mu)\} \tag{4.7a}$$

$$i_{rea} = \frac{3\hat{e}}{2\omega L_c \pi}\{\mu - \sin\mu\cos(2\alpha+\mu)\} \tag{4.7b}$$

In many practical situations, the ripple on the direct current may be neglected, so that the above description may be used for the steady state. The description may also be used for slow changes in the amplitude or the frequency of the phase voltages and the average value of the direct current.

The dynamic model

The dynamic model introduced in this way may be improved by enlarging the inductance in the dc-circuit with $2L_c$ [Bue 77]. This enlargement corresponds to the inductance seen from the dc-side of the rectifier when two thyristors are conducting. Using (4.5), the equivalent circuit given in figure 4.3 may be composed.

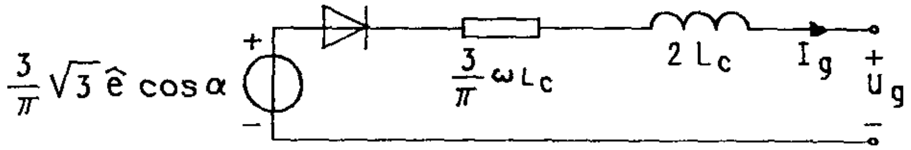

Figure 4.3 An equivalent circuit for the rectifier

4.2 The equations of the dc-link

In this section the equations corresponding with the circuit given in figure 4.4 will be given for as far they are needed in this report. For this purpose the equations from section 4.1 and the equivalent circuit in figure 4.3 will be used. However, ωt will be replaced by $\omega t-\epsilon$.

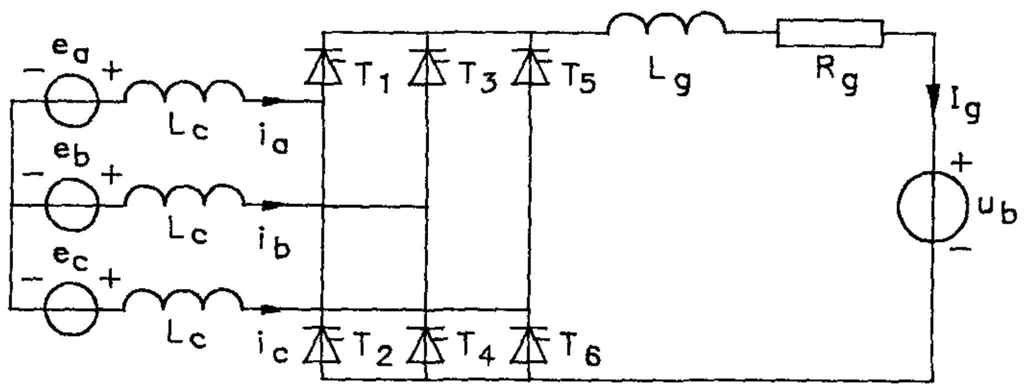

Figure 4.4 The dc-link

Hence (4.1) becomes:

$$e_a = \hat{e}\cos(\omega t-\epsilon) \tag{4.8a}$$

$$e_b = \hat{e}\cos(\omega t-\epsilon-\tfrac{2}{3}\pi) \tag{4.8b}$$

$$e_c = \hat{e}\cos(\omega t-\epsilon-\tfrac{4}{3}\pi) \tag{4.8c}$$

The expressions for the fundamental components (4.6) become:

$$i_{a1}(\omega t) = \hat{i}_{act}\cos(\omega t-\epsilon) + \hat{i}_{rea}\sin(\omega t-\epsilon) \tag{4.9a}$$

$$i_{b1}(\omega t) = \hat{i}_{act}\cos(\omega t-\epsilon-\tfrac{2}{3}\pi) + \hat{i}_{rea}\sin(\omega t-\epsilon-\tfrac{2}{3}\pi) \tag{4.9b}$$

$$i_{c1}(\omega t) = \hat{i}_{act}\cos(\omega t-\epsilon-\tfrac{4}{3}\pi) + \hat{i}_{rea}\sin(\omega t-\epsilon-\tfrac{4}{3}\pi) \tag{4.9c}$$

For reasons of surveyability, the equations (4.3) and (4.7) and the conditions mentioned before are repeated here:

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_c I_g}{\sqrt{3}\hat{e}} \tag{4.10}$$

$$\hat{i}_{act} = \frac{3\hat{e}}{2\omega L_c \pi}\sin\mu\sin(2\alpha+\mu) = \frac{\sqrt{3}}{\pi} I_g (\cos\alpha + \cos(\alpha+\mu)) \tag{4.11a}$$

$$\hat{i}_{rea} = \frac{3\hat{e}}{2\omega L_c \pi}(\mu - \sin\mu\cos(2\alpha+\mu)) \tag{4.11b}$$

$$\mu < \frac{\pi}{3} \qquad ; \qquad 0 \le \alpha < \pi-\mu \tag{4.12}$$

Combining the equivalent circuit of the rectifier according to figure 4.3 and the circuit given in figure 4.4 results into the equivalent circuit for the dc-link according to figure 4.5.

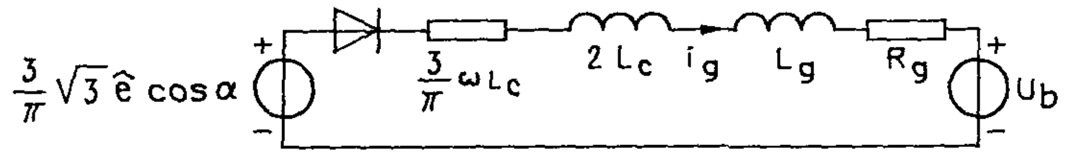

Figure 4.5 An equivalent circuit for the dc-link

For the description of this circuit ($i_g>0$) the differential equation

$$(L_g+2L_c)\frac{di_g}{dt} = \frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - (\frac{3}{\pi}\omega L_c+R_g)i_g - U_b \tag{4.13}$$

may be used.

The equations in this section will be sufficient for the description of the dc-link.

5 THE STEADY-STATE MODEL OF THE SYNCHRONOUS MACHINE WITH RECTIFIER

5.1 The coupling of the synchronous machine model and the rectifier model

In the previous chapters, the synchronous machine and the rectifier have been treated separately. However, the models developed in these chapters cannot simply be connected. In this chapter a method will be given to develop a model of the combination. Considering figure 5.1, this will be done for the steady-state.

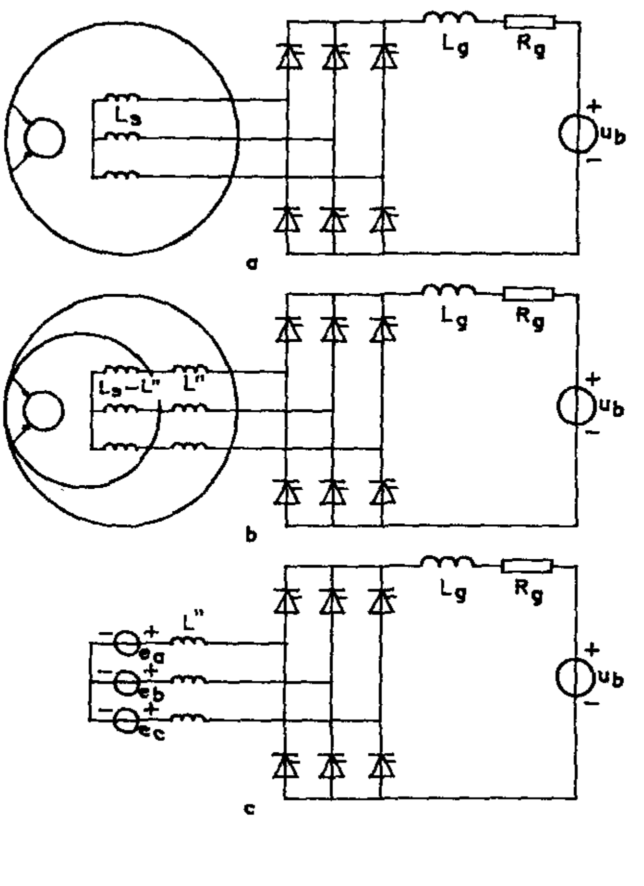

Figure 5.1 Splitting off the subtransient inductance

In order to investigate the interaction between the machine and the rectifier, we shall consider the current harmonics in the phase currents. Because of the symmetry in the circuit in figure 5.1 and the way of triggering the thyristors (see section 4.1), the armature phase currents also produce a symmetrical three-phase system. Hence, in steady-state operation, the armature phase currents may be expressed as Fourier series:

$$i_a = \sum_{n=1}^{\infty} \hat{i}_n \cos(np\omega_m t - \beta_n) \tag{5.1a}$$

$$i_b = \sum_{n=1}^{\infty} \hat{i}_n \cos(n(p\omega_m t - \tfrac{2}{3}\pi) - \beta_n)) \tag{5.1b}$$

$$i_c = \sum_{n=1}^{\infty} \hat{i}_n \cos(n(p\omega_m t - \tfrac{4}{3}\pi) - \beta_n)) \tag{5.1c}$$

Thanks to the property $i(p\omega_m t-\pi)=-i(p\omega_m t)$ all even harmonics are zero. Moreover, as the star connection terminal of the machine is not used, the armature phase currents do not contain harmonics with an angular frequency which is an integer multiple of $3p\omega_m$. Hence, the expressions (4.1) can be wrtten as:

$$i_a = \hat{i}_1 \cos(p\omega_m t - \beta_1) + \sum_{k=1}^{\infty} [\hat{i}_{6k-1} \cos((6k-1)p\omega_m t - \beta_{6k-1}) + \\ + \hat{i}_{6k+1} \cos((6k+1)p\omega_m t - \beta_{6k+1})] \tag{5.2a}$$

$$i_b = \hat{i}_1 \cos(p\omega_m t - \tfrac{2}{3}\pi - \beta_1) + \sum_{k=1}^{\infty} [\hat{i}_{6k-1} \cos((6k-1)p\omega_m t + \tfrac{2}{3}\pi - \beta_{6k-1}) + \\ + \hat{i}_{6k+1} \cos((6k+1)p\omega_m t - \tfrac{2}{3}\pi - \beta_{6k+1})] \tag{5.2b}$$

$$i_c = \hat{i}_1 \cos(p\omega_m t - \tfrac{4}{3}\pi - \beta_1) + \sum_{k=1}^{\infty} [\hat{i}_{6k-1} \cos((6k-1)p\omega_m t + \tfrac{4}{3}\pi - \beta_{6k-1}) + \\ + \hat{i}_{6k+1} \cos((6k+1)p\omega_m t - \tfrac{4}{3}\pi - \beta_{6k+1})] \tag{5.2c}$$

Choosing

$$\gamma = p\omega_m t + \frac{\pi}{2} \tag{5.3}$$

and the Park transformation according to (2.42), the following expressions for i_d, i_q, and i_0 are found:

$$i_q = \frac{\sqrt{3}}{\sqrt{2}} \hat{i}_1 \cos(\beta_1) + \frac{\sqrt{3}}{\sqrt{2}} \sum_{k=1}^{\infty} [\hat{i}_{6k-1} \cos(6kp\omega_m t - \beta_{6k-1}) + \\ + \hat{i}_{6k+1} \cos(6kp\omega_m t - \beta_{6k+1})] \tag{5.4a}$$

$$i_d = -\frac{\sqrt{3}}{\sqrt{2}} \hat{i}_1 \sin(\beta_1) + \frac{\sqrt{3}}{\sqrt{2}} \sum_{k=1}^{\infty} [-\hat{i}_{6k-1} \sin(6kp\omega_m t - \beta_{6k-1}) + \\ + \hat{i}_{6k+1} \sin(6kp\omega_m t - \beta_{6k+1})] \tag{5.4b}$$

$$i_0 = 0 \tag{5.4c}$$

As can be seen in these expressions, the fundamental components in the armature phase currents are transformed into the dc-components of i_d and i_q. For this moment, we suppose that these fundamental components see the (synchronous) inductance L_s on the machine terminal points

(plus a sinusoidal voltage).
The harmonics in the armature phase currents result into components of i_d and i_q with an angular frequency which is an integer multiple of $6p\omega_m$. In practice, these are relatively very high frequencies, so that these harmonics in the armature currents hardly cause changes in the rotor fluxes ψ_{1Q}, ψ_f, and ψ_{1D}. As has been explained in section 3.4, this means that the voltages e_d and e_q may be considered constant, so that the voltage sources in figure 3.3 are sinusoidal. Hence the armature current harmonics only see the (subtransient) inductance $L''=(L_q''+L_d'')/2$.
This inductance will be splitted off by subtracting it from the synchronous inductances (see figures 5.1a and 5.1b). Here arises the so-called internal machine: the original machine minus the subtransient inductances $L''=(L_q''+L_d'')/2$.
This is a normal synchronous machine again, with the difference that it is a short-circuit for the armature phase current harmonics. So that the armature phase voltages of this machine are always sinusoidal. These voltages only depend on the excitation current and the fundamental components of the armature phase currents (or the dc-components of i_d and i_q).
Hence, the internal machine may be represented by a sinusoidal three-phase voltage source (figure 5.1c), which is controlled by the excitation current and the fundamental components of the armature phase currents.
Using figure 3.3, these voltages sources may be described by

$$e_a = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma)+e_q\sin(\gamma)\} \tag{5.5a}$$

$$e_b = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{2}{3}\pi)+e_q\sin(\gamma-\tfrac{2}{3}\pi)\} \tag{5.5b}$$

$$e_c = \frac{\sqrt{2}}{\sqrt{3}}\{e_d\cos(\gamma-\tfrac{4}{3}\pi)+e_q\sin(\gamma-\tfrac{4}{3}\pi)\} \tag{5.5c}$$

Using (5.3), these may be written as:

$$e_a = \hat{e}\cos(p\omega_m t-\epsilon) \tag{5.6a}$$

$$e_b = \hat{e}\cos(p\omega_m t-\epsilon-\tfrac{2}{3}\pi) \tag{5.6b}$$

$$e_c = \hat{e}\cos(p\omega_m t-\epsilon-\tfrac{4}{3}\pi) \tag*{(5.6b)}$$

where:

$$\hat{e} = \frac{\sqrt{2}}{\sqrt{3}} \sqrt{e_d^2+e_q^2} \tag{5.7a}$$

$$\epsilon = -\arctan\left(\frac{e_d}{e_q}\right) \tag{5.7b}$$

Figure 5.1c corresponds with figure 4.4 and the expressions (5.6) correspond with the expressions (4.8), so that we may compute the fundamental components in the phase currents by using the equations given in section 4.2, when the voltage amplitude $\hat{e}$ is given and $\omega = p\omega_m$ is used. Besides, we have to choose:

$$L_c = \frac{L_d'' + L_q''}{2} \tag{5.8}$$

Using the equations given in section 3.5, the voltage amplitude $\hat{e}$ may be computed again. Via an iteration process the steady-state may be computed.

In section 5.2 a set of equations for the description of the steady-state will be given; in section 5.3 a method to solve this set will be given.

5.2 The equations

Because we are considering the steady-state situation and the dc-components in i_d and i_q, the flux derivatives in (3.46) are zero. Hence, it follows from this set of equations:

$$\psi_q'' = S_q L_{1Qu} i_q \tag{5.9a}$$

$$\psi_d'' = \frac{u_f}{R_f}(C_F L_f' + K_{f1D} S_d L_{1Du}) + i_d(C_F^2 L_f' + S_d L_{1Du}) \tag{5.9b}$$

$$\psi'' = \sqrt{\psi_d''^2 + \psi_q''^2} \tag{5.9c}$$

$$S_q = S_d = \frac{1}{1 + a\psi''^6} \tag{5.9d}$$

$$e_q = R_{1Q} i_q + p\omega_m \psi_d'' \tag{5.9e}$$

$$e_d = \{C_F^2 R_f + (1 - C_F K_{f1D})^2 R_{1D}\} i_d - p\omega_m \psi_q'' \tag{5.9f}$$

$$m = p\{i_d(L_q'' i_q + \psi_q'') - i_q(L_d'' i_d + \psi_d'')\} \tag{5.9g}$$

The resistance terms in the expressions for e_d and e_q may be neglected in most cases. However, when the steady-state model is used to compute the initial conditions for a dynamic simulation with the model treated in chapter 6, it may be necessary to take these terms into account in order to prevent a little jump at the initial moment.
Using (5.3) and $\omega=p\omega_m$, substituting (4.9) into (3.46a) and (3.46b) results into

$$i_d = -\frac{\sqrt{3}}{\sqrt{2}}\,(\hat{i}_{act}\sin\epsilon + \hat{i}_{rea}\cos\epsilon) \tag{5.10a}$$

$$i_q = \frac{\sqrt{3}}{\sqrt{2}}\,(\hat{i}_{act}\cos\epsilon - \hat{i}_{rea}\sin\epsilon) \tag{5.10b}$$

In steady-state, $di_g/dt=0$ is valid. Using this, (4.13) may be written as ($\omega=p\omega_m$):

$$\frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha = (\frac{3}{\pi}p\omega_m L_c + R_g)i_g + U_b \tag{5.11}$$

For surveyability, the equations (4.10), (4.11), and (5.7) and the conditions (4.12) are repeated here ($\omega=p\omega_m$):

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2p\omega_m L_c i_g}{\sqrt{3}\hat{e}} \tag{5.12}$$

$$\hat{i}_{act} = \frac{3\hat{e}}{2p\omega_m L_c \pi}\sin\mu\sin(2\alpha+\mu) \tag{5.13a}$$

$$\hat{i}_{rea} = \frac{3\hat{e}}{2p\omega_m L_c \pi}(\mu - \sin\mu\cos(2\alpha+\mu)) \tag{5.13b}$$

$$\hat{e} = \frac{\sqrt{2}}{\sqrt{3}}\sqrt{e_d^2+e_q^2} \tag{5.14a}$$

$$\epsilon = -\arctan\left(\frac{e_d}{e_q}\right) \tag{5.14b}$$

$$\mu < \frac{\pi}{3} \quad ; \quad 0 \le \alpha < \pi-\mu \tag{5.15}$$

The equations (5.9), (5.10), (5.11), (5.12), (5.13), and (5.14) form a set of 16 equations with 16 unknowns: e_d, e_q, i_d, i_q, S_d, S_q, ψ''_d, ψ''_q, ψ'', ϵ, $\hat{e}$, $\hat{i}_{act}$, $\hat{i}_{rea}$, i_g, m, and μ. A possible solution method for this set will be given in the next section.

5.3 A solution method

An analytic way of solving the set of equations in the previous section has not been found until now. However, there are many possibilities to solve this set iteratively. In this section one possibility will be discussed.

In this method a value of μ is choosen between 0 and μ_M. The latter value follows from (5.15): $\mu_M=\pi/3$ for $\alpha<2\pi/3$ and $\mu_M=\pi-\alpha$ for $2\pi/3<\alpha<\pi$. Using the value of μ choosen and the equations in the previous section, we can successively compute a number of quantities. The remaining equation will be used as an error criterion. This criterion may be used to choose a new value of μ. In the computer program (see appendix 1), the Newton-Raphson method is used for the iteration process.

Eliminating i_g from (5.11) and (5.12), we may find:

$$\hat{e} = \frac{\frac{2}{\sqrt{3}}U_b}{\left(\frac{3}{\pi}-\frac{R_g}{p\omega_m L_c}\right)\cos\alpha + \left(\frac{3}{\pi}+\frac{R_g}{p\omega_m L_c}\right)\cos(\alpha+\mu)} \tag{5.16}$$

This is an expression for $\hat{e}$ as a function of μ. Next, using (5.13), we may compute:

$$\hat{i}_{act} = \frac{3\hat{e}}{2p\omega_m L_c \pi}\sin\mu\sin(2\alpha+\mu) \tag{5.17a}$$

$$\hat{i}_{rea} = \frac{3\hat{e}}{2p\omega_m L_c \pi}(\mu - \sin\mu\cos(2\alpha+\mu)) \tag{5.17b}$$

From (5.14), if follows:

$$e_d = -\frac{\sqrt{3}}{\sqrt{2}}\,\hat{e}\,\sin\epsilon \tag{5.18}$$

Substituting (5.10a), (5.18) and (5.9a) with (5.10b) into (5.9f), we may find an equation for ϵ:

$$\begin{aligned}\hat{e}\sin\epsilon = {} & p\omega_m S_q L_{1Qu}(\hat{i}_{act}\cos\epsilon - \hat{i}_{rea}\sin\epsilon) + \\ & + (C_F^2 R_f + (1-C_F K_{f1D})^2 R_{1D})(\hat{i}_{act}\sin\epsilon + \hat{i}_{rea}\cos\epsilon)\end{aligned} \tag{5.19}$$

Since S_q is not known at this moment, we shall make an estimation. This will result in an extra iteration process. Now, (5.19) is written in the following form:

$$\epsilon = \arctan\frac{p\omega_m S_q L_{1Qu}\hat{i}_{act} + (C_F^2 R_f + (1-C_F K_{f1D})^2 R_{1D})\hat{i}_{rea}}{\hat{e} + p\omega_m S_q L_{1Qu}\hat{i}_{rea} - (C_F^2 R_f + (1-C_F K_{f1D})^2 R_{1D})\hat{i}_{act}} \tag{5.20}$$

Next, we may compute i_d and i_q by means of (5.10):

$$i_d = -\frac{\sqrt{3}}{\sqrt{2}}(\hat{i}_{act}\sin\epsilon + \hat{i}_{rea}\cos\epsilon) \tag{5.21a}$$

$$i_q = \frac{\sqrt{3}}{\sqrt{2}}(\hat{i}_{act}\cos\epsilon - \hat{i}_{rea}\sin\epsilon) \tag{5.21b}$$

Using S_d=S_q and these values of i_d and i_q, we may compute S_q again by means of (5.9):

$$\psi''_q = S_q L_{1Qu} i_q \tag{5.22a}$$

$$\psi''_d = \frac{u_f}{R_f}(C_F L'_f + K_{f1D} S_d L_{1Du}) + i_d(C_F^2 L'_f + S_d L_{1Du}) \tag{5.22b}$$

$$\psi'' = \sqrt{\psi''^2_d + \psi''^2_q} \tag{5.22c}$$

$$S_q = S_d = \frac{1}{1 + a\psi''^6} \tag{5.22d}$$

We may now adapt the initial value of S_q by means of an iteration process.

When the right value of S_q is found, we have to check the choosen value of μ by substituting ψ''_d, $\hat{e}$, ϵ, and i_q into (5.9e) with (5.14):

$$p\omega_m\psi''_d + R_{1Q}i_q - \frac{\sqrt{3}}{\sqrt{2}}\hat{e}\cos\epsilon \overset{?}{=} 0 \tag{5.23}$$

During the whole iteration process, $0<\mu<\mu_M$ should be valid.

6 THE DYNAMIC MODEL OF THE SYNCHRONOUS MACHINE WITH RECTIFIER

6.1 Introduction

As in chapter 5, in the phase currents of the synchronous machine only the basic harmonics are taken into account for modelling the synchronous machine with rectifier. However, the amplitude, phase and angular frequency of these basic harmonics may vary now. In order to make it possible to use the description of the rectifier in section 4.2, these variations should be slow compared to the commutation phenomena.
As has been shown in section 5.1, the basic harmonics in the phase currents are transformed to dc-components in the currents i_d and i_q. In the machine model only these dc-components were considered. When the basic harmonics vary "slowly", these "dc" components will vary slowly too. These "dc" components, a kind of short-term averaged parts, will be used in the machine model.
In the steady-state case, we only consider the dc-components in i_d and i_q and neglect the components with an angular frequency which is an integer multiple of $6p\omega_m$ (see (5.4)). Hence, the angular frequncy of the variation of the "dc"-component should be much smaller than $6p\omega_m$. So, if, for example, the frequency of the basic component of the phase current equals 50 Hz, the frequency of the variation of this basic harmonic should be much smaller than 6x 50 Hz = 300 Hz.
Besides, like in chapter 4, the ripple on the current in the dc-link is neglected.

6.2 The equations

We may use the equations (4.10), (4.11), and (4.13) with $\omega = p\omega_m$ for the description of the rectifier ($i_g > 0$):

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2p\omega_m L_c i_g}{\sqrt{3}\hat{e}} \tag{6.1}$$

$$\hat{i}_{act} = \frac{3\hat{e}}{2p\omega_m L_c \pi}\sin\mu\sin(2\alpha+\mu) \tag{6.2a}$$

$$\hat{i}_{rea} = \frac{3\hat{e}}{2p\omega_m L_c \pi}(\mu - \sin\mu\cos(2\alpha+\mu)) \tag{6.2b}$$

$$(L_g+2L_c)\frac{di_g}{dt} = \frac{3}{\pi}\sqrt{3}\hat{e}\cos\alpha - (\frac{3}{\pi}p\omega_m L_c+R_g)i_g - u_b \tag{6.3}$$

The "dc"-components of i_d and i_q are found by means of equation (5.10):

$$i_d = -\frac{\sqrt{3}}{\sqrt{2}}(\hat{i}_{act}\sin\epsilon + \hat{i}_{rea}\cos\epsilon) \tag{6.4a}$$

$$i_q = \frac{\sqrt{3}}{\sqrt{2}}(\hat{i}_{act}\cos\epsilon - \hat{i}_{rea}\sin\epsilon) \tag{6.4b}$$

For the description of the synchronous machine, the equations (3.46c), (3.46d), (3.46e), (3.46f), (3.46g), (3.46h), (3.46m), (3.46n), (3.46o), and (3.46p) may be used directly:

$$\psi''_q = \psi_{1Q} \tag{6.5a}$$

$$\psi''_d = C_F\psi'_f + \psi_{1D} \tag{6.5b}$$

$$\psi'' = \sqrt{\psi''^2_d+\psi''^2_q} \tag{6.5c}$$

$$S_q = S_d = \frac{1}{1 + a\psi''^6} \tag{6.5d}$$

$$e_q = \frac{R_{1Q}}{S_q L_{1Qu}}\psi_{1Q} + p\omega_m\psi''_d \tag{6.5e}$$

$$e_d = -C_F u_f + (1-C_F K_{f1D})\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} + \{C_F R_f+(C_F K_{f1D}-1)R_{1D}K_{f1D}\}\frac{\psi'_f}{L'_f} - p\omega_m\psi''_q \tag{6.5f}$$

$$\frac{d\psi_{1Q}}{dt} = -R_{1Q}\left\{\frac{\psi_{1Q}}{S_q L_{1Qu}} - i_q\right\} \tag{6.5g}$$

$$\frac{d\psi'_f}{dt} = u_f - (R_f+K^2_{f1D}R_{1D})\frac{\psi'_f}{L'_f} + \{R_f C_F-K_{f1D}R_{1D}(1-K_{f1D}C_F)\}i_d + K_{f1D}\frac{R_{1D}}{S_d L_{1Du}}\psi_{1D} \tag{6.5h}$$

$$\frac{d\psi_{1D}}{dt} = -R_{1D}\left\{\frac{\psi_{1D}}{S_d L_{1Du}} - (1-K_{f1D}C_F)i_d - K_{f1D}\frac{\psi'_f}{L'_f}\right\} \tag{6.5i}$$

$$m = p\{i_d(L''_q i_q+\psi''_q) - i_q(L''_d i_d+\psi''_d)\} \tag{6.5j}$$

Further, we need equation (5.7):

$$\hat{e} = \frac{\sqrt{2}}{\sqrt{3}}\sqrt{e^2_d+e^2_q} \tag{6.6a}$$

$$\epsilon = -\arctan\left(\frac{e_d}{e_q}\right) \tag{6.5b}$$

For the commutation inductance, we need (5.8):

$$L_c = \frac{L_d'' + L_q''}{2} \tag{6.7}$$

The equations (6.1), (6.2), (6.3), (6.4), (6.5), and (6.6) with (6.7) describe a model of the synchronous machine with rectifier incorporating saturation with ψ_{1Q}, ψ_f', ψ_{1D}, and i_g as state variables, α, u_f, and u_b as input quantities and p, a, L_d'', L_q'', L_{1Qu}, K_{f1D}, L_f', L_{1Du}, C_F, R_{1Q}, R_f, R_{1D}, R_g and L_g as parameters for the case that ω_m is known. This set may easily be solved by means of a simulation program. For this purpose, we may start from the state variables, and perform successively the following operations:

- computation of ψ_d'' and ψ_q'' by means of (6.5a) and (6.5b);
- computation of ψ'' by means of (6.5c);
- computation of $S_d = S_q$ by means of (6.5d);
- computation of e_q and e_d by means of (6.5e) and (6.5f);
- computation of $\hat{e}$ and ϵ by means of (6.6);
- computation of μ by means of (6.1);
- computation of $\hat{i}_{act}$ and $\hat{i}_{rea}$ by means of (6.2);
- computation of i_d and i_q by means of (6.4);
- integration of the equations (6.3), (6.5g), (6.5h), and (6.5i);
- computation of m by means of (6.5j) (not always necessary).

Solving this set of equations, the condition (4.12) must be satisfied:

$$\mu < \frac{\pi}{3} \qquad ; \qquad 0 \le \alpha < \pi - \mu \tag{6.8}$$

Sometimes, we also want to know the rotor currents. These currents may be computed from the state variables by means of (3.33b), (3.36c), and (3.36d):

$$i_{1Q} = \frac{\psi_{1Q}}{S_q L_{1Qu}} - i_q \tag{6.9a}$$

$$i_f = \frac{\psi_f'}{L_f'} - C_F i_d \tag{6.9b}$$

$$i_{1D} = \frac{\psi_{1D}}{S_d L_{1Du}} - (1 - K_{f1D} C_F) i_d - K_{f1D} \frac{\psi_f'}{L_f'} \tag{6.9c}$$

In appendix 2, an example of the use of the model described here is given.

7 THE MODEL PARAMETERS

7.1 The parameters needed for the model

As has been mentioned in section 6.2, the parameter p, a, L''_d, L''_q, L_{1Qu}, K_{f1D}, L'_f, L_{1Du}, C_F, R_{1Q}, R_f, R_{1D}, R_g and L_g are necessary for the dynamic model of the synchronous machine with rectifier incorporating saturation as it is described in this report. For practical reasons, the parameters L_{1Qu}, K_{f1D}, L_{1Du}, R_{1Q}, and R_{1D} will be computed from more known parameters.
From (3.5), it follows:

$$L_{1Qu} = L_{qu} - L''_q \tag{7.1}$$

where L_{1Qu} and L_{qu} are the unsaturated values of, respectively, L_{1Q} and L_q.
From (3.19), it follows:

$$L_{1Du} = L_{du} - L''_d - C_F^2 L'_f \tag{7.2}$$

where L_{1Du} and L_{du} are the unsaturated values of, respectively, L_{1D} and L_d.
From (3.17b), it follows:

$$K_{f1D} = \frac{L_{afdu} - C_F L'_f}{L_{1Du}} \tag{7.3}$$

where L_{afdu} is the unsaturated value of L_{afd}.
The damper winding resistances will be characterized by their time constants T_{1Q} and T_{1D}:

$$T_{1Q} = \frac{L_{1Qu}}{R_{1Q}} \tag{7.4}$$

$$T_{1D} = \frac{L_{1Du}}{R_{1D}} \tag{7.5}$$

Using the equations (7.1), (7.2), (7.3), (7.4), and (7.5), we may use the set of parameters p, a, L''_d, L''_q, L_{qu}, L_{afdu}, L'_f, L_{du}, C_F, T_{1Q}, R_f, T_{1D}, R_g and L_g.

7.2 The saturation constant

In this section a method for the determination of the saturation constant a will be given. In this method the no-load saturation test of the synchronous machine is used.

In the steady-state no-load test, the armature phase currents and the damper-winding currents are zero. Hence, using (3.10a), (3.24a), and (3.28), it may be seen that:

$$\psi_q = \psi''_q = 0 \quad ; \quad \psi_d = \psi''_d = (C_F L'_f + S_d L_{1Du} K_{f1D}) i_f \tag{7.6}$$

Since the flux derivatives are zero too, (3.29) results into:

$$u_q = p\omega_m \psi_d \quad ; \quad u_d = 0 \tag{7.7}$$

Using (2.56) and, for example, $\gamma = p\omega_m t + \pi/2$, the phase voltage for phase a may be found:

$$u_a = \frac{\sqrt{2}}{\sqrt{3}} u_q \cos(p\omega_m t) = \frac{\sqrt{2}}{\sqrt{3}} p\omega_m \psi_d \cos(p\omega_m t) \tag{7.8}$$

Hence, the root-mean-square value of the line voltage, which is measured in the no-load saturation test, is given by:

$$U_L = p\omega_m \psi_d \tag{7.9}$$

Combining the equations (3.26), (3.27), (7.6), and (7.9) may result into:

$$\frac{U_L}{p\omega_m} + a\left(\frac{U_L}{p\omega_m}\right)^7 = \left(C_F L'_f + C_F L'_f a\left(\frac{U_L}{p\omega_m}\right)^6 + L_{1Du} K_{f1D}\right) i_f \tag{7.10}$$

or by using (7.3)

$$\frac{U_L}{p\omega_m} + a\left(\frac{U_L}{p\omega_m}\right)^7 = \left(L_{afdu} + C_F L'_f a\left(\frac{U_L}{p\omega_m}\right)^6\right) i_f \tag{7.11}$$

After measuring U_L as a funcion of i_f, the parameter a may be determined by using (7.11) and a least-squares estimation process.

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APPENDIX 1 A SUBROUTINE FOR THE COMPUTATION OF THE STEADY STATE

In this appendix a Fortran77 subroutine for the computation of the steady state of the synchronous machine with rectifier is given. This subroutine , which is based on the description in section 5.3, has as input variables:

a : a
Ldu : L_{du}
Lafdu: L_{afdu}
Lqu : L_{qu}
Ldll : L''_d
CF : C_F
Lfl : L'_f
Rd : $C_F^2 R_f + (1-C_F K_{f1D})^2 R_{1D}$
Rq : R_{1Q}
Lqll : L''_q
Omega: $p\omega_m$
Alfa : α
Ub : U_b
Iv : U_f/R_f
Rg : R_g
EE : a standard for precision
NI : the maximum number of iteration.

The output qantities are:

EF : Error Flag; it should be 1.0; in case of an error, it is 0.0
Id : i_d
Iq : i_q
Ig : i_g
S : $S_d = S_q$

```
      Subroutine SS(EF,Id,Iq,Ig,S,a,Ldu,Lafdu,Lqu,Ldll,CF,Lfl,Rd,Rq,
     *              Lqll,Omega,Alfa,Ub,Iv,Rg,EE,NI)
        Real Iact,Irea,Id,Iq,Ig,Iv,KflD,LlDu,Ldu,Lafdu,Lqu,Lc,Ldll,
     *       Lqll,LlQu,Lfl,Mu,Mu0,Mu2,MuM,Omega
        EF  = 0.0
        Pi  = 4.0*ATan(1.0)
        Lc  = (Ldll+Lqll)/2.0
        LlQu= Lqu - Lqll
        LlDu= Ldu - CF*CF*Lfl - Ldll
        KflD= (Lafdu-CF*Lfl)/LlDu
        Wa  = Sqrt(1.5)
```

```
C Mu = 0
        Psid0 = 0
       F0    = Lafdu*Iv
        Psid11= Lafdu*Iv
        Write(6,'(''No load'')')
        Write(6,'(''Psid11='',F10.7)') Psid11
        Do 1 K=1,NI
           S = 1.0/(1.0+a*Psid11**6)
           F = Iv*(Kf1D*S*L1Du+CF*Lf1) - Psid11
           If (Abs(F).LE.(EE/100.0)) GoTo 2
           FF    = (F-F0)/(Psid11-Psid0)
           Psid2 = Psid11 - F/FF
           Write(6,'('' Psid2='',F10.7)') Psid2
           F0    = F
           Psid0 = Psid11
           Psid11= Psid2
1       Continue
2       Continue
        F0  = Omega*Psid11 - Ub/3.0/Sqrt(3.0)*Pi/Cos(Alfa)*Wa
        If (F0.LE.0.0) Then
          Write(6,'(''F0 <= 0'')')
          GoTo 99
        EndIf
        Write(6,'(''Mu0=0           F0='',F13.7,'' S=SS='',F10.7)') F0,S
C Mu is maximum with simple approximation for S
        Write(6,'(''Maximum value Mu with simple approximation S'')')
        MuM = Pi/3.0
        If (Alfa.GT.(2.0*Pi/3.0)) MuM = Pi - Alfa
        E      = 2.0*Ub/Sqrt(3.0)/(Cos(Alfa)*(3.0/Pi-Rg/Omega/Lc)
     *           +Cos(Alfa+MuM)*(3.0/Pi+Rg/Omega/Lc))
        Iact   = 1.5/Pi/Omega/Lc*E*Sin(MuM)*Sin(2.0*Alfa+MuM)
        Irea   = 1.5/Pi/Omega/Lc*E*(MuM - Sin(MuM)*Cos(2.0*Alfa+MuM))
        S      = 1.0/(1.0+a*(Wa*E/Omega)**6)
        Y      = Omega*S*L1Qu*Iact+Rd*Irea
        X      = E + Omega*S*L1Qu*Irea-Rd*Iact
        SinEps= Y/Sqrt(X*X+Y*Y)
        CosEps= X/Sqrt(X*X+Y*Y)
        Id     = - Wa*(Iact*SinEps+Irea*CosEps)
        Iq     = Wa*(Iact*CosEps-Irea*SinEps)
        Psid11= CF*Lf1*(CF*Id+Iv) + S*L1Du*(Id+Kf1D*Iv)
        F      = Omega*Psid11 + Rq*Iq - Wa*E*CosEps
        If (F.GE.0.0) Then
          Write(6,'(''FM >= 0'')')
          GoTo 99
        EndIf
        Write(6,'(''MuM='',F10.7,''  FM='',F13.7,''   SS='',F10.7)')
     *       MuM,F,S
C Iteration with Mu with simple approximation for S
        Write(6,'(''Iteration with Mu with simple approximation S'')')
        Mu  = MuM/2.0
        Mu0 = 0.0
        Do 3 K=1,5
           E      = 2.0*Ub/Sqrt(3.0)/(Cos(Alfa)*(3.0/Pi-Rg/Omega/Lc)
     *              +Cos(Alfa+Mu)*(3.0/Pi+Rg/Omega/Lc))
           Iact   = 1.5/Pi/Omega/Lc*E*Sin(Mu)*Sin(2.0*Alfa+Mu)
           Irea   = 1.5/Pi/Omega/Lc*E*(Mu-SIN(Mu)*Cos(2.0*Alfa+Mu))
           S      = 1.0/(1.0+a*(Wa*E/Omega)**6)
           Y      = Omega*S*L1Qu*Iact+Rd*Irea
           X      = E + Omega*S*L1Qu*Irea-Rd*Iact
```

```
            SinEps= Y/Sqrt(X*X+Y*Y)
            CosEps= X/Sqrt(X*X+Y*Y)
            Id     = - Wa*(Iact*SinEps+Irea*CosEps)
            Iq     = Wa*(Iact*CosEps-Irea*SinEps)
            Psidl1= CF*Lf1*(CF*Id+Iv) + S*L1Du*(Id+Kf1D*Iv)
            F      = Omega*Psidl1 + Rq*Iq - Wa*E*CosEps
            Write(6,'(''Mu ='',F10.7,''  F ='',F13.7,''SS='',F10.7)')
     *           Mu,F,S
            FF  = (F-F0)/(Mu-Mu0)
            Mu2 = Mu - F/FF
            IF (Mu2.GT.MuM) Mu2 = (MuM+Mu)/2.0
            IF (Mu2.LT.0.0) Mu2 = Mu/2.0
            F0  = F
            Mu0 = Mu
            Mu  = Mu2
3       Continue
C Iteration with Mu with correct value for S
        Write(6,'(''Iteration with Mu with correct value for S'')')
        E      = 2.0*Ub/Sqrt(3.0)/(Cos(Alfa)*(3.0/Pi-Rg/Omega/Lc)
     *           +Cos(Alfa+Mu0)*(3.0/Pi+Rg/Omega/Lc))
        Iact   = 1.5/Pi/Omega/Lc*E*Sin(Mu0)*Sin(2.0*Alfa+Mu0)
        Irea   = 1.5/Pi/Omega/Lc*E*(Mu0-SIN(Mu0)*Cos(2.0*Alfa+Mu0))
        S0     = 1.0
        Y      = Omega*L1Qu*Iact+Rd*Irea
        X      = E + Omega*L1Qu*Irea-Rd*Iact
        SinEps= Y/Sqrt(X*X+Y*Y)
        CosEps= X/Sqrt(X*X+Y*Y)
        Id     = - Wa*(Iact*SinEps+Irea*CosEps)
        Iq     = Wa*(Iact*CosEps-Irea*SinEps)
        Psiql1= L1Qu*Iq
        Psidl1= CF*Lf1*(CF*Id+Iv) + L1Du*(Id+Kf1D*Iv)
        G0     = 1.0/(1.0+a*(Psiql1*Psiql1+Psidl1*Psidl1)**3) - 1.0
        S      = 1.0/(1.0+a*(Wa*E/Omega)**6)
        Write(6,'('' S='',F10.7)') S
        Do 5 M=1,NI
            Y      = Omega*S*L1Qu*Iact+Rd*Irea
            X      = E + Omega*S*L1Qu*Irea-Rd*Iact
            SinEps= Y/Sqrt(X*X+Y*Y)
            CosEps= X/Sqrt(X*X+Y*Y)
            Id     = - Wa*(Iact*SinEps+Irea*CosEps)
            Iq     = Wa*(Iact*CosEps-Irea*SinEps)
            Psiql1= S*L1Qu*Iq
            Psidl1= CF*Lf1*(CF*Id+Iv) + S*L1Du*(Id+Kf1D*Iv)
            G      = 1.0/(1.0+a*(Psiql1*Psiql1+Psidl1*Psidl1)**3) - S
            If (Abs(G).LE.(EE/100.0)) GoTo 6
            GG = (G-G0)/(S-S0)
            S2 = S - G/GG
            Write(6,'(''  S2='',F10.7)') S2
            G0 = G
            S0 = S
            S  = S2
5       Continue
6       Continue
        F0     = Omega*Psidl1 + Rq*Iq - Wa*E*CosEps
        Write(6,'(''Mu0='',F10.7,''  F0='',F13.7,''    S='',F10.7)')
     *       Mu0,F0,S
```

```
         Do 8 K=1,NI
            E      = 2.0*Ub/Sqrt(3.0)/(Cos(Alfa)*(3.0/Pi-Rg/Omega/Lc)
     *               +Cos(Alfa+Mu)*(3.0/Pi+Rg/Omega/Lc))
            Iact   = 1.5/Pi/Omega/Lc*E*Sin(Mu)*Sin(2.0*Alfa+Mu)
            Irea   = 1.5/Pi/Omega/Lc*E*(Mu-SIN(Mu)*Cos(2.0*Alfa+Mu))
            S0     = 1.0
            Y      = Omega*L1Qu*Iact+Rd*Irea
            X      = E + Omega*L1Qu*Irea-Rd*Iact
            SinEps= Y/Sqrt(X*X+Y*Y)
            CosEps= X/Sqrt(X*X+Y*Y)
            Id     = - Wa*(Iact*SinEps+Irea*CosEps)
            Iq     = Wa*(Iact*CosEps-Irea*SinEps)
            Psiql1= L1Qu*Iq
            Psidl1= CF*Lf1*(CF*Id+Iv) + L1Du*(Id+Kf1D*Iv)
            G0     = 1.0/(1.0+a*(Psiql1*Psiql1+Psidl1*Psidl1)**3) - 1.0
            S      = 1.0/(1.0+a*(Wa*E/Omega)**6)
            Write(6,'('' S='',F10.7)') S
            Do 9 M=1,NI
               Y      = Omega*S*L1Qu*Iact+Rd*Irea
               X      = E + Omega*S*L1Qu*Irea-Rd*Iact
               SinEps= Y/Sqrt(X*X+Y*Y)
               CosEps= X/Sqrt(X*X+Y*Y)
               Id     = - Wa*(Iact*SinEps+Irea*CosEps)
               Iq     = Wa*(Iact*CosEps-Irea*SinEps)
               Psiql1= S*L1Qu*Iq
               Psidl1= CF*Lf1*(CF*Id+Iv) + S*L1Du*(Id+Kf1D*Iv)
               G      = 1.0/(1.0+a*(Psiql1*Psiql1+Psidl1*Psidl1)**3) - S
               If (Abs(G).LE.(EE/100.0)) GoTo 10
               GG = (G-G0)/(S-S0)
               S2 = S - G/GG
               Write(6,'(''  S2='',F10.7)') S2
               G0 = G
               S0 = S
               S  = S2
9           Continue
10          Continue
            F      = Omega*Psidl1 + Rq*Iq - Wa*E*CosEps
            Write(6,'(''Mu ='',F10.7,''  F ='',F13.7,''    S='',F10.7)')
     *           Mu,F,S
            If (Abs(F).LE.EE) Then
              EF = 1.0
              GoTo 99
            EndIf
            If (Mu.EQ.Mu0) Then
              WRITE(6,'(''Mu=Mu0='',F10.4,'' F='',F11.7,''  S='',F10.7)')
     *           Mu,F,S
              GoTo 99
            EndIf
            FF  = (F-F0)/(Mu-Mu0)
            Mu2 = Mu - F/FF
            IF (Mu2.GT.MuM) Mu2 = (MuM+Mu)/2.0
            IF (Mu2.LT.0.0) Mu2 = Mu/2.0
            F0  = F
            Mu0 = Mu
            Mu  = Mu2
8        Continue
99       Continue
         Ig = Sqrt(3.0)*E/2.0/(Omega*Lc)*(Cos(Alfa)-Cos(Alfa+Mu))
         If (EF.LT.0.5) Write(*,'(''****  Error in Stat  ****'')')
      End
```

APPENDIX 2 AN ACSL PROGRAM AS AN EXAMPLE

In this appendix, as an example, an ACSL-program is given, which is based on the equations in section 6.2. This program may be used for the simulation of a wind-energy conversion system with a synchronous machine of 375 kVA according to [Hoe 87b; Hoe 88a], where the mechanical coupling between the turbine and the generator has been supposed to be infinitely stiff.
First, the steady state is computed by means of the subroutine SS, which has been dealt with in appendix 1, for a given frequency. Next, the system is excited by means of a change of the voltage in the dc-link U_b. The delay angle of the rectifier α is controlled by a proportional current controller. The torque/speed characteristic of the turbine is a straight line through the steady-state point, the slope of which may be chosen.
Users of the program should notice that this example program is not safe for the case that the direct current i_g becomes negative.
Reasonable results may be obtained by choosing Alfal=0.2 and KP=0.01.

```
PROGRAM SYNCHRONOUS MACHINE WITH RECTIFIER; Delft; 89-08-23
'375 kVA; wind turbine with stiff transmission, starting from'
'         steady-state'

  Integer NI

  INITIAL
    CONSTANT KM     = -5.0       $' (Nms/rad); deriv. torque/angul sp'
    CONSTANT JT     = 90.0       $' (kgmm); inertia turbine'
    CONSTANT JG     = 12.5       $' (kgmm); inertia generator'
    CONSTANT P      =  2.0       $' (-); number of pole pairs'
    CONSTANT Freq   = 50.0       $' (Hz); frequency'
    CONSTANT a      =  0.125     $' (-); saturation constant'
    CONSTANT Ldu    = 0.00305975 $' (H); unsat. synchr. d-inductance'
    CONSTANT Lafdu  = 0.002986   $' (H); unsaturated'
    CONSTANT CF     = 0.375      $' (-)'
    CONSTANT Uf     = 1.064      $' (V); excitation voltage'
    CONSTANT Rf     = 0.0017     $' (Ohm); excitation resistance'
    CONSTANT Lf1    = 0.000368   $' (H)'
    CONSTANT T1D    = 0.3966574  $' (Ohm); d-damper time constant'
    CONSTANT Ld11   = 0.00016    $' (H); subtransient d-inductance'
    CONSTANT Lqu    = 0.00196    $' (H); unsat. synchr. q-inductance'
    CONSTANT T1Q    = 0.4        $' (s); q-damper time constant'
    CONSTANT Lq11   = 0.00016    $' (H); subtransiente q-inductance'
    CONSTANT Alfa1  = 0.0        $' (rad); initial delay angle'
    CONSTANT KP     = 0.0        $' (rad/A); proport. current control'
    CONSTANT Lg     = 0.0015     $' (H); self inductance dc-link'
    CONSTANT Rg     = 0.06       $' (Ohm); resistance dc-link'
    CONSTANT Ub     = 400.0      $' (V); voltage dc-link'
    CONSTANT UStep  = -10.0      $' (V); dc-link voltage step'
    CONSTANT TUStep = 0.1        $' (s); moment of step'
    CONSTANT TULeng = 10.0       $' (s); length of step'
    CONSTANT EE     = 1.0E-4     $' (V); maximal error in Stat'
    CONSTANT NI     = 15         $' (-); max. number of iterations Stat'
    CONSTANT TFIN   = 1.99       $' (s); finishing time'
    L1Qu   = Lqu - Lq11
    L1Du   = Ldu - Ld11 - CF*CF*Lf1
    Kf1D   = (Lafdu-CF*Lf1)/L1Du
    R1Q    = L1Qu/T1Q
    R1D    = L1Du/T1D
    Lc     = (Ld11+Lq11)/2.0
    Pi     = 4.0*ATAN(1.0)
    W3     = SQRT(3.0)
    Wa     = SQRT(1.5)
    OmegM1= 2.0*Pi*Freq/P
    Call SS(EF,Id,Iq,Ig1,S,a,Ldu,Lafdu,Lqu,Ld11,CF,Lf1, ...
            (CF*CF*Rf+(1-Kf1D*CF)**2*R1D),R1Q,Lq11, ...
            (P*OmegM1),Alfa1,Ub,(Uf/Rf),Rg,EE,NI)
    Psif10 = Lf1*(Uf/Rf+CF*Id)
    Psi1D0 = S*L1Du*(Id+Kf1D*Uf/Rf)
    Psi1Q0 = S*L1Qu*Iq
    Ig0    = Ig1
    Psiq11 = Psi1Q0
    Psid11 = CF*Psif10 + Psi1D0
    MT1    = - P*(Id*(Lq11*Iq+Psiq11) - Iq*(Ld11*Id+Psid11))
    OmegM0  = OmegM1
  END $ " INITIAL "
```

```
DYNAMIC
  DERIVATIVE
    CINTERVAL CInt = 0.02
    MT     = MT1 + KM*(OmegM-OmegM1)
    Alfa   = Alfa1 + KP*(Ig-Ig1)
    CosAlf= COS(Alfa)
    Psiql1= Psi1Q
    Psidl1= CF*Psif1 + Psi1D
    S      = 1.0/(1.0+a*(Psidl1*Psidl1+Psiql1*Psiql1)**3)
    Ed     = - P*OmegM*Psiql1- CF*Uf + (1-CF*Kf1D)*R1D/S/L1Du*Psi1D ...
             + (CF*Rf+(CF*Kf1D-1)*R1D*Kf1D)*Psif1/Lf1
    Eq     = P*OmegM*Psidl1 + R1Q/S/L1Qu*Psi1Q
    Sq     = SQRT(Ed*Ed+Eq*Eq)
    SinEps = -Ed/Sq      $    CosEps = Eq/Sq     $      E = Sq/Wa
    Mu     = - Alfa + ACOS(CosAlf - 2.0*P*OmegM*Lc*Ig/W3/E)
    SinMu = Sin(Mu)
    Con    = 1.5*E/(Pi*P*OmegM*Lc)
    Iact   = Con*SinMu*Sin(2.0*Alfa+Mu)
    Irea   = Con*(Mu-SinMu*Cos(2.0*Alfa+Mu))
    Id     = - Wa*(Iact*SinEps + Irea*CosEps)
    Iq     = Wa*(Iact*CosEps - Irea*SinEps)
    MG     = - P*(Id*(Lql1*Iq+Psiql1) - Iq*(Ldl1*Id+Psidl1))
    Ub1    = Ub + UStep*(STEP(TUStep)-STEP(TUStep+TULeng))
    OmegMD = (MT-MG)/(JT+JG)
    Psif1D = Uf-(Rf+Kf1D*Kf1D*R1D)*Psif1/Lf1+Kf1D*R1D/S/L1Du*Psi1D ...
             + (Rf*CF-Kf1D*R1D*(1-Kf1D*CF))*Id
    Psi1DD = - R1D*(Psi1D/S/L1Du -(1-Kf1D*CF)*Id - Kf1D*Psif1/Lf1)
    Psi1QD = - R1Q*(Psi1Q/S/L1Qu - Iq)
    IgD    = (3.0*W3/Pi*E*CosAlf-(Rg+3.0/Pi*P*OmegM*Lc)*Ig-Ub1)/ ...
              (Lg+2.0*Lc)
    OmegM = Integ(OmegMD,OmegM0)
    Psif1 = Integ(Psif1D,Psif10)
    Psi1D = Integ(Psi1DD,Psi1D0)
    Psi1Q = Integ(Psi1QD,Psi1Q0)
    Ig     = Limint(IgD,Ig0,0.0,100000.0)
  END $ " DERIVATIVE "
  MShaft = (JG*MT + JT*MG)/(JT+JG)
  TERMT((T.GT.TFIN).OR.(EF.LT.0.5))
END $ " DYNAMIC "

TERMINAL
END $  " TERMINAL "

END $ " PROGRAM "
```

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